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A Three-Dimensional FDTD Subgridding Algorithm Based on Interpolation of Current Density

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Abstract—**A three-dimensional subgridding algorithm for the Finite Difference Time Domain (FDTD) method is proposed in this paper. The method is based on interpolation of electric and magnetic current densities. The coarse-fine mesh ratio can be either 1:2 or 1:3. Results of a test model utilizing a lossless cavity excited with a dipole show no tendency of instability after 500,000 time steps. The reflection in time domain at the subgridding interface was calculated to test the accuracy of the subgridding algorithm.**

Keywords-FDTD; subgridding; linear interpolation; symmetry; stability; electric and magnetic current density

I. INTRODUCTION

The FDTD method is one of the prevailing numerical tools in electromagnetic modeling for its simplicity and capability of handling complex geometries [1][2]. However, the standard FDTD method is inefficient if geometry details need to be modeled, since a global fine mesh will be required and total number of cells increases dramatically. At the same time, the time step must be reduced to satisfy the well-known Courant criterion for stability. As a result, the computational time will increase significantly.

A few methods can be used to improve the efficiency of the FDTD method, such as non-uniform meshing, sub-cell algorithm, non-orthogonal meshing, and subgridding. A stable subgridding algorithm is very useful for an FDTD code, since it can refine the mesh locally and improve the accuracy of the result without significantly increasing the computational efforts. The main difficulty in a subgridding algorithm is the late time stability. It is worth noticing that the result of a subgridded mesh cannot have the same accuracy of a global fine mesh. The higher efficiency is achieved at the cost of accuracy. For a subgridding scheme, stability and accuracy are the two primary concerns.

Many papers have been published with different field coupling schemes [3]-[8]. The majority of the existing subgridding methods use the field interpolation schemes at the subgridding interface. The general idea is to use the known field values in the base-grid to estimate the missing field values in the sub-grid at the coarse-fine mesh interface by either interpolation or extrapolation. The sub-grid field values then can be coupled back to base-grid at the collocated points. Several overlapped layers are sometimes used to make a smooth transition from the base-grid to the sub-grid. This soft transition boundary, however, may result in late time instability.

Nicolas Chavannes proposed a subgridding method with a mesh ratio of 1:2 based on cubic spline interpolation. A time step smaller than that allowed by Courant criterion is used to increase the stability [3]. Mikel White published a general scheme for both odd and even mesh ratios in [4] using a weighted current method for materials transverse at the subgridding interface. Michael Chevalier proposed a scheme allowing material transverse for all odd numbers of mesh ratios, and this method only interpolates the H-field on the coarse-fine mesh interface. Michal Okoniewski in [6] uses a pulsing, overlapping scheme and spline interpolation and extrapolation in a subgridding algorithm with a mesh ratio of 1:2. By shrinking and restoring the subgridding interface, a local time step in the sub-grid is used.

Oliver Podĕbrad and Frank Mayer proposed a stable subgridding scheme with a mesh ratio of 1:2 based upon flux conservation in the framework of Finite Integral Technique (FIT) [7][9][10]. A transition grid is introduced, where field coupling is implemented. Their work set a good basis for the FDTD subgridding algorithm. However, the interpolation used is based on an area-weighted scheme, which is suitable for the case with a ratio of 1:2 and is not easily extended to other mesh ratios. Wlodarczyk proposed a subgridding scheme for the Transmission Line Method (TLM) [11]. As the TLM is based on the transmission line theory and uses V and I as the updating variables, it is natural to use a transformer model. From a physical view, energy conservation condition holds.

The interpolation scheme for a subgridding algorithm should be physically motivated. Only focusing on the smooth distribution of fields is not enough and cannot ensure the late time stability. For example, in a Yee cell shown in Figure 1. , an E-field value on an edge is actually the average value along the edge, not the exact value at the very center. If the E-field value is used as the exact value at the center in an interpolation, the magnetic current conservation condition may be violated, as well as the flux conservation condition.

II. SUBGRIDDING SCHEME BASED ON INTERPOLATION OF ELECTRIC AND MAGNETIC CURRENT

A. *Reciprocity of the Yee's scheme*

The famous Yee scheme discretizes the Maxwell equation in a very intuitional way, showing explicitly how the E-field and H-field are coupled. Figure 1. shows a Yee cell. The E-field components are located on the edges of the cubic, and the H-field components are in the center of the surfaces.

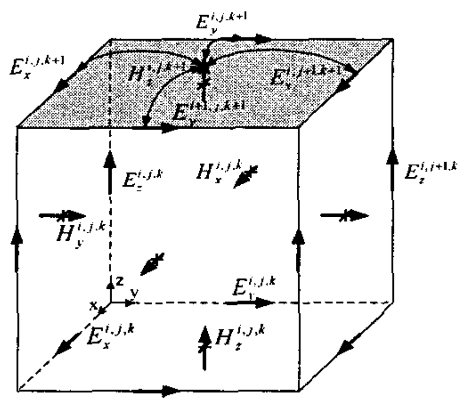

Figure 1. A Yee Cell.

In the standard FDTD field update scheme, the H-field component is calculated based on the four neighboring E-field components. In the following step, this H-field component will participate in the calculation of all neighboring E-field components. This implements the reciprocity of the Maxwell equations. In other words, an E-field (H-field) component is involved in the calculation of an H-field (E-field) component, then this H-field (E-field) must be involved in the calculation of the related E-field (H-field). From a physical point of view, it ensures the consistency of energy conversion between the H-field and the E-field.

For a stable subgridding algorithm, this property must also be preserved, which means the interpolation should not only be used for the field coupling from base-grid to sub-grid, but also for the coupling from sub-grid to base-grid. Moreover, the forward and backward coupling schemes should be symmetric.

B. *The interpolation scheme of current density*

In a source free region, the Maxwell equations in the integral form are,

$$\oint \vec{E}\cdot d\vec{l} = -\int_S \mu \frac{\partial \vec{H}}{\partial t}\cdot d\vec{S}$$
$$\oint \vec{H}\cdot d\vec{l} = \int_S \varepsilon \frac{\partial \vec{E}}{\partial t}\cdot d\vec{S} + \int_S \sigma\vec{E}\cdot d\vec{S} \qquad (1)$$

Discretize Eq. (1) to get the update equations,

$$H^{n+\frac{3}{2}} = H^{n+\frac{1}{2}} - \frac{\Delta t}{\mu}\delta_m^{n+1}$$
$$E^{n+1} = \frac{\varepsilon/\Delta t - \sigma/2}{\varepsilon/\Delta t + \sigma/2}E^n + \frac{1}{\varepsilon/\Delta t + \sigma/2}\delta_e^{n+\frac{1}{2}} \qquad (2)$$

where δ_e and δ_m are electric and magnetic currents, and defined as $\delta_e = \frac{\sum H\cdot\Delta l}{\Delta S}$ and $\delta_m = \frac{\sum E\cdot\Delta l}{\Delta S}$, respectively. For simplicity reason, we assume $\sigma = 0$. Then, Eq. (2) can be rewritten as,

$$H^{n+\frac{3}{2}} = H^{n+\frac{1}{2}} - \frac{\Delta t}{\mu}\delta_m^{n+1}$$
$$E^{n+1} = E^n + \frac{\Delta t}{\varepsilon}\delta_e^{n+\frac{1}{2}} \qquad (3)$$

If the electric and magnetic currents are known, the two update equations can be continued. Therefore, the field coupling scheme on the coarse-fine mesh interface can be done via interpolation of current density. Another reason for using currents as the interpolation variables is the easy extension to subgridding in time. It is obvious that the most efficient subgridding method is to use a local time step in the sub-grid. In this case, field update in the sub-grid will performed multiple times during in one time step in the base-grid. If the interpolation is based field values, the interpolated field components for sub-grid will be incomplete in time. The current interpolation scheme, however, can simply overcome this problem, by assuming that the currents stay constant in one coarse time step.

The current density distribution scheme is based on 2-D linear interpolation, as shown in Figure 2. A mesh ratio of 1:3 is used.

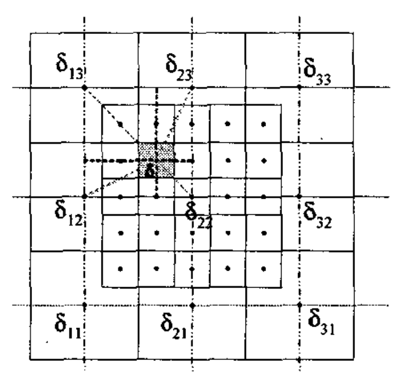

Figure 2. Current density interpolation scheme.

For example, to calculate the current density δ in the grey colored fine cell, the current density of the four neighboring coarse cells, δ_{12} δ_{13} δ_{22}, and δ_{23}, are used, and the calculation has the form of,

$$\delta = \frac{2}{3}\left(\frac{2}{3}\delta_{22} + \frac{1}{3}\delta_{23}\right) + \frac{1}{3}\left(\frac{2}{3}\delta_{12} + \frac{1}{3}\delta_{13}\right) \qquad (4)$$

If the cell is square, then the current density distribution scheme for a coarse cell can be simplified as shown in Figure 3.

1/9	2/9	1/3	2/9	1/9
2/9	4/9	2/3	4/9	2/9
1/3	2/3	1	2/3	1/3
2/9	4/9	2/3	4/9	2/9
1/9	2/9	1/3	2/9	1/9

Figure 3. Distribution of the current density.

This interpolation scheme can be viewed as a two-dimensional basis function with a pyramidal shape for current distribution. It can be easily extended to the three-dimensional case by implementation of linear interpolation along the additional axial in a similar way. Figure 2. and Figure 3. show the interpolation scheme for a 1:3 mesh ratio, but can be modified for other mesh ratios.

C. *Subgridding scheme*

In the FIT method [9][10], the sub-grid interface is located on the dual mesh. Such construction is suitable for a scheme of 1:2. However, when extended to other mesh ratios, the mesh needs to be modified. Figure 4. shows the construction of a two-dimensional sub-grid mesh for a ratio of 1:3. The upper-case letter E and H indicate the E-field and H-field of base-grid at the coarse-fine cell interface, and the lower-case letter e and h indicate the E-field and H-field of sub-grid.

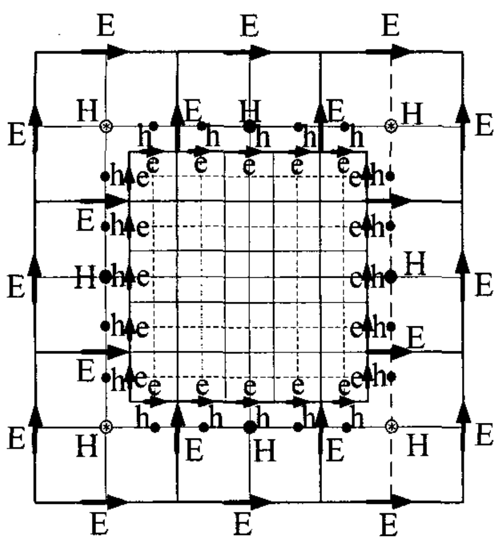

Figure 4. Construction of the sub-grid mesh.

At the subgridding interface, the sub-grid E-field e cannot be updated, since the H-field h is located outside the domain. At the same time, base-field H-field H cannot be updated in the standard FDTD scheme, because those coarse cells at the interface are not cubic and the integral paths are incomplete. Therefore, an interpolation scheme must be introduced to couple the field values between base-grid and sub-grid. Noting that, for such mesh construction, the coarse cells at the interface are no longer cubic, it is difficult is apply the differential form of update scheme.

For the forward coupling scheme (from base-grid to sub-grid), the electrical current of base cells tangential to the interface is interpolated using the described scheme to calculate the missing H-field in the sub-grid as shown in Figure 5. For the backward coupling scheme (from sub-grid to base-grid), the magnetic current of the sub-grid cells at the interface is interpolated to calculate the missing E-field of the base-grid as shown Figure 6. The missing E-field of the base-grid is indicated with E' in Figure 6.

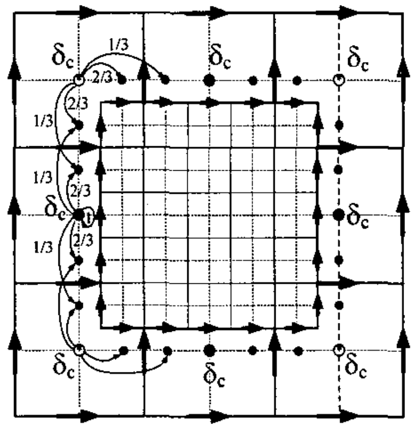

Figure 5. Forward coupling scheme

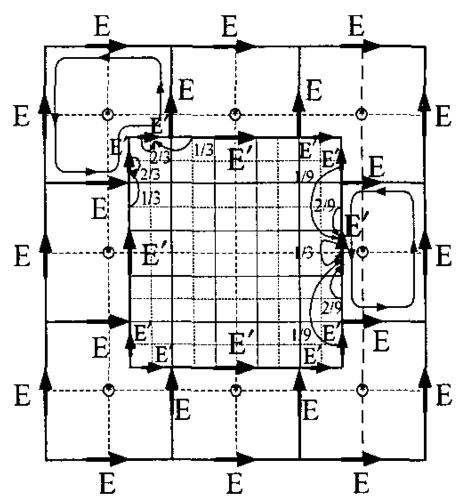

Figure 6. Backward coupling scheme

Figure 7. gives the flow chart for the subgridding scheme. Field components other than those at the subgridding interface in both base-grid and sub-grid are calculated by the standard FDTD scheme. The update of the missing H-field components for the sub-grid is based upon the interpolation of magnetic current density through the interface coarse cells, and the missing E-field of the base-grid is obtained by a symmetric backward interpolation of electric current calculated in sub-grid.

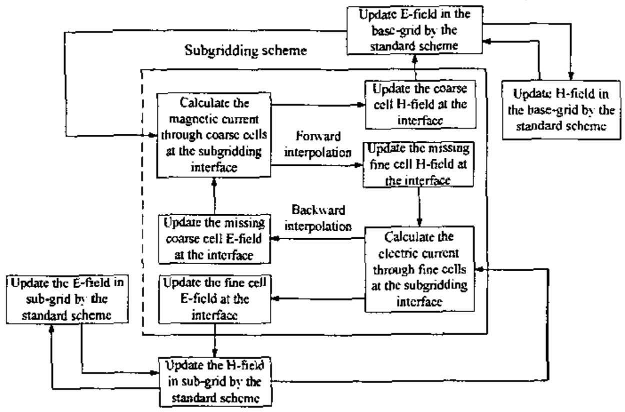

Figure 7. Flow chart of the FDTD scheme with subgridding

Note that the forward and backward interpolation scheme should be symmetric. This can be shown in Figure 8. The H-field component at the corner is involved in the update of four E-field components in the base-grid marked with E and the four E-field in the sub-grid marked with e. When the H-field is updated, all these E-field components must contribute to the integral path for magnetic current.

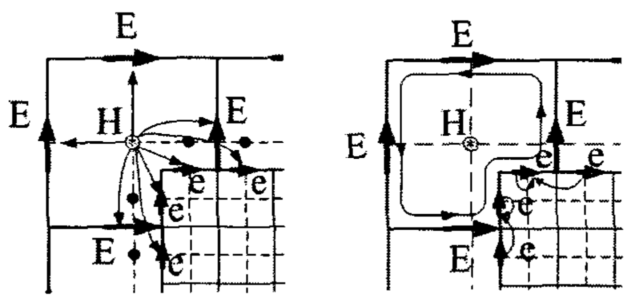

Figure 8. Symmetry of the interpolation scheme

The interpolation scheme shown in Figure 5. and Figure 6. can be easily extended to three-dimension by introduction of linear interpolation in the additional axial direction. However, for the implementation in three-dimension, special treatment is needed for the cells located on the termination edges. This is to ensure the interpolation is done locally in space, and keep the forward and backward interpolation scheme symmetric.

In Figure 9. , for the fine cells located at the termination edges of the subgridding interface, the four neighboring magnetic current densities are needed in the interpolation, normally, for example δ_{z1}, δ_{z2}, δ_{z3}, and δ_{z4}. The integral path for δ_{z3} and δ_{z4}, however, is complete. If these two values are used, the interpolation scheme will no longer be symmetric and the reciprocity will be broken, which may lead to late time stability. To keep the interpolation locally in space, special treatment for the interpolation of the magnetic current of the edge cells is needed. The modified scheme is given in Figure 9. Such modification will compromise the accuracy of the results. However, this is a trade-off between efficiency and stability.

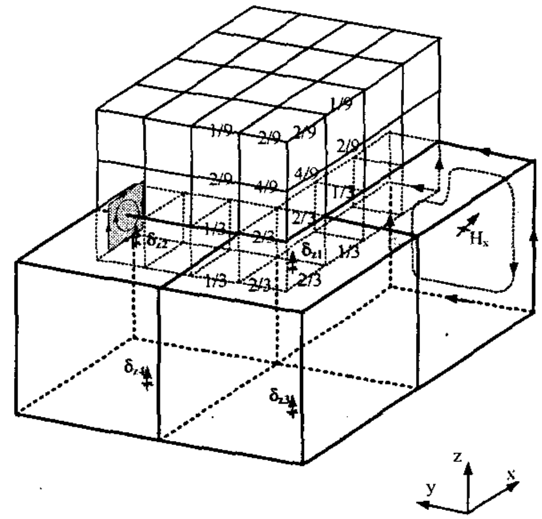

Figure 9. The subgridding scheme in 3-D with special treatment at the edges for a mesh ratio of 1:3

The construction of the mesh for the subgridding with a mesh ratio of 1:2 is different from that with a mesh ratio of 1:3. The sub-grid cells at the coarse-fine mesh interface have a reduced size of a quarter or a half of a normal cell for updating E-field, as shown in Figure 10. As a result, the time step also needs to be reduced by half, which compromises the subgridding efficiency. However, this reduction is necessary, since only in this case the interpolation scheme can be symmetric and the subgridding algorithm is stable.

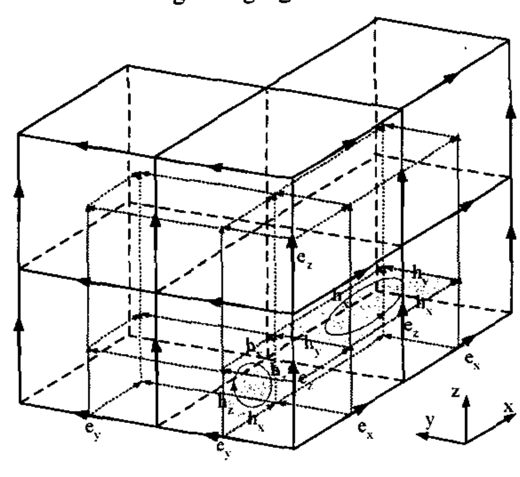

Figure 10. The 3-D sub-mesh construction for a mesh ratio of 1:2

If the subgridding is only implemented in space, and the same time step is used both in base-grid and sub-grid. The maximum gain of efficiency for the 1:3 scheme is 27 times the global fine mesh, while only 4 times for the 1:2 scheme.

For mesh ratios other than 1:2 and 1:3, the sub-mesh needs to be constructed differently, and the interpolation scheme can be obtained from the proposed two-dimensional linear interpolation. However, cascading sub-grids with combination of mesh ratio 1:2 and 1:3, other mesh ratios can be achieved, which can avoid the complexity of implementation of too many mesh schemes.

At this point, we believe that it is possible to prove the stability of the subgridding method mathematically, and we will publish this part in a separate paper.

III. NUMERICAL EXAMPLES

An FDTD code with the described subgridding method was developed. Currently, the same time step is used in the base-grid and the sub-grid.

To test the stability of the proposed method, a cavity model is used, as shown in Figure 11. The model consists of a dipole source, which is implemented with two PEC stabs connected with a lumped 50-ohm voltage source excited with a Gaussian pulse. PEC boundary conditions are used. A subgridded domain is set in the vacuum. The whole computational domain has 40 * 40 * 40 cells, and the dimensions for one cell are 5cm by 5cm by 5cm. The sub-domain has 36 * 36 * 36 fine cells, and the mesh ratio is 1:2. The only loss for this model is the 50-ohm source resistance. This is a stringent stability test model.

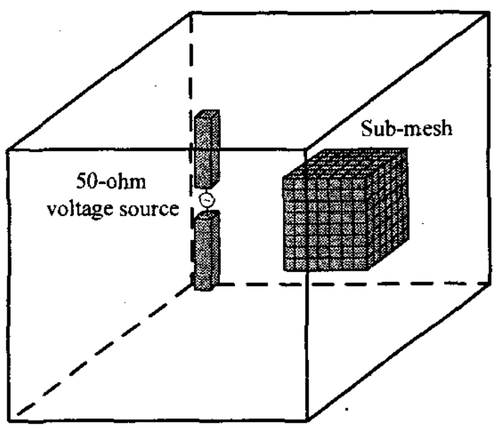

Figure 11. A cavity model for stability test.

Figure 12. gives the time domain E-field values at the center of the sub-grid. After 500,000 time steps, no sign of instability is observed. The envelop of the E-field curve clearly shows a convergence tendency.

To test the accuracy of the subgridding algorithm, a model with an incident plane wave was calculated. The plane wave is implemented by putting a dipole source far away from the sub-mesh. In the first calculation, no sub-grid was used, and E-field was recorded. Then, the calculation was repeated with the presence of a sub-grid. The probe was set in the base-grid at a point right in front of the sub-grid interface facing the incident direction.

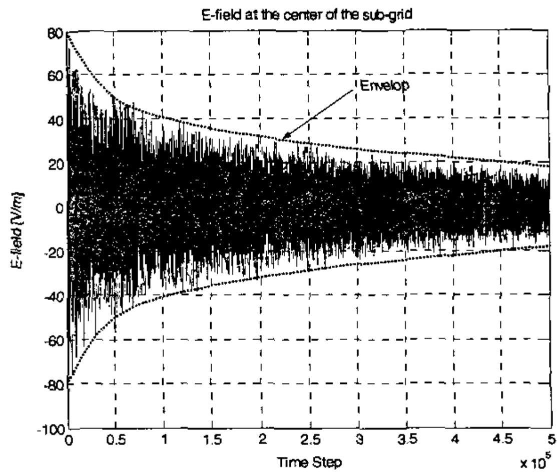

Figure 12. E-Field values in time domain at the center of the sub-mesh

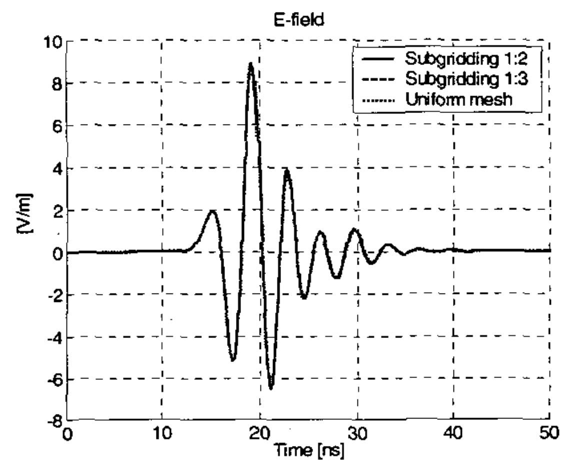

Figure 13. Comparison of results with and without subgridding

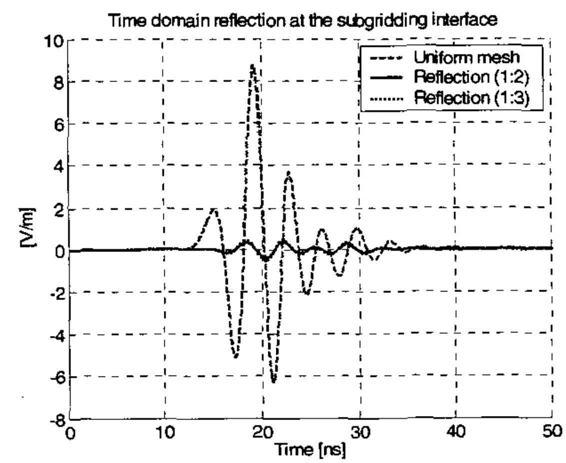

Figure 14. Reflection in time domain

The results are compared in Figure 13. It shows that the curves match very well. The difference between the curves with and without subgridding gives the reflection in time domain at the subgridding interface, as shown in Figure 14. The reflection can be caused by both the imperfect coupling scheme and different numerical dispersion in the two grids. Figure 14. also shows that the 1:2 and 1:3 subgridding schemes have the similar reflection level, and the reflection is low compared to the field values.

IV. CONCULSIONS

A three-dimensional subgridding algorithm allowing coarse-fine mesh ratio of 1:2 or 1:3, is described in this paper. The method is based on the interpolation of electric and magnetic currents. The missing H-field components of sub-grid at the subgridding interface are calculated by an interpolation of the magnetic current obtained in base-grid, and the missing E-field components of base-grid are calculated by interpolation of the electrical current obtained in sub-grid. Symmetry is preserved for the forward and backward interpolation schemes to ensure the stability of the algorithm. Special treatments are given to the cells located on the termination edges and corners at the coarse-fine mesh interface to keep the interpolation scheme symmetric by either introducing cells with reduced size in the sub-mesh (mesh ratio of 1:2) or modification of the interpolation scheme (mesh ratio of 1:3).

A test model of a cavity excited with a dipole was run for more than 500,000 time steps to test the stability of the algorithm. No sign of instability was observed, and the field curve showed a consistent tendency of convergence.

In the future work, the electromagnetic properties will be taken into account in the current interpolation scheme for a more accurate model of material transversal. More cases will be simulated to test the accuracy of the subgridding algorithm.

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