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# A Robust Controller for the Manipulation of Micro Scale Objects

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# Decentralized Discrete-Time Neural Network Controller for a Class of Nonlinear Systems with Unknown Interconnections[1]

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*Abstract*— **A novel decentralized neural network (NN) controller in discrete-time is designed for a class of uncertain nonlinear discrete-time systems with unknown interconnections. Neural networks are used to approximate both the uncertain dynamics of the nonlinear systems and the unknown interconnections. Only local signals are needed for the decentralized controller design and the stability of the overall system can be guaranteed using the Lyapunov analysis. Further, controller redesign for the original subsystems is not required when additional subsystems are appended. Simulation results demonstrate the effectiveness of the proposed controller. The NN does not require an offline learning phase and the weights can be initialized at zero or randomly. Simulation results verify the theoretical conclusions.**

## I. INTRODUCTION

Many physical systems, such as power grid, computer and communication networks, networked dynamic systems, transportation systems, etc., are complex large-scale interconnected systems [2,6]. To coordinate the control activities of such large scale systems, centralized control schemes are proposed in the literature by assuming that global information of the overall system is available. While there are obvious theoretical advantages, control centralization is very difficult for a complex large scale system with interconnections due to technical and economic reasons [2]. Furthermore, centralized control designs are dependent upon the system structure and cannot handle the structural changes. If new subsystems are added or removed, the controller for the overall system has to be redesigned.

To overcome the problems of centralized control, decentralized schemes are currently being addressed in the literature [2,6]. Instead of an overall controller, decentralized control design aims at designing controllers for each subsystem. Thus subsystem controllers only require local information with a minimum amount of information from other subsystems.

Earlier works on decentralized control of nonlinear systems assumed that the interconnection dynamics are linear in the unknown parameters (LIP) and bounded with first order terms. To overcome this assumption on the interconnection terms and to further relax the LIP assumption on the nonlinear system, recently, neural network (NN) have been utilized to design decentralized controllers [2,6] by assuming that the interconnections can also be approximated by using the nonlinear in the parameter NNs. Further, the direct controller designs require neither the knowledge of nor the direct estimation of the unknown input gain matrix. Finally, it is very important to note that all available decentralized schemes are developed to control nonlinear continuous-time systems. To the knowledge of the author, decentralized control scheme in discrete-time is currently not available in the literature.

On the other hand, discrete-time implementation of controllers is of importance since all the controllers have to be implemented on today's embedded hardware. As indicated in [4], discrete-time adaptive control design is far more complex than continuous-time due primarily to the fact that discrete-time Lyapunov derivatives are quadratic in the state, not linear as in the continuous-case. This has led to traditional techniques where the parameter identification problem is decoupled from the control problem using so called certainty equivalence (CE) assumption.

Motivated by the advancements in the area of centralized nonlinear discrete-time NN control, this paper introduces a decentralized NN controller design for a control of a class of large-scale unknown nonlinear discrete-time systems with unknown interconnections. The NNs are used to approximate the unknown nonlinear dynamics of the subsystems and to compensate the unknown nonlinear interactions. The first or higher order

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polynomial bound assumption of earlier works [2] on the unknown interconnection terms can be treated here as special cases. The developed controller is robust to perturbations in the system dynamics and interconnections. Lyapunov analysis is demonstrated for the closed-loop system and boundedness of all the closed-loop signals is shown. Simulations results on nonlinear discrete-time systems demonstrate the effectiveness of the proposed decentralized NN controller.

## II. BACKGROUND

The following mathematical notion is required for the development of adaptive critic NN controller. First the universal approximation property of two-layer NN is presented. Then actual controller development is introduced.

### *A. Approximation property*

A general function $f(x) \in C^{(s)}$ can be approximated by using a two-layer neural network [7] as

$$f(x(k)) = W^T \varphi_2\left(V^T \varphi_1(x(k))\right) + \varepsilon(k) \tag{1}$$

where W and V are constant weights and $\varphi_2\left(V^T \varphi_1(x(k))\right)$, $\varphi_1(x(k))$ denote the vectors of activation functions at the instant k, with $\varepsilon(k)$ being NN functional reconstruction error vector and $\|\varepsilon(k)\| \le \varepsilon_M$. The net output is defined as

$$\hat{f}(x(k)) = \hat{W}^T \varphi_2\left(\hat{V}^T \varphi_1(x(k))\right) \tag{2}$$

From now on $\varphi_1(x(k))$ is denoted as $\varphi_1(k)$ and $\varphi_2\left(V^T \varphi_1(x(k))\right)$ is denoted as $\varphi_2(k)$.

### *B. Stability of Systems*

To formulate the discrete-time controller, the following stability notion is needed. Consider the nonlinear system given by [5]

$$\begin{aligned} x(k+1) &= f(x(k), u(k)) \\ y(k) &= h(x(k)) \end{aligned} \tag{3}$$

where x(k) is a state vector, u(k) is the input vector and $y(k)$ is the output vector. The solution is said to be uniformly ultimately bounded (UUB) if for all $x(k_0) = x_0$ there exists a $\mu \ge 0$ and a number $N(\mu, x_0)$ such that $\|x(k)\| \le \mu$ for all $k \ge k_0 + N$.

### *C. Nonlinear System Description*

Consider the following nonlinear system, to be controlled, given in the following form

$$\begin{aligned} x_{i1}(k+1) &= x_{i2}(k) \\ &\;\;. \\ &\;\;. \\ x_{in}(k+1) &= f_i(x_i(k)) + u_i(k) + \Delta_i(x_1, x_2 ...., x_n) + d_i(k) \\ y_i &= x_{i1}(k); \quad i = 1, 2, ....m, \end{aligned} \tag{4}$$

with $x_i(k) = \left[x_{i1}(k),\ x_{i2}(k),\ \cdots,\ x_{in}(k)\right]^T$ the state vector at time instant k, $f_i(x_i(k))$ the unknown nonlinear function, $\Delta_i(x_1(k), x_2(k), ....., x_n(k))$ the interconnection term, $u_i(k)$ the input and $d_i(k)$ is the unknown but bounded disturbance vector, whose bound is assumed to be a known constant, $\|d_i(k)\| \le d_{iM}$.

The nonlinear system presented in (3) is in general form. For the sake of convenience, the nonlinear system in general form can be expressed into several classes. Equation (4) presents a simplified scenario of a specific class of nonlinear system.

Given a trajectory, $x_{ind}(k)$, and its delayed values for the ith subsystem, define the tracking error

$$e_{in}(k) = x_{in}(k) - x_{ind}(k), \tag{5}$$

and the filtered tracking error, $r_i(k)$, as

$$r_i(k) = [\lambda_i \quad I] e_i(k), \tag{6}$$

with $e_i(k) = \left[e_{i1}(k),\ e_{i2}(k),\ \cdots,\ e_{in}(k)\right]^T$, $e_{i1}(k+1) = e_{i2}(k)$, where $e_{i1}(k+1)$ is the future value for the error $e_{i1}(k)$, $e_{in-1}(k),\ \cdots,\ e_{i1}(k)$ are delayed values of the error $e_{in}(k)$ for the ith subsystem and $\lambda_i = \left[\lambda_{i,n-1},\ \lambda_{i,n-2},\ \cdots,\ \lambda_{i,1}\right]^T$ is a constant diagonal positive definite matrix selected such that the eigen values are within the unit disc. Equation (6) can be expressed as

$$r_i(k+1) = f_i(x_i(k)) - x_{ind}(k+1) + \lambda_{i,1} e_{in}(k) + \cdots + \lambda_{i,n-1} e_{i2}(k) + u_i(k) + d_i(k) + \Delta_i(.) \tag{7}$$

*Assumption 1:* The interconnection terms of the *i*th subsystem $\Delta_i(x_1, x_2, ..., x_n)$ are bounded, whose bounds are given by

$$\left|\Delta_i(x_1, x_2, ..., x_n)\right| \le \delta_{i0} + \sum_{j=1}^{m} \delta_{ij}(|r_j|)$$

where $\delta_{i0}$ are unknown positive constants and $\delta_{ij}(|r_j|)$ are unknown smooth functions.

Define the control input $u_i(k)$ as

$$u_i(k) = x_{ind}(k+1) - \hat{f}_i(x_i(k)) - \hat{\Delta}_i(x_1(k), x_2(k), ...., x_n(k)) + k_{i,v} r_i(k) - \lambda_{i,1} e_{in}(k) - \cdots - \lambda_{i,n-1} e_{i2}(k) \quad (8)$$

where $\hat{f}_i(x(k))$ is an estimate of the unknown function $f_i(x_i(k))$, $\hat{\Delta}_i(x_1(k), x_2(k), ...., x_n(k))$ is an estimate of $\Delta_i(x_1(k), x_2(k), ...., x_n(k))$, and $k_{i,v}$ is a diagonal gain matrix for the ith subsystem. Then, the closed-loop system becomes

$$r_i(k+1) = k_{i,v}\ r_i(k) + \tilde{f}_i(x_i(k)) + \tilde{\Delta}_i(.) + d_i(k), \quad (9)$$

where the functional estimation errors are given by $\tilde{f}_i(x_i(k)) = f_i(x_i(k)) - \hat{f}_i(x_i(k))$ and $\tilde{\Delta}_i(.) = \Delta_i(.) - \hat{\Delta}_i(.)$.

Equation (9) relates the filtered tracking error with both the functional estimation error resulting from uncertain dynamics and unknown interconnection estimation error. In the remainder of this paper, (9) is used to focus on selecting NN tuning algorithms that guarantee the stability of the filtered tracking error $r_i(k)$. Then since (6) with the input considered as $r_i(k)$ and the output as $e_i(k)$ describes a stable system, standard techniques [1,4] guarantee that $e_i(k)$ exhibits a stable behavior.

Define $k_v = [k_{1,v}, ...., k_{m,v}]^T$, $\tilde{f}(.) = [\tilde{f}_1(.), ...., \tilde{f}_m(.)]^T$, $\tilde{\Delta}(.) = [\tilde{\Delta}_1(.), ...., \tilde{\Delta}_m(.)]^T$ and $\mathrm{d}(k) = [d_1(k), ...., d_m(k)]^T$. Then the closed-loop tracking error system (9) can also be expressed as

$$r(k+1) = k_v r(k) + \delta_0(k), \quad (10)$$

where $\delta_0(k) = \tilde{f}(x(k)) + \tilde{\Delta}(.) + d(k)$. If the unknown dynamic, interconnection estimation errors, and bounded distuebances, $\tilde{f}(x(k)), \tilde{\Delta}(.)$ and d(k) are bounded above such that $\left\|\tilde{f}(x(k))\right\| \le f_M$, $\left\|\tilde{\Delta}(.)\right\| \le \Delta_M$ and $\left\|d(k)\right\| \le d_M$, for some known bounding function $f_M$, $\Delta_M$, and $\mathrm{d_M}$, then next stability results hold.

*Theorem 2.1:* Consider the system given by (4). Let the control action for ith subsystem be provided by (8). The closed-loop tracking error system (10) is stable provided

$$k_v^T k_v < I. \quad (11)$$

*Proof:* Let us consider the following Lyapunov function candidate

$$J = r^T(k) r(k). \quad (12)$$

The first difference is

$$\Delta J = r^T(k+1) r(k+1) - r^T(k) r(k). \quad (13)$$

Substituting the tracking error dynamics (9) along with the auxiliary input results in

$$\Delta J = (k_v r(k) + \tilde{f}(x(k)) + \tilde{\Delta}(.) + d(k))^T (k_v r(k) + \tilde{f}(x(k)) + \tilde{\Delta}(.) + d(k)) - r^T(k) r(k).$$

This further implies that $\Delta J \le 0$ provided

$$\left\|(k_v r(k) + \tilde{f}(x(k)) + \tilde{\Delta}(x(k)) + d(k)\right\| \le k_{v\max}\left\|r(k)\right\| + f_M + \Delta_M + d_M < \left\|r(k)\right\|.$$

or

$$\left\|r(k)\right\| \le \frac{f_M + \Delta_M + d_M}{(1 - k_{v\max})} \quad (14)$$

The closed-loop system is bounded.

## III. Adaptive NN Controller design

In the remainder of this paper, a three-layer NN is considered both for the uncertain dynamics and unknown interconnections. Stability analysis using a Lyapunov direct method is carried out for the closed-loop system (9) using novel weight tuning updates. Assume that some constant weight updates, V and W such that

$$f_i(x(k)) = W^T \varphi_2\left(V^T \varphi_1(x_i(k))\right) + \bar{\varepsilon}_i(k) \quad (15)$$

where $\varphi_2(k)$ is the vector of hidden layer activation functions, $\varphi_1(k)$ is a vector linear function and the NN functional estimation errors are given by $\bar{\varepsilon}_i(k)$. Here for the sake of convenience, the estimation errors for the the NN approximating the uncertain dynamics and the NN approximating the interconnection term are combined into one term and its bound is given by $\left\|\varepsilon_i(k)\right\| \le \varepsilon_{iN}$ with the bounding constant $\varepsilon_{iN}$ known. This error is referred to as reconstruction error or functional approximation error.

For suitable approximation properties, it is necessary to select a large enough number of hidden-layer neurons. It is not known how to compute this number for general multi-layer NN. Typically, the number of hidden layer neurons is selected by a trail and error procedure. To overcome this limitation, the first layer of weights is selected initially at random to provide a basis [3] and then they are kept constant through out the tuning process as they are not dependent upon time.

### A. Adaptive NN Structure

The actual NN output is given by

$$\hat{f}_i(.) = \hat{w}_{i1}^T(k)\sigma_{i1}(v_{i1}^T x_i(k)) = \hat{w}_{i1}^T(k)\sigma_{i1}(.), \quad (16)$$

and

$$\hat{\Delta}_i(.) = \hat{w}_{i2}^T(k)\sigma_{i2}(v_{i2}^T x(k)) = \hat{w}_{i2}^T(k)\sigma_{i2}(.) \quad (17)$$

where $\hat{w}_{i1}(k)$, $v_{i1}$ and $\hat{w}_{i2}(k)$, $v_{i2}$ represent the matrix of weight estimates. Here only the hidden layer NN weights are a function of time whereas the input layer weights are selected initially at random and held constant. The according to [3], the NNs can approximate the nonlinear functions over the compact set. Tuning one layer of weights will ensure that the computational complexity is tractable.

Let $w_{i1}$ and $w_{i2}$ are the unknown target NN weights for the NNs in the ith subsystem and assume that they are bounded so that

$$\|w_{i1}\| \le w_{i1\max}, \ \|w_{i2}\| \le w_{i2\max}. \quad (18)$$

where $w_{i1\max}$ and $w_{i2\max}$ are the maximum bound on the unknown weights. Then the error in weights during estimation is given by

$$\tilde{w}_i(k) = w_i - \hat{w}_i(k), \quad (19)$$

where $w_i = \begin{bmatrix} w_{i1} & 0 \\ 0 & w_{i2} \end{bmatrix}$, and $\hat{w}_i = \begin{bmatrix} \hat{w}_{i1} & 0 \\ 0 & \hat{w}_{i2} \end{bmatrix}$.

*Fact:* The activation functions are bounded by known positive values so that

$$\|\sigma_{i2}(.)\| \le \sigma_{i2\max} \text{ and } \|\sigma_{i1}(.)\| \le \sigma_{i1\max}. \quad (20)$$

*Remark:* Though the development is done for the ith subsystem, similar analysis can be done for the other subsystems.

Let the control input, $u_i(k)$, is selected as

$$u_i(k) = x_{ind}(k+1) - \hat{f}_i(x_i(k)) - \hat{\Delta}_i(.) + k_{i,v} r_i(k) - \lambda_{i,1} e_{in}(k) - \cdots - \lambda_{i,n-1} e_{i2}(k) \quad (21)$$

yields the tracking error dynamics for the ith subsystem as

$$r_i(k+1) = k_{i,v} r(k) + \zeta_{i1}(k) + \zeta_{i2}(k) + \varepsilon_i(k) + d_i(k). \quad (22)$$

where the functional estimation *errors* are defined by

$$\zeta_{i1}(k) = \tilde{w}_{i1}^T(k)\sigma_i(k), \quad (23a)$$

for the unknown nonlinear dynamics and

$$\zeta_{i2}(k) = \tilde{w}_{i2}^T(k)\sigma_{i2}(k). \quad (23b)$$

for the unknown interconnections with the combined NN approximation error for both NN is denoted by $\varepsilon_i(k)$. The bound on the approximation error is defined above.

An inner action generating NN loop eliminates the nonlinear dynamics of the system as well as compensates for the interactions. The outer loop designed via Lyapunov guarantees the stability and accuracy in following the desired trajectory. The next step is to determine the weight updates so that the performance of the closed-loop tracking error dynamics are guaranteed.

### *B. Weight Updates for Guaranteed Performance*

It is required to demonstrate that the performance criterion in terms of tracking error, $r_i(k); i = 1, 2, ...m$, is suitably small and that the NN weights, $\hat{w}_{i1}(k)$, $\hat{w}_{i2}(k); i = 1, 2, ...m$, remain bounded. In the following theorem, a discrete-time weight tuning algorithm based on the tracking error is given, which guarantees that both the tracking error and the NN weight estimates are bounded not only for the ith subsystem but also to the overall nonlinear discrete-time system.

*Theorem 3.1: (with PE Condition):* Let the desired trajectory vector, $x_{ind}(k); i = 1, ..., m$, and its delayed values be bounded. Also, let the NN approximation error be bounded above by $\varepsilon_{iN}$ for the ith subsystem, and the disturbance bound $d_{iM}$ a known constant. Let the NN weight tuning for approximating the unknown subsystem dynamics be provided by

$$\hat{w}_{i1}(k+1) = \hat{w}_{i1}(k) + \alpha_{i1}\sigma_{i1}(.) r_i^T(k+1); i = 1, ..., m, \quad (24)$$

and the NN weight tuning for approximating the interconnection terms is provided by

$$\hat{w}_{i2}(k+1) = \hat{w}_{i2}(k) - \alpha_{i2}\sigma_{i2}(.)(\hat{y}_{i2}(k) + B_{i2} k_{i,v} r_i(k))^T; i = 1, ..., m, \quad (25)$$

where $\hat{y}_{i2}(k) = \hat{w}_{i2}^T(k)\sigma_{i2}(k)$, $\|B_{i,2}\| \le \kappa_{i2}$ and $\alpha_{i1}$, $\alpha_{i2}$ ;i=1,..,m are NN adaptation gains. Then the filtered tracking error, $r_i(k), \nabla i = 1, ...., m$, and the NN weight estimates, $\hat{w}_{i1}(k)$, $\hat{w}_{i2}(k), \nabla i = 1, ..., m$ are UUB provided (a) output vectors of the hidden layers for the NN $\sigma_{i1}(k)$ and $\sigma_{i2}(k)$ are persistently exciting (PE) and the bounds specifically given by (35) through (37) (b) the design parameters are selected as:

(1) $\alpha_{i1}\|\sigma_{i1}(k)\|^2 < 1$, i=1,…,m (26)

(2) $\alpha_{i2}\left\|\sigma_{i2}(k)\right\|^2 < 2$, i=1,…,m (27)

(3) $k_{v\max} < \frac{1}{\sqrt{\eta}}$, (28)

where

$$\eta = \sum_{i=1}^{m}[\frac{1}{(1-\alpha_{i1}\left\|\sigma_{i1}(k)\right\|^2)} + \frac{\kappa_{i2}^2}{(2-\alpha_{i2}\left\|\sigma_{i2}(k)\right\|^2)}] \quad (29)$$

and $k_{v\max}$ is the maximum singular value of the gain matrix, $k_v$.

*Remark:* Note it is very easy to verify conditions (1) through (3) and therefore the proof is omitted. And the upper bound on $k_{v\max}$ is independent of the choices of NN activation functions.

*Remark:* The selection of the gain matrix is done on an individual subsystem basis and the controller design uses local signals only. However, the overall gain matrix must be satisfy (28) in order to keep the overall system stable.

*Proof: Define the Lyapunov function candidate*

$$J = \sum_{i=1}^{m}[r_i^T(k)r_i(k) + \frac{1}{\alpha_{i1}}tr(\tilde{w}_{i1}^T(k)\tilde{w}_{i1}(k)) + \frac{1}{\alpha_{i2}}tr(\tilde{w}_{i2}^T(k)\tilde{w}_{i2}(k))], \quad (29)$$

whose first difference is calculated as $\Delta J_1 + \Delta J_2$, where

$$\Delta J_1 = \sum_{i=1}^{m}[r_i^T(k+1)r_i(k+1) - r_i^T(k)r_i(k) + \frac{1}{\alpha_{i1}}tr(\tilde{w}_{i1}^T(k+1)\tilde{w}_{i1}(k+1) - \tilde{w}_{i1}^T(k)\tilde{w}_{i1}(k))] \quad (30)$$

$$\Delta J_2 = \sum_{i=1}^{m}[\frac{1}{\alpha_{i2}}tr(\tilde{w}_{i2}^T(k+1)\tilde{w}_{i2}(k+1) - \tilde{w}_{i2}^T(k)\tilde{w}_{i2}(k))]. \quad (31)$$

The first difference $\Delta J$ is obtained by using the tracking error dynamics (22) and weight tuning updates from (24) and (25) as

$$\Delta J \le -(1-\eta k_{v\max}^2)[\left\|r(k)\right\|^2 - 2\frac{\gamma k_{v\max}}{(1-\eta k_{v\max}^2)} - \frac{\rho}{(1-\eta k_{v\max}^2)}]$$
$$-\sum_{i=1}^{m}(1-\alpha_{i1}\sigma_{i1}^T(k)\sigma_{i1}(k))$$
$$\left\|(\varsigma_{i1}(k)+\varsigma_{i2}(k)) - \frac{k_{vi}r_i(k)+\alpha_{i1}\sigma_{i1}^T(k)\sigma_{i1}(k)(w_{i1}^T\tilde{\sigma}_{i1}(k)+w_{i2}^T\tilde{\sigma}_{i2}(k)+\varepsilon_i(k)+d_i(k))}{(1-\alpha_{i1}\sigma_{i1}^T(k)\sigma_{i1}(k))}\right\|^2$$
$$-\sum_{i=1}^{m}(2-\alpha_{i2}\sigma_{i2}^T(k)\sigma_{i2}(k))$$
$$\left\|(\hat{w}_{i2}^T(k)\sigma_{i2}(k)) - \frac{(1-\alpha_{i1}\sigma_{i1}^T(k)\sigma_{i1}(k))(w_{i2}^T\sigma_{i2}(k)+k_{vi}r_i(k))}{(2-\alpha_{i1}\sigma_{i1}^T(k)\sigma_{i1}(k))}\right\|^2 \quad (32)$$

where $\eta$ is given in (29) with $k_{v\max}$ the maximum singular value of $k_v$ and

$$\gamma = \sum_{i=1}^{m}[\frac{1}{(1-\alpha_{i1}\left\|\sigma_{i1}(k)\right\|^2)}(w_{i1\max}\sigma_{i1\max} + w_{i2\max}\sigma_{i2\max} + \varepsilon_{iN} + d_{iM}) + \frac{\kappa_{i2}\left\|\sigma_{i2}(k)\right\|w_{2\max}}{(2-\alpha_{i2}\left\|\sigma_{i2}(k)\right\|^2)}] \quad (33)$$

and

$$\rho = \sum_{i=1}^{m}[\frac{1}{(1-\alpha_{i1}\left\|\sigma_{i1}(k)\right\|^2)}(w_{i1\max}\sigma_{i1\max} + w_{i2\max}\sigma_{i2\max} + \varepsilon_{iN} + d_{iM})^2 + \frac{\left\|\sigma_{i2}(k)\right\|^2 w_{2\max}^2}{(2-\alpha_{i2}\left\|\sigma_{i2}(k)\right\|^2)}] \quad (34)$$

This further implies that the first difference $\Delta J \le 0$ as long as (26) through (28) holds and

$$\left\|r(k)\right\| > \frac{1}{(1-\eta k_{v\max}^2)}[\gamma k_{v\max}^2 + \sqrt{\gamma^2 k_{v\max}^2 + \rho(1-\eta k_{v\max}^2)}] \quad (35)$$

In general $\Delta J \le 0$ in a compact set as long as (26) through (28) are satisfied and (35) holds. According to a standard Lyapunov extension theorem [5], this demonstrates that the filtered tracking error vector is bounded and hence individual subsystem filtered tracking errors, $r_i(k);\Delta i = 1,....,m$ are bounded. It remains to show that the weight estimates $\left\|\tilde{w}_1(k)\right\|$ and $\left\|\tilde{w}_2(k)\right\|$; i=1,….,m are bounded.

The dynamics relative to error in weight estimates using (25) are given by

$$\tilde{w}_{i2}(k+1) = [I-\alpha_{i2}\sigma_{i2}(k)\sigma_{i2}^T(k)]\tilde{w}_{i2}(k) + \alpha_{i2}\sigma_{i2}(k)[w_{i2}^T\sigma_{i2}(k) + B_{i2}k_{i,v}r_i(k)]^T \text{;i=1,….,m} \quad (36)$$

where the filtered tracking error for the ith subsystem is bounded. Applying the PE condition [4] and using the tracking error bound (35), the boundedness of $\tilde{w}_{i2}(k)$ and hence $\hat{w}_{i2}(k)$ is assured. Now the dynamics relative to the weight estimates using (24) are given by

$$\tilde{w}_{i1}(k+1) = [I-\alpha_{i1}\sigma_{i1}(k)\sigma_{i1}^T(k)]\tilde{w}_{i1}(k) - \alpha_{i1}\sigma_{i1}(k)[w_{i1}^T\tilde{\sigma}_{i1}(k) + k_{i,v}r_i(k) + \varsigma_{2i}(k) + \varepsilon_i(k) + d_i(k)]^T \quad (37)$$

Applying the PE condition, and using the filtered tracking error bound (35) and the weight estimation error bound for the interconnection subsystem from (36), the boundedness of $\tilde{w}_{i2}(k)$ and hence $\hat{w}_{i2}(k)$ is assured.

*Theorem 3.1: (without PE Condition):* Assume the hypothesis presented in Theorem 3.1 and take the NN weight tuning for approximating the unknown subsystem dynamics as

$$\hat{w}_{i1}(k+1) = \hat{w}_{i1}(k) + \alpha_{i1}\sigma_{i1}(.)r_i^T(k+1),$$
$$-\Gamma\left\|I-\alpha_{i1}\sigma_{i1}(.)\sigma_{i1}^T(,)\right\|\hat{w}_{i1}(k); i=1,...,m \quad (38)$$

and the NN weight tuning for approximating the interconnection terms are provided by

$$\hat{w}_{i2}(k+1) = \hat{w}_{i2}(k) - \alpha_{i2}\sigma_{i2}(.)(\hat{y}_{i2}(k) + B_{i2}k_{i,v}r_i(k))^T,$$
$$-\Gamma\left\|I-\alpha_{i2}\sigma_{i2}(.)\sigma_{i2}^T(.)\right\|\hat{w}_{i2}(k); i=1,...,m \quad (39)$$

where $\hat{y}_{i2}(k) = \hat{w}_{i2}^T(k)\sigma_{i2}(k)$, $\left\|B_{i,2}\right\| \le \kappa_{i2}$ and $\alpha_{i1}$, $\alpha_{i2}$ ;i=1,..,m are NN adaptation gains. Then the filtered tracking error, $r_i(k), \nabla i = 1,....,m$, and the NN weight estimates, $\hat{w}_{i1}(k),\ \hat{w}_{i2}(k), \nabla i = 1,...,m$ are UUB.

## IV. SIMULATION RESULTS

The nonlinear system is described by:
*Subsystem 1:*

$$\begin{aligned} x_{11}(k+1) &= x_{12}(k) \\ x_{12}(k+1) &= f_1(x_1(k)) + (x_{21}^2 + x_{22}^2) + u_1(k) \end{aligned}, \quad (40)$$

*Subsystem 2:*

$$\begin{aligned} x_{21}(k+1) &= x_{22}(k) \\ x_{22}(k+1) &= f_2(x_2(k)) + (x_{11}^2 + x_{12}^2) + u_2(k) \end{aligned} \quad (41)$$

where $x_1 = [x_{11} \quad x_{12}]^T$ and $x_2 = [x_{21} \quad x_{22}]^T$ with

$$f_1(.) = -\frac{3}{16}\left[\frac{x_{11}(k)}{1 + x_{12}^2(k)}\right] + x_{12}(k) \cdot \quad (42)$$

and

$$f_2(.) = -\frac{3}{16}\left[\frac{x_{21}(k)}{1 + x_{22}^2(k)}\right] \quad (43)$$

The objective is to track a reference signal using the proposed adaptive NN controller. The reference signal used for the first subsystem is $x_{1nd} = \sin(\omega kT),\ \omega = 2,$ with a sampling interval of T=1sec. The reference signal for the second system is $x_{1nd} = \sin(\omega kT),\ \omega = 3$. The gains of the PD controller are taken as k=0.15 with $\lambda = 0.2$.

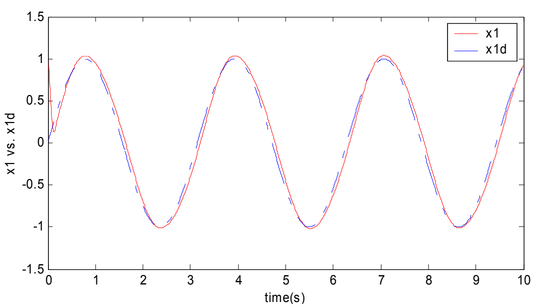


**Figure 1:** Actual vs. desired trajectory.

Both the NNs contain 10 nodes in the hidden layer. For weight updating, the learning rate is selected as $\alpha_{11} = \alpha_{12} = \alpha_{21} = \alpha_{22} = 0.1$. All the initial weights are selected randomly from [0,1] and all the activation functions are hyperbolic tangent sigmoid functions. The NN weights are initialized at random but they can be initialized at zero. No offline training is performed. Figures 1 and 2 illustrate the performance of the adaptive NN controller. From the figure, it is very obvious that the system tracking performance is satisfactory with the NN controller even though local information is used by the subsystem controllers. Moreover, the overall system stability is guaranteed even in the presence of unknown interconnections.

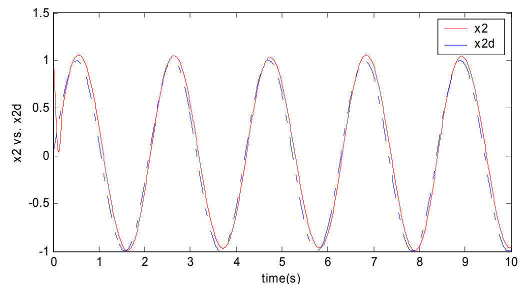


**Figure 2:** Actual vs. desired trajectory.

## V. CONCLUSIONS

This paper proposes an adaptive decentralized NN based controller for a class of nonlinear systems. This adaptive NN-based approach neither requires the information about the system dynamics nor the interconnection dynamics. No initial learning phase is needed. Novel weight tuning methods are derived using a rigorous mathematical analysis. The adaptive NN controller includes two inner loops one for compensating the unknown dynamics of the subsystem and the other for the unknown interconnections and an outer PD control loop for tracking. The tuning of the NNs is performed online and guarantees performance as shown through the Lyapunov analysis.

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