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Down-Aisle Stability of Rack Structures

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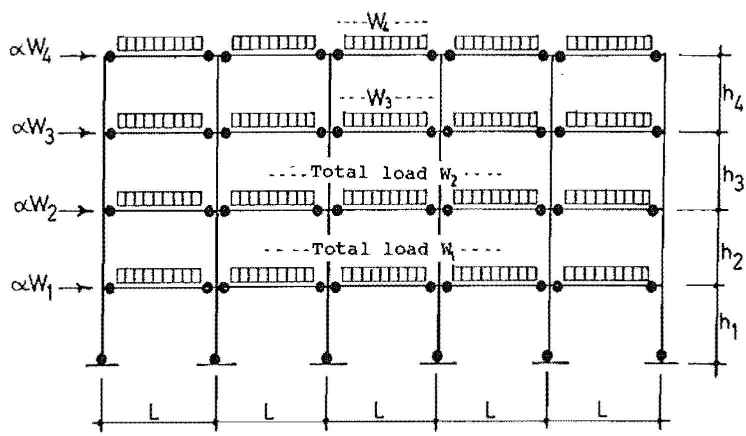
DOWN-AISLE STABILITY OF RACK STRUCTURES

by J Michael Davies[1]

Introduction

The problem under consideration is illustrated in Fig.1. A rack structure may have a large number of bays and many beam levels. It is typified by the use of cold-formed section members and by the semi-rigid joints between the beams and the uprights which is a factor that dominates the design. In many cases, it is also necessary to recognise the partial rigidity at the bases of the uprights where steel plates rest on a (concrete) floor.

Although rack structures are designed to carry predominantly the vertical loads from the stored material, most Codes of Practice include the requirement for a nominal horizontal load. In addition to ensuring adequate stiffness in the down-aisle direction, this takes into account of any lack of plumb in the uprights which can be aggravated by initial looseness in the semi-rigid joints. The nominal side load at each beam level is usually specified to be some proportion α of the total vertical load at that level as shown in Fig.1.



Flexible connections thus:- ●

Fig.1. Design loads on a typical rack structure

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The essential analytical requirement for the consideration of down-aisle stability is a suitable second-order elastic analysis for a large plane frame with semi-rigid joints. Conventional computer analysis using the matrix displacement method, suitably modified to include joint flexibility, is evidently applicable(1) and serves as a yardstick whereby other methods may be judged. However, as evidenced by the approximate treatments in the various Codes, this is not always practicable for the large assemblies that often arise and there is a need for a simplified treatment which takes advantage of the regular nature of the construction.

This paper first reviews the procedures that are incorporated in some current Codes of Practice. It then proposes an improved approach which gives rise to explicit design expressions. Comparison with exact analyses reveals that these are sufficiently accurate for all practical purposes.

Treatment of down-aisle stability in current Codes

The author's interest in this subject was reawakened when he was asked to take part in a working group commissioned by the "Federation Europeenne de la Manutention" (FEM), an association of European manufacturers of storage equipment, to appraise and update their draft Recommendations for the Design of Steel Static Pallet Racking(2).

The procedure for down-aisle stability incorporated in the draft FEM document has been described by Stark and Tilburgs(3) and the model that they used is shown in Fig.2. The points to note are:

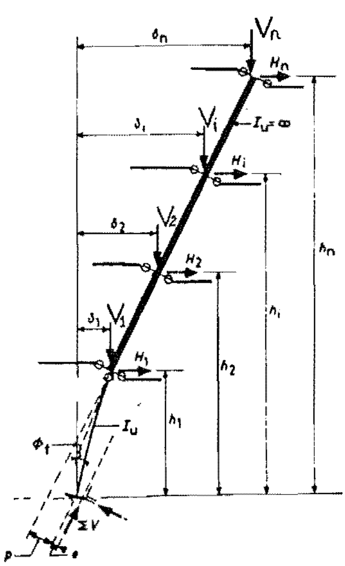


Fig.2. Stark and Tilburgs model

- It is based on a single internal upright, regardless of the number of bays in the structure. This is safe but may become inaccurate for a small number of bays.
- It only allows for column flexibility below the level of the first beam and the remainder of the column is treated as rigid. This assumption becomes increasingly unsafe as the number of storey levels increases. It is particularly inappropriate when the first beam is near the ground.
- It takes into account second-order effects.
- The stiffness of unbolted bases with thin steel baseplates is included using an "eccentricity method" in which the vertical reaction ΣV is offset by an amount e which depends on the base rotation ϕ_f. This eccentric reaction exerts a variable base restoring moment.

A calibration of this method, the results of which will be given later, revealed that it is rather cumbersome, though this can be overcome by programming the equations, and also that there are some typing errors common to both the paper and the draft code which must be corrected before it can give sensible answers. More seriously, however, it appears that the method was only verified by comparison with 'exact' theoretical analysis and test results for low rise racks. As expected, it could become unacceptably unsafe as the height of the structure increased.

Notwithstanding the above deficiencies, Stark and Tilburgs method is probably the best of the simplified treatments that have been codified to date. It should also be noted that, in common with the other codes considered in this section, the draft FEM code allows a full second-order frame analysis as an alternative to the simplified treatment.

The American RMI Specification[4] states in clause 5.3.1.1

> "For pallet racks and for the portion of the column between the bottom beam and the floor as well as between the beam levels, the effective length factor K is defined as 1.7 or as otherwise determined by rational analysis or tests."

This approach is also potentially unsafe because the value of K depends critically on the stiffness of the beam to upright connections as well as on a number of other factors such as the geometry of the rack and the relative stiffness of the members. Exact analyses show that values of K well in excess of 1.7 are not uncommon.

The provisions of the British SEMA code[5] are more comprehensive than those of the RMI code but a primary requirement for down-aisle stability is still the effective length factor of the uprights. This is restricted to the rather low value of K = 1.25 except for the lowest length where K must be increased to 1.5 unless there are two floor fixings, one on each side of the upright in the down-aisle direction.

In addition, there are some further requirements for floor fixings depending on the height of the rack and an additional requirement for overall stability based on a quasi-plastic theory approach, namely:

$$0.25 \sum M_j \leq \sum W_i(R_0 + 0.005)h_i \qquad .....(1)$$

where ΣM_j = the sum of the safe moments of resistance of all connections to a single upright

W_i = the total load entering the upright at height h_i

R_0 = The looseness of a connector

Although equation (1) implies a factor of safety of eight, the primary need is to ensure adequate *stiffness* to resist elastic buckling so that this latter requirement cannot be relied upon to give consistently safe results.

Effective length and elastic critical load

The concept of effective length has been a source of great confusion to Engineers who have tended to regard it as a property of the member and its end conditions whereas, in reality, it reflects the behaviour of the complete structure. The problem probably arises because engineering students are often taught about effective lengths early in their studies in the context of the buckling of individual columns and before they are in any position to appreciate the wider theory of the stability of complete structures. These early ideas are then difficult to displace.

Effective lengths obtained by considering individual members or small substructures are only relevant in the few cases where buckling of an individual member can reasonable be considered in isolation from the elastic buckling of the complete structure. Examples of this situation are the columns of *braced* frames and the internal members of trusses. Unbraced rack structures do *not* fall into this category.

In general, the only way to calculate an effective length is to first calculate the elastic critical load factor λ_{crit} of the complete structure. The effective length of any member of length L and second moment of area I is then given by

$$L_{eff} = \sqrt{\frac{\pi^2 EI}{\lambda_{crit} P}} \qquad .....(2)$$

where P is the axial force at unit load factor. The critical length factor K is then

$$K = \frac{L_{eff}}{L} \qquad .....(3)$$

It is salutary to note that, as the stiffness of the semi-rigid joints is reduced, the effective length of the uprights in the down-aisle direction of a rack structure tends to infinity.

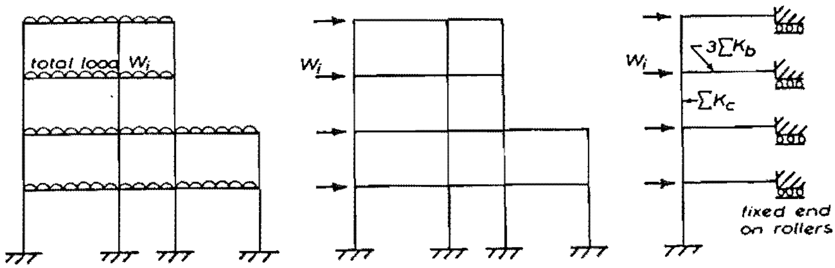
For the above reasons, further discussion of down-aisle stability will be primarily concerned with estimating λ_{crit} and then using it as part of a rational design procedure. The effective length factor K will be conspicuously absent from this discussion.

Analysis for down-aisle stability

The principles of an improved analysis for down-aisle stability have already been described by the author[1]. This paper takes the process a stage further to yield explicit design expressions. The method arises initially out of the work of Horne[6] who demonstrated a simple procedure for estimating the elastic critical loads of multi-storey frames. An elastic analysis is carried out in which the total vertical load on each storey is applied as a side load at that level as shown in Figs. 3(a) and (b). If, in a given storey of height h_i, the relative sway of the beams above and below is u_i, the sway index $\phi_i = u_i/h_i$ and an accurate value of the elastic critical load factor λ_{crit} is given by

$$\lambda_{crit} = \frac{1}{\phi_{max}} \qquad .....(4)$$

where ϕ_{max} is the maximum value of ϕ when all storeys are considered.



(a) Actual frame and loads

(b) Substitute load system (Horne)

(c) Substitute frame and load system

Fig.3. Models for approximate down-aisle stability analysis

It is of interest to note that the Horne method has now been incorporated into both the current British Standard (BS 5950: Part 1) and the European Standard (Eurocode 3) as an appropriate method for the determination of the elastic critical load of a multi-storey frame.

The author showed(1) that, particularly with pallet rack structures in mind, the method can be improved by reducing the full frame to the "Grinter" substitute frame shown in Fig 3(c). The equivalent frame has, in each storey, one column of stiffness equal to the total stiffness of all of the columns in that storey and one beam of stiffness equal to three times the total beam stiffness at that level. The multiplier of three occurs because each beam restrains two columns and has a 50% increase of stiffness if the end rotations are constrained to be equal.

The practical justification for this suggestion is that the complete substitute frame can be solved manually in a single moment distribution process using Naylor's no shear method. The sway indices and hence the elastic critical load then followed using slope deflection equations.

For rack structures, the member stiffnesses require modification to take account of flexible joints at the ends of the beams and at the base of the columns. The method then tends to be even more accurate than for rigid-jointed multi-storey frames. The theoretical reason for this is that the semi-rigid joints ensure that the sway mode under side load and the relevant buckling mode are virtually identical. The relevant equations, which will be required later, are as follows:

(a) Modified beam stiffness

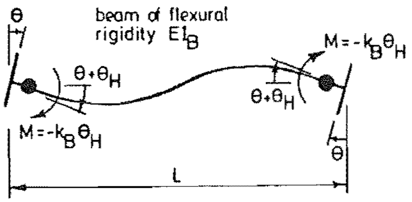


Fig.4. Typical beam

The situation to be considered is shown in Fig.4. In the absence of flexible joints, the rotational stiffness of the beam is $M/\theta = 6EI_B/L$. When joints of rotational stiffness k_B are introduced at both ends, the rotational stiffness of the beam becomes

$$\frac{M}{\theta} = \frac{6EI_B k_B}{6EI_B + k_B L} \qquad .....(5)$$

(b) Modified stiffness of lower length of upright

The situation to be considered is shown in Fig.5. In the absence of base flexibility, the no shear stiffness $M/\theta = EI_C/h$. When a semi-rigid joint of stiffness k_C is introduced at the base,

$$\frac{M}{\theta} = \frac{EI_C k_C}{EI_C + k_C h} \qquad .....(6)$$

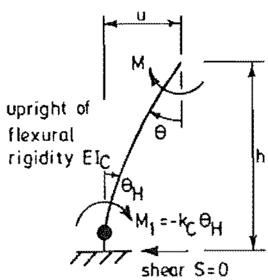


Fig.5. No shear case for bottom storey upright

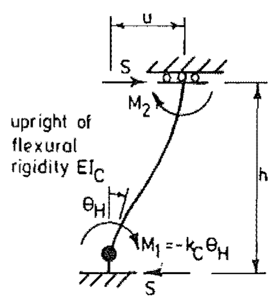


Fig.6. Fixed end moments for bottom storey upright

(c) Fixed-end moments for lower length of upright

The situation to be considered is shown in Fig.6 where S is the storey shear which will generally be the sum of all side loads above the storey under consideration. The expressions for the end moments are:

$$M_2 = \frac{-Sh}{2}\left[\frac{2EI_C + k_C h}{EI_C + k_C h}\right] \qquad .....(7)$$

$$M_1 = \frac{-Sh}{2}\left[\frac{k_C h}{EI_C + k_C h}\right] \qquad .....(8)$$

(d) Analysis of the substitute frame

A manual solution of the substitute frame has already been described[1]. More attractive in the age of the micro-computer is to set up the equations in matrix form for computer solution. Using the notation of Fig.7, the procedure is as follows:

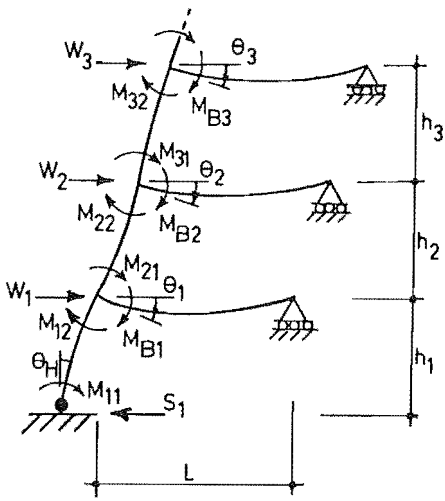


Fig.7. Notation for matrix analysis of the substitute frame

Each beam in the substitute frame is representative of two beams framing into the upright so that, from equation (5), at level i,

$$M_{Bi} = \frac{12EI_B k_B}{6EI_B + k_B L}\theta_i = K_{Bi}\theta_i \qquad .....(9)$$

Each column above the first beam is in a standard no-shear situation so that for the length above level i with end rotations θ_i and θ_{i+1},

$$M_{i2} = \frac{-S_i h_i}{2} + \frac{EI_{Ci}}{h_i}(\theta_i - \theta_{i-1}) = \frac{-S_i h_i}{2} + K_{Ci}\theta_i - K_{Ci}\theta_{i-1} \qquad .....(10)$$

$$M_{i+1,1} = \frac{-S_{i+1}h_{i+1}}{2} + \frac{EI_{C,i+1}}{h_{i+1}}(\theta_i - \theta_{i+1}) = \frac{-S_{i+1}h_{i+1}}{2} + K_{C,i+1}\theta_i - K_{C,i+1}\theta_{i+1} \quad .....(11)$$

where S_i is the storey shear.

The lowest column length requires special consideration because of the semi-rigid connection at the base. Here, from equations (6) and (7),

$$M_{12} = \frac{-S_1 h_1}{2}\left[\frac{2EI_{CI} + k_C h_1}{EI_{Ci} + k_C h_1}\right] + \frac{EI_{Ci} k_C}{EI_{Ci} + k_C h_1}\theta_1 = \frac{-S_1^1 h_1}{2} + K_{Ci}^1 \theta_1 \quad ....(12)$$

The required matrix equations for the evaluation of the unknown rotations θ_i can now be set up by considering the equilibrium of each joint in turn, for example,

$$M_{i,2} + M_{i+1,1} + M_{Bi} = 0 \quad .....(13)$$

and denoting

$$K_{Ci} + K_{C,i+1} + K_{Bi} = \Sigma K_i \quad .....(14)$$

gives

$$\begin{bmatrix} \Sigma K_1{}^1 & -K_{C2} & & \\ -K_{C2} & \Sigma K_2 & -K_{C3} & \\ & -K_{C3} & \Sigma K_3 & -K_{C4} \\ & & \text{etc} & \\ \vdots & & & \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \text{etc} \\ \vdots \end{bmatrix} = \begin{bmatrix} 0.5(S_1{}^1 h_1 + S_2 h_2) \\ 0.5(S_2 h_2 + S_3 h_3) \\ 0.5(S_3 h_3 + S_4 h_4) \\ \text{etc} \\ \vdots \end{bmatrix} \quad .....(15)$$

These equations can then be solved for the unknown rotations and the bending moments follow from equations (9) to (11). The sway index for any storey can then be determined as follows using the notation of Fig.8.

$$\phi_i = \frac{u_i}{h_i} = \frac{M_{Bi}}{K_{Bi}} - \frac{2M_{i2} - M_{i1}}{6K_{Ci}} \quad .....(16)$$

and the elastic critical load then follows from equation (4).

Explicit expressions for the elastic critical load

Although the methodology described above is attractively simple, it is still best done on a micro-computer and requires special programming. It is therefore of interest to explore whether further simplification may be possible without significant loss of accuracy. The first possibility is to apply the fundamental assumption of Stark and Tilburgs and to restrict bending of the column to the lowest storey as shown in Fig.9.

This is, of course, likely to lead to unsafe answers for tall structures but it leads to simple explicit design expressions for low-rise structures.

Let n_b = the number of bays in the rack
n_s = the number of storeys

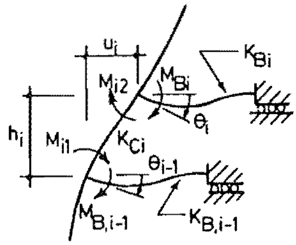


Fig.8. Typical storey of substitute frame

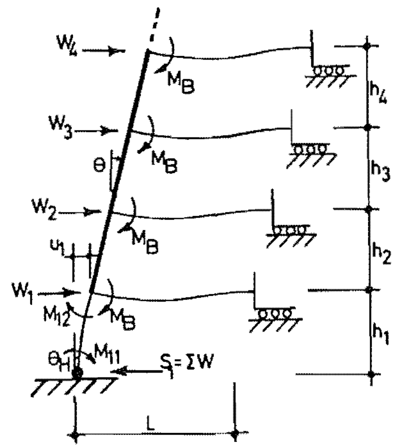


Fig.9. First simplified model for rack analysis

Then the equivalent second moment of area of the column and the equivalent base stiffness in the substitute frame are:

$$I_C^1 = (n_b + 1)I_C \qquad .....(17)$$

$$k_C^1 = (n_b + 1)k_C \qquad .....(18)$$

The beam end moment is given by equation (9) which, for a series of identical beams, can be more conveniently expressed as

$$M_B = F\theta \qquad .....(19)$$

where

$$F = \frac{12EI_B n_b k_b}{6EI_B + k_B L} \qquad .....(20)$$

Considering moment equilibrium of the column about the base gives

$$\sum(Wh) - \sum M_B + M_{11} = 0 \qquad .....(21)$$

But

$$\sum M_B = n_s F\theta \qquad .....(22)$$

and, from Fig.5 and equations (6) and (8),

$$M_{11} = \frac{-S_1 h_1}{2}\left[\frac{k_C^1 h_1}{EI_C^1 + k_C^1 h_1}\right] - \frac{EI_C^1 k_C^1}{EI_C^1 + k_C^1 h_1}\theta \qquad .....(23)$$

and therefore

$$\theta = \frac{\sum (Wh) - \frac{S_1 h_1}{2}\left[\frac{k_C^1 h_1}{EI_C^1 + k_C^1 h_1}\right]}{n_s F + \frac{EI_C^1 k_C^1}{EI_C^1 + k_C^1 h_1}} \qquad .....(24)$$

Having found θ, M_{11} is given by equation (23) above and

$$M_{12} = \frac{-S_1 h_1}{2}\left[\frac{2EI_C^1 + k_C^1 h_1}{EI_C^1 + k_C^1 h_1}\right] + \frac{EI_C^1 k_C^1}{EI_C^1 + k_C^1 h_1}\theta \qquad .....(25)$$

The bottom storey sway index ϕ is then given by

$$\phi = \theta - \frac{(2M_{12} - M_{11})h_1}{6EI_C^1} \qquad .....(26)$$

Noting that, within the restrictions of the model, only the bottom storey can buckle, it follows that:

$$\lambda_{crit} = \frac{1}{\phi} = \frac{EI_C^1 + k_C^1 h_1}{\left[EI_C^1 + \frac{k_C^1 h_1}{2}\right]\theta + \frac{\sum Wh^2}{12EI_C^1}\left[4EI_C^1 + k_C^1 h_1\right]} \qquad .....(27)$$

Although giving rise to a simple explicit expression for the elastic critical load, this model has the same limitations as that of Stark and Tilburgs. Therefore, before considering how an estimate of the elastic critical load can be incorporated in a complete design procedure for down-aisle stability, it is appropriate to first investigate a potential improvement in the model.

Improved model for down-aisle stability

The only significant limitation of the simplified model shown in Fig.9 is that it neglects the flexibility of the upper storey columns. However, it is sufficiently accurate to suggest that it is not necessary to take into account the flexibility of all storeys and that an adequate estimate of the critical load will be obtained if the columns in just the two lowest storeys are allowed to bend. This argument also takes into account the observation that, when using either the full Horne method or the author's original simplification, the critical storey with the highest sway index is usually the first or second.

An improved model is therefore shown in Fig.10. The derivations of the design expressions corresponding to this model follow a similar course to the above though the equations are a little more cumbersome. Using a similar notation to the previous section, augmented by Fig.10,

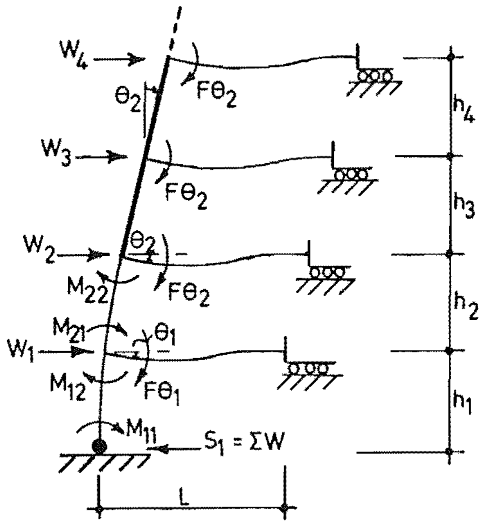


Fig.10. Improved model for rack analysis

$$M_{11} = \frac{-S_1 h_1}{2}\left[\frac{k_C^1 h_1}{EI_C^1 + k_C^1 h_1}\right] - \frac{EI_C^1 k_C^1}{EI_C^1 + k_C^1 h_1}\theta_1 \qquad .....(28)$$

$$M_{12} = \frac{-S_1 h_1}{2}\left[\frac{2EI_C^1 + k_C^1 h_1}{EI_C^1 + k_C^1 h_1}\right] + \frac{EI_C^1 k_C^1}{EI_C^1 + k_C^1 h_1}\theta_1 \qquad .....(29)$$

$$M_{21} = \frac{-S_2 h_2}{2} + \frac{EI_C^1 \theta_1}{h_2} - \frac{EI_C^1 \theta_2}{h_2} \qquad .....(30)$$

Joint equilibrium at the first beam level can be expressed as

$$M_{12} + M_{21} + F\theta_1 = 0 \qquad .....(31)$$

After substituting for M_{12} and M_{21} from above, this can be expressed in the form

$$\theta_1 = A + B\theta_2 \qquad .....(32)$$

where

$$A = \frac{S_1 h_1}{2C}\left[\frac{2EI_C^1 + k_C^1 h_1}{EI_C^1 + k_C^1 h_1}\right] + \frac{S_2 h_2}{2C} \qquad .....(33)$$

$$B = \frac{EI_C^1}{h_2 C} \qquad .....(34)$$

$$C = F + \frac{EI_C^1 k_C^1}{EI_C^1 + k_C^1 h_1} + \frac{EI_C^1}{h_2} \qquad .....(35)$$

Moment equilibrium of the entire upright gives

$$\sum (Wh) - (n_s - 1)F\theta_2 - F\theta_1 + M_{11} = 0 \qquad .....(36)$$

Then, substituting for θ_1 and M_{11} and rearranging gives

$$\theta_2 = \frac{G}{D} \qquad .....(37)$$

where

$$G = \sum (Wh) - FA - \frac{S_1 h_1}{2}\left[\frac{k_C^1 h_1}{EI_C^1 + k_C^1 h_1}\right] - \frac{EI_C^1 k_C^1 A}{EI_C^1 + k_C^1 h_1} \qquad .....(38)$$

$$D = \theta_2 \left[(n_s - 1 + B)F + \frac{EI_C^1 k_C^1}{EI_C^1 + k_C^1 h_1}\right] \qquad .....(39)$$

The bending moments M_{11}, M_{12} and M_{21} can then be determined by substituting for θ_1 and θ_2 in equations (28) to (30). The sway index for the bottom storey is then

$$\phi_1 = \theta_1 - \frac{(2M_{12} - M_{11})h_1}{6EI_C^1} \qquad .....(40)$$

The calculation of the sway index ϕ_2 for the first storey also requires

$$M_{22} = \frac{-S_2 h_2}{2} - \frac{EI_C^1 \theta_1}{h_1} + \frac{EI_C^1 \theta_2}{h_2} \qquad .....(41)$$

and then

$$\phi_2 = \theta_2 - \frac{h^2}{6EI_C^1}(2M_{22} - M_{21}) = \frac{S_2h_2^2}{12EI_C^1} + \frac{\theta_1 + \theta_2}{2} \qquad .....(42)$$

Finally, the elastic critical load λ_{crit} is given by

$$\lambda_{crit} = \frac{1}{\phi_{max}} \qquad .....(43)$$

where ϕ_{max} is the greater of ϕ_1 and ϕ_2.

Actual bending moments under notional horizontal loads

The various procedures described above for the determination of the elastic critical load λ_{crit} can also be used to estimate the sway of the complete frame and its bending moment distribution under the combination of vertical and notional horizontal loads shown in Fig.1 for load factors less than λ_{crit}. This is because the "amplified sway" method is remarkably accurate for racking structures with semi-rigid joints. The sway deflections and bending moments given by the Horne method and its various derivatives described above merely have to be multiplied by an amplification factor β to give the corresponding second order values arising from the notional side loads αW, where

$$\beta = \frac{\alpha\lambda\lambda_{crit}}{\lambda_{crit} - \lambda} \qquad .....(44)$$

To obtain the design bending moments in individual members, it is necessary to apportion the sway moments in the substitute frame to the uprights and to add the bending moments due to vertical (pattern) load.

Bending moments due to pattern loading

It is in the nature of pallet racking that an unlimited number of possible loading patterns exist and it is an impossible task to predict which pattern will generate the critical combination of axial load and bending moment which will determine the design of a particular upright. However, a limited investigation suggests that it is generally sufficient to consider the rack to be fully loaded except for a single beam near the middle of the structure at the lowest level. If the bottom beam is near the ground, a check should also be made with the load omitted from a single beam near the middle of the second level, otherwise there is only one load case to consider.

This load case is most easily generated by adding to the second-order sway case considered above (a) a pattern of bending moments corresponding to uniform downward loading and (b) a pattern of bending moments corresponding to a single beam load applied in the upward direction.

Near the middle of the structure, full downward loading gives rise to equal fixed end moments in all beams of magnitude

$$M_{BF} = \frac{W_B L}{12}\left[\frac{k_B L}{2EI_B + k_B L}\right] \qquad .....(45)$$

where W_B is the total load per beam.

The moments generated by removing the load from a beam at the lowest level can be estimated with sufficient accuracy using the sub-frame shown in Fig.11. Taking advantage of symmetry, the bending moments indicated on the figure are given by the following expressions:

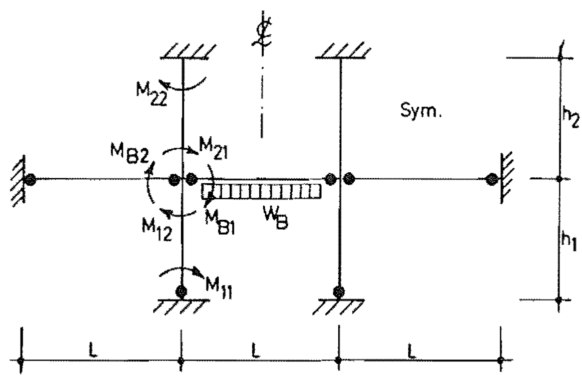


Fig.11. Sub-frame for the removal of a single beam load

Beam stiffness, symmetrical case

$$K_{B1} = \frac{2EI_B k_B}{2EI_B + k_B L} \qquad .....(46)$$

Beam stiffness, general case

$$K_{B2} = \frac{4EI_B k_B (3EI_b + k_B L)}{(2EI_B + k_B L)(6EI_B + k_B L)} \qquad .....(47)$$

Stiffness of lower upright

$$K_{C1} = \frac{4EI_C(3EI_C + k_C h_1)}{h_1(4EI_C + k_C h_1)} \qquad .....(48)$$

Stiffness of upper upright

$$K_{C2} = \frac{4EI_C}{h_2} \qquad .....(49)$$

Total stiffness of joint

$$\Sigma K = K_{B1} + K_{B2} + K_{C1} + K_{C2} \qquad .....(50)$$

Then, with the fixed-end moment M_{BF} according to equation (45),

$$M_{B1} = M_{BF}(1 - \frac{K_{B1}}{\Sigma K}) \qquad .....(51)$$

$$M_{B2} = M_{BF}\frac{K_{B2}}{\Sigma K} \qquad .....(52)$$

$$M_{12} = M_{BF}\frac{K_{C1}}{\Sigma K} \qquad .....(53)$$

$$M_{11} = \frac{M_{12}}{2}\frac{k_C h_1}{3EI_C + k_C h_1} \qquad .....(54)$$

$$M_{21} = M_{BF}\frac{K_{C2}}{\Sigma K} \qquad .....(55)$$

$$M_{22} = \frac{M_{21}}{2} \qquad .....(56)$$

It has been shown by the use of stability functions that it is not necessary to consider second-order effects in this sub-frame. The same sub-frame can also be used to omit a beam load at the second level by inserting the appropriate values for h_1 and h_2 and letting $k_C = \infty$.

Design procedure

The methods described in this paper allow a complete pattern of axial loads, bending moments and sway deflections to be calculated for a typical rack structure such as is shown in Fig.1. This calculation takes account of:

- second-order ($P\Delta$) effects
- notional horizontal loads
- semi-rigid joints
- pattern loading

These bending moments and axial loads can be used for member design in the usual way. Here it should be noted that the analysis includes for all buckling effects in the down-aisle direction. This includes member buckling (by enhancing the bending moments) and it is not necessary to include for this effect twice by using an arbitrary effective length factor.

In addition to checking member stability, it is necessary to limit the sway deflection. This ensures that the rack is adequately stiff in the down-aisle direction and also ensures an adequate reserve of safety with respect to sway failure of the entire structure. An appropriate criterion is to limit the sway index in any storey under the notional horizontal loads (enhanced by second-order effects)to 0.005 at the serviceability limit state and 0.02 at the ultimate limit state.

Verification of the method

The methods described above have been subject to detailed calibration against the results of exact analysis and, where possible, test results. Space precludes a full account here and it is only possible to give a small selection of the comparisons with exact analyses. The family of frames shown in Fig.12 will be used for this purpose with the following values:

Uprights:	second moment of area	I_C = 700 000 mm^4
Beams:	second moment of area	I_B = 550 000 mm^4
Beam/column connection stiffness		k_B = 70 000 kNmm/radian
Base stiffness (when relevant)		k_C = 90 000 kNmm/radian
Ratio of notional horizontal to vertical load		α = 0.01

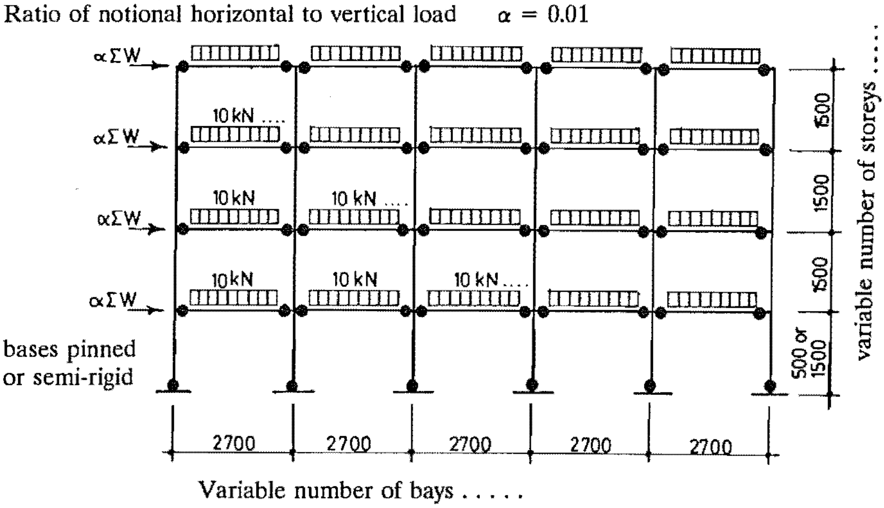


Fig.12. Frames for the verification of the design procedures

As well as varying the number of storeys and the number of bays, other variations were included. The various analyses are denoted by either Aij or Bij with or without (p) where:

Series A: lower storey = 1500 mm (all storey heights equal)
Series B: lower storey = 500 mm (lower beam close to ground)
i = number of storeys
j = number of bays
(p) denotes pinned bases (otherwise analysis includes k_C)

The most sensitive test of the basic methodology is the value predicted for the elastic critical load factor λ_{crit}. The results given by the different available methods are summarised for frames up to 6 storeys (8 metres) high in Table 1.

Frame Type	Values of λ_{crit}					
	Exact	Horne	Horne/ Davies	Stark & Tilburgs	Eq.(27)	Design Method
A33	4.004	3.785	3.802	5.571	4.337	3.910
A33(p)	2.193	1.947	1.960	2.058	2.553	2.035
A44	2.770	2.597	2.608	3.797	3.190	2.764
A44(p)	1.576	1.362	1.369	1.700	1.961	1.476
A55	2.085	1.959	1.966	2.856	2.509	2.131
A55(p)	1.219	1.037	1.043	1.411	1.590	1.154
A35	3.820	3.582	3.596	5.571	4.180	3.698
A35(p)	2.111	1.876	1.886	2.058	2.470	1.956
B33	6.350	5.913	5.944	Not applicable	Not applicable	6.315
B33(P)	4.437	4.127	4.153			4.365
B44	4.035	3.702	3.718			4.220
B44(p)	3.019	2.758	2.774			3.071
B55	2.868	2.638	2.648			3.159
B55(p)	2.230	2.045	2.055			2.554
B64	2.229	2.027	2.038			2.570
B64(p)	1.767	1.608	1.616			1.941

Table 1. Elastic critical loads of a family of frames

The various analyses included in this table are as follows:

Exact analysis uses a second-order plane frame analysis with semi-rigid joints and is the yardstick by which other methods may be judged.

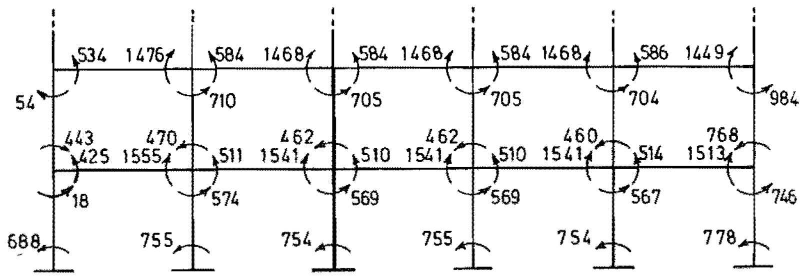
Horne is the application of the Horne side load method[6] using a first-order elastic analysis of the complete frame.

Horne/Davies is the simplification of the Horne method based on the Grinter substitute frame shown in Fig.7.

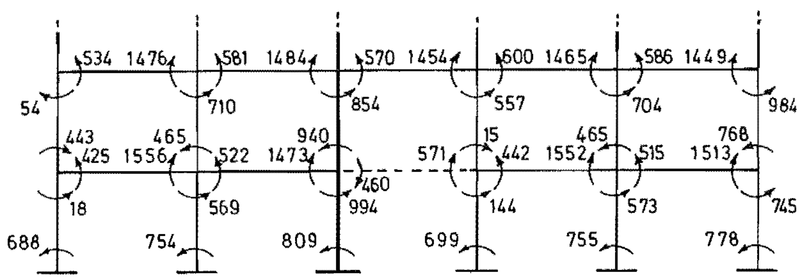
Stark and Tilburgs is the method given in Reference 3 which was proposed for the draft FEM recommendations[3].

Equation (27) is the first simplification of the Grinter substitute frame with column flexibility in the first storey only. This gives rise to an explicit expression for λ_{crit}.

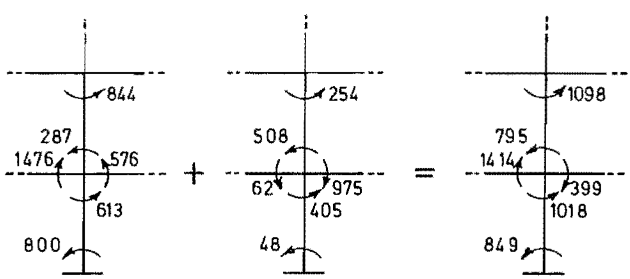
Design method is the second simplification of the substitute frame with column flexibility in the first two storeys. This method is considered to be the most appropriate available for a rational design procedure.



(a) second-order analysis with full vertical and notional side loads



(b) second-order analysis with load omitted on lowest central beam



fully loaded frame + negative beam load = pattern loading

(c) values obtained using the proposed design expressions

Fig.13. Exact and 'design' bending moments for frame A55 (kNmm)

As anticipated, both the Stark and Tilburgs method and equation (27) can be unconservative as the frame height increases. Furthermore, they are not applicable to the 'B' series in which the first beam is near the floor. All other methods for which the results are included in Table 1 give reasonable agreement over the whole range of frames considered and any of them can be used to give results of practical accuracy.

The success of the proposed procedure for the determination of the second-order bending moment distribution under both uniform and pattern loading is illustrated in Fig.13. Figs. 13(a) and (b) show the complete distributions of bending moment in the lower storeys for uniform and pattern loading respectively. the critical member to the leeward of the unloaded beam is highlighted. Fig.13(c) shows the bending moments in this critical member according to the proposed design procedure. Bearing in mind the nature of the exercise, the procedure is considered to be sufficiently accurate to provide a sound basis for member design. It may be noted that, provided pattern loading is considered, the internal uprights will always be more critical than the outer uprights because the latter carry only half the axial load.

Conclusions

This paper has described three simplified methods for the determination of the second-order behaviour of pallet racks with semi-rigid joints. This includes an estimate of the elastic critical load and the bending moment distribution under an appropriate pattern load. These methods have been verified by comparison with exact analysis and one, in particular, gives rise to a convenient design procedure using explicit equations.

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