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# A Fuzzy Based Load Model for Power System Direct Load Control

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## Abstract

Demand side management programs are strategies designed to alter the shape of the load curve. In order to successfully implement such a strategy, customer acceptance of the program is vital. It is thus desirable to design a model for direct load control which may accommodate customer preferences. This paper presents a methodology for optimizing both customer satisfaction and utility unit commitment savings, based on a fuzzy load model for the direct load control of appliances.

## 1. Introduction

In the competitive operation and business climate, load management programs will become more prevalent as customers demand more pricing and usage options. Many utilities will need to be more proactive in the stature they take in implementing load management programs. The most common load management program is end-use equipment control, which is also known as direct load control (DLC). The purpose of DLC is to shape the load curve by cycling customers' large current drawing appliances, such as air conditioners and water heaters. One critical area which will be of paramount importance in the new, competitive marketplace, is customer input and satisfaction. Also, in order to achieve maximum cost benefits, a DLC dispatch schedule must be coordinated with utility economic considerations such as unit commitment and economic dispatch. In this paper, a new approach to DLC is proposed in which customer preferences are accommodated while concurrently maximizing the savings of the utility.

In the competitive marketplace, any load model which is used as a basis for establishing a DLC dispatch schedule must consider the customers' preferences up front, and not as a secondary issue. The load model should be versatile enough to capture the spectra of preferences, and simple enough for successful implementation and easy interpretation of the results. It should also contain a mechanism for accounting for feedback from the customer as comfort and economic levels evolve and change.

Currently used models for DLC do not consider customer demographics at all. The method proposed by Hsu [1] classifies customers into  $N$  cycling groups,

each with a fixed cycling capacity. The method proposed by Cohen [2] models the DLC cycling as a change in energy demand. These methods assume that all customer groups are identical and homogeneous. They do not account for customer variation in preferences, such as maximum temperature tolerances, maximum temperature deviations, and differences in cycling group capacities. This paper presents a new load model and approach to direct load control based on fuzzy logic techniques which optimizes the trade-off between customer preferences, utility resources, and uncertainties in the load. The first part of this paper is devoted to deriving a fuzzified load model for use in direct load control. The remainder will discuss the implementation of this load model.

## 2. The Load Model

Many utilities summer peak due to the large contribution of central air conditioning loads. Controlling the operation of central air conditioners is one means of reducing the peak load. The controlled air conditioners are segmented into groups in which one or more groups are off, while the remainder are on. At the conclusion of the "offtime," the disabled air conditioners are switched back to an active state, while a different group is disabled. This arrangement permits the total utility load to remain effectively uniform.

The load control period usually lasts between four to ten hours per day, depending on the duration of the utility peak load. Following the load control period, the air conditioner is permitted to run until the house temperature reaches the thermostat setting. This postcontrol period is referred to as the "payback" period.

During the load control period, the house interior temperature may rise several degrees higher than if the air conditioner were not controlled. This implies that the customer must endure a certain degree of discomfort during the cycling of the air conditioning load. Thus, in order to effectively capture all aspects of the customer preferences, there are a number of parameters which must play a dominant role in the evolution of a load model. They are:

- The normal temperature or ambient energy content that the customer prefers (ambient criteria),

- The maximum temperature deviation or energy content that the customer is willing to tolerate (comfort criteria),
- The distribution of the cyclable load,
- Residential thermal loss, and
- Payback amount.

### 2.1. The Ambient and Comfort Criteria

In order to quantify customer preferences, two criteria are defined. The first is the ambient criteria, which is a measure proportional to the ambient internal temperature a customer or group of customers prefer. The second is the comfort criteria, which is a measure proportional to the maximum temperature a customer will comfortably tolerate. These preferences tend to be non-specific and vary from customer to customer. These preferences may overlap and may vary over time due to various outside influences. Thus, characterizing these preferences is well suited to a fuzzified environment which may account for non-specific quantities, or a range of quantities.

To achieve a load model which may account for customer preferences, a global distribution is first designated in which all customer preferences will lie. Similar to the approach in [2], this distribution is defined in terms of energy requirements. According to the nature of the load, global maximum and minimum levels for both ambient and comfort energy are defined. These energy levels are then divided into a number of fuzzy templates. These fuzzy subsets are given linguistic names like SMALL, MEDIUM, LARGE, etc. A SMALL ambient energy level would most likely correspond to those customers who prefer very cool ambient temperatures, perhaps in the range of 65°F to 69°F. These ranges will probably vary from utility to utility depending on geographic differences such as normal outside high temperature, humidity levels, and time zone. These fuzzy subsets define the Global Ambience Fuzzy Subset and the Global Comfort Fuzzy Subset.

The total customer area under DLC may then be broken into cycling groups based on criteria such as geographic (feeder) location or the nature of the load. The customers in each cycling group are then characterized by their their ambience and comfort levels. The reason for doing so is to allow for a certain degree of uncertainty that the customer may have if asked to specify exact figures. It should be noted that the distribution specification obtained for the ambient energy level need not be the same as that for the comfort energy level. For example, a customer may prefer a high thermostatic setting (LARGE ambient) but will not tolerate large deviations (SMALL comfort).

The results of the individual preferences may be obtained by truncating the global fuzzy subsets in accordance with the obtained percentage levels. These truncated fuzzy subsets are the Local Fuzzy Subsets, which are unique for each group under DLC.

### 2.2. The Distribution of Cyclable Loads

The distribution of the cyclable load within a utility defined DLC area is not a specific quantity, but also depends on the number of residences in the defined area, the types of units in the residences, the thermal energy transfer levels of the residences, and other outside influences as well. Many DLC approaches in the literature [1][2], have assumed the customer load under DLC to be homogeneous, with fixed capacities and preferences under all operating and weather conditions. The load model proposed herein attempts to rectify these shortcomings with a more flexible load model which may account for both customer preferences and variances in the load itself.

In each group of customers, the devices under DLC will encompass a range of power ratings. In addition, each specific load type may have a different frequency of occurrence. For example, within a group  $N$ , there may be  $N_1$  units corresponding to a power rating of  $PL_1$ ,  $N_2$  units corresponding to  $PL_2$ , and  $N_n$  units corresponding to  $PL_n$ . If  $N_{max}$  is the largest number of units corresponding to a specific rating, then all ratings may be normalized with respect to  $N_{max}$ . The resultant load template is then defined by:

$$(PL_i | \mu_{PL_i}) = \left( PL_i \mid \frac{N_i}{N_{max}} \right) \quad \forall i \in [1, \dots, n] \quad (1)$$

where  $0 \leq \mu_{PL_i} \leq 1$  is the membership value of the load  $PL_i$  and  $PL_1, \dots, PL_n$  is the range of the cyclable load. Note that the membership function  $\mu_{PL_i}$  denotes the strength of the membership of the load  $PL_i$  in this range. A high value of  $\mu_{PL_i}$  implies that  $PL_i$  has a high frequency of occurrence.

### 2.3. Fuzzy Rules For Load Transitions

Each DLC group is now described by three fuzzy templates which comprise the load model for that group. These templates are: the local ambience fuzzy subset, the local comfort fuzzy subset, and the load template. In the section 2.5, fuzzy rules will be used to map these subsets onto another fuzzy template for cycling period, or offtime. This template will then be used in coordination with a similar template for payback to establish the cycling times and commitment order for the DLC groups.

Each group within the DLC area will have a unique offtime  $T_{off}$  associated with it. This offtime will depend on the transition between the local fuzzy ambient and comfort templates. These transitions are defined as a series of if-then rules which govern the transition from one template to another. A typical fuzzy rule to calculate offtime is:

If ( $E_a = \text{SMALL}$ ) & ( $E_c = \text{SMALL}$ ) then ( $\mu_{T_{off}} = \text{SMALL}$ )

This particular rule implies that if a customer prefers a cooler ambient temperature ( $E_a = \text{SMALL}$ ) and will not tolerate large temperature deviations ( $E_c = \text{SMALL}$ ) then the subsequent offtime should be small ( $T_{off} = \text{SMALL}$ ). These fuzzy rules are

common to all groups. As customer preferences vary, the application of these rules to different groups will yield different fuzzy offtime templates.

#### 2.4. Effect of Thermal Losses

The load distribution model derived in Section 2.2 accounts for the range of cyclable load within a group. In this section, this model will be modified to account for thermal losses. Although detailed space conditioning models are generally available for steady-state and transient building analysis, a simplified model is often adequate to account for heat loss. Thermal losses from residences depend on a number of factors, but the two significant contributing factors are size and insulation. One straightforward method to account for thermal losses in the previous model is to introduce a bias into the base load rating of the device, based on size and age of the residence, where it is assumed that the level of insulation is inversely proportional to the age of the structure. This assumption of correlating age and insulation factor may not be valid in some specific cases, but over the large number of residences within a group, it is a valid generalization.

The bias in the load is accomplished through a series of additional fuzzy rules. After defining size and age templates similar to the ambient and comfort templates, and a template corresponding to the coefficient of thermal losses ( $T_\gamma$ ), the effective coefficient of thermal losses  $\Gamma_L$ , is defined as a fuzzy function of the application of the fuzzy rules to the templates. A crisp value of  $\Gamma_L = 1$  corresponds to the case where the effect of thermal loss is neglected. A typical fuzzy rule to determine the effect of thermal losses is:

IF (AGE is NEW) & (SIZE is SMALL) then ( $T_\gamma$  is SMALL)

This implies that if the structure has a small floor area and is newly constructed, there are very low thermal losses. This means that the load in this case has an effective rating lower than the base load rating. This process is repeated for all possible fuzzy rules to yield a range of coefficients of thermal loss for all combinations of age and size for all groups under consideration. Mathematically stated, this is

$$\Gamma_L = \frac{\sum_{i=1}^n \gamma_i \times \mu_{\gamma_i}}{\sum_{i=1}^n \mu_{\gamma_i}}$$

where  $\gamma_i$  is an element of the fuzzy template corresponding to the coefficient of thermal losses,  $\mu_{\gamma_i} \leq \min(\mu_{a_j}, \mu_{s_k})$  is the membership value of  $\gamma_i$ , and  $\mu_{a_j}$  and  $\mu_{s_k}$  are the corresponding membership values of the age and size elements of the fuzzy templates.

#### 2.5. Offtime Calculation

The offtime is dependent on the load distribution, customer preferences, and the loss demographics. The templates defined above and the fuzzy rules may be merged by a weighted normalization of the offtime on the basis of the fuzzy templates. This is given by the following relationship:

$$T_{\text{off},j} = \frac{\sum_{k=1}^n \frac{1}{PL_k} \left( \sum_{l=1}^m \frac{E_{c_l} - E_{a_j}}{\Gamma_L} \right) \times \mu_{PL_k}}{m \sum_{k=1}^n \mu_{PL_k}} \quad (2)$$

where  $m$  represents the number of different scenarios corresponding to the fuzzy rules, and  $\Gamma_L$  is the weighting factor of the load  $PL_k$  corresponding to the thermal conductivity. The value  $T_{\text{off},j}$  is an element of the offtime template which reflects the transition from state  $i$  to state  $j$ .

Once the cycling time intervals are established, an appropriate membership value is assigned to the individual offtimes that indicates the strength of specific transitions. This is dependent on the membership values of the individual energy instances between which these transitions occurs. In the simplest case, this is:

$$\mu_{T_{\text{off},i,j}} = \frac{\max(\mu_{E_{a_i}}, \mu_{E_{c_j}})}{1 + |\mu_{E_{a_i}} - \mu_{E_{c_j}}|} \quad (3)$$

Equations (2) and (3) define the elements of the fuzzy template for offtime. It is also possible to place upper and lower bound on the offtime, which allows the utility more flexibility in choosing an appropriate cycling time. For example, a utility may desire to specify that all offtimes should be between 15 and 45 minutes. This then places an upper and lower bound on the fuzzy template.

Once the fuzzy template for offtime is obtained, a crisp value for offtime for each group is obtained using the centroid method:

$$T_{\text{off}} = \frac{\sum_{j=1}^n T_{\text{off},i,j} \times \mu_{T_{\text{off},i,j}}}{\sum_{j=1}^n \mu_{T_{\text{off},i,j}}} \quad (4)$$

#### 2.6. Payback

Following the load control period, the air conditioners are permitted to catch up and reduce the residences ambient temperature back to the desired setting. This postcontrol period is the payback period, in which the deferred energy must be paid back into the system. Reported values of energy payback percentages are lower in the northern states (Detroit Edison 25%, American Electric Power 50%) and higher in the southern utilities (Arkansas P&L and Mississippi P&L report almost 100%) [5]. In this study, a payback fraction of 100% was assumed for all calculations. This could be generalized easily for lower payback fractions. It is also assumed that this payback starts immediately after the control period and lasts approximately three time intervals beyond the control period. A typical payback pattern over these three intervals is 60%, 30%, and 10% [3] [4]. This implies that 60% of the deferred energy is paid back in the first interval following the control period, 30% is the second interval and 10% in the third interval. This payback pattern may be altered in a straightforward manner to account for specific utilities patterns in the fuzzy algorithm.

Since 100% payback of deferred energy is assumed, the payback template will correspond directly to the offtime template. The payback intervals are fractions

of the offtime corresponding to each group. The payback template is given as:

$$E_{\beta DLC,i,j} = \beta \times (E_{c,i} - E_{a,j}) \times N \quad (5)$$

where  $N$  is the number of devices in the group and  $\beta$  is the fraction of energy (0.6, 0.3, 0.1) being repaid in that specific time interval. The template corresponding to payback is identical to the fuzzy template for offtime with time on the  $x$ -axis replaced by energy. The payback at stage  $j$  for group  $i$  is obtained by dividing the defuzzified energy by the offtime of the group being cycled. Since this offtime is usually different than the offtime for group  $i$ , the payback will probably be different than the load being cycled off. This difference is typically small and does not significantly impact the overall solution.

Using equation (4), the crisp value of the energy template can be obtained for the time interval under consideration. The value of cycle time and energy may then be input directly into a modified unit commitment algorithm as discussed in the next section.

### 3. The DLC Dispatch Schedule

In DLC, it is desired to cycle the load to reduce the peak load in such a way as to minimize some objective function. This function is typically chosen in coordination with a unit commitment or economic dispatch strategy. The groups under direct load control are typically cycled on and off in stages which span the entire DLC interval, which is typically several hours. The control period may be divided into  $M$  stages which start at stage  $(K+1)$  and terminate at stage  $(K+M)$ . In most applications, each of the  $M$  stages is of equal duration (typically 15 or 30 minutes) [1]-[4]. In this paper, it is proposed that the duration of these stages be optimized for customer satisfaction, and may therefore not be equal.

The load area under DLC is divided into a number of groups. Each group is assumed to have a different cycling capacity depending on the customer demographics. If  $G_N$  is the group under direct load control at stage  $N$ , the load reduction  $L_{DLC}(N)$  is the cyclable load corresponding to this group. Note that when  $N \leq K$  or  $N \geq K+M+1$ , then the load reduction is zero ( $L_{DLC}(N) = 0$ ).

As discussed in the previous section, each group has a unique cycling time corresponding to the preferences as defined by the customers of that group. When the control period for a group is over, the energy difference is paid back. The net restoring demand for this group is determined by the fuzzy template corresponding to the energy difference and the defuzzified cycling time of the next group. A payback schedule based on the typical 60, 30, 10% payback pattern is modified to account for differences in offtime. Thus, the payback corresponding to  $L_{DLC}(N)$  is:

$$L_{PB}(N+k) = \frac{E_{\beta DLC}(N)}{T_{off}(N+k)} \quad (6)$$

Table 1: Classification of Global Energy Levels

Fuzzy Template	Ambient (kWh)		Comfort (kWh)	
	Min	Max	Min	Max
SMALL	0.52	0.94	1.67	2.08
MEDIUM	0.83	1.35	1.98	2.50
LARGE	1.25	1.56	2.33	2.71

where  $E_{\beta DLC}(N)$  corresponds to  $\beta$  payback of the energy deficiency, where  $\beta$  may take on values 0.6 for  $k=1$ , 0.3 for  $k=2$ , and 0.1 for  $k=3$ . Although 100% energy payback is attempted, the actual energy paid back may not be exactly the same as the energy difference after load control. This is due to the nature of the problem in which the payback periods vary and the  $N+1$  stage will most likely be of different length than the  $N$  and  $N+2$  stages. This difference is typically small and does not significantly impact the dispatch schedule.

Including the effects of the payback schedule, the modified system load as a consequence of direct load control at any stage  $N$  is given:

$$L_{net}(N) = L_{actual}(N) - L_{DLC}(N) + L_{PB}(N-1) + L_{PB}(N-2) + L_{PB}(N-3) \quad (7)$$

This load model may now be used, along with the crisp offtime values, as input to a unit commitment strategy. The approach used in this paper is similar to the approach proposed in [1].

### 4. Illustrative Example

The proposed methodology for DLC is illustrated in this section for a simple system which is given in [6]. In this system, the total system peak demand is 600 MW, in which the cyclable load has been divided into five groups, where each group is assumed to have 5000 devices ranging from 1 to 10 kW. Each group is also demographically diverse; the residences range from 0 to 50 years old, and from 1000  $ft^2$  to 3000  $ft^2$  floor space. The demographic differences are used to weight the range of load to yield an effective cyclable load as discussed in section 2.4.

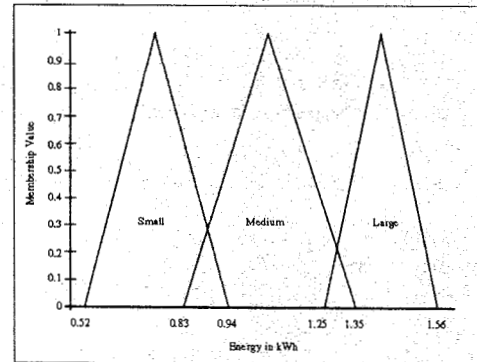


Figure 1: Global Fuzzy Ambience Template

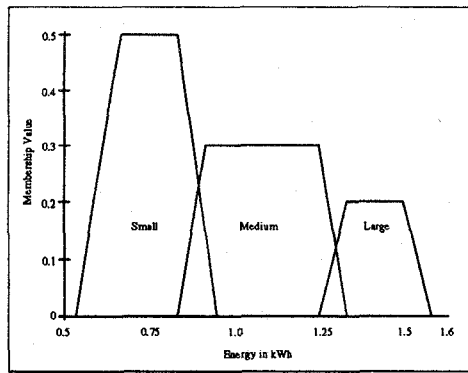


Figure 2: Local Fuzzy Templates

Table 2: Classification of Customer Preferences

Group No.	Ambient Percentage		Comfort Percentage	
1	SMALL	50	SMALL	30
	MEDIUM	30	MEDIUM	40
	LARGE	20	LARGE	30
2	SMALL	40	SMALL	30
	MEDIUM	50	MEDIUM	50
	LARGE	10	LARGE	20
3	SMALL	35	SMALL	35
	MEDIUM	45	MEDIUM	45
	LARGE	20	LARGE	20
4	SMALL	30	SMALL	40
	MEDIUM	50	MEDIUM	50
	LARGE	20	LARGE	10
5	SMALL	30	SMALL	50
	MEDIUM	40	MEDIUM	30
	LARGE	30	LARGE	20

Table 1 represents the classification of energy templates for ambience and comfort into three fuzzy templates: SMALL, MEDIUM and LARGE. For the purpose of illustration, these fuzzy templates are assumed to be triangular in shape with the maximum membership value corresponding to the mid-point of the energy template for the base case. These then define the global fuzzy subsets for the example system.

Table 2 illustrates how the local fuzzy subsets are created from the global fuzzy subsets. To create the local fuzzy subsets specific to each group, the global fuzzy subsets are truncated in accordance with the customers stated preferences. Figures 1 and 2 illustrate the global fuzzy subsets and the ambient local fuzzy subset for group 1. Figure 1 is a representation of Table 1. This is common for all groups under consideration. Figure 2 shows the global fuzzy subsets with respect to the ambiency preferences of group 1.

As previously indicated, the transitions between the energy levels defined by the ambience and comfort criteria are governed by a set of fuzzy rules. These fuzzy rules are then used to calculate the offtime. In this example, there exist three possible transitions that can define the offtime template corresponding

Table 3: Crisp Values of Overtime and Capacity

Group No.	Cycle Time (Minutes)	Cycling Capacity (MW)
1	34.9	15.7
2	31.1	27.6
3	29.2	36.2
4	27.6	38.5
5	26.5	41.1

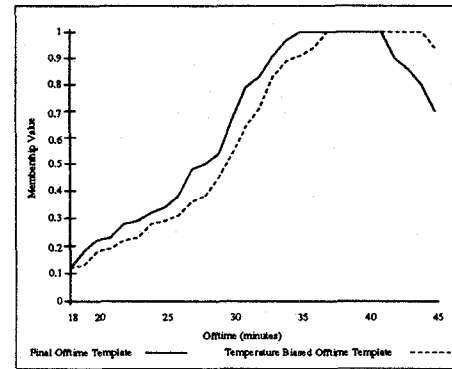


Figure 3: Overtime Templates

to SMALL. Similarly, there are two possible transitions that define MEDIUM and only one transition that defines LARGE. If the energy templates were classified into a larger number of fuzzy subsets, each fuzzy subset of offtime would be defined by a larger number of transitions. The final template for group 1 is shown as the solid line in Figure 3. Upon defuzzification, the cycle time of group 1 is 34.9 minutes. The crisp values of cycle time and cyclable load for all groups in the example are given in Table 3.

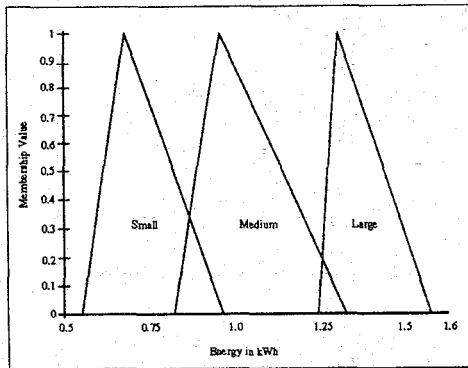
#### 4.1. Effect of External Temperature

The fuzzy subsets are defined for a specific reference temperature, say 90°F. As the outside temperature deviates from the reference temperature, the subsets must also reflect this change. Deviation from the reference temperature may be reflected by biasing the global fuzzy templates either to the left or the right depending on lower or higher temperature conditions.

For example, if the external temperature were lower than the reference temperature, the fuzzy subsets would be biased to the left to account for this difference. The new fuzzy subsets, which correspond to 0.3 on the temperature template, are as shown in Figure 4. The biased offtime template for these subsets is given as the dashed line in Figure 3. This template shows a stronger bias towards intervals of longer duration. Thus the effect of temperature is reflected by longer duration intervals. Note that the x-axis does not change. This is because the minimum and maximum energy levels do not change. However, the distribution of these elements in the fuzzy template is modified. The effect is then reflected in the membership values of the individual time intervals. The defuzzified cycle time is 35.7 minutes.

**Table 4: Comparison of Production Cost (in R) for Unit Commitment with and without DLC**

Time	Without DLC		With DLC				
	Load (MW)	Production Cost	Load (MW)	$L_{DLC}$ (MW)	Group No.	Payback (MW)	Production Cost
13:00:00	600	6986	559	41.1	5	0.0	5946
13:26:30	600	6986	585	38.5	4	23.6	6386
13:54:06	540	6049	537	36.2	3	33.0	5894
14:23:18	540	6049	547	27.6	2	34.1	6285
14:54:24	400	4789	427	0	—	26.9	5031
15:24:24	400	4789	412	0	—	12.1	4898
15:54:24	280	3077	283	0	—	2.9	3154
Total Fuel Costs Over 8 Hours		82698 R					81333 R
Fuel Cost Savings							1.64%



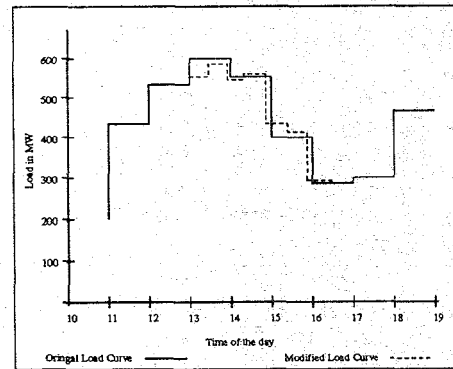
**Figure 4: Global Fuzzy Subsets Biased for Lower Temperature Conditions**

#### 4.2. Dispatch Schedule

To demonstrate the effectiveness of the proposed DLC approach, the production cost savings are compared with unit commitment without DLC. The results are tabulated in Table 4. For the case without DLC, the total fuel cost for a period of 8 hours is 82698 monetary units (R). The production cost with the proposed methodology is 81333 R where the actual control period extends from 13:00 to 15:00 hours. Note that the energy payback extends for approximately 90 minutes more. The net savings obtained using the proposed methodology is 1.64%. Figure 5 compares the original load pattern with the modified load pattern.

#### 5. Conclusions

A new load model is proposed for the dispatch of direct load control. In the proposed load model, provisions are made for customer preferences such as minimum and maximum acceptable temperature to increase customer acceptance of the load management program. These preferences are quantified and represented using fuzzy logic. The load model also accounts for the range of devices and thermal differences within cycling groups. This load model is then used in computation of the cycling time and net restored energy corresponding to each group. The crisp cycling time and net restored energy are incorporated



**Figure 5: Original vs. Modified Load Curve**

into an optimization procedure to yield a strategy to schedule the groups for minimum production cost.

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