International Specialty Conference on Cold-Formed Steel Structures

(1978) - 4th International Specialty Conference on Cold-Formed Steel Structures

Jun 1st, 12:00 AM

Applicability of Limit Design to Cold Formed Box Beams

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APPLICABILITY OF LIMIT DESIGN TO COLD-FORMED BOX BEAMS

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Introduction

Elastic design procedures are presently used to design the majority of rigid frames used in prefabricated metal buildings. The rigid frame cross-section is typically a wide-flange shape, often fabricated from plate and sheet steel using automatic welding equipment. Frame members are either prismatic or tapered; shop connections are typically welded, while most field connections use high-strength bolts.

Substitution of a box-shaped member of cold-formed steel can provide advantages over the use of a wide-flange section. Some of those advantages are (1) the enhanced feasibility of using plastic design procedures, (2) the elimination of lateral bracing, and (3) the potential saving of material in the rigid frame members. The purpose of this paper is to determine the ultimate behavior of such box sections when subjected to moment.

Plastic Design Theory

Plastic design is a limit method which can be applied to a ductile structural member or frame subject primarily to bending. The method is based on the maximum load which the structure will carry as determined from an analysis of strength in the plastic range. It also consists of

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consideration of certain limitations which might prevent the structure from withstanding the computed maximum load. The ultimate moment is reached at a cross-section when all fibers have yielded; this condition is termed plastic hinge. As the load on a statically indeterminate beam or frame increases toward the ultimate, some plastic hinges may form before others. In this case, the earlier-forming plastic hinges must rotate so that later hinges may form. When all plastic hinges have formed, a mechanism exists and the ultimate load is reached. A necessary condition which must be satisfied in order to justify the use of plastic design procedures is that the actual ultimate moment at each plastic hinge location maintain a value close to its theoretical maximum until all other plastic hinges have formed. Fig. 1 depicts the idealized relationship between moment and rotation at a plastic hinge. The quantity M_p is the full plastic moment. The rotations θ_p and θ_m pertain to the proportional limit and to the utlimate, respectively. The rotational capacity, R, is given by

$$R = \frac{\theta_m}{\theta_p} - 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

As a result of several studies (3,5) most researchers have accepted values of R between 3 and 4 as being sufficient for redistribution of moments in typical applications.

Stability Considerations

For the case of wide-flange beams subject to moment, two modes of instability are of interest: lateral-torsional buckling and local buckling. In beams not properly braced against lateral-torsional buckling, the moment capacity is sharply reduced as soon as buckling occurs. Therefore, stringent lateral bracing requirements are part of plastic design specifications. However, elastic design specifications require much less lateral

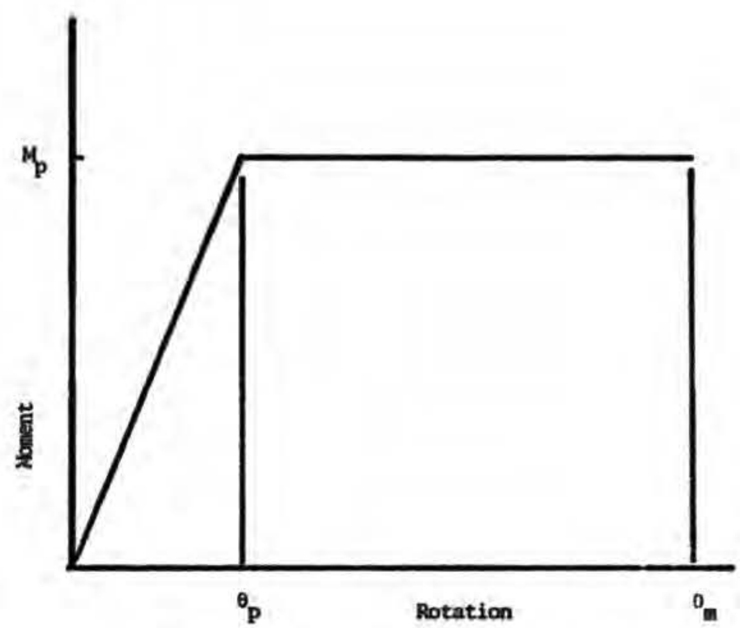

Fig. 1 - Idealized Moment-Rotation Curve

bracing. The economies realized by the use of plastic design may be offset by the cost of the additional bracing.

In a properly braced wide-flange beam, local buckling of its thin plate elements causes a reduction in moment capacity. The effects of local buckling upon the moment-curvature characteristics of wide flange beams have been studied previously (6,7,8).

Tse (11) has demonstrated that a box beam with a laterally unsupported length conceivable in practical applications will not buckle in a lateral-torsional mode under the action of pure bending moments. Also, unlike the wide-flange shape, a box beam does not lose its resistance against lateral-torsional buckling due to penetration of yielding over the cross-section.

While local buckling limitations cannot be completely eliminated, they can be mitigated by the use of closed box sections. This is due to the fact that the flange of a box section is considered to be a stiffened compression element while the flange of a wide-flange beam is unstiffened. An unstiffened compression element is defined as a flat element which is stiffened at only one edge parallel to the direction of stress. In the case of a wide flange member, the web is the stiffener and each side of the flange is an unstiffened compression element. A stiffened compression element is a flat element which is stiffened on both edges parallel to the direction of stress. The stiffeners may be webs, intermediate stiffeners, or simple lips. For example, the compression flange of a box beam is a stiffened compression element.

Effective Width Theory

In 1932 Von Kármán (12) proposed the effective width approach to the problem of ultimate plate strength. He reasoned that when the ultimate strength is reached, two strips of width $b_e/2$ at each edge carry the yield stress, while the central region remains unstressed. This condition is depicted in Fig. 2. Using Von Kármán's approach, one may express the effective width of a simply supported plate as

$$b_e = 1.9t \sqrt{E/\sigma_y} \quad \text{.........................} \quad (2)$$

where t is the plate thickness, E is the modulus of elasticity, and σ_y is the yield stress.

Based upon some 150 tests on stiffened compression elements whose width-thickness ratios ranged from 14.3 to 440, Winter (13) proposed the following relationship

$$b_e = 1.9 \sqrt{E/\sigma_y} \left(1 - \frac{0.415 \sqrt{E/\sigma_y}}{b/t}\right) t \quad \text{................} \quad (3)$$

As a result of the aforementioned research, the ultimate strength of the compression flange of a box beam can be very closely determined. However, the determination of the ultimate strength of the webs is very complex. In the case of a box beam with an effective compression flange width, b_e, which is less than the tension flange width, b, the neutral axis will be lowered. Also at loads near the ultimate, the stress distribution in the compression portion of the web is non-linear due to the large deflection of the buckles which occur there. The behavior of the compression portion of the web is not unlike the uniformly loaded plate in which material is considered to be removed. The efforts of many researchers over the years to experimentally check web buckling theories based upon small deflection theory were relatively unsuccessful (4). Basler (1), in studying plate girders with thin webs, suggests an effective section in which a portion of the compression zone of the web is assumed to be non-participating.

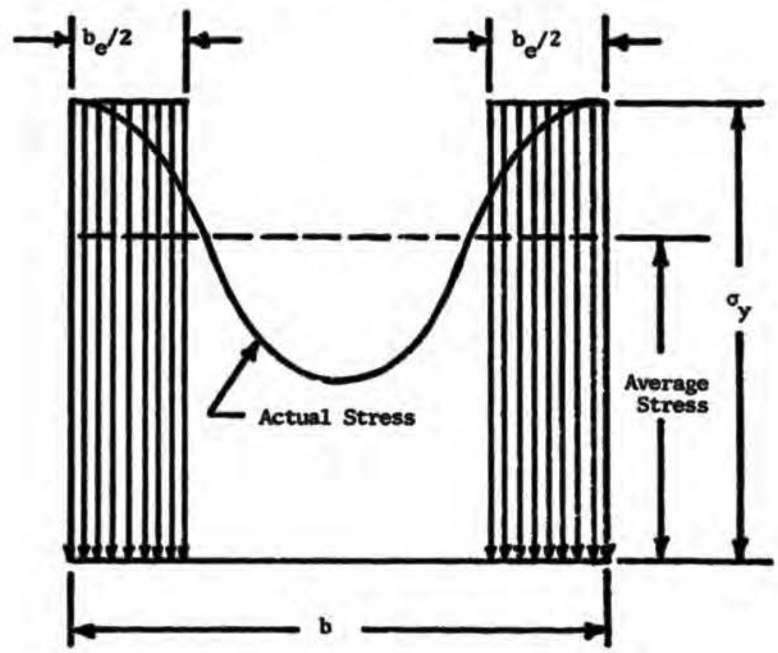

Fig. 2 - Effective Width of a Buckled Plate

The effective section is shown in Fig. 3. Any removal or nonconsideration of material in the web serves to lower the neutral axis farther.

Ultimate Moment Theory

The ultimate moment of a box shape of steel in which the depth-thickness ratio of the webs is less than that given by the formula (14)

$$d/t = 412/\sqrt{\sigma_y} \quad \text{.......................} \quad (4)$$

where σ_y is in ksi (1 ksi = 6.9 MPa), and the width-thickness ratio of the compression flange is less than

$$b/t = 190/\sqrt{\sigma_y} \quad \text{.......................} \quad (5)$$

can be easily calculated by the principles of plasticity. Eqs. 4 and 5 describe the upper dimension-thickness ratios of shapes defined as "plastic design shapes." Under the provisions of Section 2.7 of the 1969 edition of the AISC code, these ratios represent the upper limits beyond which presently accepted plastic or ultimate design procedures may not be used.

In order to predict the ultimate moment capacity of box beams whose width-thickness and depth-thickness ratios exceed the provisions of AISC-2.7, a procedure using effective widths, somewhat similar to that described in Section 2.3 of the AISI code, is proposed. Fig. 4 shows the effective stresses in the compression flange of a beam with no intermediate stiffeners. This effective removal of material from the flange lowers the neutral axis to some distance from the top, c. If the compression portion of the web does not buckle (i.e. is fully effective), the final stress state is as shown in Fig. 5.

If the web does buckle, its compression side may be considered as

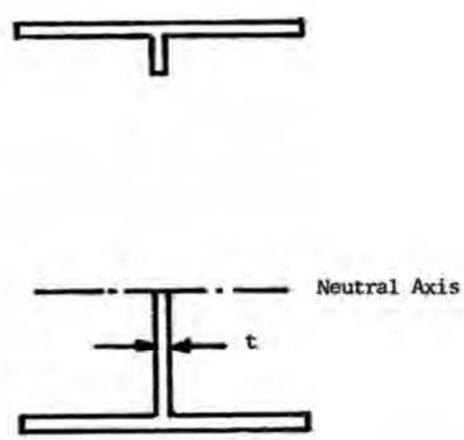

Fig. 3 - Effective Section of Plate Girder in Bending

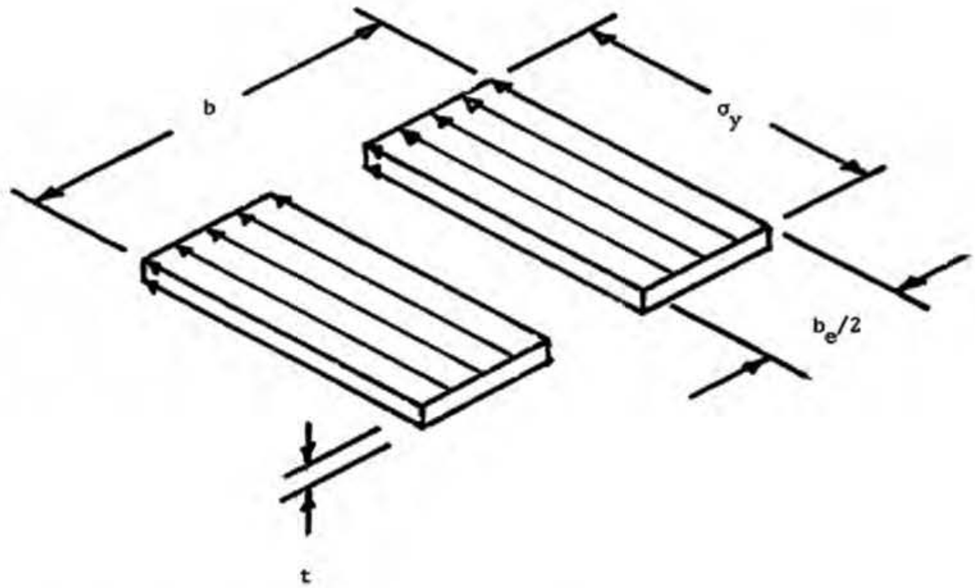

Fig. 4 - Effective Stresses in Compression Flange

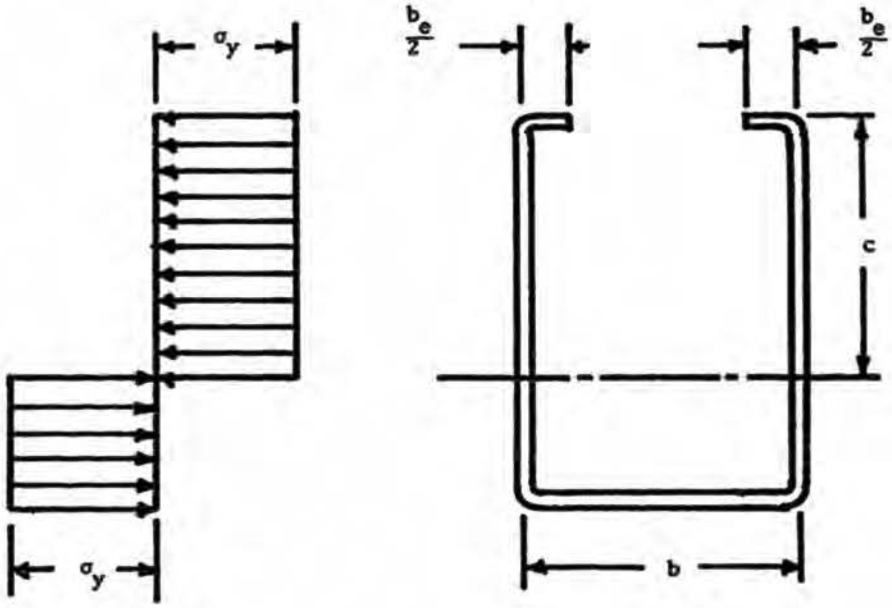

Fig. 5 - Final Stress State with Web Fully Effective

a uniformly loaded plate with an effective width of b_o' in the fully plastic state. This state of stress is shown in Fig. 6.

The location of the neutral axis for the fully yielded state is found by equating the area above the axis to the area below:

$$b_e + 2c = b + 2(d - c) \quad \text{......................} \quad (6)$$

for the case where the web does not buckle (i.e. the distance to the neutral axis, c, does not exceed $190t/\sqrt{\sigma_y}$, where σ_y is in ksi), and

$$b_e + 2b_e' = b + 2(d - c) \quad \text{......................} \quad (7)$$

for the case where the web does buckle.

The theoretical ultimate moment, M_u, may then be found by the use of the principles of plasticity on the effective section.

This analysis assumes that no unloading of the effective compression flange occurs. Reck has shown (10) that box beams, whose flange width-thickness ratios exceed $221/\sqrt{\sigma_y}$, are incapable of maintaining a sustained moment while undergoing large rotations.

The theoretical ultimate moment capacity for box beams with longitudinal web stiffeners may be found in a similar manner. The effect of the stiffeners is to impede the progression of the buckle down the web and therefore to increase the effective area. The assumed effective section is shown in Fig. 7. The effective width of the top portion of the web is therefore based upon the distance between stiffeners, d/2.

Instrumentation and Equipment

Fourteen experiments were performed to study the behavior of box beams made from thin plate elements having longitudinal stiffeners. The structure

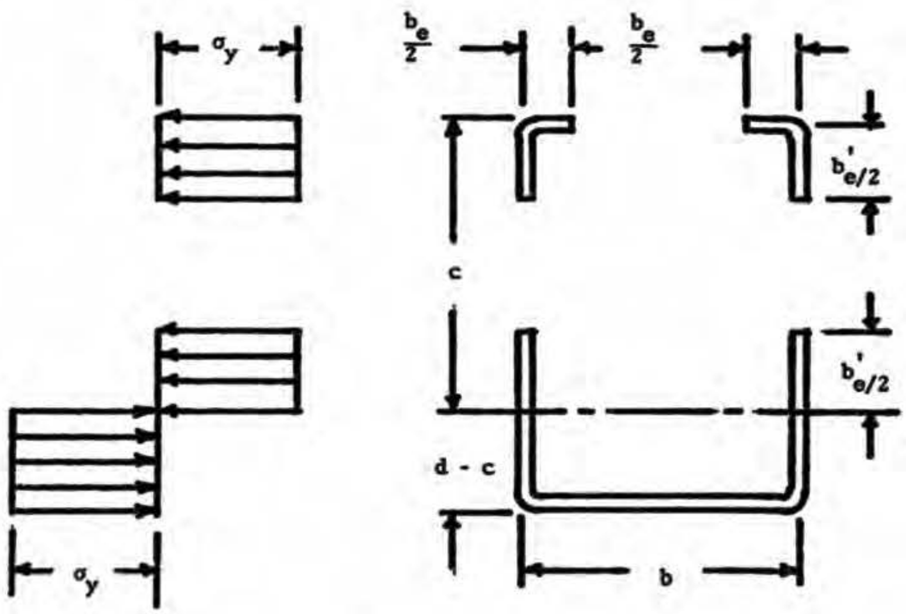

Fig. 6 - Final Stress State with Web Partially Effective

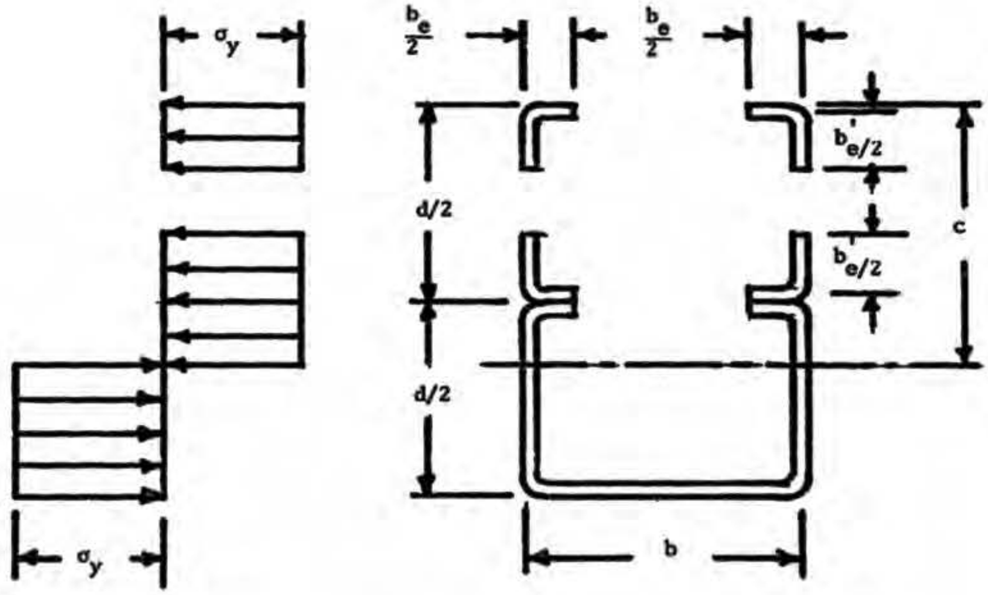

Fig. 7 - Final Stress State with Stiffened Web, Partially Effective

used in these tests was a simply supported beam which was loaded by two symmetrically placed vertical loads so that the central portion of the beam was subjected to constant moment. A schematic view of the test structure is shown in Fig. 8.

The test beams were cold-formed in a press-brake. Transverse stiffeners were welded to the beams at points of load application in order to prevent web crippling.

The test machine used in this program was a 450,000 lb. (2002.5 kN) screw-fed, universal testing machine manufactured by Riehle.

Deflections in the loading direction were measured during each test by three dial gages positioned as shown in Fig. 8. In the elastic range readings were taken at convenient increments of load, while in the inelastic range increments of deformation were used. Loading was stopped each time readings were taken.

In the inelastic range the readings were not taken until sufficient time had elapsed to permit the system to come to rest, and therefore the effects of the rate of loading do not influence the results. The test points in the curves shown later herein represent stable deflection configurations for static loading.

Eleven test sections twelve feet (3.66m) long were fabricated from 14 gage (1.90mm) material, while three sections twelve feet (3.66m) long were made from 11 gage (3.04mm) material. The yield stress of the metal used in these tests was found to be 33.4 ksi (230 MPa) by standard coupon tests. Each box section was fabricated from two C-sections as shown in Fig. 9. Fig. 9a shows a beam with the stiffeners located in the webs. Fig. 9b shows a beam with the stiffeners located in the flanges. The designation of each beam is descriptive in that the first two numbers

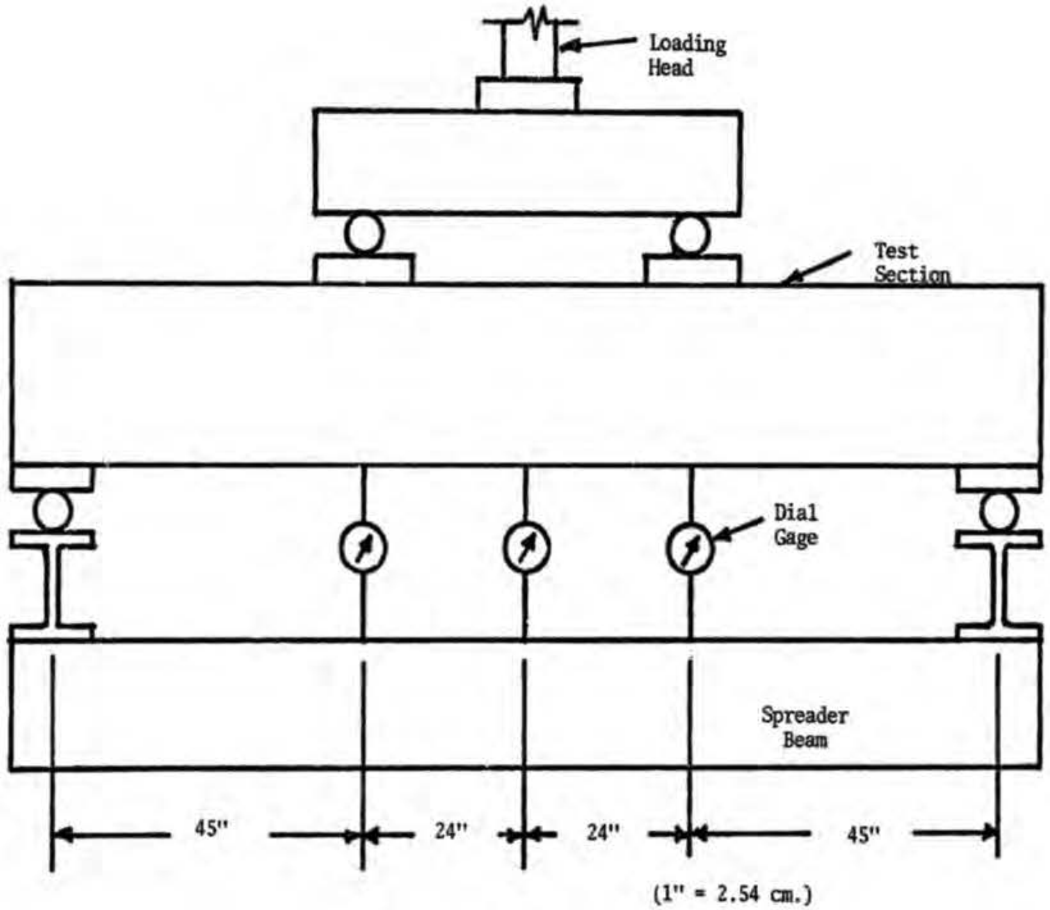

Fig. 8 - Schematic View of Test Setup

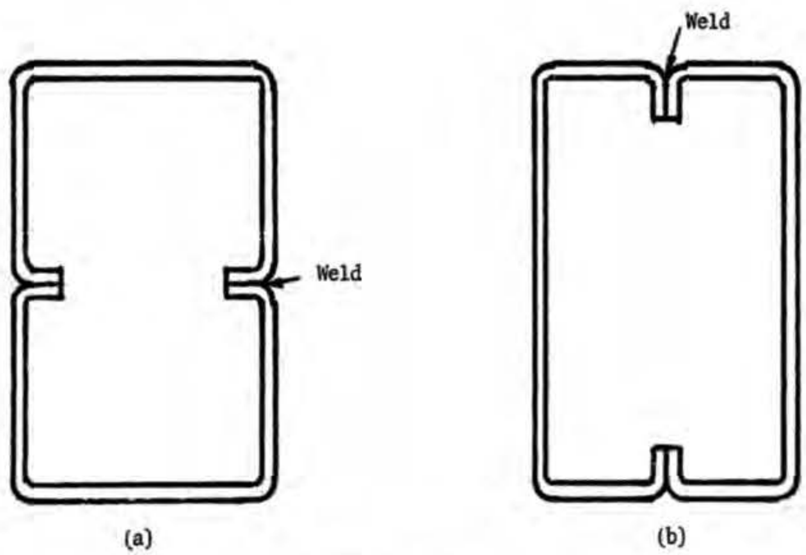

Fig. 9 - Test Sections

represent the nominal width of the beam in inches, and the second two numbers represent the nominal depth. The letter at the end of the description indicates the location of the longitudinal stiffener--F for flange and W for web. The beams made from 11 gage (3.04mm) material are followed by "(11)"; all others were made from 14 gage (1.90mm) material. All the test sections studied in this program, along with the pertinent dimensions, are listed in Table 1. To aid in formulating a means to predict the beam's ultimate moment carrying capacity, strain gages were used in two tests--the 1218W and the 1610W. The strain gages used were attached as shown in Fig. 10.

Twenty such gages were used in order to have them at or very near the section where the out-of-plane buckling was a maximum. The buckling pattern of a stiffened compression element is a series of dish-dome type deformations as shown in Fig. 11. At sections "a" and "c" the deformations of the compression flange are at a maximum being the dish or inward and dome or outward deformation respectively. At section "b" there are no out-of-plane deformations of the plate elements comprising the beam. Due to minor variations which might exist in the location of the neutral axis and stress distribution along the beam, gages were used at several sections in order to determine the strains at a section of maximum buckle deformations.

The results of the strain gage readings at sections of maximum buckle deflection yield two important results - the stresses at the various locations and the location of the neutral axis. The results of the strain readings are shown for various loadings in Figs. 12 and 13.

Table 1. Test Section Dimensions

Section	Width in. (mm)	Depth in. (mm)	Stiffener Length in. (mm)	Thickness in. (mm)
0406F	4.1(104.1)	5.9(149.9)	0.8(20.3)	0.0747(1.90)
0406F(11)	3.9(99.1)	5.8(137.2)	0.8(20.3)	0.1196(3.04)
0408F	4.1(104.1)	7.9(200.7)	0.8(20.3)	0.0747(1.90)
0408F(11)	3.9(99.1)	7.8(198.1)	0.8(20.3)	0.1196(3.04)
0609F(11)	5.8(147.3)	8.9(226.1)	0.8(20.3)	0.1196(3.04)
0612F	6.1(154.9)	12.0(304.8)	0.8(20.3)	0.0747(1.90)
0616F	6.0(152.4)	16.0(406.4)	0.8(20.3)	0.0747(1.90)
1008F	10.2(259.1)	8.0(203.2)	0.9(22.9)	0.0747(1.90)
1012F	10.3(261.6)	12.0(304.8)	0.8(20.3)	0.0747(1.90)
0610W	6.0(152.4)	9.9(251.5)	0.8(20.3)	0.0747(1.90)
0815W	8.0(203.2)	15.1(303.5)	0.8(20.3)	0.0747(1.90)
1214W	11.9(302.2)	14.0(355.6)	0.8(20.3)	0.0747(1.90)
1218W	12.0(304.8)	18.0(457.2)	0.8(20.3)	0.0747(1.90)
1610W	16.0(406.4)	10.2(259.1)	0.8(20.3)	0.0747(1.90)

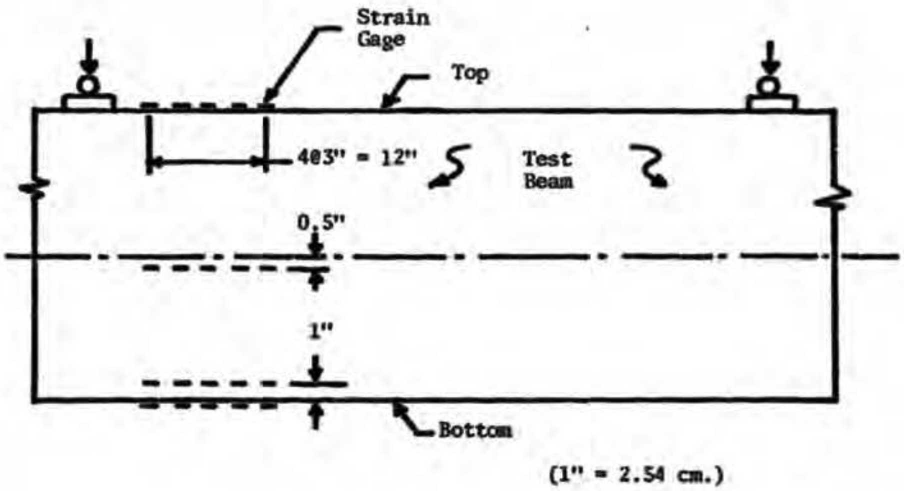

Fig. 10 - Strain Gage Locations

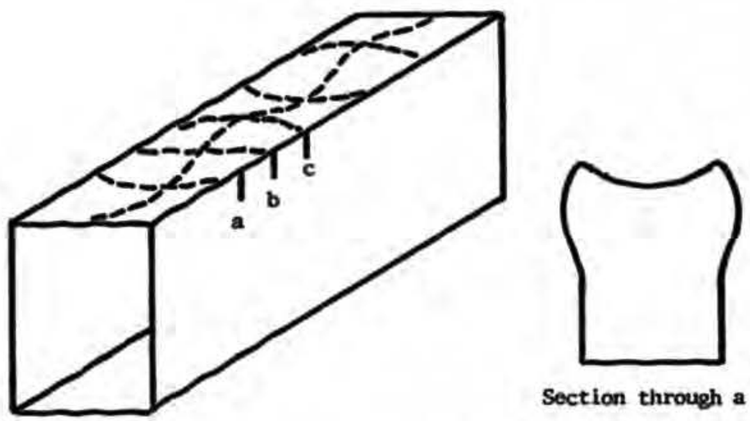

Fig. 11 - Buckled Configuration of Compression Flange

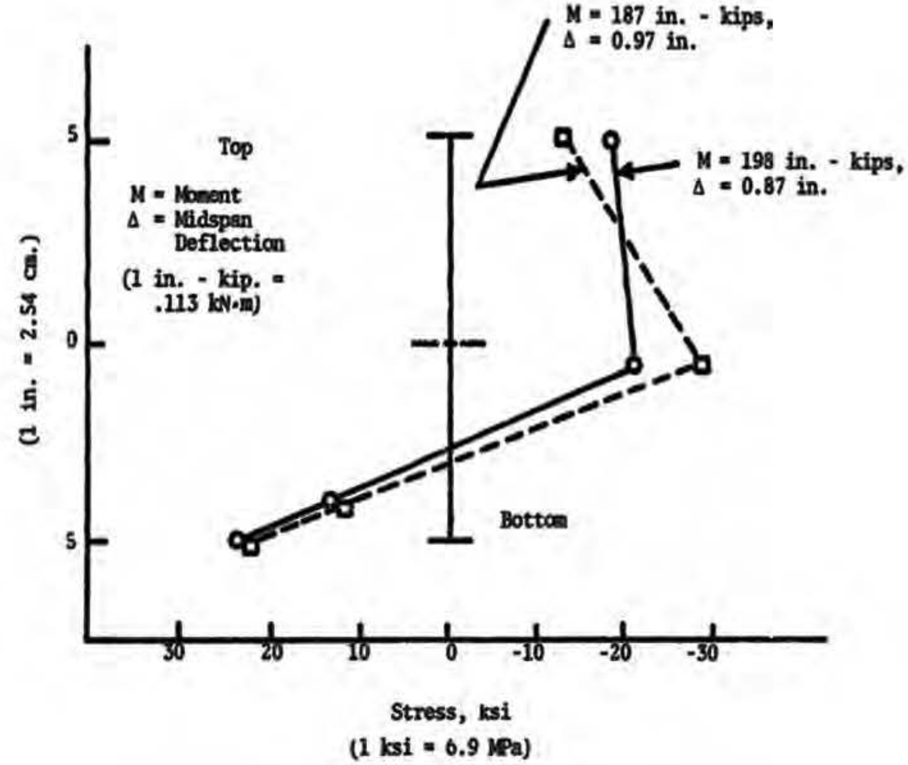

Fig. 12 - Stress Versus Depth - 1610W

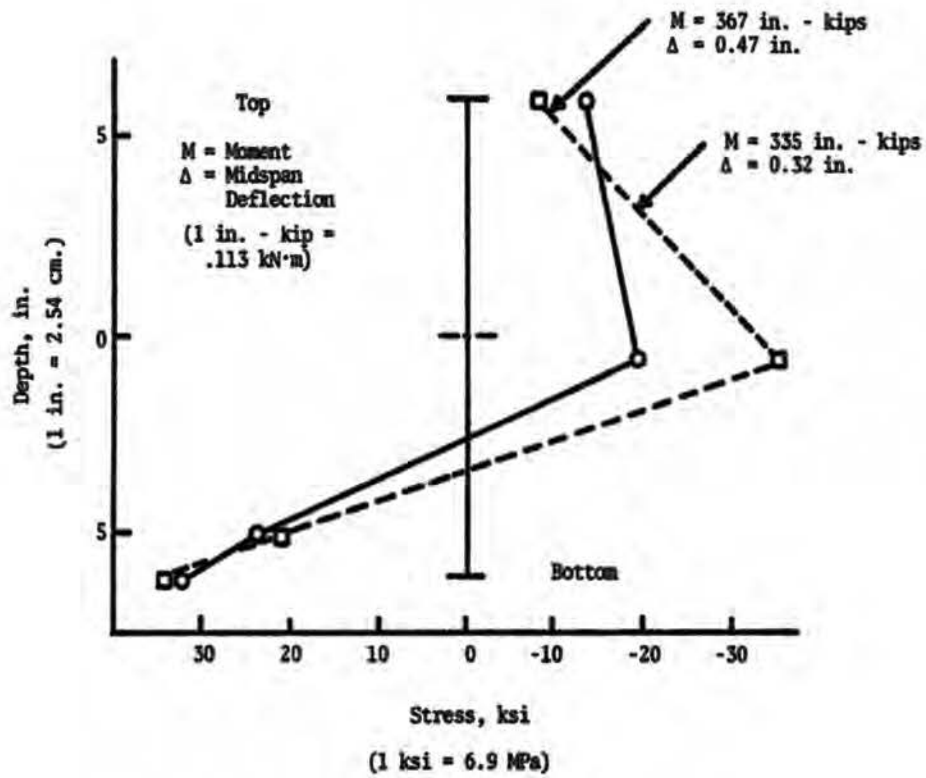

Fig. 13 - Stress Versus Depth - 1218W

Table 2. Summary of Theoretical and Experimental Results

Section	b/t*	d/t*	$.9M_u$ in.-kips	$.9M_u$ kN·m	M_t in.-kips	M_t kN·m
0406F	22.1	73.6	112.3	12.7	121	13.7
0406F(11)	13.0	45.2	173.5	19.6	208***	23.5
0408F	22.1	100.4	162.4	18.4	170	19.2
0408F(11)	13.0	61.9	259.0	29.3	308***	34.8
0609F(11)	20.9	71.1	368.0	41.6	411	46.4
0612F	35.5	155.3	294.7	33.3	320	36.1
0616F	34.8	208.8	399.7	45.2	381	43.0
1008F	62.9	101.7	235.7	26.6	192	21.7
1012F	63.6	155.3	360.4	40.7	293	33.1
0610W	75.0	127.2	154.6**	17.5	149	16.8
0815W	101.7	196.8	239.2**	27.0	262	29.6
1214W	153.9	182.1	230.4**	26.0	288	32.5
1218W	155.3	235.6	308.9**	34.9	363	41.0
1610W	208.8	131.2	163.5**	18.5	201***	22.7

* b, d are flat widths

** calculated by elastic-plastic theory

*** acceptable yield plateau reached

M_u = theoretical ultimate moment

M_t = ultimate moment from test

Discussion of Results

Moment-curvature tests on wide-flange members which qualify as "plastic design shapes" show a variance from the theoretical plastic moment of as much as 10% (9). For the purpose of the evaluation of the tests in this program, any beam whose capacity falls below 90% of the theoretical ultimate moment before reaching the specified rotation will be considered not to have formed a plastic hinge. A rotation of three times the rotation at the intersection of the elastic and plastic curves will be used as the minimum required rotation (3). Table 2 lists width-thickness ratios, 90% of M_u, the theoretical ultimate moment, and M_t, the ultimate test moment. Curves showing the observed variation of applied moment with maximum deflection appear in Figs. 14 to 27.

For flange-stiffened beams, M_u was calculated assuming a fully plastic condition. It was observed that one of the web-stiffened sections maintained an acceptable yield plateau, even though it did not qualify as a plastic design shape. This section, 1610W, fortuitously was one of the two selected for strain gaging. The strain gage readings indicated (a) some unloading of the compression flange at the corners and (b) a linear stress distribution at maximum moment in the tension portion of the web (see Figs. 12 and 13). Therefore the theoretical ultimate moment for each of the five web-stiffened sections was calculated assuming a fully plastic effective section above the web stiffener, and a linear elastic stress distribution below the web stiffener which satisfied static equilibrium.

Conclusions

Tests of fourteen cold-formed box beams were conducted to determine width-thickness ratios beyond which acceptable plastic behavior did not occur. Nine sections were flange-stiffened, and five were web-stiffened. Of the nine flange-stiffened sections tested, only two qualified as shapes acceptable for use in plastic design, using the criteria described previously. This was a surprising result, because five of the nine flange-stiffened sections were compact shapes (14), and three of these five also met plastic design criteria; the two which exhibited acceptable plastic behavior were in this last group. Based on this limited number of tests, it can be concluded that presently accepted plastic design criteria may not be sufficient to ensure plastic behavior in a box section with flanges and webs of equal thickness. The presently accepted plastic design criteria are based on tests of plate girder sections whose flanges were very stiff compared to the webs (2), which provided nearly the equivalent of a fixed edge to the web plate.

The remaining five sections of the fourteen tested were web stiffened. None of these were compact shapes, and therefore none met the more stringent plastic design criteria (14). However, as can be seen in Figs. 23 through 27, four of the web-stiffened sections nearly satisfied the criteria described previously ($M_t \geq 0.9\ M_u$ for $1 \leq R \leq 3$), and one, 1610W, did. The behavior of these five sections indicates that the web stiffener plays an important role. The web stiffener inhibits the downward progression of the buckled web area and, therefore, the neutral axis. It also provides additional compression area after the onset of buckling.

The authors agree with Reck (10) that the limiting width-thickness ratio of the flange is $221/\sqrt{\sigma_y}$, for box sections to be used in plastic design. It is also concluded that, based on these tests, a limiting depth-thickness ratio for the webs of such sections be $375/\sqrt{\sigma_y}$. These ratios apply to the flat widths of these elements between stiffeners.

If stiffeners are to be used, their placement in the flanges results in a greater section modulus and, therefore, a greater allowable elastic moment. However, based upon these tests, a more efficient placement of stiffeners is in the webs for sections which are to undergo ultimate behavior.

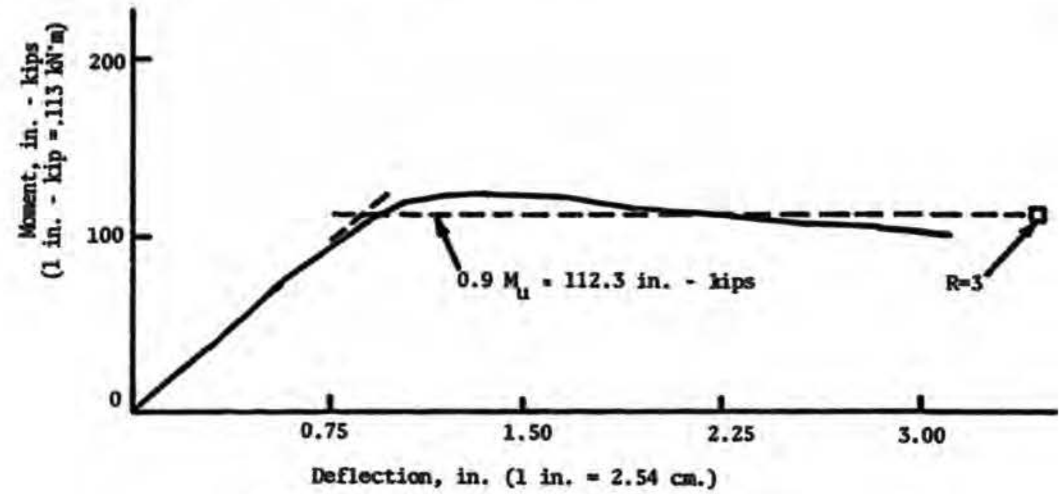

Fig. 14 - Moment Deflection Relationship - 0406F

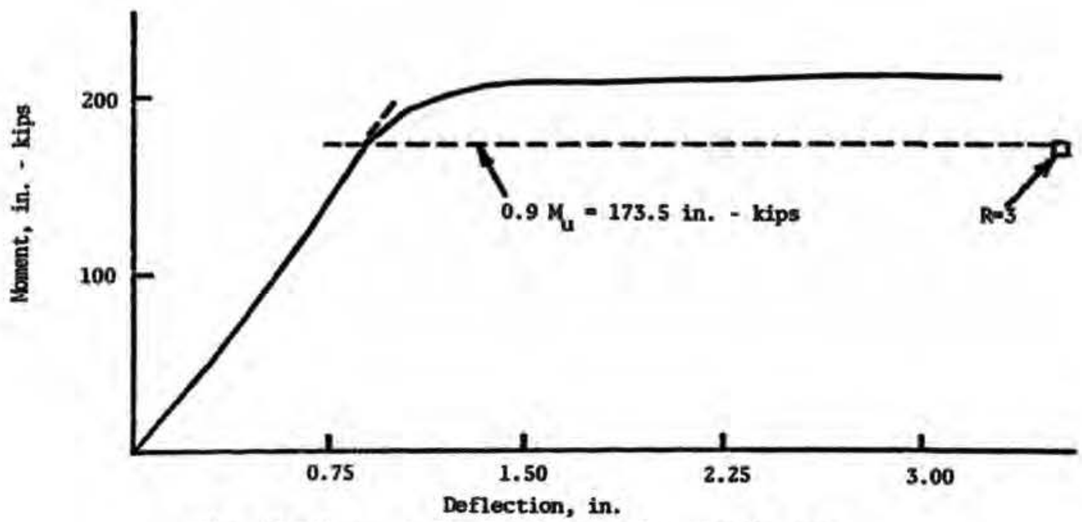

Fig. 15 - Moment-Deflection Relationship - 0406F (11)

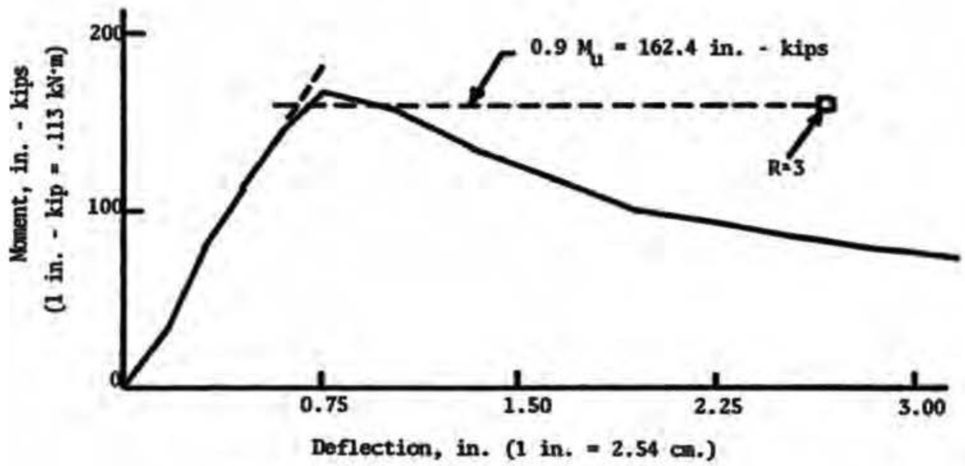

Fig. 16 - Moment-Deflection Relationship - 0408F

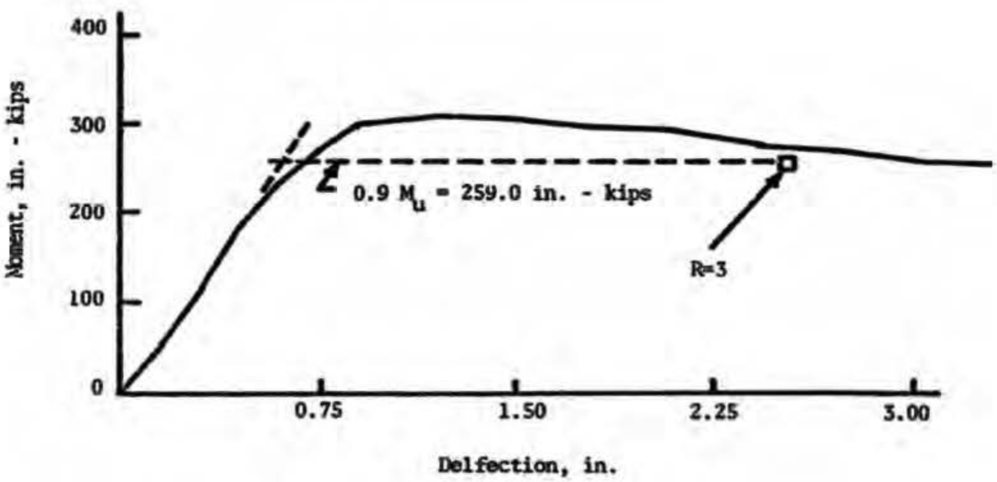

Fig. 17 - Moment-Deflection Relationship - 0408F(11)

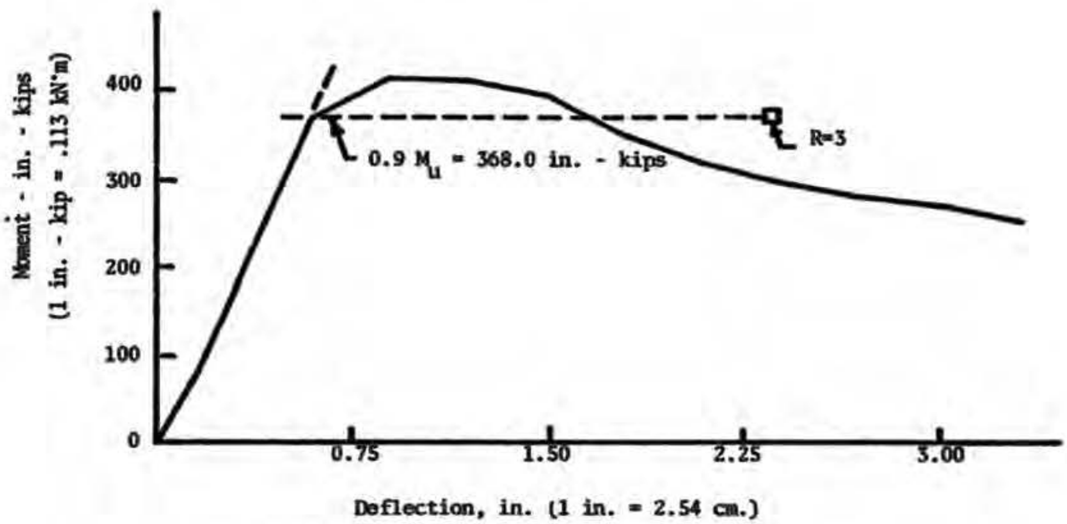

Fig. 18 - Moment-Deflection Relationship - 0609F(11)

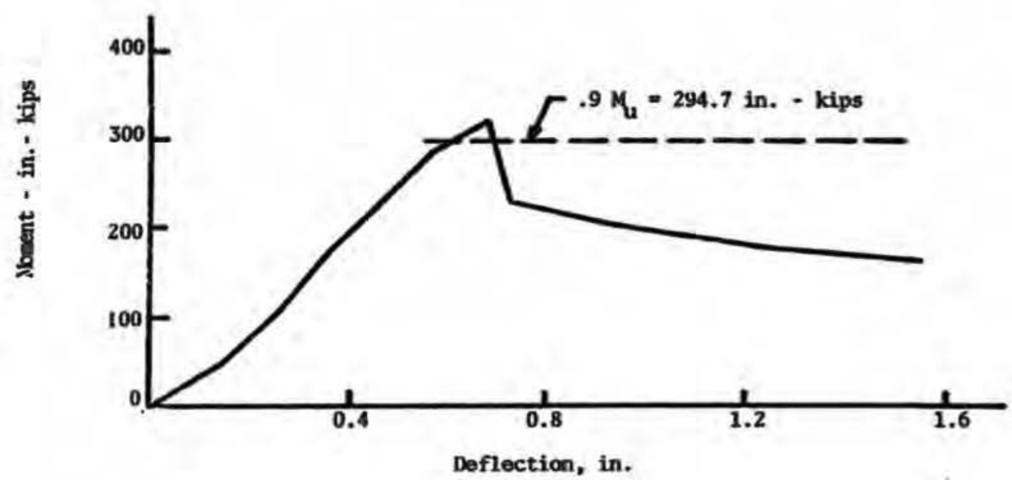

Fig. 19 - Moment-Deflection Relationship - 0612F

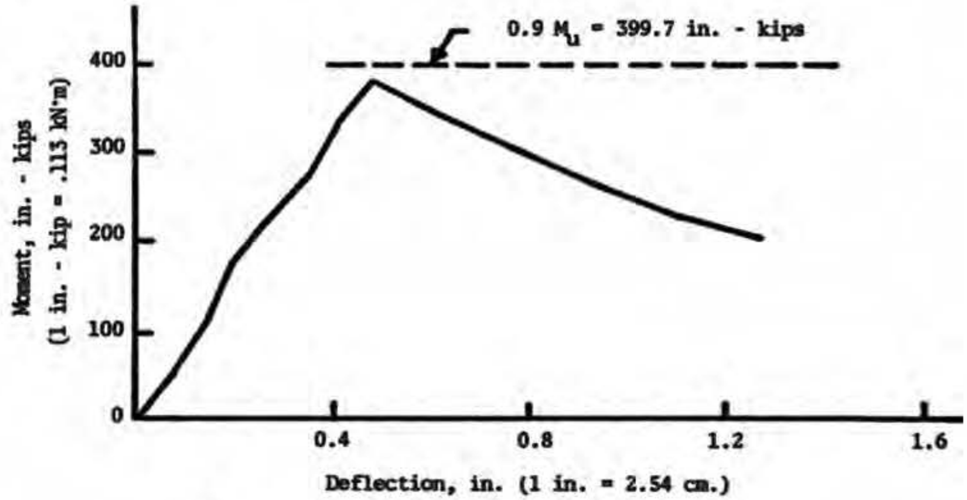

Fig. 20 - Moment-Deflection Relationship - 0616F

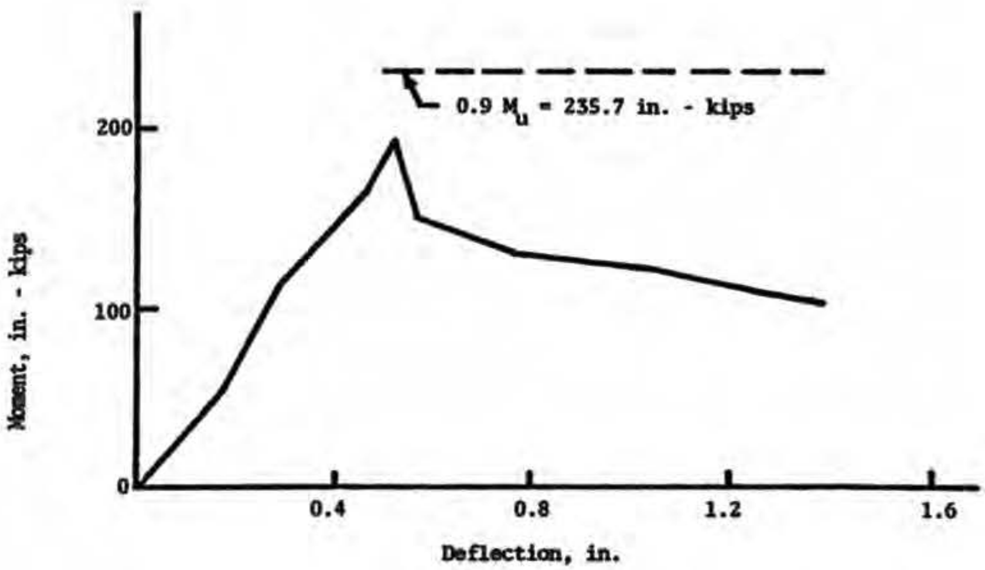

Fig. 21 - Moment-Deflection Relationship - 1008F

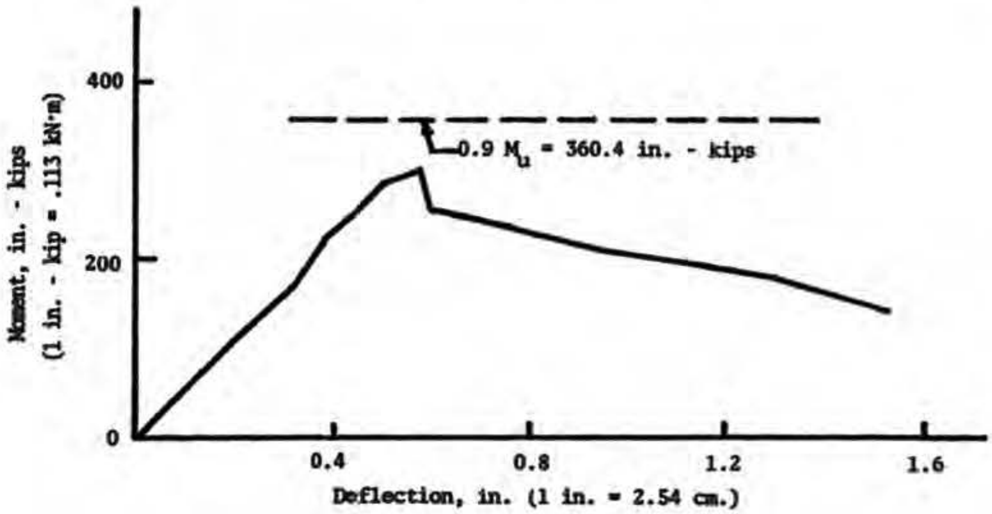

Fig. 22 - Moment-Deflection Relationship - 1012F

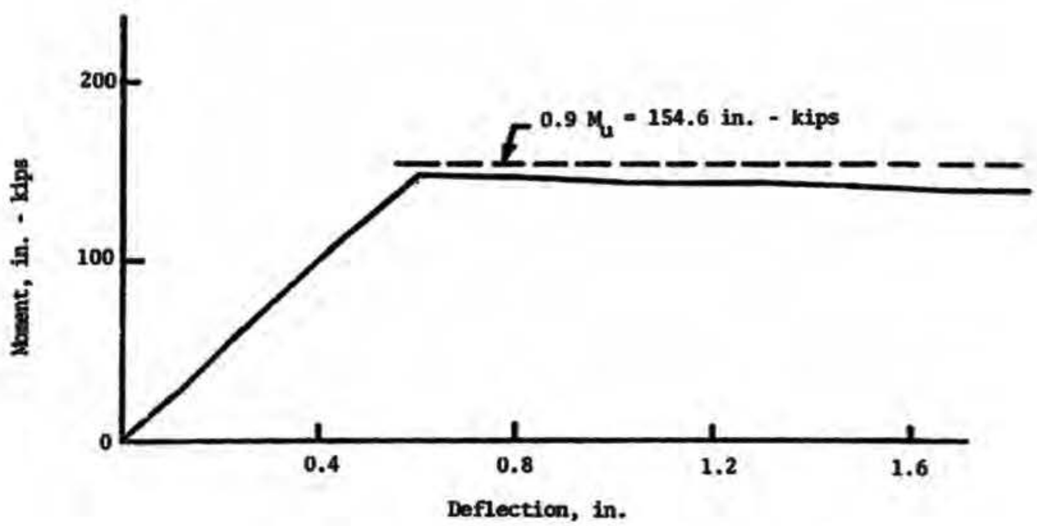

Fig. 23 - Moment-Deflection Relationship - 0610W

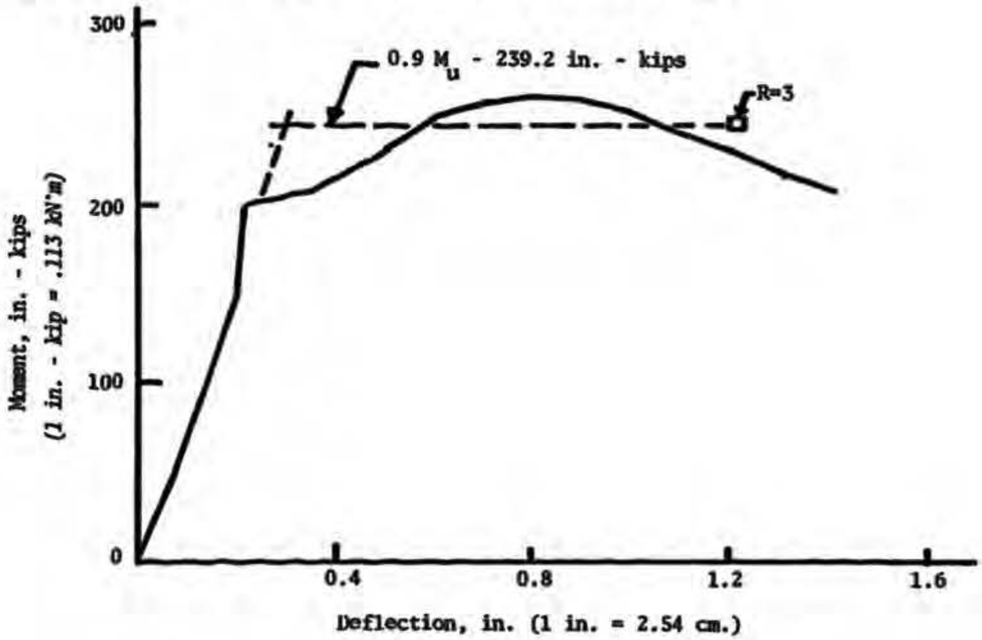

Fig. 24 - Moment-Deflection Relationship - 0815W

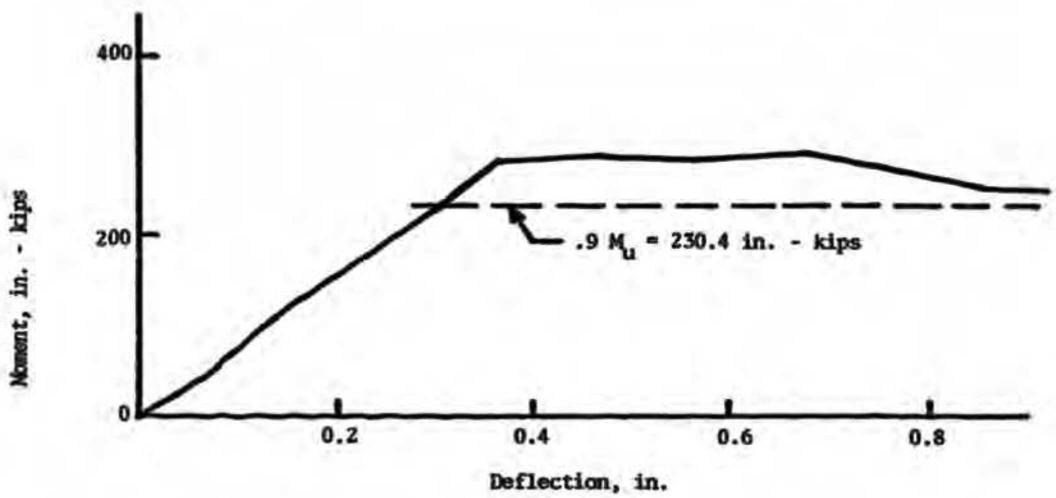

Fig. 25 - Moment-Deflection Relationship - 1214W

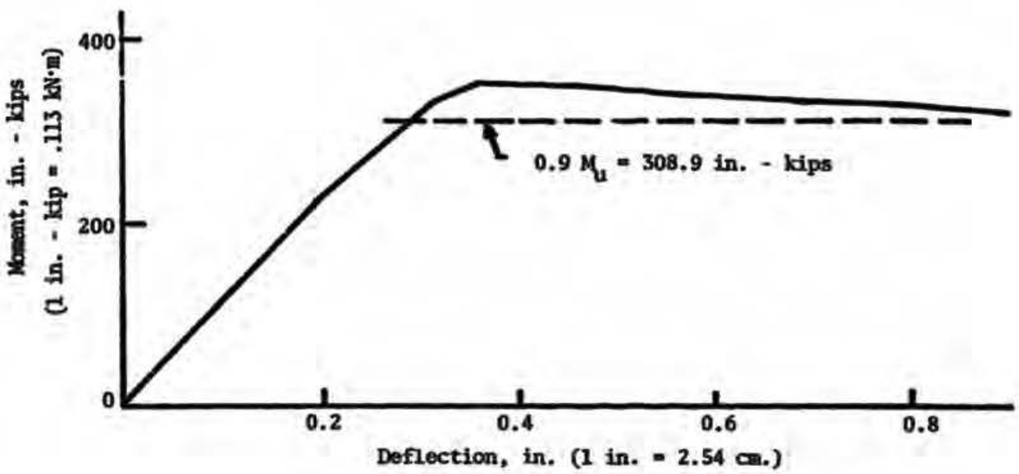

Fig. 26 - Moment-Deflection Relationship - 1218W

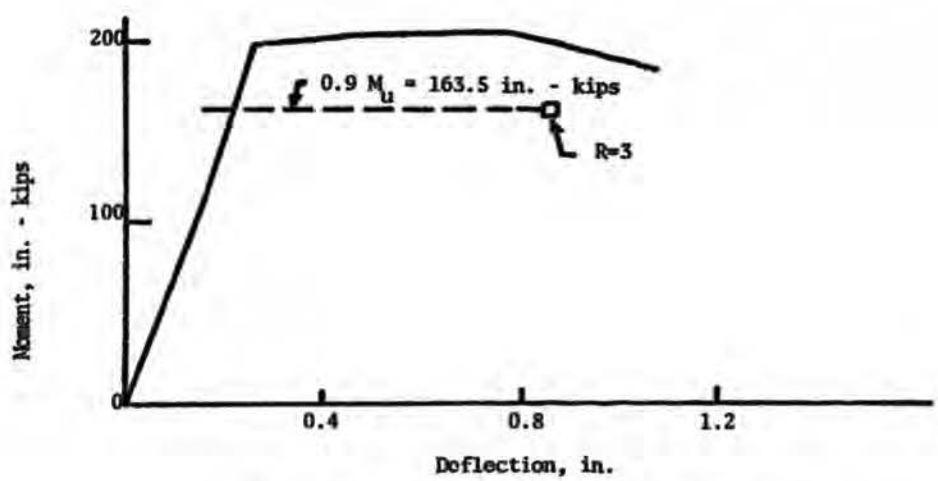

Fig. 27 - Moment-Deflection Relationship - 1610W

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APPENDIX II - Notation

b = width

b_e = effective width of unstiffened plate

b'_e = effective width of portion of stiffened plate

c = distance to neutral axis

d = depth

E = modulus of elasticity

M_p = full plastic moment

M_t = ultimate moment from test

M_u = ultimate moment

R = rotation capacity at a plastic hinge

t = thickness

Δ = deflection

θ_m = rotation at ultimate moment

θ_p = rotation at proportional limit

σ_y = yield stress