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DESIGN FORMULAS FOR BIAXIALLY LOADED
THIN-WALLED STEEL BOX COLUMNS

N.E. Shanmugam¹, J.Y. Richard Liew² and S.L. Lee³

SYNOPSIS

Simplified design equations for predicting the ultimate load capacity of thin-plated steel box columns under combined action of axial stress and biaxial end moments is proposed. These equations allow for local buckling of component plates and different levels of welding residual stresses. Results obtained by using the proposed design equations are compared with those computed by numerical procedure for columns having different loading conditions and various column and plate parameters. The limitations of the design equations are then discussed and their accuracy examined. Comparisons between the predicted results and experimental results are also made.

INTRODUCTION

Design of thin-walled compression members requires the consideration of local plate buckling, overall column buckling and interactive local and overall buckling. The authors have previously (11) presented an analytical method for predicting the ultimate strength of thin-walled steel box columns simply supported and subjected to axial load and biaxial end moments. The numerical method is carried out in two stages; in the first stage, Moment-curvature-thrust ($M-\phi-P$) relationships of a locally buckled column segment are established using tangent stiffness (9) method. Local buckling of component plate is accounted for in terms of effective width (14) using the effective width formulae (5,7). The $M-\phi-P$ relationships are then incorporated into the column analysis, in the second stage, in which the differential equations of bending are integrated numerically. The ultimate strength is then obtained by using Horne's stability criterion (4).

The method takes into account the effect of residual stresses of different magnitude and column initial deflection on the load carrying capacity of columns. The accuracy of the method has been established by comparing the analytically predicted values with the corresponding experimental loads of large-scale steel box column models subjected to biaxial eccentric loading (8).

Several design methods (3,6,10,12) have been suggested for the design of welded thin-walled members subjected to eccentric loading; they are restricted to uniaxial bending and the effect of residual stresses is not

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included explicitly in the calculations. The object of the present paper is, therefore, to propose simple design formulas to predict the strength of thin-walled steel box columns subjected to biaxial loading. The design formulas are based on the informations derived from the computer analysis presented in Reference 11 and it follows closely the AISC-Q factor method (1).

Q-FACTOR

The expression for the Q-factor can be derived based on the following assumptions; (1) the cross section is considered to be a box-section of breadth b , depth d and thickness t . 'b' is assumed to be smaller than 'd'; (2) the component plates are assumed to be simply supported along the welded edges; (3) the maximum strength of the cross section is the sum of maximum strength of component plates. The Q-factor is written as

$$Q = \frac{\zeta b_o/b + d_o/d}{1 + \zeta} \quad (1)$$

in which $\zeta = \frac{b}{d} \leq 1.0$; b_o and d_o are the effective width and effective depth of the flange and web plates of a box section, respectively (Fig. 1).

ULTIMATE LOAD CAPACITY OF AXIALLY LOADED COLUMNS

The formula for local and overall interaction buckling strength of axially loaded columns can be written, based on the AISC-Q factor method (1) as

$$\begin{aligned} P_u &= Q P_y & \lambda \sqrt{Q} < 0.15 \\ &= \frac{P_y}{2\lambda_y^2} (\gamma - \sqrt{\gamma^2 - 4Q\lambda_y^2}) & \lambda \sqrt{Q} \geq 0.15 \end{aligned} \quad (2)$$

$$\text{where } \lambda_y = \frac{L}{\pi r_y} \sqrt{\epsilon_y} \quad (3)$$

$$\text{and } \gamma = 1 + 0.293 (\lambda_y \sqrt{Q} - 0.15) + Q\lambda_y^2 \quad (4)$$

For $Q = 1$, the above Perry-Robertson type formula fits, closely, with the SSRC curve '2' and ECCS curve 'b' which are commonly proposed for use in designing welded box columns.

Figure 2 shows a plot of eqn. (2). Numerical points obtained by using the computer method (11) are also plotted for comparison. Columns are assumed to be square in shape and to have sinusoidal initial out-of-straightness with a maximum mid-span deflection of $L/1000$. Numerical results for σ'_{rc} equal to 0.1 and 0.2 are presented. It can be observed that the computed data for welded plates with $\sigma'_{rc} = 0.1$ and 0.2 and β_{yb} ranging from 0.7 and 1.3 fall within a narrow scatter band which can be predicted by eqn. (2) with reasonable degree of accuracy.

ULTIMATE LOAD CAPACITY OF BIAXIALLY LOADED COLUMNS

A general form of biaxial bending interaction equation is proposed as follows:

$$\frac{P}{P_u} + \frac{M_x}{M_{ux} (1 - P/P_{ex})} + \frac{M_y}{M_{uy} (1 - P/P_{ey})} \leq 1 \quad (5)$$

This equation can be used in practical design when overall stability governs the limit state. For members with low column slenderness ratios or at support locations, local buckling rather than overall buckling, may govern the limit state. In such cases, the following equation is adopted

$$\frac{P}{Q P_y} + \frac{M_x}{M_{ux}} + \frac{M_y}{M_{uy}} \leq 1.0 \quad (6)$$

where P , M_x and M_y are the applied axial force and bending moments about x and y axes, respectively. P_u is the axially loaded column strength defined by eqn. (2). P_{ex} and P_{ey} are the Euler's buckling strength for fully effective section defined by P_y/λ_x^2 and P_y/λ_y^2 , respectively.

$$M'_{ux} = M_{ux} (1.07 - 0.172 \lambda_y) \leq M_{ux} \quad (7)$$

M_{ux} and M_{uy} are the ultimate moment capacity of the effective cross section about x and y axes, respectively.

M_{ux} for a thin-plated box section bent about the x axis (shown in Fig. 1) can be obtained as follows. The effective width of the compression flange at first yielding is computed by substituting $\beta = \beta_{yb}$ in the equations

$$\begin{aligned} \frac{b_e}{b} &= \frac{C_1}{\beta^2} + \frac{C_2}{\beta} + C_3 + C_4 \beta & \beta &\geq 0.526 \\ &= R_{rb} & & \end{aligned} \quad (8)$$

where

$$\begin{aligned} C_1 &= 0.2766 (R_{rb} - 1.901 C_2 - C_3 - 0.526 C_4) \\ C_2 &= -3 C_4 \beta_{yb}^2 - 2 C_3 \beta_{yb} \\ C_3 &= \frac{0.526 R_{rb}}{(0.526 - \beta_{yb})} - 1.5 C_4 (0.526 + \beta_{yb}) \\ C_4 &= \frac{2R_{rb} (0.526 \beta_{yb} - 0.2766)}{(0.526 - \beta_{yb})^3} \end{aligned} \quad (9)$$

and

$$\beta = \frac{b}{t} \sqrt{\frac{12(1-\nu^2)}{\pi^2 K}} \epsilon, \quad \beta_{yb} = \frac{b}{t} \sqrt{\frac{12(1-\nu^2)}{\pi^2 K}} \epsilon_y, \quad (10)$$

ϵ and ϵ_y are the compressive normal strain and yield strain. R_{rb} is the strength reduction factor caused by residual stresses defined by

$$\begin{aligned} R_{rb} &= 1 - \sigma'_{rc} \phi_1 & \beta_{yb} &> 1.413 \\ &= 1 - \sigma'_{rc} \phi_1 \phi_2 & 0.526 &\leq \beta_{yb} \leq 1.413 \\ &= 1 - \sigma'_{rc} \phi_2 & \beta_{yb} &< 0.526 \end{aligned} \quad (11)$$

where

$$\begin{aligned} \phi_1 &= \frac{\beta_{yb}^2}{(1.052 \beta_{yb} - 0.2766)} \\ \phi_2 &= \frac{171.27 \beta_{yb}^4}{(13.1 + 3.268 \beta_{yb}^4)^2} \end{aligned} \quad (12)$$

σ'_{rc} is the normalized compressive residual stress with the idealized distribution as shown in Fig. 3.

The position of neutral axis \bar{y} of the effective cross section, as shown in Fig. 1 is calculated as

$$\bar{y} = \frac{d(b_{e1} + 2d_{e1}) + (d_{e2}^2 - d_{e1}^2)}{b + b_e + 2(d_{e1} + d_{e2})} \quad (13)$$

where d_{e1} and d_{e2} are the effective width of web plate at first yielding under combined compression and bending. They can be obtained by using the equations

$$\begin{aligned} \frac{b_{e1}}{b} &= 0.5 \left(\frac{C_1}{\beta^2} + \frac{C_2}{\beta} + C_3 + C_4 \beta \right) & \beta &\geq 0.526 \\ &= 0.5 R_{rb} & \beta &< 0.526 \end{aligned} \quad (14)$$

and

$$\begin{aligned} \frac{b_{e2}}{b} &= (1 + 0.44\alpha) \frac{b_{e1}}{b} & \alpha &\leq 1.0 \\ &= 1.44 \frac{b_{e1}}{b} + \frac{\alpha-1}{\alpha} & \alpha &> 1.0 \end{aligned} \quad (15)$$

in which $\alpha = 1 - \sigma_2/\sigma_1$, where σ_1 and σ_2 are the maximum and minimum edge stresses. b_{e1} and b_{e2} are the effective widths defined in Fig. 4.

The ratio of d_{e2}/d can be obtained as

$$\frac{d_{e2}}{d} = \frac{(-g_1 + \sqrt{g_1^2 + 4g_2})}{2} \quad (16)$$

where $g_1 = \xi \left(1 + \frac{b}{b_e}\right) - 0.88 \frac{d_{e1}}{d}$

$$g_2 = 1.88 \left(\frac{d_{e1}}{d}\right)^2 + \left(1.44 \xi \left(1 + \frac{b}{b_e}\right) + 2\right) \frac{d_{e1}}{d} + \xi \frac{b}{b_e} \quad (17)$$

If $\left(\frac{d_{e1}}{d} + \frac{d_{e2}}{d}\right) \leq 1.0$ (i.e. web buckling occur) then

$$I_{ex} = t \left[b\bar{y}^2 + b_e (d - \bar{y})^2 + \frac{(d_{e1}^3 + d_{e2}^3)}{6} + 2d_{e1} \left(d - \bar{y} - \frac{d_{e1}}{2}\right)^2 + 2d_{e2} \left(\frac{d_{e2}}{2} - \bar{y}\right)^2 \right] \quad (18)$$

where \bar{y} is defined by eqn. (13).

If $\left(\frac{d_{e1}}{d} + \frac{d_{e2}}{d}\right) > 1.0$. (i.e. web plates do not buckle) then eqns. (13) and (18) reduce to

$$\bar{y} = \frac{d b_e + d^2}{2d + b + b_e} \quad (19)$$

and

$$I_{ex} = t \left[b\bar{y}^2 + b_e (d - \bar{y})^2 + \frac{d^3}{6} + 2d \left(\frac{d}{2} - \bar{y}\right)^2 \right] \quad (20)$$

I_{ex} is the second moment of the effective cross-sectional area. The moment capacity of the effective cross section about x-axis, M_{ux} can thus be obtained by the expression

$$M_{ux} = \frac{I_{ex} \sigma_y}{(d - \bar{y})} \quad (21)$$

Similarly, the moment capacity about the minor axis, M_{uy} can be obtained using the same procedures as given for calculating M_{ux} , simply by interchanging b and d values.

In Fig. (5), the cross-sectional moment capacity about the x axis normalized by the respective plastic moment M_{px} , are plotted against the nondimensional flange plate slenderness ratio β_{yb} for different values of b/d ratios. It is observed that the moment capacity of the cross section decreases as the component plate slenderness increases. Further reduction in strength can occur when the web plates become too slender. The welding residual stresses have significant effect on the moment capacity. Web

buckling tends to occur at smaller β_{yd} value for plates having higher compressive residual stresses. For example, web buckling tends to occur when the web slenderness ratios exceed 1.52 and 1.257 for σ'_{rc} equal to 0.1 and 0.2 respectively. For heavily welded box columns, web buckling may occur at lower plate slenderness ratio hence, their effect on overall column buckling strength cannot be neglected. The interaction formula for uniaxially loaded box columns proposed by Usami and Fukumoto (12) has ignored the local buckling of the web plates in deriving the cross-sectional moment capacity; this may lead to an overestimation of the ultimate strength.

ACCURACY OF THE PROPOSED FORMULAE

Comparison with more exact solutions

The computer method given in Reference 11 and the proposed design formulae (eqns. 5 and 6) were applied to analyse pin-ended, thin-plated square box columns of various proportions subjected to biaxial end loads and the results are presented in Fig. 5. For all these results, a compressive residual stress of $0.1 \sigma_y$ with the rectangular type of distribution shown in Fig. 3 was assumed. For the results obtained by using the computer program, sinusoidal deflections of $L/1000$ were assumed in the computation of long columns.

Biaxial interaction curves shown in Fig. 5 are for slender square box columns having nondimensional plate slenderness ratio, $\beta_{yb} = 0.7, 0.9$ and 1.1 . In this figure, curves are presented for various values of p and two different values of nondimensional column slenderness ratio viz. 0.4 and 0.8 . It can be observed that the biaxial bending capacity of column decreases significantly as p increases. The interaction curves obtained by the computer method exhibit a nonlinear behaviour whereas those obtained by using the proposed interaction formula are linear. The computer solution as shown in Fig. 5 for columns having $\beta_{yb} = 1.1$ indicates a reduction in strength for columns under biaxial bending when load path θ tends towards 45° (i.e. $M_x = M_y$). This observation is true for $\lambda_y = 0.4$ and 0.8 . The reason for this is that when M_x and M_y are applied in equal magnitude, both flange and web plates may buckle under the combined compressive edge strain. Whereas, if one of the bending moment is dominating, only one plate element is susceptible to local buckling. The proposed formulae, again, fail to show this phenomenon because the terms P_u , M_{ux} and M_{uy} in eqn. (5) are calculated separately and hence the true column interaction behaviour cannot be accurately accounted for. In Fig. 6, column strengths are presented in terms of normalized resultant moment m_T and nondimensional column slenderness, λ_y for loading path defined by $\theta = 45^\circ$. Curves are plotted for $\beta_{yb} = 1.0$ with p varying from 0.05 to 0.5 . Residual stress is chosen to be $0.1 \sigma_y$. It can be observed that, the proposed design formula gives a satisfactory prediction of column strength and in most cases, it is conservative as compared to the more exact computer solution.

Comparison with experimental results

35 large-scale test on square and rectangular box columns were

carried out by Usami and Fukumoto (12,13) (Tables 1 and 2). Columns having slenderness ratio (L/r) varying from 35 to 65 and plate width-thickness ratio (b/t) ranging from 22 to 59 were tested to failure. Chiew et al. (2) reported tests on 20 small-scale box columns in which higher plate width-thickness ratios, up to 80, were used (Table 3). 28 large-scale welded box columns tested under biaxial loading were reported by Richard Liew et al. (8) (Table 4). The parameters in the test program included plate slenderness ratios, column slenderness ratios, cross sectional shapes and eccentricities of loading. In addition to these, extensive measurement of residual stresses and initial column imperfections were also made.

Failure loads of all these column specimens were computed by using (i) the computer method (ii) the proposed design formulas and (iii) BS5950: Part 1: 1985 British Standard for structural use of steelwork in building. The predicted failure loads, along with those experimentally observed are presented in Tables 1-4 as ratios of the collapse load to the respective squash loads.

In Table 3, measured values of σ'_{rc} were used for numerical predictions since no stub-columns tests were carried out. It has also been pointed out (13) that the use of stub-column test, rather than direct measured values of residual stresses, to determine the stress reduction factors is theoretically more reasonable since the effective width formulae usually have indirect means to allow for plate imperfections.

The predicted results using BS5950 were based on the stress reduction factors specified in Table 8 of Section 3.6 for built-up sections of the code. For rectangular sections, the stress reduction factors were obtained based on an area weighted values of σ_y for the web and flanges.

A further reduction of 20 MPa was allowed to account for the effect of welding residual stresses. Robertson constant equal to 3.5 was chosen for Table 27(b) of the code. The interaction equation based on a simplified approach given in Section 4.8.3.3.1 was used to compute the strength of columns subject to eccentric loading. In the calculation of the buckling resistance moment capacity M_b , the lateral torsional buckling has been ignored since most rectangular sections had b/d ratios greater than or equal to 0.75.

In Tables 1-4, the mean values and standard deviation of the comparisons between the observed and predicted values are given. These comparisons with design formulae are also presented in graphical form in Fig. 7. The comparison shows clearly that the computer method gives reasonable correlations and the predictions in most cases lie within ± 15 percent. The formulae presented in this paper give conservative results, nevertheless, it has maintained a reasonable degree of accuracy. In all cases, BS5950 shows highly conservative predictions. The proposed formulae are simpler to use unlike the computer method which usually involves complicated calculation procedures which is too expensive and undesirable at the design stage.

CONCLUSIONS

Although design charts for initially imperfect columns can be generated by using the more vigorous computer technique, the complexity involved makes this approach very undesirable at the design stage. Simplified design equations are, therefore, proposed to facilitate the

design of such columns. These simplified equations allow for local buckling of component plates and account explicitly for the effect of welding residual stresses. This is desirable in the design of fabricated members, where the degree of locked-in residual stresses in the cross section depend highly on the speed and intensity of the applied heat source and also on the plate width-thickness ratio. The accuracy of these equations is verified by comparing the predicted failure loads with available test data and the computer solutions. The results show that the proposed formulae are accurate enough for the design office where repetitive analyses are required and at preliminary stages of practical design.

REFERENCES

1. American Institute of Steel Construction, Specification for the design, fabrication and erection of structural steel for buildings, AISC, 8th Ed., New York, N Y, 168 pp.
2. Chiew, S.P., Lee, S.L. and Shanmugam, N.E., (1987), "Experimental Study of Thin-walled Steel Box Columns", J. Struct. Engrg., ASCE, 113(10), 2208-2220.
3. Dewolf, J.T., Pekoz, T. and Winter, G., (1974), "Local and Overall Buckling of Cold-formed Members", J. Struct. Div., ASCE. 100(10), 2017-2036.
4. Horne, M.R., (1956), "The Elastic-plastic Theory of Compression Members", J. Mechanics and Physics of Solid, 4 (1956) 104-120.
5. Mulligan, G.P. and Pekoz, T., (1984), "Analysis of Locally Buckled Thin-walled Columns", Proc. 7th Int. Natl. Specialty Conf. on Cold-Formed Steel Structures, University of Missouri-Rolla, 93-126.
6. Pekoz, T., (1986), "Development of a Unified Approach to the Design of Cold-formed Steel Members", Report SG 86-4, American Iron and Steel Institute, 1000, 16th Street, NW, Washington, DC, 200 pp.
7. Richard Liew, J.Y., Shanmugam, N.E. and Lee, S.L., (1989), "Local Buckling of Thin-walled Steel Box Columns", Thin-walled Structures, 8 (1989) 119-145.
8. Richard Liew, J.Y., Shanmugam, N.E. and Lee, S.L., (1989), "Behaviour of Thin-walled Steel Box Columns under Biaxial Loading", J Struct. Engrg., ASCE, 115(12), 3076-3094.
9. Santathadaporn, S. and Chen, W.F., (1972), "Tangent Stiffness Method for Biaxial Bending", J. Struct. Div., ASCE, 98(1), 153-163.
10. Shanmugam, N.E., Chiew, S.P. and Lee, S.L., (1988), "A Design Formula Thin-walled Steel Box Columns", IABSE Proceedings, P-130/88, 105-116.
11. Shanmugam, N.E., Richard Liew, J.Y. and Lee, S.L., (1989), "Thin-walled Steel Box Columns under Biaxial Loading", J Struct. Engrg., ASCE, 115(11), 2706-2726.
12. Usami, T. and Fukumoto, Y., (1982), "Local and Overall Buckling of Welded Box Columns", J. Struct. Div., ASCE, 108(3), 525-542.

13. Usami, T. and Fukumoto, Y., (1984), "Welded Box Compression Members", J. Struct. Div., ASCE, 110(10), 2457-2470.
14. Winter, G., (1947), "Strength of Thin Steel Compression Flanges", Trans. Am. Soc. Civ. Engrs., 112, 527-554.

NOTATIONS

The following symbols are used in this paper:

- A = area of cross section
- b, d = width and depth of column cross section measured between median planes of walls
- b_e, d_e = effective width of the flange and web plates
- b_{e1}, b_{e2} = effective widths of plates subject to compression and bending, defined by eqns. (14) and (15)
- C_1 to C_4 = constants defined by eqn. (9)
- e_x, e_y = load eccentricities in x and y directions
- I_{ex} = second moment of effective area about the x axis
- K = elastic coefficient of buckling of a simply-supported plate
- L = length of column
- M_x, M_y = applied end moment about x and y axes
- M_{px} = plastic moment capacity about the x axis
- M_{ux}, M_{uy} = ultimate moment capacity of a cross section bent about x and y axes
- M'_{ux} = reduced ultimate bending strength of a beam bent about the strong axis
- m_x, m_y = nondimensional moments defined by $M_x/M_{px}, M_y/M_{py}$
- m_T = resultant moment defined by $\sqrt{m_x^2 + m_y^2}$
- P = applied axial load
- P_{ex}, P_{ey} = Euler buckling loads about the strong and weak axes
- P_u = axially loaded column strength

- P_y = squash load
 p = nondimensional axial load defined by P/P_y
 Q = factor defined by eqn. (1)
 R_{rb}, R_{rd} = strength reduction factors for flange and web plates due to the effect of residual stress
 \bar{y} = position of neutral axis of bending
 Z_{px} = plastic section modulus about x axis
 β = $\frac{b}{t} \sqrt{\frac{12(1-\nu^2)}{\pi^2 K} \epsilon}$
 β_{yb}, β_{yd} = nondimensional slenderness ratios for the flange and web plates defined by $\frac{b}{t} \sqrt{\frac{12(1-\nu)^2}{\pi^2 K} \epsilon_y}$ and $\frac{d}{t} \sqrt{\frac{12(1-\nu)^2}{\pi^2 K} \epsilon_y}$, respectively
 σ_{rc} = compressive residual stress
 σ'_{rc} = σ_{rc} / σ_y
 ϵ_y, σ_y = yield strain and stress
 ν = Poisson's ratio
 λ_x, λ_y = $\frac{L}{\pi r_x} \sqrt{\epsilon_y}$, $\frac{L}{\pi r_y} \sqrt{\epsilon_y}$ are the nondimensional column slenderness ratios about x and y axes, respectively
 ξ = width to depth ratio

Table 1 - Comparison of Predicted Results with Test Results¹²

Specimen	β_{yb}	β_{yd}	λ_y	Effective strength factor		Experimental	Computer method	Proposed Formulae				BSS950		
				R_{rb}^*	R_{rd}^*			$\frac{P_{exp}}{P_y}$	$\frac{P_{cal}}{P_{exp}}$	$\frac{P_{cal}}{P_y}$	$\frac{P_{cal}}{P_{exp}}$		$\frac{P_{cal}}{P_y}$	$\frac{P_{cal}}{P_{exp}}$
S-35-22	0.686	0.686	0.640	1.000	1.000	0.852	0.902	1.058	0.810	0.950	0.625	0.734		
S-35-27	0.840	0.840	0.642	1.000	1.000	-	0.818	-	-	-	-	-		
S-35-33	1.014	1.014	0.647	0.988	0.988	0.722	0.727	1.007	0.644	0.892	0.386	0.535		
S-35-38	1.177	1.177	0.648	0.945	0.945	0.621	0.634	1.021	0.573	0.923	0.329	0.530		
S-35-44	1.360	1.360	0.648	0.926	0.926	0.544	0.543	0.998	0.513	0.943	0.278	0.511		
S-50-22	0.683	0.683	0.948	1.000	1.000	0.740	0.845	1.142	0.653	0.882	0.548	0.741		
S-50-27	0.840	0.840	0.918	1.000	1.000	0.672	0.768	1.143	0.601	0.894	0.445	0.662		
S-50-33	1.026	1.026	0.927	0.988	0.988	0.670	0.674	1.006	0.551	0.822	0.356	0.531		
R-50-22	0.686	0.515	0.913	1.000	1.000	0.743	0.855	1.151	0.682	0.918	0.639	0.860		
R-50-27	0.840	0.630	0.927	1.000	1.000	0.731	0.753	1.030	0.637	0.871	0.515	0.705		
R-50-33	1.026	0.770	0.920	1.000	1.000	0.709	0.702	0.991	0.585	0.825	0.414	0.584		
R-50-38	1.180	0.885	0.923	0.948	0.977	0.639	0.611	0.957	0.531	0.987	0.354	0.554		
R-50-44	1.363	1.022	0.925	0.929	0.961	0.579	0.546	0.943	0.497	0.858	0.302	0.522		
R-65-22	0.683	0.512	1.186	1.000	1.000	0.593	0.650	1.096	0.500	0.843	0.498	0.840		
R-65-27	0.840	0.630	1.201	1.000	1.000	0.637	0.611	0.959	0.477	0.749	0.428	0.672		
R-65-33	1.026	0.770	1.203	1.000	1.000	0.585	0.566	0.968	0.451	0.771	0.360	0.615		
ER-50-22	0.683	0.512	0.913	1.000	1.000	0.557	0.560	1.005	0.498	0.894	0.519	0.932		
ER-50-27	0.837	0.628	0.917	1.000	1.000	0.557	0.534	0.959	0.472	0.847	0.415	0.745		
ER-50-33	1.023	0.767	0.920	1.000	1.000	0.542	0.486	0.968	0.438	0.808	0.335	0.618		
MEAN							1.018	1.018	0.871	0.871	0.666	0.666		
STANDARD DEVIATION							0.062	0.062	0.062	0.062	0.124	0.124		

* Values of R_{rb} and R_{rd} were obtained from stub column test results

Table 2 - Comparison of Predicted Results with Test Results¹³

Specimen	β_{yb}	β_{yd}	λ_y	Effective strength factor		Exper- mental	Computer method		Proposed Formulae		BSS950	
				R_{rb}^*	R_{rd}^*		$\frac{P_{exp}}{P_y}$	$\frac{P_{cal}}{P_y}$	$\frac{P_{cal}}{P_{exp}}$	$\frac{P_{cal}}{P_y}$	$\frac{P_{cal}}{P_{exp}}$	$\frac{P_{cal}}{P_{exp}}$
R-40-29	0.813	0.610	0.651	1.000	1.000	0.798	0.865	1.085	0.766	0.960	0.600	0.752
R-40-44	1.229	0.922	0.658	0.916	0.962	0.644	0.668	1.037	0.619	0.961	0.375	0.582
R-40-58	1.618	1.214	0.654	0.890	0.919	0.498	0.520	1.044	0.486	0.976	0.264	0.530
R-65-29	0.816	0.612	1.051	1.000	1.000	0.619	0.736	1.189	0.549	0.887	0.481	0.777
R-65-44	1.235	0.926	1.069	0.916	0.962	0.521	0.524	1.006	0.467	0.896	0.316	0.607
R-65-58	1.620	1.215	1.062	0.890	0.919	0.441	0.427	0.968	0.406	0.921	0.234	0.531
ER-40-29e1	0.813	0.610	0.651	1.000	1.000	0.610	0.621	1.018	0.575	0.943	0.474	0.777
ER-40-44e1	1.218	0.914	0.658	0.916	0.961	0.510	0.486	0.953	0.459	0.900	0.289	0.567
ER-40-58e1	1.615	1.211	0.658	0.890	0.919	0.391	0.382	0.977	0.384	0.982	0.207	0.529
ER-40-44e2	1.232	0.924	0.658	0.916	0.967	0.411	0.396	0.963	0.379	0.922	0.236	0.574
ER-65-29e1	0.818	0.614	1.059	1.000	1.000	0.435	0.476	1.094	0.406	0.933	0.395	0.908
ER-65-44e1	1.218	0.914	1.067	0.916	0.962	0.406	0.385	0.948	0.356	0.877	0.256	0.631
ER-65-58e1	1.615	1.211	1.062	0.890	0.919	0.312	0.313	1.003	0.315	1.010	0.189	0.606
ER-65-44e2	1.218	0.914	1.067	0.916	0.967	0.325	0.320	0.984	0.298	0.917	0.217	0.668
ER-65-58e2	1.615	1.211	1.062	0.890	0.919	0.268	0.269	1.004	0.265	0.989	0.157	0.586
ES-40-44e1	1.227	0.920	0.651	0.867	0.867	0.441	0.420	0.952	0.408	0.925	0.241	0.546
ES-40-58e1	1.615	1.211	0.658	0.878	0.878	0.363	0.344	0.948	0.351	0.970	0.174	0.479
MEAN								1.010		0.939		0.626
STANDARD DEVIATION								0.063		0.037		0.114

* Values of R_{rb} and R_{rd} were obtained from stub column test results

Table 3 - Comparison of Predicted Results with Test Results²

Specimen	β_{yb}	β_{yd}	λ_y	Residual stress		Effective strength factor		Experimental	Computer method		Proposed formulae		BSS950	
				σ'_{rc}	R_{rb}^*	R_{rd}^*	$\frac{P_{exp}}{P_y}$		$\frac{P_{cal}}{P_{exp}}$	$\frac{P_{cal}}{P_y}$	$\frac{P_{cal}}{P_{exp}}$	$\frac{P_{cal}}{P_y}$	$\frac{P_{cal}}{P_{exp}}$	
A-S-40-10	0.739	0.739	0.11	0.114	0.968	0.968	1.119	0.887	0.792	0.877	0.784	0.608	0.543	
A-S-40-25	0.739	0.739	0.28	0.114	0.968	0.968	1.036	0.868	0.838	0.846	0.817	0.576	0.556	
A-S-40-23	0.739	0.739	0.38	0.114	0.968	0.968	0.897	0.860	0.959	0.820	0.914	0.557	0.621	
A-S-40-45	0.739	0.739	0.52	0.126	0.965	0.965	0.785	0.844	1.075	0.776	0.989	0.530	0.675	
A-S-40-56	0.739	0.739	0.64	0.095	0.973	0.973	0.802	0.832	1.037	0.739	0.921	0.505	0.630	
A-S-57-56	1.050	1.050	0.64	0.280	0.734	0.734	0.562	0.519	0.923	0.494	0.879	0.325	0.578	
A-S-62-10	1.155	1.155	0.11	0.127	0.846	0.846	0.687	0.595	0.865	0.596	0.868	0.341	0.496	
A-S-62-25	1.155	1.155	0.28	0.127	0.846	0.846	0.590	0.585	0.983	0.584	0.990	0.323	0.547	
A-S-62-30	1.155	1.155	0.35	0.127	0.846	0.846	0.581	0.582	1.002	0.575	0.990	0.316	0.544	
A-S-80-10	1.507	1.507	0.11	0.116	0.799	0.799	0.534	0.460	0.861	0.469	0.878	0.248	0.464	
A-S-80-25	1.507	1.507	0.28	0.104	0.820	0.820	0.507	0.476	0.939	0.475	0.937	0.234	0.462	
A-S-80-33	1.507	1.507	0.38	0.104	0.820	0.820	0.496	0.465	0.937	0.465	0.938	0.227	0.458	
A-S-80-56	1.507	1.507	0.64	0.281	0.512	0.512	0.321	0.283	0.882	0.286	0.891	0.209	0.651	
A-R-40-40	0.739	0.739	0.46	0.113	0.968	0.987	0.884	0.886	1.002	0.829	0.938	0.624	0.706	
A-R-57-40	1.050	0.840	0.46	0.153	0.854	0.930	0.740	0.690	0.933	0.660	0.892	0.448	0.605	
A-R-57-52	1.050	1.206	0.60	0.215	0.795	0.968	0.669	0.683	1.021	0.642	0.960	0.398	0.595	
A-R-80-40	1.507	1.206	0.46	0.109	0.811	0.856	0.563	0.508	0.903	0.500	0.888	0.256	0.455	
B-S-40-33	0.739	0.739	0.38	0.089	0.975	0.975	0.768	0.616	0.880	0.635	0.827	0.431	0.561	
B-S-57-33	1.050	1.050	0.38	0.102	0.903	0.903	0.556	0.515	0.926	0.510	0.917	0.272	0.489	
B-S-80-33	1.507	1.507	0.38	0.124	0.785	0.785	0.412	0.350	0.850	0.365	0.886	0.173	0.420	
MEAN									0.931		0.905		0.553	
STANDARD DEVIATION									0.086		0.055		0.081	

* Values of R_{rb} and R_{rd} were obtained by direct substitution of measured value of σ'_{rc} into equation (11)

Table 4 - Comparison of Predicted Results with Test Results⁶

Specimen	λ_y	Eccentricities		Effective strength factor		Experi- mental		Computer method		Proposed Formulae		BSS950	
		e_x (mm)	e_y (mm)	$R_{t,b}^*$	$R_{t,d}^*$	$\frac{P_{exp}}{P_y}$	$\frac{P_{cal}}{P_y}$	$\frac{P_{cal}}{P_{exp}}$	$\frac{P_{cal}}{P_y}$	$\frac{P_{cal}}{P_{exp}}$	$\frac{P_{cal}}{P_y}$	$\frac{P_{cal}}{P_{exp}}$	$\frac{P_{cal}}{P_y}$
X-S-30-85	1.137	15	0	0.937	0.937	0.448	0.425	0.948	0.337	0.752	0.349	0.779	
XY-S-30-85a	1.137	5	10	0.937	0.937	0.522	0.449	0.860	0.331	0.634	0.355	0.680	
XY-S-30-85b	1.109	5	5	0.937	0.937	0.600	0.503	0.838	0.359	0.598	0.383	0.638	
X-S-45-57	0.768	15	0	0.891	0.891	0.523	0.480	0.917	0.443	0.847	0.289	0.553	
XY-S-45-57a	0.768	5	15	0.891	0.891	0.502	0.435	0.867	0.401	0.799	0.275	0.548	
XY-S-45-57b	0.768	15	15	0.891	0.891	0.461	0.403	0.874	0.355	0.770	0.241	0.523	
X-S-52-48	0.648	20	0	0.860	0.860	0.441	0.417	0.946	0.400	0.907	0.234	0.531	
XY-S-52-48a	0.648	15	10	0.860	0.860	0.460	0.391	0.850	0.379	0.824	0.223	0.485	
XY-S-52-48b	0.655	10	10	0.860	0.860	0.482	0.415	0.860	0.399	0.703	0.238	0.494	
Y-S-64-64	0.743	0	10	0.870	0.870	0.436	0.406	0.931	0.401	0.920	0.220	0.505	
XY-S-64-64a	0.743	10	5	0.870	0.870	0.414	0.363	0.877	0.371	0.896	0.199	0.481	
XY-S-64-64b	0.743	15	15	0.870	0.870	0.316	0.300	0.950	0.304	0.962	0.162	0.513	
X-S-75-55	0.631	15	0	0.912	0.912	0.390	0.372	0.954	0.381	0.977	0.170	0.436	
XY-S-75-55a	0.631	15	10	0.912	0.912	0.362	0.331	0.914	0.337	0.931	0.150	0.414	
XY-S-75-55b	0.631	15	15	0.912	0.912	0.341	0.314	0.921	0.319	0.935	0.141	0.413	
Y-S-85-48	0.560	0	20	0.897	0.897	0.349	0.331	0.949	0.342	0.980	0.141	0.404	
XY-S-85-48a	0.557	20	10	0.897	0.897	0.342	0.300	0.877	0.299	0.874	0.111	0.325	
XY-S-85-48b	0.560	15	15	0.897	0.897	0.331	0.300	0.924	0.300	0.906	0.124	0.375	
X-R-53-64	0.705	19	0	0.941	0.870	0.485	0.487	1.004	0.442	0.911	0.264	0.544	
XY-R-53-64a	0.705	20	10	0.941	0.870	0.484	0.436	0.901	0.393	0.812	0.234	0.483	
XY-R-53-64b	0.705	20	20	0.941	0.870	0.440	0.398	0.904	0.359	0.816	0.213	0.484	
XY-R-53-64c	0.705	10	20	0.941	0.870	0.500	0.464	0.928	0.404	0.808	0.240	0.480	
Y-R-53-64	0.705	0	20	0.941	0.870	0.578	0.535	0.926	0.467	0.808	0.274	0.474	
X-R-86-62	0.575	20	0	0.920	0.904	0.339	0.339	1.000	0.338	0.997	0.146	0.431	
XY-R-86-62a	0.575	20	10	0.920	0.904	0.313	0.306	0.947	0.308	0.984	0.131	0.419	
XY-R-86-62b	0.575	20	20	0.920	0.904	0.302	0.280	0.927	0.283	0.937	0.119	0.394	
XY-R-86-62c	0.575	10	20	0.920	0.904	0.331	0.310	0.936	0.317	0.958	0.134	0.405	
Y-R-86-62	0.575	0	20	0.920	0.904	0.369	0.364	0.986	0.361	0.978	0.154	0.417	
MEAN								0.918		0.865		0.487	
STANDARD DEVIATION								0.043		0.104		0.096	

⁶ Values of $R_{t,b}$ and $R_{t,d}$ were obtained from stub-column test results

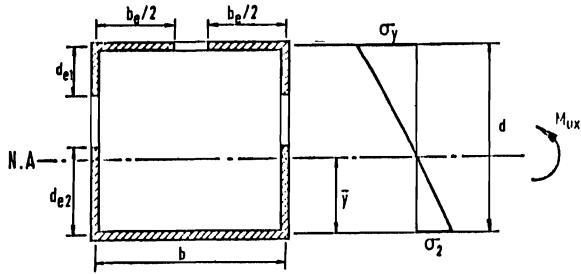


Fig. 1 Thin-walled box section in bending

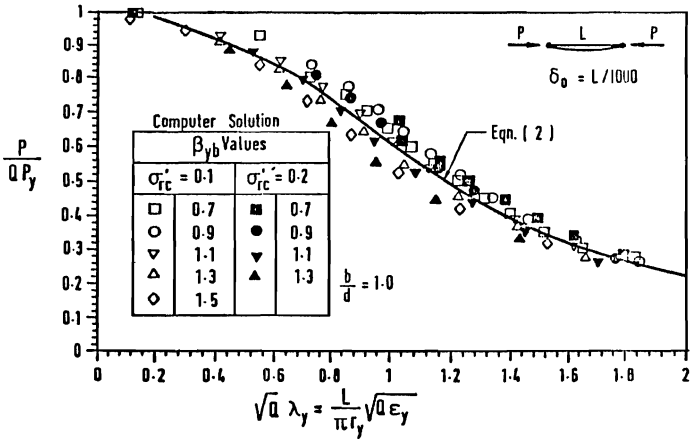


Fig. 2 Strength of axially loaded columns

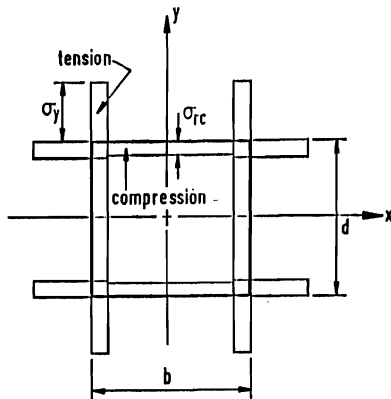


Fig. 3 Idealized residual stress distribu of box section edge welds

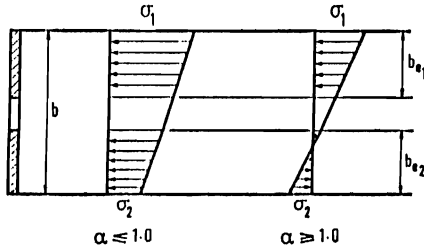


Fig. 4 Effective widths of plate subject to compression and bending

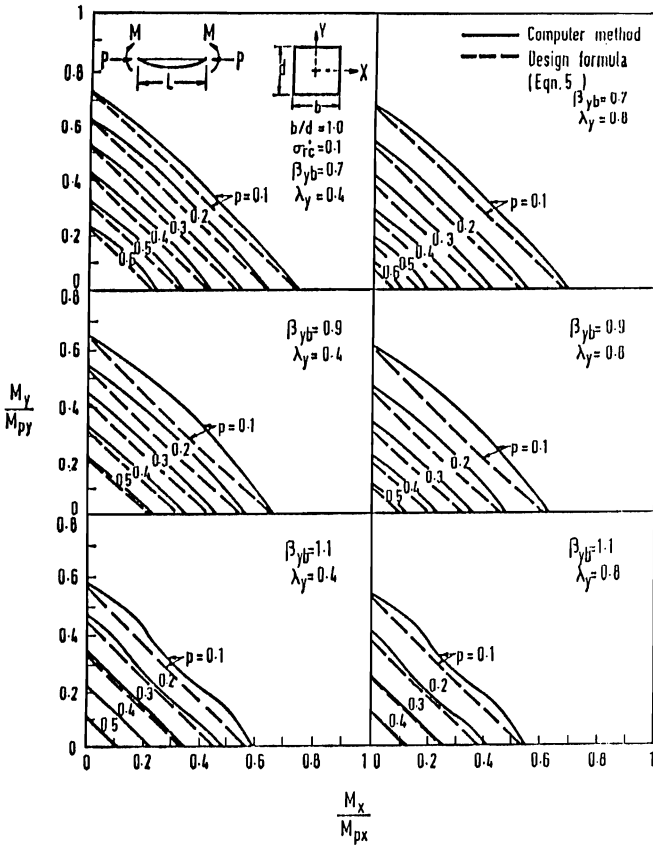


Fig. 5 Comparison of interaction curves for biaxially loaded square box columns

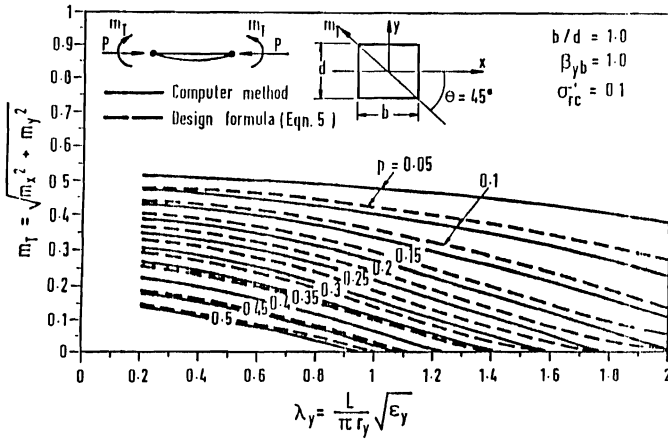


Fig. 6 $m_T - \lambda_\gamma$ curves for biaxially loaded square box columns with $\theta = 45^\circ$

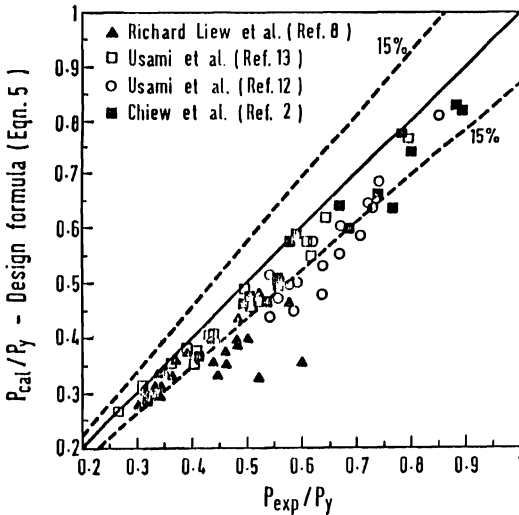


Fig. 7 Comparison of experimental values and theoretical values obtained by design formulae

