A Model for the Behavior of Thin-walled Flexural Members under Concentrated Loads

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by Monique Bakker (1), Teoman Peköz (2), and Jan Stark (3).

SUMMARY

This paper presents a mechanism approach for analyzing the web crippling behavior of thin-walled members subjected to the combined action of a concentrated load and a bending moment. This approach also applies to the determination of redistribution of bending moments in continuous multi-span members due to web crippling.

The approach is based on yield line analysis of failure mechanisms. To investigate the web crippling failure mechanisms, a series of web crippling tests on cold-formed hat sections was performed. It was found that the corner radius largely influences the type of mechanism that takes place.

INTRODUCTION

Thin-walled steel members are frequently used as structural elements in buildings. When such a member is subjected to a concentrated load or reaction it may fail by web crippling. In 1985 the existing (Interior One Flange) web crippling prediction formulas were compared with test results reported in literature (see Fig. 1) and it was found that these formulas gave inconsistent and sometimes unsafe results (Bakker and Peköz, 1985). Furthermore, all these formulas were based on curve-fitting of test results. Of course each equation correlated with the test results it was based on. The correlation is much worse for test results from other sources. The present research was prompted by the lack of a general analytical model that can explain the significance of principal parameters. The primary object of the research was to develop a theoreticl model to describe the web crippling behavior. This model should then be used to develop more reliable design formulas.

CURRENT APPROACH

In practice web crippling may occur for different loading conditions in different types of members. In this paper only Interior One Flange loading conditions will be considered. For this condition the force is applied through one flange and resisted by shear forces in the web (see Fig. 3). This situation occurs at the interior supports of continuous, multi-span members, or at a concentrated load applied somewhere in the middle of a span.

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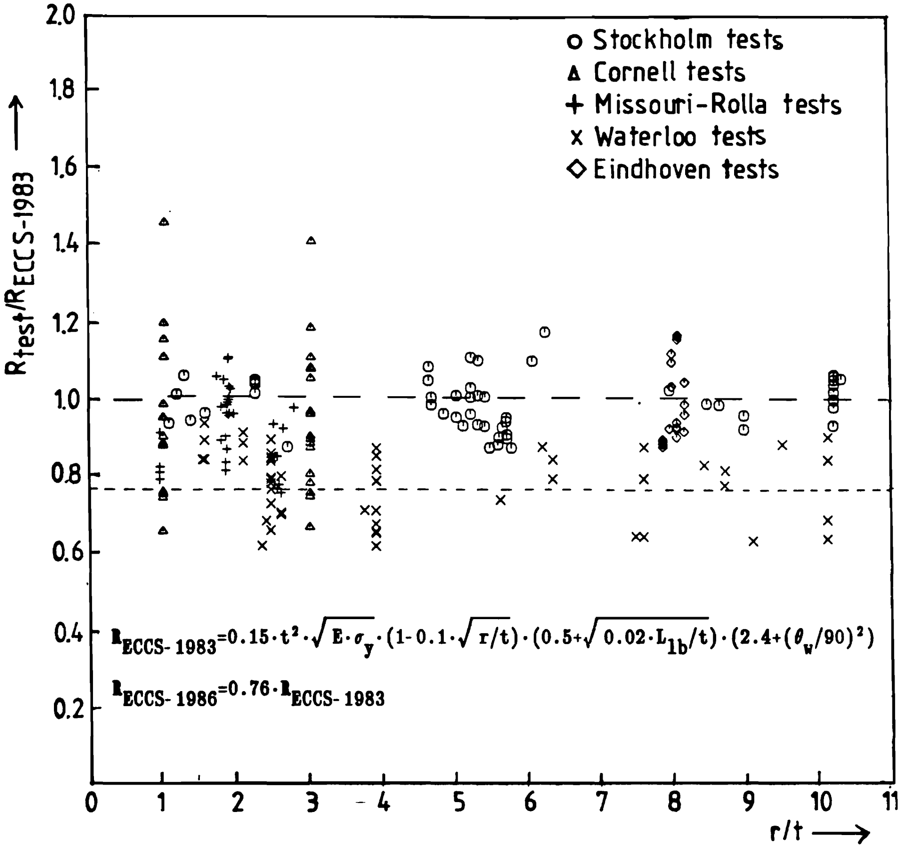


Figure 1. Comparison of ECCS web crippling prediction formula with test results

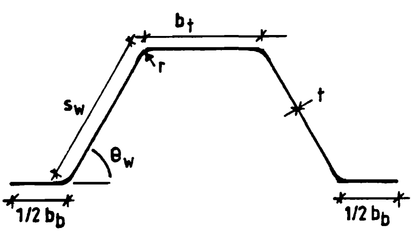


Figure 2. Geometry of members

The current design formulas (AISI Specification 1986, ECCS 1986) for IOF web crippling of sections having single, unreinforced webs have the following general format:

$$R_d = C \cdot \sigma_y \cdot t^2 \cdot C(s_w/t) \cdot C(L_{1b}/t) \cdot C(r/t) \cdot C(\theta_w) \cdot C(\sigma_y),$$

where C(i) are nondimensional factors accounting for the influence of the web slenderness ratio, bearing length ratio, inside corner radius ratio, angle of web inclination, and yield capacity of the steel respectively (see Fig. 2). These web crippling prediction formulas predict the ultimate limit load of a member whose span length is so short that the influence of the bending moment on the web crippling failure is thought to be negligibly small. For members with a larger span, the influence of the bending moment on the web crippling capacity is taken into account by means of an interaction formula in the form:

$$\alpha \cdot R/R_d + \beta \cdot M/M_d \leq 1,$$

where R is the applied concentrated load or reaction, M is the bending moment acting at the place where the concentrated load is applied to the member, M_d is the design value for the bending moment capacity, and α and β are empirically determined coefficients.

In continuous, multi-span members, a web crippling deformation at the interior support results in a permanent rotation, which contributes to a redistribution of bending moments. This phenomenon was described by von Unger (1973), Reinsch (1983), Bryan and Leach (1984) and Tsai and Crisinel (1986). To predict the ultimate capacity of continuous members, the load-deformation behavior of the member at the interior support is determined from small scale tests, similar to combined web crippling-bending tests (see Fig.3).

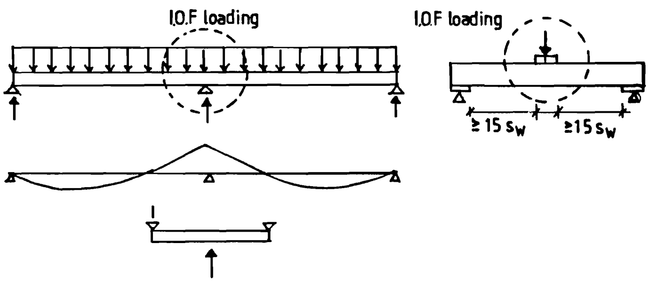


Figure 3. Interior One Flange loading condition and small scale tests simulating the behavior of the interior support of a continuous multi-span member

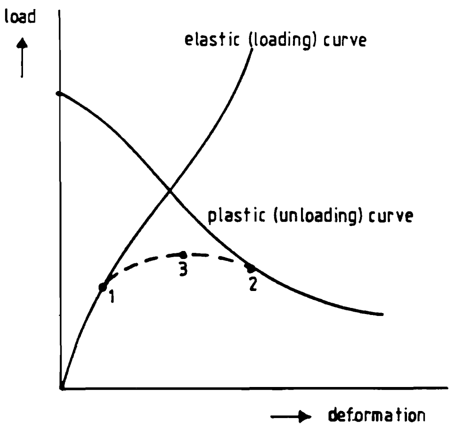
THEORY

Analysis of load-deformation behavior

The theoretical model is developed assuming that the behavior of a loaded steel structure can be described approximately by using two different material models: a model of ideal linear-elastic material behavior for the first phase of loading, and a model of ideal rigid-plastic material behavior for the final or post-collapse phase of loading.

To clarify this, the load-deformation behavior of a loaded structure is considered (see Fig. 4). As long as the load and deformations are small, the steel will behave elastically. The load-deformation behavior of the structure, calculated assuming linear-elastic material behavior, can be represented by an elastic loading curve in the load-deformation diagram. With increasing load the steel will start to yield locally. The areas in which yielding occurs will expand until a mechanism develops. After the formation of a mechanism, the load-deformation behavior can be analyzed by applying rigid-plastic theory to the (yield line) mechanism, and be represented by a plastic curve in the load-deformation diagram. This curve can be either a loading or an unloading curve, depending on the type of mechanism.

The actual load-deformation curve of the structure, indicated by a dotted line in Fig. 4, will start to deviate from the elastic loading curve at first yield, and will coincide with the plastic curve only after the formation of a mechanism.



1. Elastic limit load (first yield)
2. Plastic limit load (mechanism)
3. Ultimate limit load (maximum load)

Figure 4. Load-deformation behavior

Strength

In the analysis of structures it is useful to distinguish between response functions and resistance functions. A response function describes the behavior of the structure subjected to loading, whereas a resistance function describes the strength of the structure. A resistance function can be determined from a response function by defining a limit state, that is a just acceptable state of the structure, beyond which the structure is believed to be unable to perform its function (to carry loads), or service (to provide an acceptable environment).

In the behavior of the structure as described above different limit loads (strengths), corresponding to different limit states can be recognized, for example:

1. the elastic limit load, corresponding to the state of first yield;
2. the plastic limit load, corresponding to the formation of a plastic mechanism;
3. the ultimate limit load, corresponding to the maximum load carrying capacity of the structure;

In developing strength prediction formulas, and in comparing test results with these prediction formulas, it is important to realize what strength definition is used.

Generalized yield line theory

The plastic curve of a structure can be analyzed by using a generalized yield line theory (Bakker, 1989). In yield line analysis the structure to be analyzed is thought to consist of rigid plane elements joined by yield lines. All (plastic) deformation is postulated to occur in the yield lines. Generalized yield line theory was developed from classical yield line theory, an upper bound limit analysis technique for the analysis of concrete slabs loaded by forces perpendicular to the plate. In classical yield line theory in the yield lines only bending moments are active. In generalized yield line theory also normal forces and in-plane shear forces can be active.

Classical yield line theory is an upper bound limit analysis technique. An arbitrary mechanism will result in an upper bound of the limit load of the (undeformed) structure. The decisive mechanism can be determined by minimizing the limit load of the structure. Generalized yield line theory is not used to determine the limit load of the undeformed structure, but to analyze the complete load-deformation behavior of the structure. An arbitrary mechanism will not result in an upper bound for the load-deformation behavior of the structure. It results in an upper bound for the limit load of the structure with the deformation state as specified, but this deformation state may never be attained in the actual mechanism.

In generalized yield line theory it is not simple to determine the decisive mechanism. The form of the mechanism may be governed by the elastic behavior of the structure. Murray (1987) for instance showed that in box columns the form of the (local) yield line mechanism depends on the place of first yield in the column. In practice the occurring yield line pattern will often have to be determined from observations in tests. The first step in yield line analysis is then to check whether the load deformation behavior of the structure can be determined from the observed yield line pattern. The second step is to predict the observed yield line pattern.

It can be concluded that generalized yield line analysis is a complex and rather intuitive method for analyzing the load-deformation behavior of a structure. From trial and error it has to be determined whether an assumed (simplified) mechanism leads to reasonable results.

TEST PROGRAM

To study the web crippling load-deformation behavior and the occurring yield line patterns it was found necessary to carry out new tests. For most web crippling and combined bending-web crippling tests described in literature only the ultimate limit loads were registered, while for the development of the model also information on the deformations was needed. A total of 70 tests was performed on hat sections formed by press braking. In these tests, besides all the parameters occurring in the current web crippling prediction formulas, the width of the top flange and the span length were varied. In this paper emphasis will be given to the method used to interpret the test results. Detailed test results can be found in the test report (Bakker, 1990).

To analyze the load-deformation behavior it has to be decided what deformations should be considered. For describing the web crippling behavior the web crippling deformation e, the deflection f, and the rotation φ at the end supports were found to be relevant parameters (see Fig. 5).

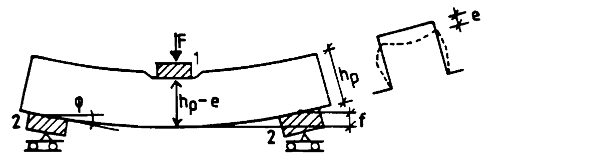
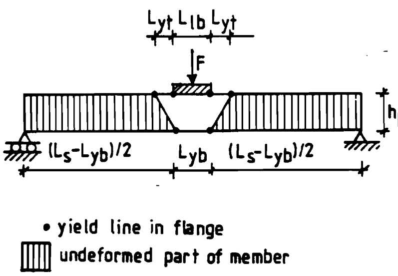


Figure 5. Deformation parameters for analyzing web crippling behavior.

The test setup for the (Interior One Flange loading) tests is shown in Fig. 7. The measurement of the deformations is shown in Fig. 8. The deflection f was not measured directly but determined from the measured displacement of the load bearing plate with respect to the reaction bearing plates:
$f = \delta_r - e$.

INTERPRETATION OF TEST RESULTS

In the interpretation of the test results it was assumed that web crippling results in the formation of a kind of plastic hinge, as shown in Fig. 6.



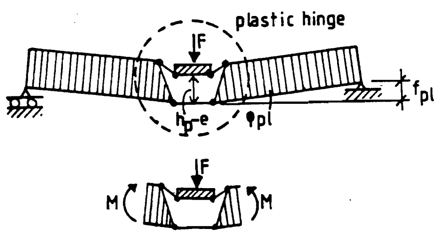
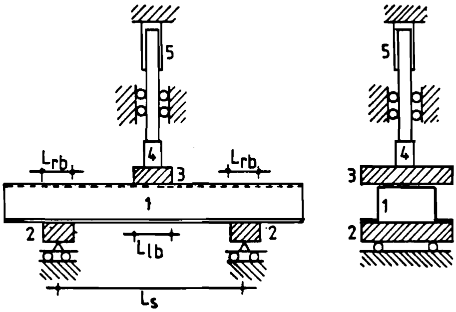
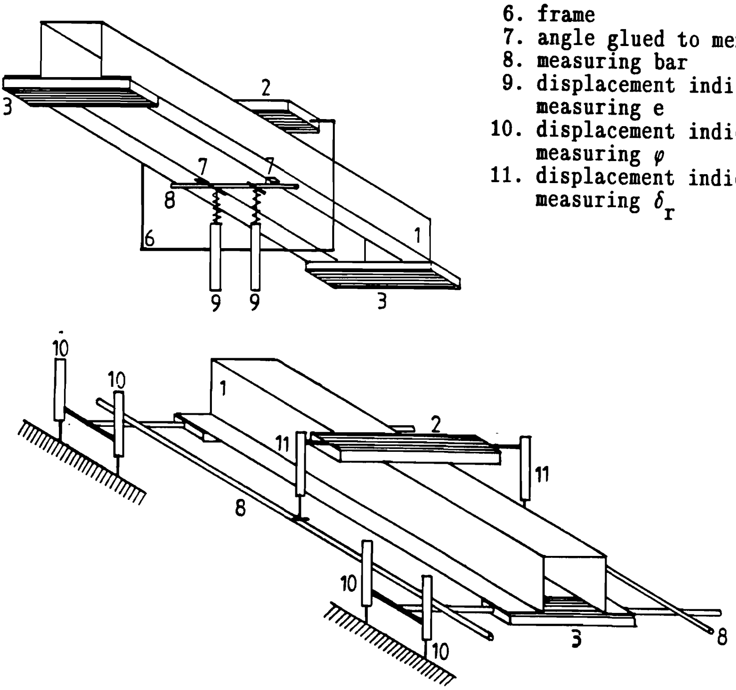


Figure 6. Idealized plastic hinge



1. test specimen
2. load bearing plate
3. reaction bearing plate
4. load cell
5. hydraulic jack

Figure 7. Test setup



6. frame
7. angle glued to member
8. measuring bar
9. displacement indicator measuring e
10. displacement indicator measuring φ
11. displacement indicator measuring δ_r

Figure 8. Measurement of deformations

Plastic deflection and rotation

In a web crippling or combined web crippling-bending test there are three different phenomena which may cause nonlinear load-deflection behavior:
1. partial plastification of the tension flange,
2. local buckling of web and flange elements,
3. web crippling.

In this paper the deflection caused by web crippling is called plastic deflection because it is caused by a kind of plastic hinge.
In the performed tests the plastic deflection was determined as the nonlinear component of the deflection (see Fig. 9). Calculations showed that for all test specimens yielding first occurred in the compression flange, so that plastification of the tension flange did not contribute to the nonlinear deflection. Local buckling of flange and web elements did occur, but the resulting nonlinear deflections were calculated to be negligible small compared to the measured nonlinear deflections.

Analogous to the plastic deflection, the plastic rotation was determined as the nonlinear component of the measured rotations. The plastic rotation is important in the determination of redistribution of bending moments in continuous, multi-span members.

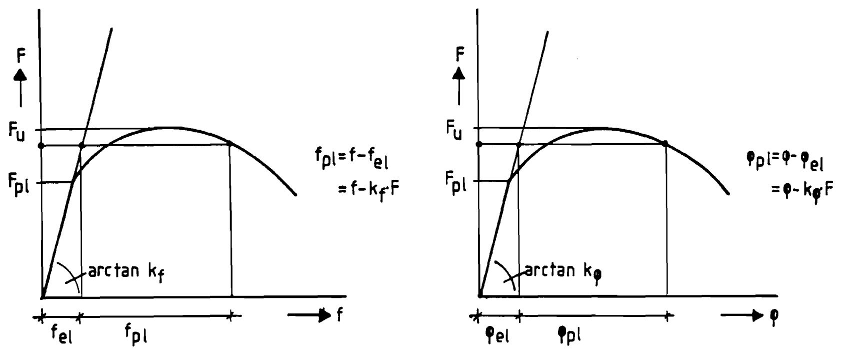


Figure 9. Determination of plastic deflection and rotation

Plastic limit load and ultimate limit load

Earlier in this paper the ultimate limit load was defined as the maximum load carrying capacity of the member, the plastic limit load as the load corresponding to the formation of a mechanism. In the tested hat sections a web crippling mechanism resulted in a plastic deflection of the member. It was therefore decided to define the plastic limit load as the load initiating a plastic deflection of the member (see Fig. 9).

Determining the yield line pattern

In the idealized hinge as shown in Fig. 6 both yield lines are assumed in the top flange and in the bottom flange of the member. The position of these yield lines can be determined (approximately) from the tests results.

Assuming that the plastic rotation is concentrated in two yield lines in the bottom flange the distance between these two yield lines can be calculated from the relation between the plastic deflection and the plastic rotation (corresponding to the same web crippling deformation e):

$$L_{yb}(e) = L_s - 2 \cdot f_{pl}(e) / \sin \varphi_{pl}(e)$$

In the tests it was found that the distance between the yield lines in the bottom flange did not change with increasing web crippling deformation. Therefore it was concluded that the yield lines in the bottom flange are stationary yield lines. So far no simple rule to predict the distance between these yield lines has been found.

Considering the plastic hinge in more detail, for every web crippling deformation the distance between the yield lines in the top flange can also be calculated from purely geometrical considerations. This calculation is based on the following assumptions:

- the length of the top and bottom flange do not change
- the bending deformations of the top flange and the bottom flange are concentrated in yield lines
- the inner yield lines in the top flange coincide with the edges of the load bearing plate.

In most tests it was found that this distance changed with increasing (web crippling) deformation (see for instance Fig. 14C). This means that the outer yield lines in the top flange are moving yield lines.

OBSERVED MECHANISMS

In the test two different mechanisms were observed, a yield arc mechanism occurring in members with a small corner radius, and a rolling mechanism occurring in members with a large corner radius. These mechanism were described in literature before, but the possibility to analyze them with generalized yield line theory was not recognized.

In Figs. 10, 11, 12 and 13 the deformations of the member for the rolling and yield arc mechanism are shown. In Fig. 14 typical graphs for both mechanisms are given. In Fig. 14A is a load-web crippling diagram, and in Fig. 14 B a load-plastic rotation diagram is given. The load F corresponds to a bending moment: $M = 1/4 \cdot F \cdot (L_s - L_{lb})$. Fig. 14B can therefore also be interpreted as a moment-plastic rotation diagram, as used to determine the redistribution of bending moments in continuous multi-span members. In Fig. 14C the web crippling deformation is shown as a function of the plastic rotation, and in Fig. 14D it is shown how the distance between the yield lines in the top flange changes with increasing web crippling deformation. It is believed that together these graphs give a good description of the web crippling behavior. From Fig. 14B one might conclude that for large plastic rotations the behavior of the rolling and yield arc mechanisms is identical. From the other graphs it can be seen however, that the deformation modes of these two mechanism are quite different.

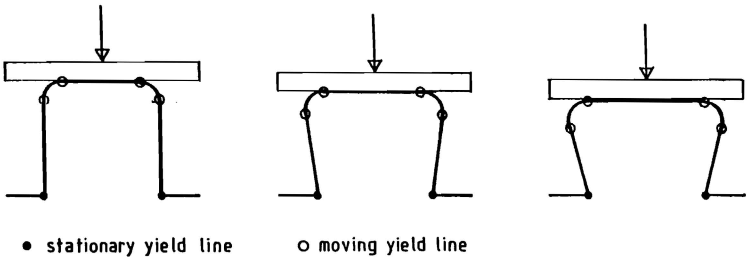


Figure 10. Web crippling deformation in rolling mechanism

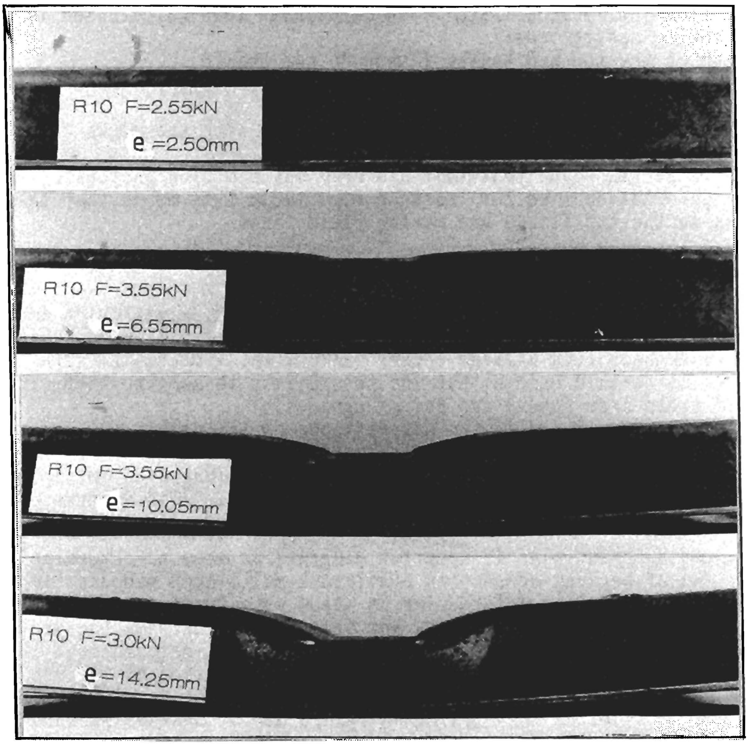


Figure 11. Deformation mode in rolling mechanism (test R10, see Fig. 14)

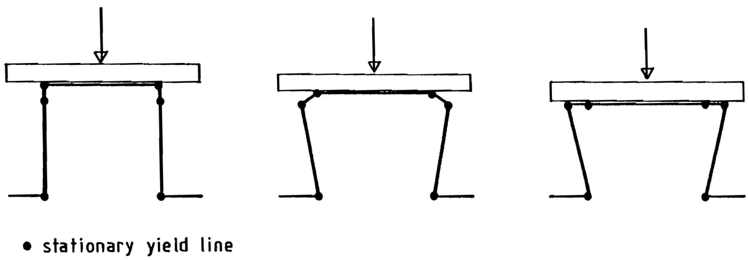


Figure 12. Web crippling deformation in yield arc mechanism

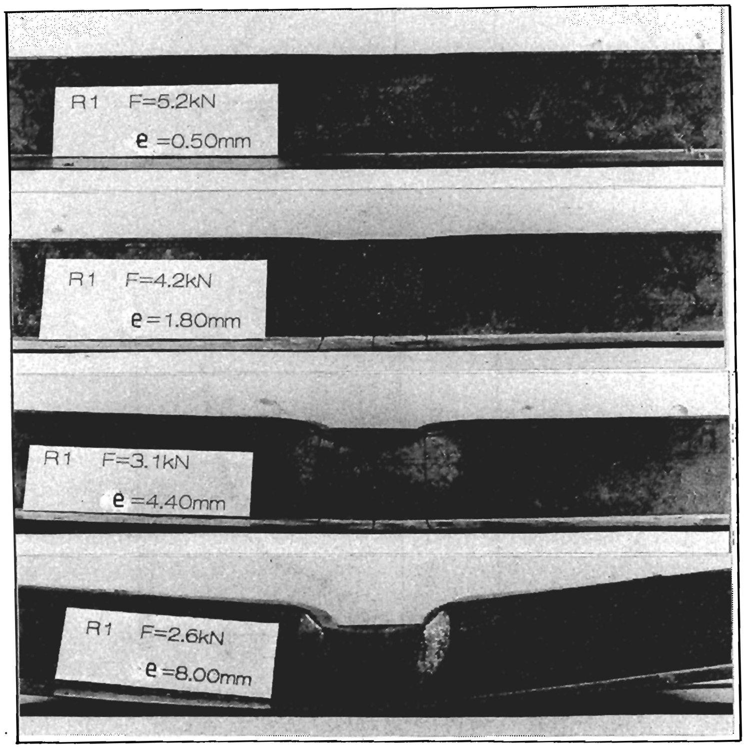
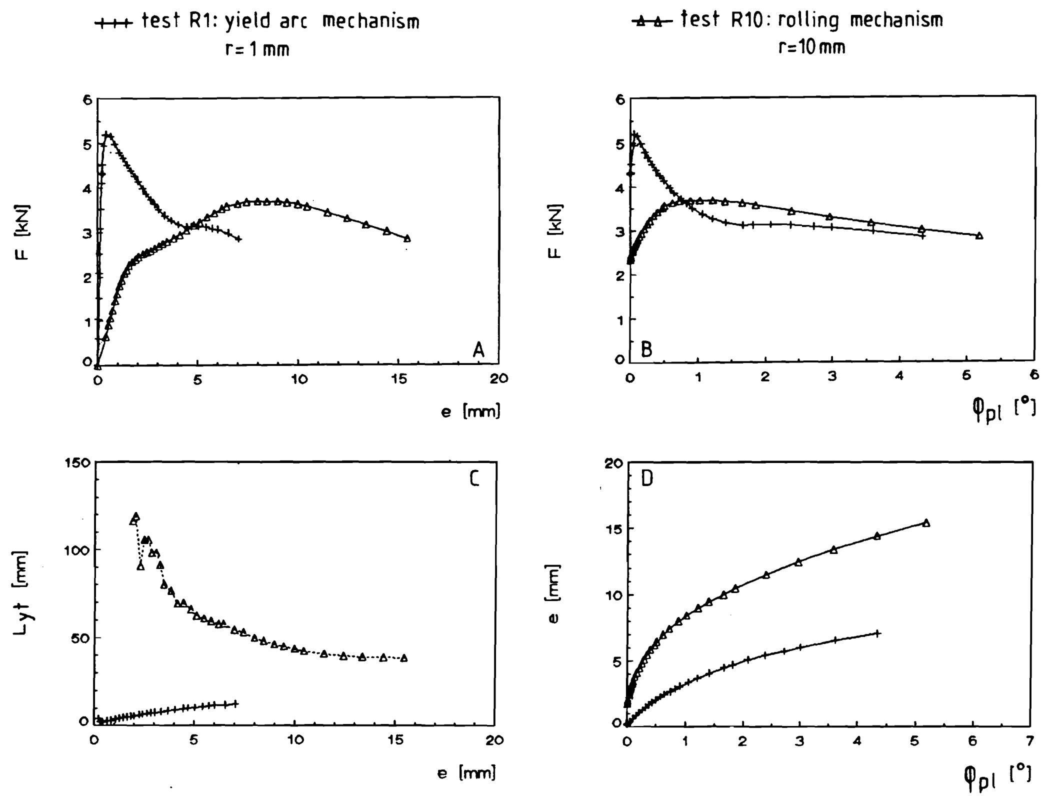


Figure 13. Deformation mode in yield arc mechanism (test R1, see Fig 14)



s_w=50 mm $b_t=b_b$=100 mm θ_w=90° — t=0.68 mm σ_y=327 N/mm² — L_s=290 mm L_{lb}=50 mm

Figure 14. Typical graphs for rolling and yield arc mechanism

Rolling mechanism

In the rolling mechanism (first described by Reinsch, 1983) the web crippling deformation is caused by a rolling process, in which the corner radius 'rolls down' through the web.(see Fig. 10). The rolling mechanism may be modeled by two moving yield lines: the first moving yield line bends the plate into a curvature and the second yield line straightens the plate again. As a result of the rolling process the contact point between the member and the load bearing plate moves to the edges of the load bearing plate.

The start of the rolling process seemed to coincide with a bend in the load- web crippling deformation curve (see Figs. 14A and 14B) and the initiation of the plastic deflection of the member. After the initiation of the plastic deflection the load steadily increased up to the ultimate limit load (attained for rather large web crippling deformations), and then slowly dropped. The distance between the yield lines in the top flange was found to decrease for increasing web crippling deformations (see Fig. 14C).

Yield arc mechanism

In the yield arc mechanism (first described by Rockey, Elgaaly and Bagchi, 1972) the web crippling deformation is caused by a yield arc (a curved yield line) in the web underneath the load bearing plate (see Fig. 15A). The development of the yield arc corresponded to the attainment of the ultimate load (at relatively small web crippling deformations) and the initiation of a plastic deflection. After the attainment of the ultimate limit load the load suddenly dropped. The distance between the yield lines in the top flange was found to increase with increasing web crippling deformations (see Fig. 14C).

For large web crippling deformations, when the web underneath the load bearing plate almost contacted the load bearing plate (see Fig. 12) the deformation process began to resemble that of the rolling mechanism. This explains why for large deformations the deformation patterns of the yield arc and the rolling mechanism look very similar.

Curved yield lines are a familiar phenomenon in thin-walled steel members (see for instance the the flip-disc mechanism described by Murray and Khoo, 1981), and can be analyzed with generalized yield line methods.

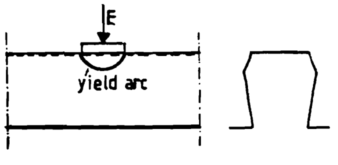


Figure 15A. Yield arc in cold-formed member

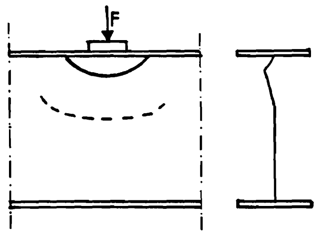


Figure 15B. Yield arc in plate girder

DISCUSSION

Parameters determining the failure mechanism

In the tests the occurrence of either the rolling mechanism or the yield arc mechanism seemed to be governed by the corner radius. This may be made plausible by regarding the corner radius as a parameter influencing the eccentricity of the load application to the web.

In rectangular hollow sections joints too, different mechanisms are encountered for different eccentricities, the eccentricity in this case being defined as the chord width to branch width ratio. Kato and Nishiyama (1984) for example, showed that in a T-joint of rectangular tubes three different failure mechanism in the chord member may occur, a web crippling failure for small eccentricities, a chord flange failure for large eccentricities and a combined web crippling-chord flange failure mechanism for medium eccentricities (see Fig. 16).

In plate girders subjected to concentrated loads, where the eccentricity of the load application to the web is very small, yield arc mechanisms similar to those in cold-formed members are observed (Rockey, 1977). In plate girders the width of the yield arc is larger than the length of the load bearing plate, due to the larger stiffness of the flange (see Fig. 15B).

Santaputra, Parks and Yu (1989) also distinguished two different types of failure mechanisms in their tests, an overstressing failure and a web buckling failure. It is believed that these mechanisms are identical to the rolling mechanism and the yield arc mechanism respectively. They derived different (empirical) web crippling prediction formulas for these two mechanisms, but did not comment explicitly on the parameters determining the failure mechanism.

Influence of bending moment

To explain the influence of the bending moment a simplified yield line analysis is given. In yield line analysis the load corresponding to a specific deformation state of a mechanism can be determined by equating the internal rate of energy dissipation in the yield lines to the rate of external work by the applied forces. In the web crippling mechanism the rate of external work is given by (see Fig. 6):

$$A_e = F \cdot \dot{e} + M \cdot \dot{\varphi}_{pl},$$

where $\dot{e}$ and $\dot{\varphi}_{pl}$ are the web crippling rate and rotation rate, and M is the bending moment acting at the place where the load is applied on the member. As explained before, there is a relation between the web crippling deformation e and the plastic rotation, depending on the distances between the yield lines in the top and bottom flange (see for instance Fig. 14C). Assuming that this relation is described by the function $\varphi_{pl} = g(e)$, the plastic rotation rate can be calculated from the plastic rotation as:

$$\dot{\varphi}_{pl} = \frac{\partial g(e)}{\partial e} \cdot \dot{e}$$

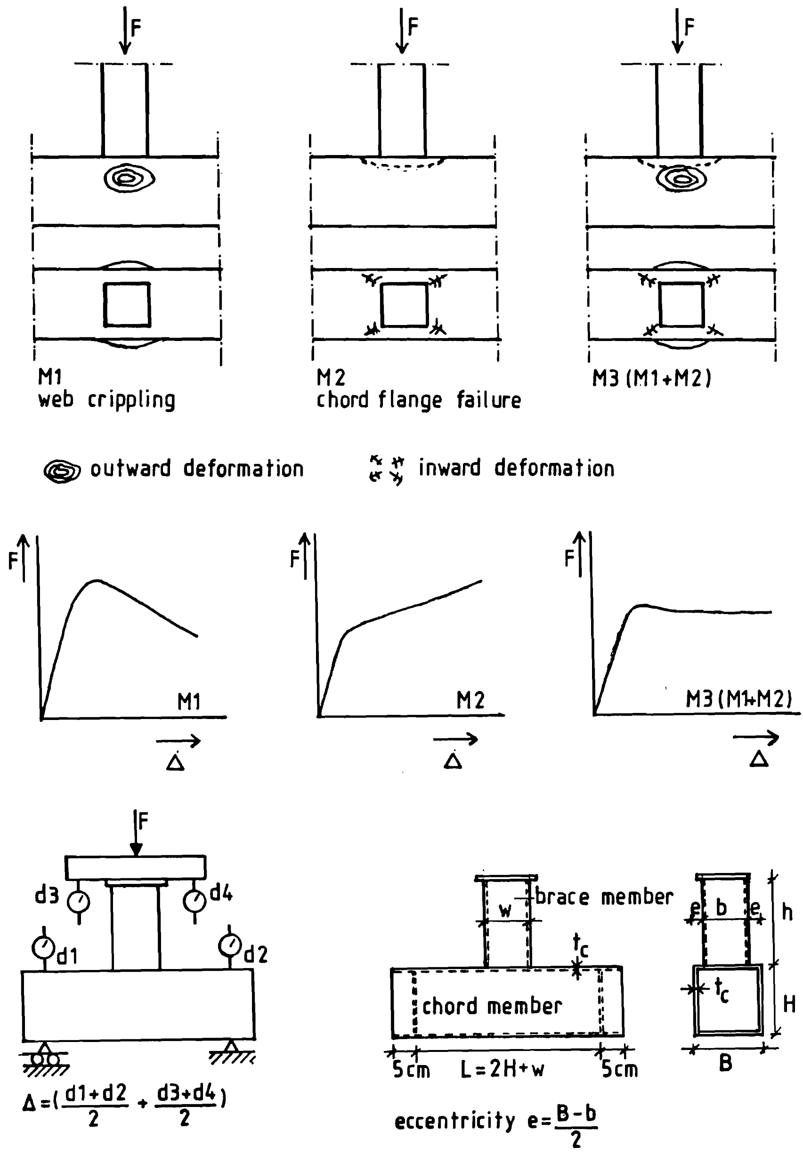


Figure 16. Failure mechanisms in rectangular hollow section joints

The internal rate of energy dissipation A_i in the yield lines also depends on the web crippling deformation e. Assuming that $A_i = h(e) \cdot \dot{e}$ and equating A_i to A_e the load on the member can be calculated as:

$$F(e) = h(e) - M \cdot \frac{\partial g(e)}{\partial e}$$

In this formula for the load on the member three influences of the bending moment can be recognized:

1. The direct influence of M on the external work: $M \cdot \dot{\varphi}_{pl}$,
2. The direct influence of M on the internal energy dissipation $h(e) \cdot \dot{e}$: the stresses caused by the bending moment result in a reduction of the plastic moment acting in the yield lines, and hence in a reduced energy dissipation in the yield lines,
3. The indirect influence of M on the external and internal work, that is, the influence of M on the form of the yield line pattern. This influence can be regarded as a kind of second order effect.

The analysis of the combined web crippling and bending behavior would be greatly simplified if the second order effect could be neglected, that is, if the form of the yield line pattern (and hence the function g(e)) were independent of the bending moment. The models by Reinsch (1983) and Tsai (1987) are based on this assumption. In the tests this assumption was not confirmed: for a larger span length (larger bending moment) the same web crippling deformation resulted in a larger plastic rotation.

CONCLUSIONS

1. Hat-sections subjected to a concentrated load may fail by either a rolling mechanism or a yield arc mechanism. What type of mechanism occurs is determined primarily by the bending radius.
2. Attempts to analyze the rolling and yield arc mechanism with generalized yield line theory are currently carried out. The results of the analysis will be used to develop more reliable web crippling prediction formulas. This will probably result in different formulas for the two mechanisms, and different formulas accounting for the influence of the bending moment (as was done by Santaputra, Parks and Yu, 1989).
3. In cold-formed steel members the plastic rotation is an important factor influencing the combined bending web-crippling behavior. In the web crippling model the load for every web crippling deformation can only be predicted accurately, if also the plastic rotation can be predicted accurately. The web crippling model can therefore also be used to determine the redistribution of bending moments in continuous, multi-span members.
4. The current web crippling formulas predict the ultimate limit load. Another approach worth considering would be to predict the plastic limit load (as for instance is done in the chord flange failure of rectangular hollow sections).

ACKNOWLEDGMENTS

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"Ein Beitrag zur Ermittlung der Traglast von querbelasteten Durchlaufträgern mit dünnwandigen Querschnitt, insbesondere von durchlaufenden Trapezblechen für Dach und Geschossdecken."
Der Stahlbau, Vol. 42, No.1, pp 20-24.

APPENDIX II. NOTATION

b_b	width of bottom flange
b_t	width of top flange
e	web crippling deformation
f	deflection
f_{pl}	plastic deflection
F	force
L_{lb}	length of load bearing plate
L_s	span length
L_{yb}	distance between yield lines in bottom flange
L_{yt}	distance between yield lines in top flange
M	bending moment
s_w	height of web
r	inside corner radius
t	thickness of plate
θ_w	angle of web inclination
φ	rotation
φ_{pl}	plastic rotation
σ_y	yield stress