01 Jun 2007

Flatness-Based Feedback Linearization of a Synchronous Machine Model with Static Excitation and Fast Turbine Valving

E. C. Anene

U. O. Aliyu

J. Levine

Ganesh K. Venayagamoorthy
Missouri University of Science and Technology

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E.C. Anene, U.O. Aliyu, *Member, IEEE*, J. Lévine and G. K. Venayagamoorthy, *Senior Member, IEEE*

***Abstract*-- This paper focuses attention on the concept of flatness-based feedback linearization and its practical application to the design of an optimal transient controller for a synchronous machine. The feedback linearization scheme of interest requires the generation of a flat output from which the feedback control law can easily be designed. Thus the computation of the flat output for reduced order model of the synchronous machine with simplified turbine dynamics is hereby presented. The corresponding linearized compensator is derived as well as the nonlinear controller for transient stabilization of a synchronous machine subjected to large disturbances. The transient behavior of a single machine equipped with the so designed nonlinear controllers feeding an infinite bus is illustrated via simulation in Matlab environment. The results obtained for transient disturbances on the single machine infinite bus system (SMIBS) are presented and compared with other control algorithms to demonstrate the effectiveness of the proposed scheme.**

***Index Terms*-- Feedback linearization, flatness, flat output, transient stability.**

I. Introduction

The need on the one hand to achieve better control objectives on the synchronous machine and its ancillary devices and on the other hand making them practically implementable, in view of limitations in observable outputs, has been a motivating factor for research effort towards more efficient control strategies. Anderson and Fouad, [1] identified three principal controls that directly affect a synchronous generator: the boiler control as well as turbine/governor and exciter controls. However, turbine speed governing, and excitation controls, both play very important roles in dynamic stability studies. The governor controls the torque or the shaft power input by a speed controller, which adjusts the steam turbine valve, and the excitation system controls the field voltage. Modern large power generating units in a complex system cannot rapidly respond to increasing demand and complex system interactions. Consequently, the problem of ensuring at all times balance between mechanical and electrical powers on the shaft of a unit turbine-generator is one of the most important challenges for power system/ control engineers [2]. Presently much work has been done in the use of various control strategies for the stability studies of the synchronous machine. Notable among them include feedback linearization schemes, optimal control, neural networks etc. Many authors [2], [3-8] have applied input-state and input-output feedback linearization schemes for SISO and MIMO systems to the synchronous machine model with good results.

Flatness-Based feedback linearization also has received a lot of attention resulting in the earlier work reported by many researchers [9-16]. Although this technique has been applied to several nonlinear and linear mechanical systems [10-14], its application to synchronous machines is yet to gain ground. The input–output scheme requires the arbitrary choice of an output whose relationship with the input is determined through differentiation leading to the problem of stability of the associated internal dynamics. But the flatness-based approach uses the characterization of system dynamics to generate a suitable output. In a situation where the output does not have a physical meaning or interpretation, the linearization could be done through a measurable system component that has a relationship to it.

This concept is herein applied to a single machine model to facilitate the design of its feedback controller. For the purpose of illustrating the proposed control algorithm, single machine infinite bus system is simulated in the Matlab environment. The simulation results for large transient disturbances at the terminal of the synchronous machine of the sample power system are presented and compared with the conventional control techniques.

This paper comprises five sections starting with introduction in Section I. Section II discusses the flat output computations and its verification. In Section III the Linearized compensator is generated from the flat output whilst Section IV presents the design of the controller for transient stabilization of the one machine infinite bus system example. Simulation results are presented in Section V and conclusions made in Section VI. References and Appendices are given at the end.

This work was partially supported by the French Embassy in Nigeria and the NSF INT # 0322894.

E. C. Anene and U. O. Aliyu are with the Electrical Engineering Programme, Abubakar Tafawa Balewa University, Bauchi-Nigeria.

J. Lévine is with Centre Automatique et Systèmes, Ecole des Mines de Paris, 35 rue Saint Honoré, F-77305 Fontainebleau-France.

G. K. Venayagamoorthy is with Real-Time and Intelligent Systems Laboratory, Department of Electrical and Computer Engineering University of Missouri-Rolla, MO 65409, USA.

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II. DYNAMICS AND FLAT OUTPUT FOR THE SYSTEM UNDER STUDY

The simplified model of the governor/steam turbine considered in this work follows from [17] and Fig. 1 shows the machine configuration connected to the infinite bus.

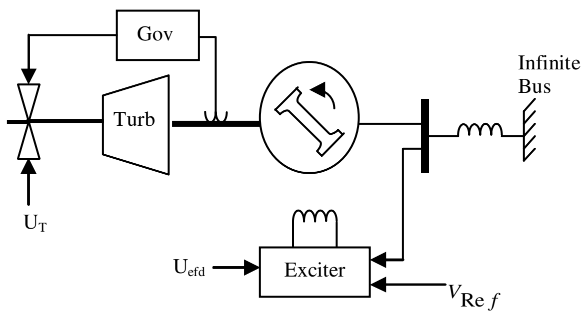

Fig. 1: Synchronous Machine connected to the infinite bus.

The state equations of a synchronous machine including its turbine dynamics only are given by:

$$\tau_{d0}\dot{e}_q^{'} = u_{efd} - e_q^{'} - (x_d - x_d^{'})i_d$$
$$\frac{2H}{w_R}\frac{d^2\delta}{dt^2} = P_m - D(\omega - \omega_0) - e_d^{'}i_d - e_q^{'}i_q$$
$$\dot{\delta} = \omega - \omega_0 \qquad (1)$$
$$\dot{P}_{gv} = \frac{1}{\tau_g}(u_T - P_{gv} - (\omega - \omega_0)/R_T)$$
$$\dot{P}_m = \frac{1}{\tau_t}(P_{gv} - P_m)$$

Where u_{efd}-excitation control, u_T-Turbine valve control, and for a machine connected to an infinite bus i_d *and* i_q are given respectively by:

$$i_d = (1/k_z)(-(r_a + R_e)(e_d^{'} - V_\infty \sin\delta) + (x_q^{'} + x_e)(e_q^{'} - V_\infty \cos\delta))$$
$$i_q = (1/k_z)(-(x_d^{'} + x_e)(e_d^{'} - V_\infty \sin\delta) + (r_a + R_e)(e_q^{'} - V_\infty \cos\delta))$$

Where $k_z = 1/(r_a + R_e)^2 + (x_d^{'} + x_e)(x_q + x_e)$.

Note that static excitation system is assumed and its dynamics neglected due to small time constants compared with the transient time frame. It is shown in the Appendix that this reduced order synchronous machine model with turbine dynamics possesses a flat output which is a function of load angle δ which can be computed via a network load flow, and mechanical power p_m which can be measured. The practical significance of the flat output of a dynamical system, which is shown in the next section, is that if the components of the flat output are measurable, then all the system variables required for feedback can be directly computed without integrating any differential equations.

Verification of the Flat Output for a Synchronous Machine Plus Turbine Model.

It is necessary to verify the flat output by showing that all system variables of (1) are functions of the flat output $Y = \gamma(\delta, p_m)$ and its derivatives. For the first output δ, it is seen that $\omega = \dot{\delta} - \omega_0$; $\dot{\omega} = \ddot{\delta}$ and from the quadratic equation (2) below $e_q^{'}$ can easily be evaluated.

$$\frac{1}{k_z}R_{eT}\, e_q^{'2} - \frac{1}{k_z}((x_{dt} - x_{qt})e_d^{'} - V_\infty(-x_{dt}\sin\delta - R_{eT}\cos\delta))e_q^{'} - P_m \qquad (2)$$
$$+ D(\omega - \omega_0) + \frac{e_d^{'}}{k_z}(R_{eT}\, e_d^{'} + V_\infty(-R_{eT}\sin\delta - x_{qt}\cos\delta)) + \frac{2H}{\omega_0}\ddot{\delta} = 0$$

$R_{eT} = (r_a + R_e)$; $x_{dt} = (x_d^{'} + x_e)$; $x_{qt} = (x_q^{'} + x_e)$.

It can be shown that other system variables and system components which are functions of the system variables are also functions of the flat output and its derivatives:

$$i_d = f_1(\delta,\dot{\delta},\ddot{\delta});\, i_q = f_2(\delta,\dot{\delta},\ddot{\delta});\, v_{dt} = f_3(\delta,\dot{\delta},\ddot{\delta});$$
$$v_{qt} = f_4(\delta,\dot{\delta},\ddot{\delta});\quad V_t = f_5(\delta,\dot{\delta},\ddot{\delta})\ ; \qquad (3)$$

For the second flat output p_m, it can be written that

$$p_{gv} = \tau_t \dot{p}_m + p_m \qquad (4)$$

II. LINEARIZED COMPENSATOR FOR NONLINEAR SYSTEMS

Fliess [9] has shown that for dynamical systems, the flat output gives us the framework to derive the endogenous feedback compensators, such that for a nonlinear system,

$$\dot{x} = f(x,u),\ x \in R^n, u \in R^m \qquad (5)$$

where $F(0,0) = 0$ and rank $\frac{\partial F}{\partial u}(0,0) = m$. Its feedback linearization means the existence of:

1) A compensator of the form:

$$\dot{z} = a(x,z,v), z \in R^q$$
$$u = b(x,z,v), v \in R^m \qquad (6)$$

Where $a(0,0,0) = 0$, $b(0,0,0) = 0$;

2) A diffeomorphism $\xi = \Xi(x,z), (\xi \in R^{n+q})$ that yields a $(n+q)$ dimensional dynamics given by:

$$\dot{x} = f(x, b(x,z,v))$$
$$\dot{z} = \beta(x,z,v) \qquad (7)$$
$$u = \alpha(x,z,v)$$

This then results in a constant linear controllable system of the form: $\dot{\xi} = F\xi + Gv$. From the foregoing, it can be shown that the reduced third order model synchronous machine plus turbine dynamics possess a multivariable compensator.

Design of Compensator for the Synchronous Machine Model.

It follows from the verification of Section II that the components of u_{efd}, u_T and $\bar{x} = F(\delta, \omega, e_q^{'}, p_{gv}, p_m)$ can be expressed as real-analytic functions of δ, *and* p_m and a finite number of their derivatives $\bar{x} = F(\delta, \dot{\delta}, \ddot{\delta}, u_{efd}, p_m, \dot{p}_m, u_T)$. It is also verified that

$u_{efd} = F_u(\delta, \dot{\delta}, \ddot{\delta});\ and\ \ u_T = F_T(p_m, \dot{p}_m)$. The feedback is shown to be endogenous since the converse hold and since y is expressed as a real-analytic function of $\delta\ and\ p_m$ -two system variables. Thus the state variables of (1) are a function of the linearizing output Y and its derivatives up to order $\alpha_1 = 2; \alpha_2 = 1$. The endogenous feedback system to the following closed loop system will be of the order $\alpha_1 + 1 = 3; \alpha_2 + 1 = 2$. So that from the linear system $\dddot{\delta} = v_1;\ \ddot{p}_m = v_2$ the compensator is derived as we perform the following state transformations yielding the equivalent normal form for the system, and from which we can compute the nonlinear controller by inverting the expressions of $\ddot{\omega}$ and u_{efd} ; $\ddot{p}_m$ and u_T .

$$\dot{z}_{11} = z_{12} = \dot{y}_1 = \dot{\delta} = \omega - \omega_0$$
$$\dot{z}_{12} = z_{13} = \ddot{y}_1 = \ddot{\delta} = \dot{\omega}$$
$$\dot{z}_{13} = \dddot{y}_1 = \dddot{\delta} = \ddot{\omega} = v_1$$
$$\dot{z}_{21} = z_{22} = \dot{y}_2 = \dot{p}_m$$
$$\dot{z}_{22} = \ddot{y}_2 = \ddot{p}_m = v_2$$

The state transformations are invertible and exist throughout the domain of stable operation $0 < \delta < 180^o$. Using the network of Fig. 1 the resulting excitation control is given by:

$$u_{efd} = \frac{\tau_{d0}}{E}\left(\frac{2H(v_1)}{k_z\omega_0} + \frac{D\dot{\omega}}{k_z} + \dot{p}_m + A\dot{e}_d^{'} + Be_d^{'} - Ce_q^{'}\right) + e_q^{'} + (x_d - x_d^{'})i_d \quad (8)$$

Where, $A = 2R_{eT}\dot{e}_d^{'} - R_{eT}V_\infty \sin\delta - x_{qt}V_\infty \cos\delta;$

$B = x_{qt}V_\infty \sin(\delta)\dot{\delta} - R_{eT}V_\infty \cos(\delta)\dot{\delta};$

$C = (x_{dt} - x_{qt})\dot{e}_d^{'} - x_{dt}V_\infty \cos(\delta)\dot{\delta} - R_{eT}V_\infty \sin(\delta)\dot{\delta};$

$E = (x_{dt} - x_{qt})e_d^{'} - x_{dt}V_\infty \sin\delta - 2R_{eT}e_q^{'} + R_{eT}V_\infty \cos\delta;$

$\dot{e}_d^{'} = \frac{1}{k_z}((x_q - x_d^{'}) + x_{dt})(x_{dt}\cos(\delta) + R_{eT}V_\infty \sin(\delta)\dot{\delta}) + R_{eT}e_q^{'};$

and the resulting Turbine valve control is given by:

$$u_T = (\tau_t v_2 + \frac{1}{\tau_t}(p_{gv} - p_m))\tau_g + \frac{(\omega - \omega_0)}{R_T} + p_{gv} \quad (9)$$

The loop closure is then done to stabilize the set point. It suffices to use (8) with $v_1 = -k_{11}(\delta - \delta_1) - k_{12}(\dot{\delta} - \dot{\delta}_1) - k_{13}(\ddot{\delta} - \ddot{\delta}_1)$, and (9) with $v_2 = -k_{21}(p_m - p_{m1}) - k_{22}(\dot{p}_m - \dot{p}_{m1})$ and to choose k_{ij} such that the linear time invariant error dynamics $e_1^{(3)} = k_{11}e + k_{12}\dot{e} + k_{13}\ddot{e}\ \ and\ \ e_2^{(2)} = k_{21}e_{21} + k_{21}\dot{e}_{21}$, where $e_i^{(j)} = y_i^{(j)} - (y_i^*)^{(j)}$ are stable. The choice of k_{ij} influences the response of the linear equivalent based on the well-known techniques for compensation of linear systems. A well-behaved system model results in adequate cancellation of the system nonlinearities, leaving only the linear (PID) equivalent to ensure system stabilization under perturbed conditions. Controller limitation may therefore become obvious when it is computed with incomplete system models such that the cancellation of the nonlinearities become inadequate and a source of instability. For synchronous machines this effect may not be too pronounced because of the stabilizing effects of system dampers.

III. System Simulations

Figure 2 summarizes the simulation block diagram implemented in the Matlab version 6.5. In all simulations, the constraints on equations (8) and (9) that is, the admissible values of the field voltage and steam valve position are included. It is assumed the generator terminals were connected to the infinite bus via a transformer and a tie line consisting of a resistance R_e and inductance X_e as shown in the single line diagram of Fig. 3.

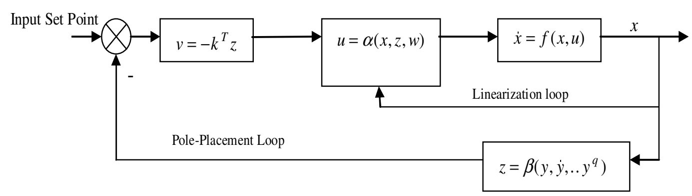

Fig. 2. Block diagram used for Simulation.

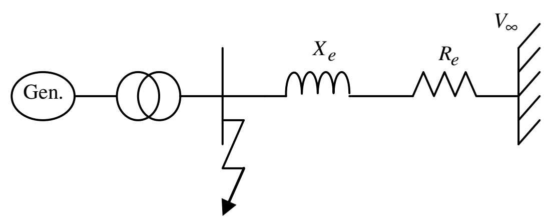

Fig. 3: 3-Φ Fault location on the SMIBS.

A three-phase short circuit fault was simulated for the study in three time periods:

- The steady state operation from 0.0 to 1.0 seconds.
- Three phase fault at transformer terminals from 1.0 sec to 1.06 seconds.
- Post fault stabilization from 1.06 to 6seconds.

It is also assumed that the parameters $X_e \& R_e$ and others used to compute the controllers remained constant during the fault transient period. Data used to initialize the system is given in the appendix.

IV. Simulation Results

The simulation results for the feedback linearized synchronous machine equipped with turbine dynamics (or multivariable dynamics feedback controller (MDFC)) under a bolted three-phase fault for a period of three cycles are presented in Figs. 4 to 8. Comparisons are offered between the proposed technique and conventional fast-valving (FV) / forced-excitation (FE) control as well as conventional Automatic Voltage Regulator (AVR) and speed regulation.

The FV/FE problem was formulated as nonlinear optimal control problem and its open loop solution obtained via multiple shooting algorithm as detailed in [17].

The MDFC trajectory at 3-cycle fault duration damps out the fault oscillations in about 2.5 seconds similar to that reported in literature for the non linear field voltage controller under similar generator data [2]. Cases reported by [22] using model predictive controller scheme, shows longer damping time of about 4 seconds.

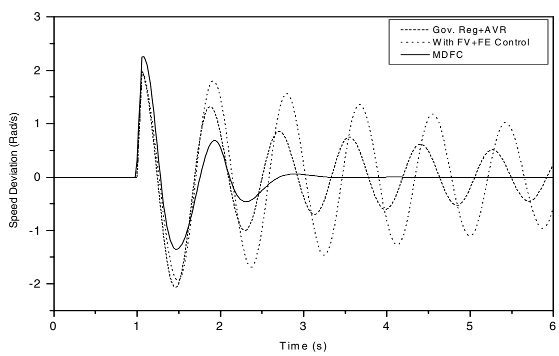

Fig. 4: Machine speed deviation response to 3-cycle fault for MDFC and FV/FE and AVR/Speed Reg..

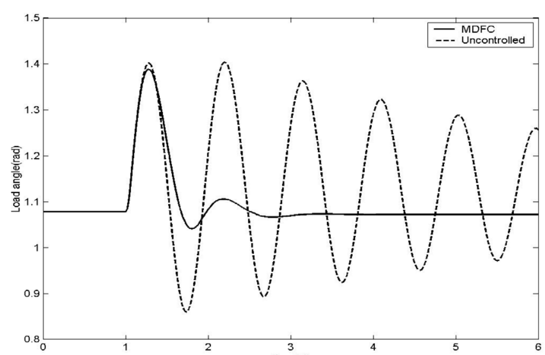

Fig. 5: MDFC Machine load angle response to 3- cycle fault.

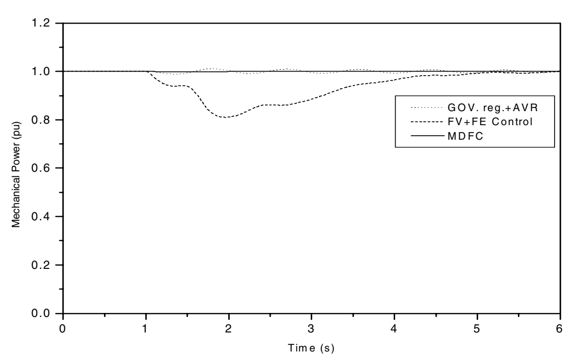

Fig. 6: Machine mechanical power response to 3-cycle fault for MDFC and FV/FE and AVR/Speed Reg..

Figure 7 compares the system with MDFC and the system with FV/FE and AVR/Speed Reg. It shows that the controller not only assures stabilization of system variables but also stabilizes system output voltage. It restores all the variables as well as the voltage to pre fault equilibrium. The field voltage under fault and post fault trajectory stabilization is shown in figure 8 for 3-cycle fault duration. It is noteworthy in the study that voltage ceiling or u_{efd} -limits, influences the dynamics of the control effort. Operating the system at high limits speeds up the system post fault stabilization but that will result in physically high dynamical stresses in the exciter and this requires a compromise solution.

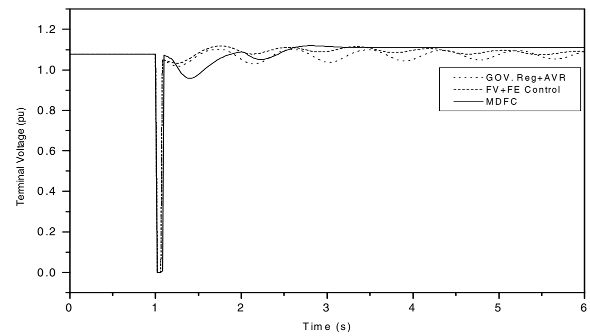

Fig. 7: Machine Terminal Voltage response to 3-cycle fault fault for MDFC and FV/FE and AVR/Speed Reg...

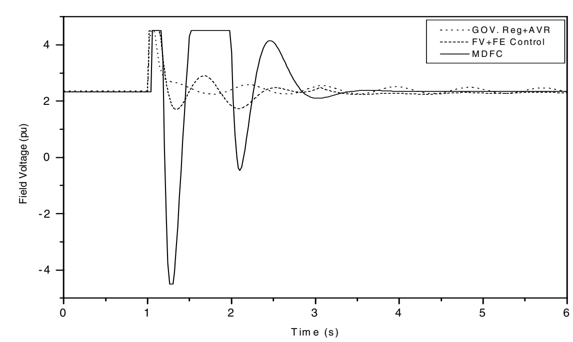

Fig. 8: Machine Excitation Control for a 3-cycle fault for MDFC and FV/FE and AVR/Speed Reg..

The field voltage control highly dominates the response trajectories of the synchronous machine during faults but the effect of turbine control is to help damp those minor oscillations that are capable of building up in the event of major disturbances. Figure 9 shows the MDFC Load Angle with and without turbine valve controller for fault duration of 9 cycles. Detailed inspection shows the trajectory without turbine control as having sustained swings. Figure 10 show results for the multivariable dynamics feedback controller (MDFC) under an eleven-cycle fault condition but similar controller gains.

The controller is seen to restore the system to equilibrium in about 5 seconds while the uncontrolled system and that without Turbine valve controller becomes unstable. This gives a good indication of the robustness of the controller. The MDFC damps the Load angle and the speed oscillations as well as stabilizes the terminal voltage to its equilibrium position.

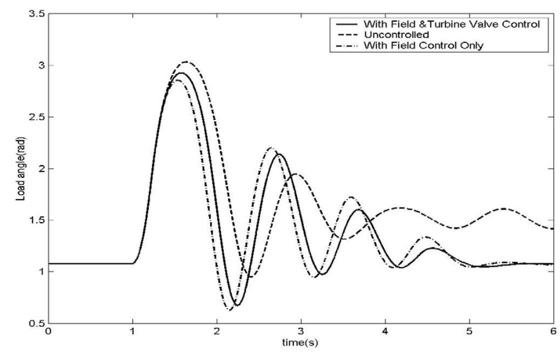

Fig 9: MDFC Load Angle with and without Turbine Control.

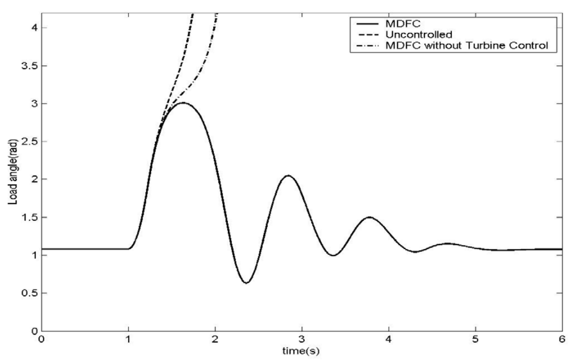

Fig.10: MDFC Machine Load angle response to 11- cycle fault.

V. CONCLUSIONS

The theory of Flatness-based feedback linearization has been successfully applied to the multivariable fifth order model of single machine infinite bus system. It was shown that for this model, a flat output $Y = \gamma(\delta, p_m)$ exists which is a function of the system state variables. The corresponding diffeomorphism led to the compensator. A nonlinear controller was subsequently designed. Simulation studies carried out showed that the nonlinear controller achieved asymptotic stabilization of the sample power system when subjected to fault conditions. The multivariable scheme based on combined stabilizing actions of excitation and fast turbine control returned better transient performance than the single variable excitation scheme usually considered in the existing literature. It was also found to be robust to fault durations and parameter variations. The ultimate goal being pursued is to extend the control design approach to multi-machine system based on decentralized control actions for system-wide transient stabilization.

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VII. APPENDIX A

FLAT OUTPUT COMPUTATION

Equation (1) admits an implicit representation without controls given by:

$$\dot{\delta} - \omega + \omega_0 = 0$$

$$\frac{2H}{w_R}\frac{d^2\delta}{dt^2} - P_m + D(\omega - \omega_0) + e_d^{'} i_d + e_q^{'} i_q = 0 \quad (10)$$

$$\dot{P}_m - \frac{1}{\tau_t}(P_{gv} - P_m) = 0 \cdot$$

The cotangent approximation of (10) as outlined in [15] becomes:

$$\begin{pmatrix} \frac{d}{dt} & -1 & 0 & 0 & 0 \\ b_{21} & D + \frac{2H}{\omega_0}\frac{d}{dt} & b_{23} & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{\tau_t} & \frac{1}{\tau_t} + \frac{d}{dt} \end{pmatrix} \quad (11)$$

Where

$$b_{21} = k_z V_\infty \dot{\delta}(e_d^{'}(-R_e\cos\delta + X_{qt}\sin\delta) + e_q^{'}(X_{dt}\cos\delta + R_e\sin\delta))$$

$$b_{23} = k_z e_d^{'}(X_{qt} - X_{dt}) + 2R_e k_z e_q^{'} + k_z V_\infty(-R_e\cos\delta + X_{dt}\sin\delta)$$

The Smith matrix manipulations will generate a hyper-regular matrix given by:

$$P(F) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -\frac{d}{dt} + \frac{2H}{\omega_0}D & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad (12)$$

The resulting R-Smith computations of the generated unimodular matrices yield:

$$U = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & \frac{d}{dt} \\ 0 & \frac{1}{b_{23}} & 0 & -\frac{1}{b_{23}} & -\frac{1}{b_{23}}(D + \frac{2H}{\omega_0}\frac{d}{dt})(-\frac{d}{dt}) - b_{21} \\ 0 & 0 & -\tau_t & \tau_t(\frac{1}{\tau_t} + \frac{d}{dt}) & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

(13)

from which $\hat{U} = U\begin{pmatrix} 0_{3,2} \\ I_2 \end{pmatrix}$, or

$$\hat{U} = \begin{pmatrix} 0 & 1 \\ 0 & \frac{d}{dt} \\ -\frac{1}{b_{23}} & -\frac{1}{b_{23}}(D + \frac{2H}{\omega_0}\frac{d}{dt})(-\frac{d}{dt}) - b_{21} \\ \tau_t(\frac{1}{\tau_t} + \frac{d}{dt}) & 0 \\ 1 & 0 \end{pmatrix} \quad (14)$$

We can then compute Q by L-Smith manipulation of (14) such that

$$Q\hat{U} = \begin{pmatrix} I_2 \\ 0_{3,2} \end{pmatrix}$$

$$Q = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ \frac{1}{b_{23}}(D + \frac{2H}{\omega_0}\frac{d}{dt})(-\frac{d}{dt}) - b_{21} & 0 & 1 & 0 & \frac{1}{b_{23}} \\ 0 & 0 & 0 & 1 & -\tau_t(\frac{1}{\tau_t} + \frac{d}{dt}) \\ -\frac{d}{dt} & 1 & 0 & 0 & 0 \end{pmatrix} \quad (15)$$

Operating Q on the vector $\begin{pmatrix} d\delta & d\omega & de_q^{'} & p_{gv} & p_m \end{pmatrix}^{'}$, yield the last three equations to be:

$$-\frac{d}{dt}\delta + d\omega = 0 \quad (16)$$

$$\frac{1}{\tau_t}dp_{gv} - \frac{1}{\tau_t}dp_m + d\dot{p}_m = 0 \quad (17)$$

$$-D\omega - \frac{2H}{\omega_0}\dot{\omega} + b_{21}d\delta + b_{23}de_q^{'} + dp_m = 0 \quad (18)$$

This identically vanishes on X_0. The remaining part of the system given by:

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} d\delta & d\omega & de_q^{'} & p_{gv} & p_m \end{pmatrix}^{'} = \begin{pmatrix} dy_1 \\ dy_2 \end{pmatrix} \quad (19)$$

is trivially strongly closed with $M = I_2$, and yields a flat output $y_1 = \delta$ and $y_2 = p_m$.

VIII. Appendix B

Synchronous Machine Parameters:

$\omega_0 = 314.159\, rad/s;\ D = 0.002\, pu;\ H = 4.74 \sec s;$

$\tau_{d0} = 5.9 \sec s;\ \tau_{q0} = 0.075 \sec s$

$x_d = 1.7\, pu;\ x_q = 1.64\ pu;\ r_a = 0.001096\, pu;$

$x_d^{'} = 0.245\, pu;\ x_q^{'} = x_d^{'}$

The operating point of the system was determined by the following initial operating conditions of the generator: $P = 1.0\, pu;\ pf = 1.0\, pu; V_\infty = 1.0\, pu \cdot$

The network parameters are given by:

$Xe = 0.4\ pu;\ \mathrm{Re} = 0.02\ pu$

The control limits of the field voltage system are given by: $e_{fd\max} = 4.5\ pu;\ e_{fd\min} = -4.5\ pu \cdot$

Turbine Model Parameters:

$P^0 = 1.0\, pu;\ \tau_g = 0.2\, s;\ \tau_t = 0.5\, s;\ R_T = 20\, pu \cdot$ The limits of the fast turbine control are given by:

$p_{gv\max} = 1.05\, pu;\ p_{gv\min} = 0.0 \cdot$

The gain coefficients used to close the loop are given by

$k_{11} = 400; k_{12} = 95.14; k_{13} = 15.86;\ k_{21} = 100;\ k_{22} = 100.$