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Comparison of Ultracapacitor Electric Circuit Models

Lisheng Shi, *Student Member, IEEE*, and M. L. Crow, *Senior Member, IEEE*

***Abstract*--Due to ultracapacitors' unique features, the electrical performances and reliabilities of electrical systems using ultracapacitors can be improved. It is important to have a good ultracapacitor model for simulation and assisting electrical system design and product development. Several kinds of ultracapacitor models are given these years. Especially, electric circuit models are the interested ones for electrical engineers. This paper concentrates on the electric circuit model. Three basic RC network models are discussed in detail, including modeling ideas, circuit formation, linear/nonlinear factors and evaluations of each model. The general electric circuit model considering the inductance and leakage current effects are given. Model selection depends on the specific applications of ultracapacitor. Based on the analysis in this paper, recommendations of ultracapacitor selection strategies are provided.**

***Index Terms* -- Ultracapacitor, RC network, Electric circuit model, model selection.**

I. Introduction

ULTRACAPACITORS, or supercapacitors, are electrochemical double layer capacitors. Due to their material composition and design structure they have a high power density and low equivalent series resistance (ESR). These characteristics lead to higher efficiency, larger current charge and/or discharge capacity, and low heating losses. Ultracapacitors are well suited for harsh environment applications due to their wider temperature range and long life cycle (in both times and discharge cycles). Finally, ultracapacitors can be deep discharged, so there is little risk of over-discharging the ultracapacitor. Additionally, ultracapacitors can be fully discharged before servicing, reducing the electrical hazard during maintenance. Due to their high power density, ultracapacitors have several potential applications: uninterruptible power systems (UPS), adjustable speed drive ride-through equipments (ASD), dynamic voltage restorer devices (DVR), flexible AC transmission systems (FACTS), wind power fluctuation buffering systems, and hybrid electric vehicle (HEV).

Since ultracapacitors are electrochemical devices, their electrical circuit model is not commonly known or understood. To fully utilize them to their full advantage, it is important for users to know the ultracapacitor's unique features during the application system design and/or product development. In recent years as ultracapacitors become used more widely, several different circuit models have been proposed in the literature [1]–[8]. Three basic modeling approaches have been used: a mathematic model, an electric circuit model, and other non-electric circuit models (such as artificial neutral network modeling method [4]). Each modeling approach has its own advantages and disadvantages respectively. The mathematical modeling approach includes complicated computations and requires too many parameters that must be experimentally identified. Additionally, the mathematical model does not usually have an explicit physical meaning and can not readily be incorporated into a circuit diagram. The non-electric circuit models have similar shortcomings. In this paper, we focus on electric circuit models, which are most commonly used by electrical engineers. The primary contribution of this paper is the comparison of the different circuit-based ultracapacitor models and the development of relationships under which the models are equivalent.

The simplest ultracapacitor circuit model RC model, which has only one RC branch. Fig.1 is the ultracapacitor simple RC model. This model is composed of a resistor R, which models the ultracapacitor's ohomic loss, usually called equivalent series resistor (ESR) and a capacitor C, which simulate the ultracapacitor's capacitance during charging and discharging effects.

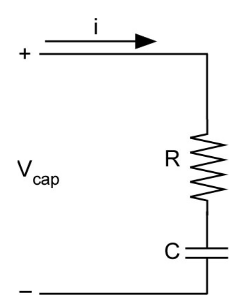

Fig.1. Simple ultracapacitor model.

Fig. 2 shows the simulation results of the simple RC model. The charging and discharging current are constant at 30A. The capacitor rated voltage, capacitance and ESR are 2.5V, 470F, and 2.5mΩ respectively.

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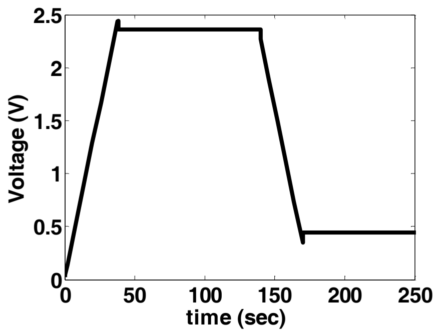


Fig.2. Ultracapacitor constant current charge and discharge simple model simulation (30A).

Fig. 3 shows the test results of an ultracapacitor [1]. The Ultracapacitor rated voltage is 2.5V. The charge and discharge current is 30 A, which is constant during charging and discharging process.

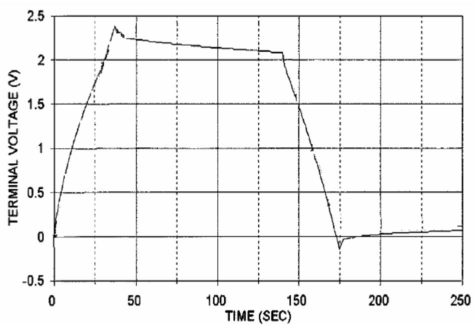


Fig.3. Ultracapacitor constant current charge and discharge experimental test results (30A) (from [1])

By comparing the simulation result and the experimental results, it can be seen that the simple RC model has several advantages and disadvantages. The primary advantage is the simplicity of the model. It is easy to incorporate into a circuit and the simulation process is computationally straightforward. The primary disadvantage is that the simple RC model is not able to capture the nonlinear rise and fall of the ultracapacitor voltage and the change in voltage after the charging and discharging stops [1]. Therefore, more detailed models are required for better accuracy.

The detailed ultracapacitor models can be categorized into three basic classes: the RC parallel branch model, the RC transmission line model, and the RC series-parallel branch model. Each class of the electric circuit model can be further extended to include both linear model and nonlinear models. The linear model has resistors and capacitors in the equivalent circuit that are linear components and are usually constants. Similarly, the nonlinear model has resistors and capacitors that are nonlinear and which may be functions of the ultracapacitor electrolyte temperature and terminal voltage.

In this paper, the three classes of electric circuit models are first reviewed and modeling methods, simulation and test results are given and then the advantages and disadvantages of each are evaluated. The number of RC branches in each model will affect the model accuracy, therefore for consistency in the modeling process three RC branches will be used in each developed model. After each class of model is analyzed, we propose a general ultracapacitor circuit model from which each class of model can be transformed.

II. Analysis of Basic RC Network Models

A. *RC parallel branch model*

The basic objective of this model is to simulate the actual ultracapacitor's behavior during charge and discharge. When the ultracapacitor is charged (discharged), the terminal voltage will increase (decrease) rapidly. If the charging (discharging) is stopped, the terminal voltage will continue to decrease (increase) gradually for several minutes and will ultimately become stable after tens of minutes. Thus, three different time constant RC branches are chosen to simulate the ultracapacitor's charge and discharge regions.

Ideally, the number RC branches should be large. However, to simplify the modeling process and to ensure satisfactory accuracy, three or two branches are the typical choice for models. The three RC parallel branch model is shown in Fig. 4. The three branches are called the fast-term branch composed of R_f and C_f, medium-term branch composed of R_m and C_m, and the long-term branch composed of R_s and C_s.

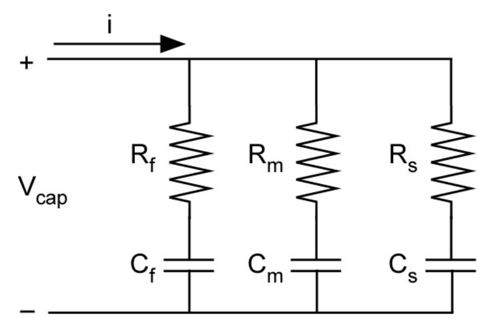


Fig.4. RC parallel branch model

Each RC branch has a different time constant. The fast-term branch dominates the charge and discharge behavior in the order of a few seconds. The medium-term branch dominates the behavior over the scale of minutes. Finally, the slow-term branch usually governs the long-term charge and discharge characteristics (longer than ten minutes).

The advantages of the RC parallel branch model are:

- The model reflects the internal charge distribution process very well within the considered time span and for voltages above 40% of the rated terminal voltage [1].
- There is good response of the ultracapacitor's dynamic behavior during charging and discharging process.
- The parameters can be extracted from a relatively simple experimental test set-up.
- The accuracy is better than the simple RC model. However, at low voltages, the error between model and actual behavior may reach 10% of the rated voltage; this is due to several assumptions that have been made to simplify the model and the parameters' identification. [1]

From the modeling process developed in [1], the parameters of this model are obtained from constant current tests and the time constant of each branch is chosen arbitrarily. This parameter extracting process affects the accuracy of the model, especially for the dynamic simulation. An improved model can be obtained by adding an additional capacitor branch in parallel with the fast-term branch capacitor where the capacitance of the added capacitor is dependent on the voltage across the C_f.

B. RC transmission line model

The RC transmission line model is based on Porous Electrode Theory developed by de Levie [10]. From this theory, the ultracapacitor model shown in Fig. 5 can be derived. This model is typically referred to as the transmission line model.

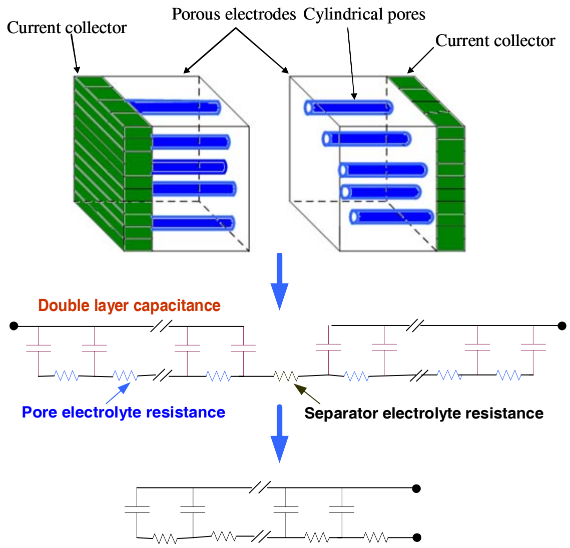


Fig.5. Porous electrode and RC ladder network.

At the physical level, each pore in a porous electrode can be modeled as a transmission line. The transmission line model attempts to capture the distributed double-layer capacitance and the distributed electrolyte resistance that extends the depth of the pore. To achieve an estimation of the double-layer capacitive effects, straight, cylindrical pores of uniform diameter and a perfectly conducting electrode are assumed, leading to a ladder network with potentially many RC elements. A three branch transmission line model is shown in Fig. 6.

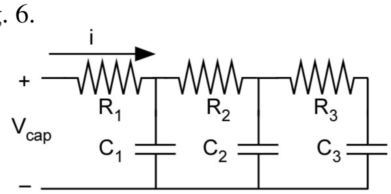


Fig. 6. RC transmission line model

This model simulates the ultracapacitor's physical structure and electromechanical characteristics directly. It has a clear physical meaning. This model takes into account both dynamic and long time behaviors. But the transmission line has a complex analytical expression which is not suitable for simulation. In addition, like in the three branch model, the parameters are obtained from constant current tests.

C. RC series-parallel branch model

The RC series-parallel branch model is proposed in [14]. Fig. 7 shows the principle circuit with three RC series-parallel branches.

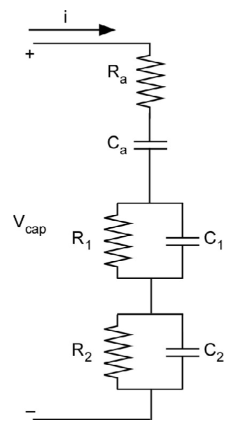


Fig. 7. RC branches series-parallel model

R_a represents the equivalent series resistance, C_a and the other parallel RC branches represent the ultracapacitor's pore impedance. For improved accuracy, the parameters can be dependent on the ultracapacitor temperature, voltage and operational frequency. In [14], the impedance spectroscopy testing method is given for obtaining the parameters.

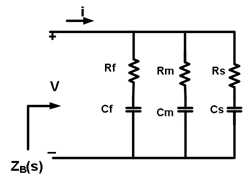
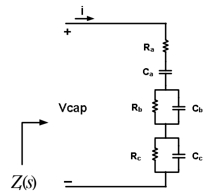
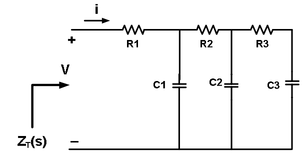
III. Relationship Between Basic RC Network Models

Since the number of RC components in each of the models is three, then regardless of model the input impedance of the three linear models can be generically expressed in the following form:

$$Z(s) = \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{B_3 s^3 + B_2 s^2 + B_1 s + B_0} \quad (1)$$

which will also result in the same bode plot if the coefficients are the same. Table I gives the conversion between the coefficients of the three basic RC networks.

Table 1 Circuit models conversion

<table>
<tr><th></th><th>I</th><th>II</th><th>III</th></tr>
<tr><td>Ultracapacitor models</td><td>
</td><td>
</td><td>
</td></tr>
<tr><td>$Z(s)=\frac{b_3s^3+b_2s^2+b_1s+1}{B_3s^3+B_2s^2+B_1s}$</td><td>$b_3=R_1C_1R_2C_2R_3C_3$
$b_2=R_1C_1R_2C_2+R_2C_2R_3C_3+R_3C_3R_1C_1+R_1C_1R_2C_3+R_1C_2R_3C_3$
$b_1=R_1C_1+R_2C_2+R_3C_3+R_1C_2+R_1C_3+R_2C_3$
$B_3=C_1R_2C_2R_3C_3$
$B_2=C_1R_2C_2+R_2C_1C_3+R_3C_3C_1+R_3C_2C_3$
$B_3=C_1+C_2+C_3$</td><td>$b_3=R_aC_aR_bC_bR_cC_c$
$b_2=R_aC_aR_bC_b+R_bC_bR_cC_c+R_cC_cR_aC_a+R_cC_cC_aR_b+R_bC_bC_aR_c$
$b_1=R_aC_a+R_bC_b+R_cC_c+R_cC_a+R_bC_a$
$B_3=C_aR_bC_bR_cC_c$
$B_2=C_aR_cC_c+C_aR_bC_b$
$B_3=C_a$</td><td>$b_3=R_1C_1R_2C_2R_3C_3$
$b_2=R_1C_1R_2C_2+R_2C_2R_3C_3+R_3C_3R_1C_1+R_1C_1R_2C_3+R_1C_2R_3C_3$
$b_1=R_1C_1+R_2C_2+R_3C_3+R_1C_2+R_1C_3+R_2C_3$
$B_3=C_1R_2C_2R_3C_3$
$B_2=C_1R_2C_2+R_2C_1C_3+R_3C_3C_1+R_3C_2C_3$
$B_3=C_1+C_2+C_3$</td></tr>
<tr><td>Convert II to I
Or
Convert III to I</td><td colspan="3">

$$R_f=\frac{1}{\frac{Y(s)}{s}(s-p_1)\big|_{s=p_1}}\qquad R_m=\frac{1}{\frac{Y(s)}{s}(s-p_2)\big|_{s=p_2}}\qquad R_s=\frac{1}{\frac{Y(s)}{s}(s-p_3)\big|_{s=p_3}}$$

$$C_f=-\frac{\frac{Y(s)}{s}(s-p_1)\big|_{s=p_1}}{p_1}\qquad C_m=-\frac{\frac{Y(s)}{s}(s-p_2)\big|_{s=p_2}}{p_2}\qquad C_s=-\frac{\frac{Y(s)}{s}(s-p_3)\big|_{s=p_3}}{p_3}$$

where:

$$A_1=(36\mathrm{b}_1\mathrm{b}_2\mathrm{b}_3-108\mathrm{b}_3^2-8\mathrm{b}_2^3+12\sqrt{3}\times\sqrt{4\mathrm{b}_1^3\mathrm{b}_3-\mathrm{b}_1^2\mathrm{b}_2^2-18\mathrm{b}_1\mathrm{b}_2\mathrm{b}_3+27\mathrm{b}_3^2+4\mathrm{b}_2^3}\times\mathrm{b}3)^{1/3}$$

$$A_2=\frac{-3\mathrm{b}_1\mathrm{b}_3+\mathrm{b}_2^2}{\mathrm{b}_3}$$

$$p_1=\frac{1}{6}\times\frac{1}{b_3}\times A_1+\frac{2}{3}\times\frac{A_2}{\mathrm{A}_1}-\frac{1}{3}\times\frac{\mathrm{b}_2}{\mathrm{b}_3}$$

$$p_2=(-\frac{1}{12}\times\frac{1}{b_3}\times A_1-\frac{1}{3}\times\frac{A_2}{A_1}-\frac{1}{3}\times\frac{\mathrm{b}_2}{\mathrm{b}_3})+i\times(\frac{\sqrt{3}}{2}(\frac{1}{6}\times\frac{1}{\mathrm{b}_3}\times A_1-\frac{2}{3}\times\frac{A_2}{A_1}))$$

$$p_3=(-\frac{1}{12}\times\frac{1}{b_3}\times A_1-\frac{1}{3}\times\frac{A_2}{A_1}-\frac{1}{3}\times\frac{\mathrm{b}_2}{\mathrm{b}_3})-i\times(\frac{\sqrt{3}}{2}\times(\frac{1}{6}\times\frac{1}{\mathrm{b}_3}\times A_1-\frac{2}{3}\times\frac{A_2}{A_1}))$$

$$z_1=\frac{-B_2+\sqrt{B_2^2-4B_3B_1}}{2B_3}\qquad z_2=\frac{-B_2-\sqrt{B_2^2-4B_3B_1}}{2B_3}$$

$$\frac{Y(s)}{s}=\frac{1}{s}\times\frac{1}{Z(s)}=\frac{B_3}{b_3}\times\frac{s^2+\frac{B_2}{B_3}s+\frac{B_1}{B_3}}{s^3+\frac{b_2}{b_3}s^2+\frac{b_1}{b_3}s+\frac{1}{b_3}}=\frac{B_3}{b_3}\times\frac{(s-z_1)(s-z_2)(s-z_3)}{(s-p_1)(s-p_2)(s-p_3)}$$

</td></tr>
</table>

Convert I to II Or Convert III to II	$R_a = \frac{b_3}{B_3}$ $\quad R_b = -\frac{b_3}{B_3} \times \frac{D_1 \times D_4 + D_3 + \frac{1}{b_3}}{D_4 \times D_2}$ $\quad R_c = \frac{b_3}{B_3} \times \frac{D_1 \times D_5 - D_3 + \frac{1}{b_3}}{D_5 \times D_2}$ $C_a = B_1$ $\quad C_b = \frac{B_3}{b_3} \times \frac{D_2}{D_1 \times D_4 + D_3 + \frac{1}{b_3}}$ $\quad C_c = \frac{B_3}{b_3} \times \frac{D_5 \times D_2}{D_1 \times D_5^2 - D_3 \times D_5 + \frac{1}{b_3}}$ where: $D_1 = \frac{b_2}{b_3} - \frac{B_2}{B_3}$ $\quad D_2 = \sqrt{(\frac{B_2}{B_3})^2 - (\frac{4B_1}{B_3})}$ $\quad D_3 = \frac{b_1}{b_3} - \frac{B_1}{B_3}$ $\quad D_4 = \frac{\frac{-B_2}{B_3} + D_2}{2}$ $D_5 = \frac{\frac{B_2}{B_3} + D_2}{2}$
Convert I to III Or Convert II to III	$R_1 = \frac{b_3}{B_3}$ $\quad R_2 = \frac{E_1}{B_2 - E_2 \times \frac{B_3}{E_1}}$ $\quad R_3 = \frac{E_2 - (B_1 - \frac{B_3}{E_1}) \times \frac{E_1}{E_3}}{(B_1 - \frac{B_3}{E_1}) - \frac{E_3}{E_2 - (B_1 - \frac{B_3}{E_1}) \times \frac{E_1}{E_3}}}$ $C_1 = \frac{B_3}{E_1}$ $\quad C_2 = \frac{E_3}{E_2 - (B_1 - \frac{B_3}{E_1}) \times \frac{E_1}{E_3}}$ $\quad C_3 = (B_1 - \frac{B_3}{E_1}) - \frac{B_2 - E_2 \times \frac{B_3}{E_1}}{E_2 - (B_1 - \frac{B_3}{E_1}) \times \frac{E_1}{B_2 - E_2 \times \frac{B_3}{E_1}}}$ where: $E_1 = b_2 - \frac{b_3}{B_3} \times B_2$ $\quad E_2 = b_1 - \frac{b_3}{B_3} \times B_1$ $\quad E_3 = B_2 - E_2 \times \frac{B_3}{E_1}$

For the indices of equation (1) of b3=5.742e005, b2=3.28e005, b1=1826, b0=1, B3=8.454e008, B2=5.262e006, B1= 3410, B0=0 the resulting bode plot is shown in Fig. 7. These coefficients are based on an ultracapacitor with the following parameters:

V: 2.5V
C: 2600 F
ESR: 0.68 mΩ

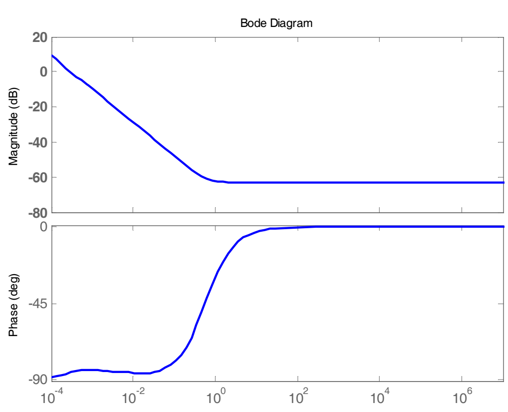


Fig. 8. Bode plot example of RC network linear models

IV. General RC Network Models

In some instances, it is not enough to simulate a real ultracapacitor only by one of the RC models presented earlier. The above models can be expanded to a more general model. The number of branches can ideally be extended to infinity. Additionally, the ultracapacitor has an inductance effect that should also be modeled, especially at high operating frequencies. Also, the effect of leakage current has been neglected in the earlier models. Fig. 9 is a general RC parallel branch circuit model. In the figure, the resistor *Rp* represents the leakage current losses and the series inductor *L* provides for the inductance effect at high frequencies.

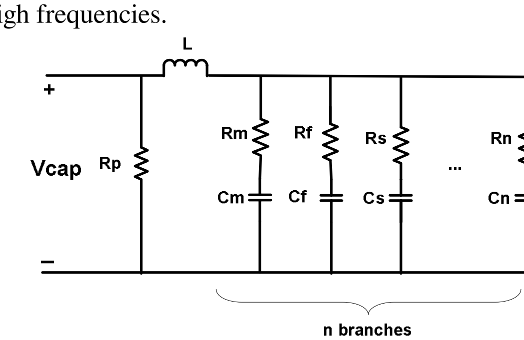


Fig.9 General RC parallel branch model

To illustrate the inductance effect, a frequency response simulation example is shown in Fig. 10. Note that in the high frequency range (greater than 1 kHz) the inductance effect can not be neglected even through the inductance value is small (~50nH)

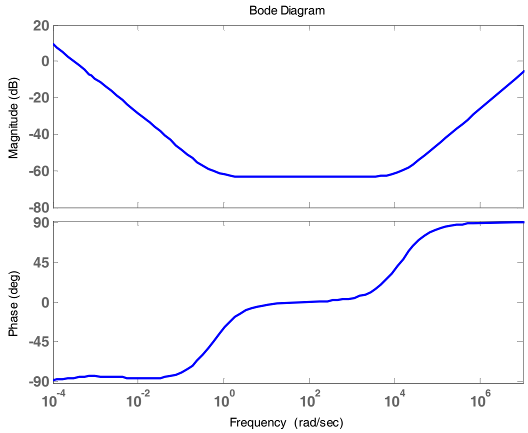


Fig.10 Frequency response of ultracapacitor

V. Conclusions

Ultracapacitor modeling is important for the electrical system analysis and equipment design. An efficient and good accurate model can help electrical engineers thoroughly understand ultracapacitor's characteristics. Different methods for parameter extraction and modeling have been proposed in the literature. This paper provides a method of translating parameters from one model to another so that the user can choose whichever model is best suited to their particular need.

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