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# EFFECTS OF DYNAMIC PROPERTIES OF ROCKFILL MATERIALS ON SEISMIC RESPONSE OF CONCRETE-FACED ROCKFILL DAMS

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## ABSTRACT

In this paper, the equivalent-linear method is used for three-dimensional seismic response analyses of concrete-faced rockfill dams (CFRDs). Different combinations of various parameters such as small-strain shear modulus, strain-dependent patterns of modulus and hysteretic damping, are considered to systematically investigate the effects of dynamic properties of rockfill-type coarse-grained materials on seismic dynamic response of CFRDs. It is concluded that the nonlinearity of embankment material has a significant effect on both vibration characteristics and seismic response behavior of CFRDs. Numerical results presented are instructive to gain a better understanding on earthquake-resistant behavior of CFRDs and the effects of dynamic properties of rockfills.

## INTRODUCTION

A number of high concrete-faced rockfill dams (CFRDs) with the height of more than 200m are being under construction or to be built in the Southwest area of China with potentially heavy seismic activity. Therefore, the safety of these high CFRDs subjected to strong earthquake shaking in addition to reservoiring becomes a crucial issue in governing their whole performance.

The equivalent-linear analysis method is still a main computing method used for seismic response analysis of CFRDs. Nonlinearity of rockfill materials is approached using an iterative procedure. A set of moduli and damping ratios is initially assumed and a series of linear analyses are conducted with each calculation using soil moduli and damping ratios compatible with the shear strain level estimated in the preceding turn. It is based on the patterns of shear moduli and damping ratios varying with shear strain obtained experimentally in laboratory. Due to the complexity of dynamic tests for coarse-grained materials, limited data is usually used to represent empirical relationships of secant shear modulus and damping ratio with shear strain (Uddin 1999; Kong *et al.* 1992). Over past decades, with application of large-scale dynamic testing apparatus which are capable to implement complex loading and improvement of in-situ freezing sampling techniques, a number of test results (Wu and Luan 2000) on gravels become available. Based on these collective data, the sensitivity of seismic response on dynamic properties of rockfills should be clarified. In addition, for high dams usually built in narrow irregular canyon and complex local geological condition, three-dimensional analyses will offer better representation on dam behavior compared with two-dimensional modelling.

In this paper, comparative studies are performed to show the effects of main parameters controlling dynamic properties of rockfills on seismic response of high CFRDs. The factors under consideration include initial shear modulus, the patterns of shear modulus and hysteretic damping ratio varying with shear strain. Both vibration characteristics of the finally-compatible linear system and dynamic responses of CFRDs are discussed. The stresses in concrete facing are emphasized on the basis of numerical results.

## NUMERICAL ANALYSIS PROCEDURE OF CFRD

The equivalent linearization procedure (Idriss *et al.* 1973) is used to carry out seismic response analysis of CFRDs. In this method, approximate nonlinear solutions can be obtained by a series of linear analyses provided the updated stiffness and damping are compatible with current effective shear strain amplitudes. The equivalent effective strain is estimated as a fraction (i.e., 0.65 or 2/3) of peak shear strain in order to define modulus and damping ratio for each iteration from the experimentally-achieved curves. Successive iterations are required until compatible dynamic parameters with strain level are acquired. The dynamic response from the final iteration, in which numerical convergence is achieved, is taken as an approximation of the nonlinear soil system. Rayleigh's concept of proportional damping is used to represent hysteretic damping of soil in order to form damping matrices of all soil elements. Wilson- $\theta$ 's numerical integration scheme is combined with equivalent linearization procedure to solve the dynamic equations of the system step-by-step in time domain.

Different finite elements are utilized to duly consider special

characteristics in geometrical configuration and material zoning of CFRDs. Rockfills are simulated by solid elements with strain-dependent shear modulus and damping ratio. Block elements and linear elasticity model are employed for concrete facing on upstream. The interface elements are set between concrete facing and rockfills in order to model the interaction of different media. The peripheral joints are placed along the interface between footwall and facing slab.

For interface elements, the dependency of shear stiffness  $K$  on shear strain  $\gamma$  and inter-relation between damping ratio  $\xi$  and shear stiffness  $K$  are expressed as following (Wu and Jiang 1992)

$$K = \frac{K_{\max}}{1 + \frac{MK_{\max}}{\tau_f} \cdot \gamma}, \quad \xi = \left(1 + \frac{K}{K_{\max}}\right) \xi_{\max} \quad (1)$$

where  $K_{\max}$  is initial shear stiffness of interface in kPa/mm and  $K_{\max} = 22\sigma_n^{0.7}$ ,  $M$  is a parameter associated with relative roughness of concrete face slab with a typical value of 2.0,  $\tau_f$  is shear strength and  $\tau_f = \tan\phi \cdot \sigma_n$ ,  $\phi$  represents friction angle of concrete face and rockfills,  $\sigma_n$  is normal stress applied on interface,  $\xi_{\max}$  is maximum damping ratio with a typical value of 20%.

In order to consider dynamic water pressure on inclined upstream slope, the modified Westergaard's formulation suggested by Xu *et al.* (1997) is used

$$P(y, t) = -\rho(1 - 0.54 \cdot \tan\alpha) \frac{7}{8} \sqrt{Hy} \ddot{u}_g(t) \quad (2)$$

where  $P(y, t)$  denotes the dynamic water pressure,  $\rho$  represents the density of water,  $y$  is the depth from water surface to the point under consideration,  $\ddot{u}_g(t)$  is input acceleration at the base-rock of dam,  $\alpha$  is inclination angle of dam face,  $H$  is dam height of the section under consideration.

Multidirectional excitations on the bedrock are taken into consideration in three-dimensional analysis.

In order to investigate the effects of dynamic parameters on the seismic response of CFRDs, a case study for Hongjiadu CFR dam is performed. The height of dam is 182.3m. Maximum cross section and its finite element discretization are shown in Fig.1. Only one type of rockfill material is used. The physical properties of main rockfill can be found elsewhere (Kong and Zou 1999). The elastic modulus of concerted-facing is 200MPa. The total number of nodes is 3801 and the total number of elements is 3463 with 263 face elements, 263 interface elements and 42 joint elements. North-South component of El Centro acceleration record of 1940, as shown in Fig.2, is selected as input ground motion. The maximum horizontal acceleration in up- and down-stream direction is 0.2g and both peak accelerations in transversal-stream and in vertical directions are 0.13g. The duration of earthquake excitation is 16s. The standard response spectrum of input acceleration is shown in Fig.3, which indicates that the predominant period of ground motion is about 0.45s.

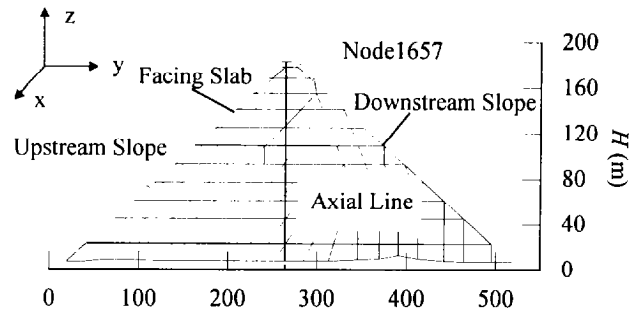


Fig. 1. Finite element discretization of Hongjiadu Dam for the maximum section

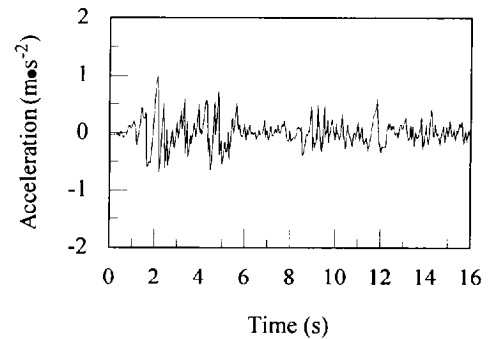


Fig. 2. Acceleration time history of El Centro record

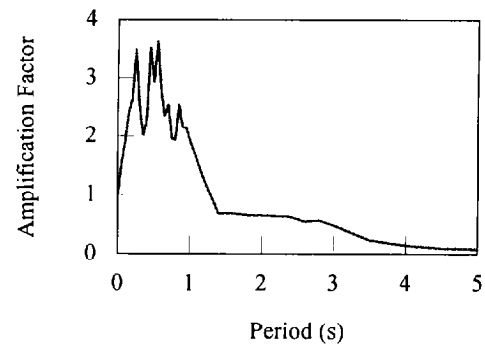


Fig. 3. Standard response spectrum of El Centro record

## EFFECTS OF DYNAMIC PROPERTIES OF ROCKFILLS ON SEISMIC RESPONSE OF CFRD

The influence of dynamic parameters of rockfill materials, such as small-strain shear modulus and variation patterns of shear modulus and hysteretic damping varying with shear strain, on seismic responses of dam will be discussed as below.

### Influence of Initial Shear Modulus of Rockfills

Dynamic shear modulus  $G_{\max}$  at very low shear strain (i.e., less than  $10^{-4}\%$ ) is associated with confining pressure and relative density of gravels. The following empirical formulae

proposed by Hardin and Drnevich (1972) is adopted here

$$G_{\max} = G_0 p_a F(e) \left( \frac{\sigma_c}{p_a} \right)^m \quad (3)$$

where  $G_0$  is modulus coefficient,  $F(e)$  denotes influencing function of void ratio and  $F(e) = \frac{(2.97 - e)^2}{1 + e}$  for angular rockfills,  $e$  is void ratio with a typical value of 0.4,  $\sigma_c$  is static effective mean principal stress which is obtained through a nonlinear incrementally-iterative numerical technique,  $m$  is modulus index with a range of 0.4-0.6.

Four cases with different values of initial shear modulus coefficients, e.g.,  $G_0 = 312.2, 416.3, 499.1, 786.0$ , are considered. The data of shear modulus and damping ratio varying with strain obtained by testing results (Kong and Zou, 1999) are directly used in the computations.

**Natural Frequency:** It is shown in Table 1 that initial shear modulus of rockfills has a noticeable influence on natural vibration behavior of linear representation of CFRD finally achieved. The fundamental frequency increases with increasing of modulus of rockfill materials. Furthermore, the natural frequency for case D approaches to the predominant frequency of input ground motion.

Table 1. Calculated fundamental natural frequencies for different values of  $G_0$

Case	$G_0$	References	Frequency(Hz)
A	312.2	Chen and Gu (1987)	1.0790
B	416.3	Chi and Lin (1998)	1.2282
C	499.1	Kong and Zou (1999)	1.3456
D	786.0	Uddin (1999)	1.7056

**Acceleration of Dam and Displacement of Peripheral Joints:** It can be observed from Fig.4 that shear modulus has little influence on peak accelerations near the bottom and the top of dam while peak accelerations near the height of  $(1/2 \sim 4/5)H$  increase with increasing initial modulus. Seismic response of

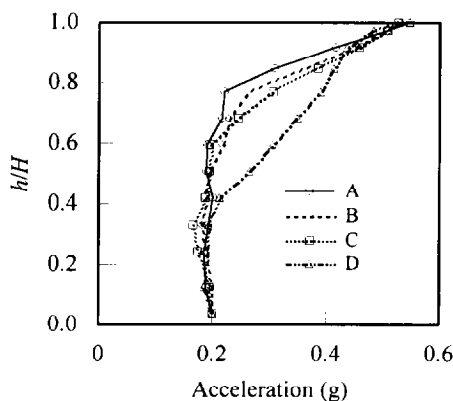


Fig. 4. Distribution of peak absolute acceleration in axial line along depth

dam in case D becomes serious since the natural frequency is close to the predominant frequency of ground motion. Dynamic displacement of the joints between slab and footwall is small with a maximum value of 0.5cm during the course of earthquake motions in all cases.

**Dynamic Stress of Concrete Facing:** It is displayed in Fig.5 and Fig.6 that the maximum value of dynamic stresses of concrete facing occurs near the height of  $(3/4 \sim 4/5)H$ . The major principal stress along slab decreases with increasing initial shear modulus of rockfills especially in the upper part of slab, and the minor principal stress along slab decreases considerably with increasing initial shear modulus. The increase of initial shear modulus of the upper part of dam materials is beneficial to reduce peak stresses of the facing. This qualitatively coincides with failure mechanism of CFR dams under earthquake observed from shaking-table tests on small-scale physical models (Han *et al.* 1990). The initial modulus can be increased by changing rockfill lithology and by adjusting grading and gravel content.

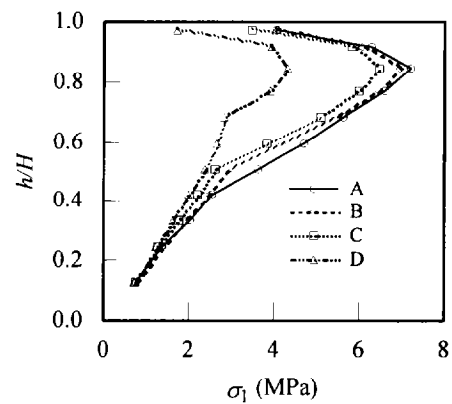


Fig. 5. Peak major principal stress along slab

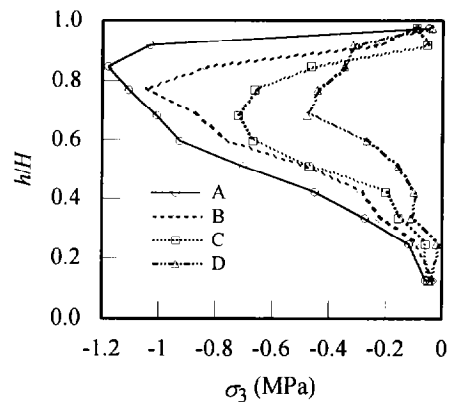


Fig. 6. Peak minor principal stress along slab

**Peak Shear Strain of Dam Rockfills:** Distributions of peak shear strains in the dam axial line and in upstream slope along depth are respectively shown in Fig.7 and Fig.8. The patterns of shear strain varying along depth are similar. The peak shear strains increase greatly with decreasing initial shear modulus, especially in the neighborhood of dam crest.

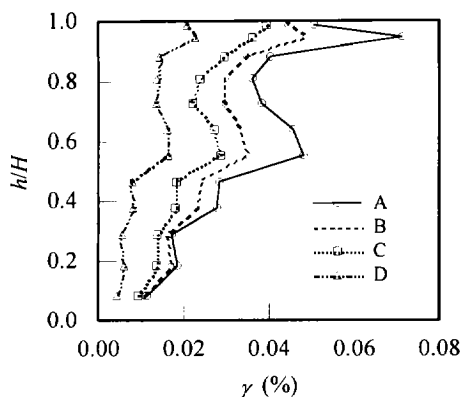


Fig. 7. Peak shear strains in axial line

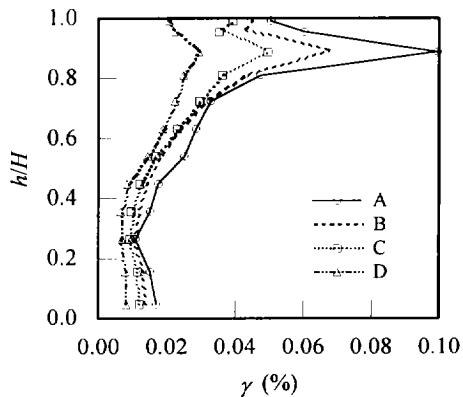


Fig. 8. Peak shear strains in upstream slope

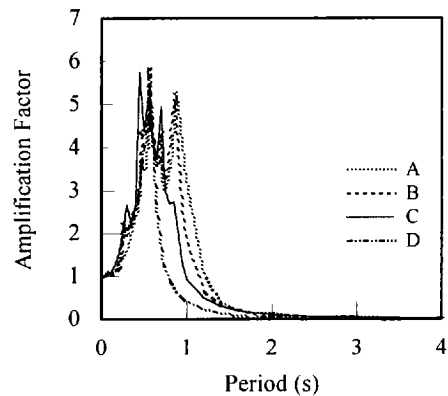


Fig. 9. Acceleration response spectra of node 1657

**Acceleration Response Spectra:** It is manifested in Fig.9 that the initial modulus affects strongly the pattern of acceleration response spectra of rockfills at the dam crest. The maximum amplification factors of all cases are almost same, however frequency components of acceleration response are rather different. The higher the initial modulus, the more narrow the frequency band. The lower the initial modulus, the more plentiful the frequency components contained.

Influence of Variation of Shear Modulus and Damping Ratio with Strain of Rockfills on Seismic Response of CFRD

The variation of shear modulus with shear strain is customarily

represented by normalized shear modulus  $\delta = G/G_{\max}$  given by dividing the shear modulus  $G$  at a given strain level by the maximum shear modulus  $G_{\max}$ . This normalization makes it possible to compare the relationships obtained from various sources. Studies on dynamic properties of coarse-grained materials have been performed (Seed *et al.* 1986; Rollins *et al.* 1998; Kong and Zou 1999). The upper and lower bounds of modulus ratio and damping ratio of rockfill materials are proposed by Seed *et al.* (1986) and Rollins *et al.* (1998) based on collected testing data, as shown in Fig.10. Using these testing data and proposed empirical curves, different cases as listed in Table 2 are considered.

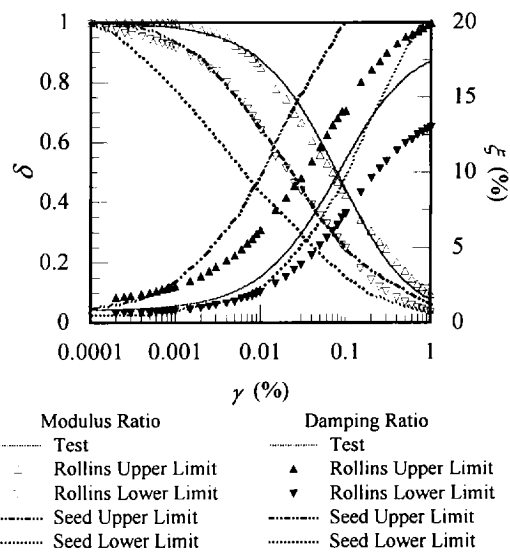


Fig. 10. Shear modulus and damping ratio curves used

Table 2. Dynamic property parameters for different cases and calculated natural frequencies

Case	$\delta$	$\xi$	Frequency (Hz)
C	Test	Test	1.3456
E	Test	Rollins Upper	1.3557
F	Test	Rollins Lower	1.3407
G	Rollins Upper	Test	1.3214
H	Rollins Lower	Test	1.1189
I	Seed Lower	Seed Upper	0.9519

**Natural Frequency:** It is implied in Table 2 that variations of shear modulus and damping ratio curves of rockfills have somewhat influence on the fundamental natural frequency of dam. The natural frequency increases with increasing modulus curve toward the upper bound. The value of damping ratio has little influence on the frequency of dam. The natural frequency decreases noticeably when modulus curves reaches the lower bound and damping ratios approach the upper bound.

**Absolute Acceleration:** It is shown in Fig.11 that response acceleration at the dam crest in case E is lower than those in other cases. The acceleration response at the depth of  $2/3H$  is the lowest in case H. The acceleration in case I with a low modulus and a high damping ratio is smaller than those in other cases.

The distribution of peak acceleration in axial line along depth is similar to that in upstream slope.

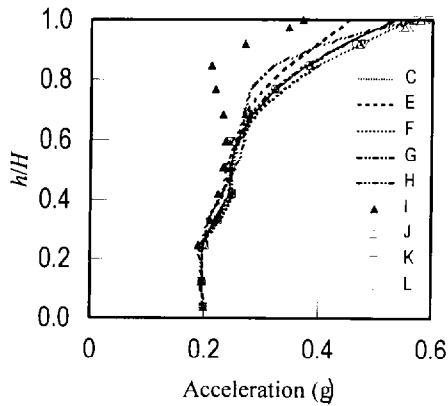


Fig. 11. Peak accelerations in upstream slope

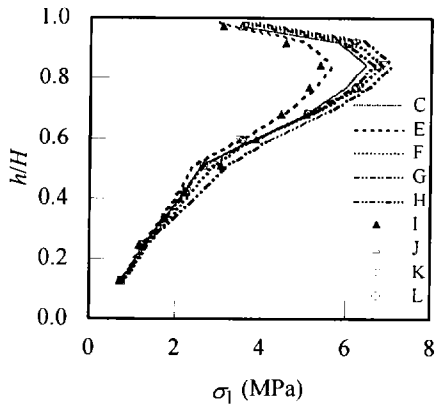


Fig. 12. Peak major principal stresses along slab

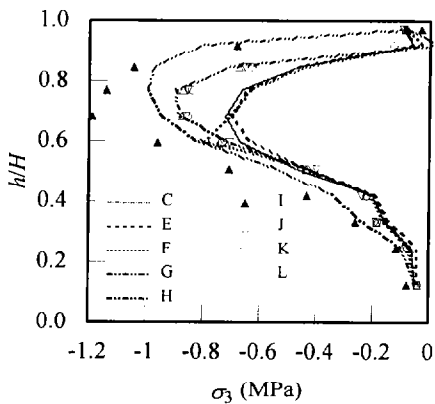


Fig. 13. Peak minor principal stresses along slab

**Dynamic Stresses of Concrete Facing:** Portrayed in Fig.12 and Fig.13 are peak dynamic stresses of concrete facing. Maximum value of peak stresses of facing occurs near the dam crest in all cases. The peak major principal stresses in case E and case I are relatively lower than those in other cases. Major principal stresses decrease with increasing damping ratios. The minor principal stresses in case H and case I are relatively higher and tensile stresses of the dam facing increase with descending modulus reduction curve of rockfills. Therefore it is necessary to

carry out optimum design especially for face slab on the basis of dynamic properties of rockfills and stress state of facing.

**Peak Shear Strains:** The variations of peak shear strains of the dam along depth are shown in Fig.14 and Fig.15. Peak shear strains increase from bedrock to crest along depth. Peak shear strain computed using experimentally-obtained shear modulus and damping ratio given by Rollins's upper bound is lower while it is higher in case H and case I with a maximum value at the height of 173.15m. It is obvious that shear strains at the dam crest increase with decreasing modulus.

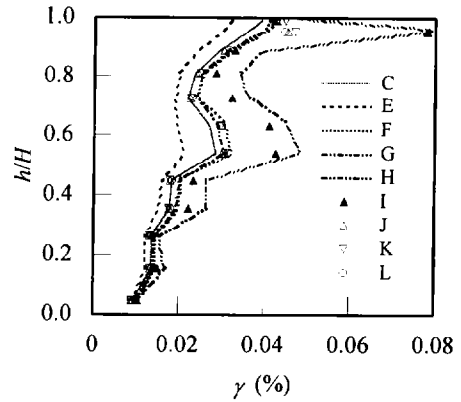


Fig. 14. Peak shear strains in axial line

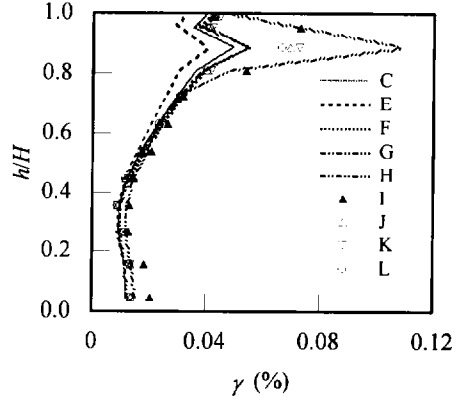


Fig. 15. Peak shear strains in upstream slope

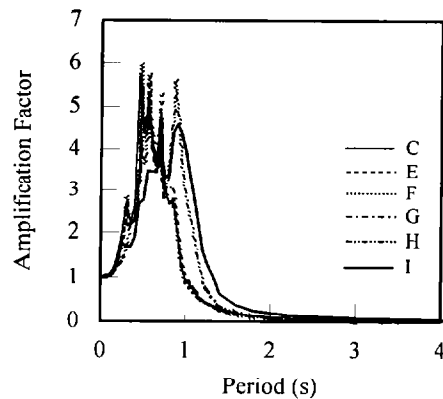


Fig. 16. Acceleration response spectra of node 1657

**Acceleration Response Spectra:** As shown in Fig.16, the frequency band of acceleration response spectra of node 1657 is rather plentiful when lower modulus curves are used in case H

and case I. The spectrum is almost independent of variations of damping ratio.

### Effect of Confining-Pressure-Dependency of Shear Modulus on Seismic Response

It is recognized (e.g., Rollins *et al.* 1998) that variation mode of shear modulus with strains depends on confining pressure. As the confining pressure increases, the relations of  $\delta$  and  $\gamma$  move from the lower bound of testing data range towards the upper bound. A set of testing data (Kong and Zou 1999) is shown in Fig.17. The relationship between normalized modulus and shear strain can be expressed by the following equation with three parameters (Wu *et al.* 2000)

$$\delta(\gamma, a, n, m) = \frac{1}{\left\{ \ln \left[ e + (\gamma / a)^n \right] \right\}^m} \quad (4)$$

Where  $a$ ,  $n$ , and  $m$  are the three parameters to be determined by curve fitting to experimental data. The dependency of shear modulus varying with strain on confining pressure can easily be simulated by adjusting the parameter  $a$ . The best-fitting curve describing modulus variations of rockfills at various confining pressures can be achieved by setting different values of  $a$ . In fact, as illustrated in Fig.18, the parameter  $a$  is linearly related to the confining pressure  $\sigma_c$  with excellent correlation

$$a = 0.05 + 0.00017\sigma_c \quad (5)$$

Considering high confining pressure, the alternating polynomial function is proposed as following

$$a = 0.045 + 0.00024\sigma_c - 1.75 \cdot 10^{-7} \sigma_c^2 \quad (6)$$

The correlation between damping ratio and shear modulus proposed by Hardin and Drnevich (1972) is used here

$$\xi = \xi_{\max} (1 - \delta) \quad (7)$$

where  $\xi_{\max}$  is maximum damping ratio when shear modulus theoretically approaches zero. It is deduced from the experimental data shown in Fig.19 (Kong and Zou 1999) that damping ratios of rockfills appear to be almost independent of confining pressure. The relation of  $\xi$  versus  $\gamma$ , estimated by Equation (7) with  $\xi_{\max} = 18\%$  and mean modulus curve, is very close to mean best-fit curve of test data.

In order to illustrate the effect of confining-pressure-dependency of dynamic shear modulus curve on seismic responses, four cases listed in Table 3 are taken account for different values of  $a$  and  $\xi$ . It can be observed that the natural frequency is almost independent of shear modulus reduction curve while the dependency on confining pressure is taken into consideration.

From Fig.11 to Fig.15, it can be concluded that when the dependency of variation mode of shear modulus with shear strain is considered, the peak acceleration of dam is almost not affected while major and minor principal stresses along slab and peak shear strain increase slightly. As portrayed in Fig.20, the distributions of peak absolute acceleration in downstream slope along depth for four cases are similar. Numerical results also indicates that the response spectra of node 1657 with different

parameters are almost identical. Therefore it is reasonable to overlook the effect of confining pressure on modulus curve.

Table 3. Dynamic property parameters and calculated natural frequencies for different cases

Case	$a$	$\delta$	$\xi$	Frequency (Hz)	Note
C	0.101	Test	Test	1.3456	$n=0.89$ ;
J	Eq.(6)	Eq.(4)	Eq.(7)	1.3422	$m=3$ ;
K	Eq.(6)	Eq.(4)	Test	1.3411	$\xi_{\max}=18\%$
L	Eq.(5)	Eq.(4)	Eq.(7)	1.3335	

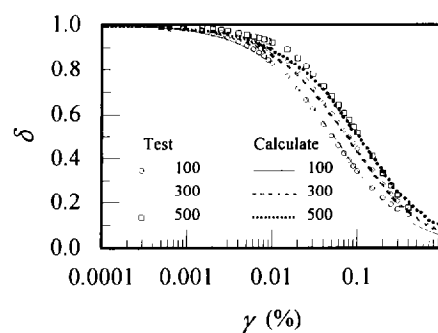


Fig. 17. Modulus ratio of main rockfill at various confining pressure

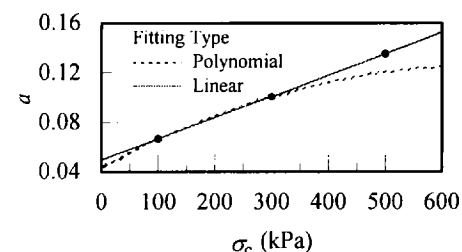


Fig. 18. Variation of  $a$  with confining pressure

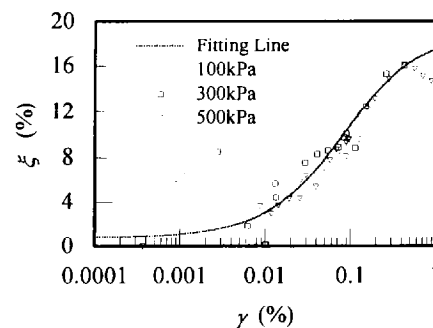


Fig. 19. Damping ratio of main rockfill at various confining pressure

In addition, it can be observed by comparing Fig.11 and Fig.20 that the difference between distribution of peak accelerations along depth in downstream slope and in upstream slope is noticeable. The amplification increases gradually from bottom to crest in upstream slope. However, another amplification region of acceleration occurs at the depth of  $1/2H$  in downstream slope due to reflection of surface waves. This feature coincides with the fact observed in model tests of CFR

dams (Han *et al.* 1990). Therefore stability of rockfills near the dam crest will play an important role in earthquake-resistant behavior of CFRDs and necessary aseismic measures should be taken.

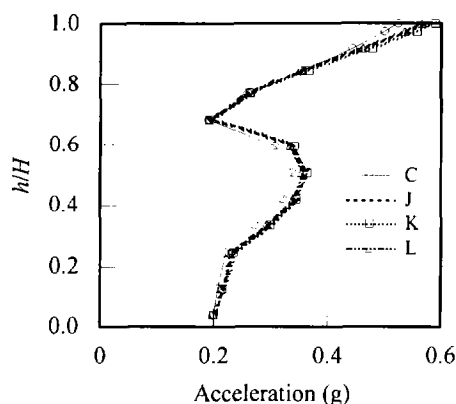


Fig. 20. Distribution of peak absolute acceleration in downstream slope along depth

## CONCLUSIONS

In this paper, the effects of dynamic property parameters of rockfills on seismic response characteristics of concrete-faced rockfill dams are systematically investigated with Hongjiadu dam as a numerical example. (1) The effect of shear modulus on the natural frequency is stronger compared with that of damping ratio curve. The natural frequency of the equivalent linear vibration system of dam increases with increasing shear modulus. Both the initial shear modulus and modulus reduction curve have considerable influence on acceleration response spectra characteristics at the dam crest. The frequency band is narrow when the higher moduli are used. However the spectra curves is almost independent of damping ratio curve. (2) Peak major principal stresses of slab decreases with the increase of damping ratios of rockfills while absolute values of minor principal stresses (tensile stresses) of slab increases with decreasing modulus curve. (3) Shear strains at the dam crest increase with descending modulus curve and peak shear strains decrease with increasing damping ratios. (4) The effect of the dependency of shear modulus reduction curve on confining pressure on seismic response can be overlooked as usual.

Reasonable selections of dynamic parameters of rockfills should be made prudently in order to confidently evaluate earthquake-resistant behavior of CFRDs from three-dimensional equivalent-linear seismic analysis.

## ACKNOWLEDGMENTS

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