01 May 2003

# Calculation of the Impedance of a Rectangular Waveguide Aperture in the Presence of a Loaded Dipole Antenna Embedded in a Generally Lossy Material

Dana M. Hughes

R. Zoughi
*Missouri University of Science and Technology*, zoughi@mst.edu

## Recommended Citation

# Calculation of the Impedance of a Rectangular Waveguide Aperture in the Presence of a Loaded Dipole Antenna Embedded in a Generally Lossy Material

Dana Hughes and Reza Zoughi

Applied Microwave Nondestructive Testing Laboratory
Department of Electrical and Computer Engineering, University of Missouri--Rolla
1870 Miner Circle, Rolla, MO 65409-0040
Phone: (573) 341-4728, Fax: (573) 341-4532, Email: zoughi@ece.umr.edu

*Abstract* – *The use of a combinded embedded modulated scattering technique, utilizing a PIN diode-loaded dipole probe, and near-field microwave nondestructive testing technique, utilizing an open-ended rectangular waveguide, has been investigated as a means for determining the complex dielectric constant in which the MST probe is embedded. For this, the formulation of the forward problem of calculating the modulated reflection coefficient, measured at the aperture of the waveguide, is presented here. This formulation is based upon the reciprocity theorem for two antennae, and utilizes the near-field radiation pattern of a rectangular waveguide aperture and current distribution along the length of a dipole antenna. This formulation is verified with measured values as a function of location, load impedance and surrounding media. Finally, potential improvements, considerations, and the use of an inversion routine is discussed.*

## I. INTRODUCTION

Microwave nondestructive testing and evaluation (NDT&E) techniques have been used extensively in the past in the investigation of structures and materials used in several industrial, aeronautic, naval and infrastructure areas [1]. These techniques have shown great success in detection of several important parameters of cement-based materials, such as mixing constituents, state of cure, and determination of the presence of water and chloride ingress [2-8]. Complex composite sandwich structures and thick composite panels have also been investigated using this technique, which has also resulted in the capability of monitoring cure state of epoxy resin, determination of the thickness or dielectric properties of composite panels and detection of defects such as impact damage, delaminations and disbonds [9-15]. Several other applications, such as detection of corrosion and surface cracks in metals, determination of dielectric properties of a materials, and analysis of complex layered structures are also possible using microwave NDT techniques [16-20].

The use of modulated scattering technique (MST) in microwave measurements has found application in microwave imaging, field mapping and antenna pattern measurements [21]. In microwave NDT domain, this technique has been used for online monitoring of conveyed products. Recent work has investigated the potential of using a combined embedded PIN diode-loaded dipole probe and an open-ended rectangular waveguide for the non-invasive measurement and monitoring of local dielectric properties of a material [22-25]. Such a combined technique provides several advantages over traditional microwave techniques. The probe may be placed in critical locations, such as near a rebar or composite joint, to allow for localized measurement around such regions. This is possible, as the modulated signal can be distinguished from signals due to surrounding reflections. The signal-to-noise ratio of the modulated signal may be increased by coherently averaging and synchronizing the measurement. Finally, arrays of MST probes may be utilized in order to more accurately or rapidly assess the structure under test.

## II. THEORETICAL FORMULATION

For this investigation, a PIN diode-loaded dipole (i.e., the MST probe) is embedded in a generally lossy half-space of material with a relatively uniform dielectric constant, and illuminated using an open-ended rectangular waveguide. The relative dielectric constant, $\varepsilon_r$, of the material is a complex value consisting of the permittivity (i.e., real part), which represents the material's ability to store microwave energy, and the loss factor (i.e., imaginary part), which represents the material's ability to absorb microwave energy. This value is based on the chemical composition of the material, and may be used to determine the material or state of material under test. Furthermore, dielectric mixing models may be used to determine an effective dielectric constant for a material which consists of a mixture of individual components, such as with cement-based material [26]. Changes in this value will result in a change in the reflection coefficient measured at the aperture of the waveguide, which may be correlated to changes in the material (e.g., ingress of water or chloride in cement based materials), or presence of a defect (e.g., delamination in a composite panel).

Figure 1 shows the geometry of the embedded MST probe illuminated by an open-ended rectangular waveguide. The PIN diode is modulated using a low frequency (e.g. 0.5 Hz to several Hz) square wave, which switches the probe between a forward- and reverse-biased state. These two states have a corresponding forward- and reverse-biased impedance, $Z_F$

and $Z_R$, respectively, associated with the PIN diode at microwave frequency. Thus, any signal incident upon the MST probe will scatter and carry with it the modulation waveform. This backscattered signal is measured as a reflection coefficient at the aperture of the waveguide, in addition to the static reflection due to the surrounding media.
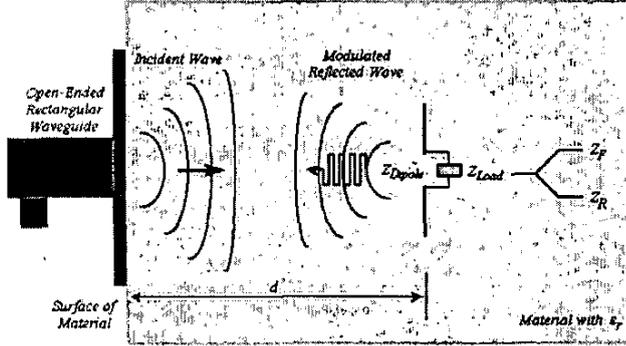


Fig. 1. Schematic of an embedded MST probe illuminated by an open-ended rectangular waveguide.

## A. Coupling Between the Waveguide Aperture and Dipole

The formulation of the reflection coefficient measured at the aperture of the waveguide can be obtained through application of the reciprocity theorem [27-29]. This theorem, which may be applied in purely field, purely circuit or combined circuit and field forms, allows for the coupling between two antennae to be modeled as a mutual impedance or admittance. For this, the field components radiated by one antenna and the current distributions on the second antenna are required. Furthermore, in order to be effective, both of these components are relative to a terminal feed point of the antenna, thus, both antennae require a well-defined port. With an open-ended waveguide, this is the case when considering modal analysis of the propagating fields within the waveguide. For the MST probe, the port is defined as the feed gap at the center of the dipole, to which the PIN diode load is attached.

The combined circuit and field form of the Reaction Theorem may be used to form an expression for the mutual coupling between the two antennae as a mutual impedance. This form is commonly referred to as the induced EMF method [30], and is given as

$$Z_{21} = \frac{V_{21}}{I_{11}} = \frac{-1}{I_{11}I_{22}} \iiint_V (\overline{E}_1 \cdot \overline{J}_2 - \overline{H}_1 \cdot \overline{M}_2) dv \qquad (1)$$

where $Z_{21}$ is the mutual impedance between the waveguide and the dipole, $I_{11}$ and $I_{22}$ are the terminal port currents of the first and second antennae, respectively, $E_1$ and $H_1$ are the fields radiated by the first antenna, and $J_2$ and $M_2$ are the

current distributions on the second antenna. The induced EMF method assumes that the radiated fields from the first antenna do not consider the presence of the second antenna, and ideal current distributions due to a voltage or current fed at the terminal port exist.

For this derivation, the waveguide aperture is taken to be antenna 1, while the dipole is taken to be antenna 2. The integral in equation (1) reduces to a line integral along the length of the dipole, with an ideal line of current present due to a unit voltage fed at the center terminal of the dipole. The expression for the current on the dipole due to this voltage, $I_{VF}(z)$, may be expressed in terms of an admittance distribution, $Y_{VF}(z)$, along the length of the dipole, given as

$$\overline{I}_{VF}(z) = V_{22}\overline{Y}_{VF}(z) \qquad (2)$$

This current distribution can be calculated using the Method of Moments [31,32], using pulse basis functions and collocation weighing functions. The details regarding this derivation is readily available and will not be discussed here [33-35].

As only electric current exists on the dipole, it is only necessary to find the radiated electric fields from the waveguide aperture. Within the waveguide, the electric and magnetic fields may be expressed as a set of modes, for which an expression for the transverse electric and magnetic fields exist. For a given mode, the aperture of the waveguide may be considered a port by expressing the overall intensity of the electric and magnetic fields as a voltage and current term, respectively, which allows for the characteristic impedance for the waveguide to be defined. It is convenient to express this electric field as an orthonormal distribution and corresponding overall electric field magnitude (i.e., $V_{11}$), for which there is a corresponding orthonormal magnetic field distribution and magnetic field magnitude (i.e., $I_{11}$). The ratio of these two values gives the characteristic impedance of the waveguide, $Z_0$.

The incident electric field along the length of the dipole can be expressed in terms of the overall magnitude of the electric field at the aperture of the waveguide, $V_{11}$, and the electric field radiated from the orthonormal electric field distribution along the aperture of the waveguide, $e^i_1(z)$

$$\overline{E}_1^i(z) = V_{11}\overline{e}_1^i(z) \qquad (3)$$

Substituting equations (2) and (3) into equation (1), the overall expression for the mutual impedance between the waveguide aperture and the dipole can be expressed as

$$Z_{21} = -Z_{11}Z_{22} \int_{-l/2}^{l/2} e_1^i(z) Y_{VF}^i(z) dz \qquad (4)$$

where $Z_{11}$ is the impedance of the waveguide aperture without the presence of the MST probe, and $Z_{22}$ is the input impedance of the dipole. The input impedance of the

waveguide aperture may either be measured from the material under test at a location away from the MST probe, or calculated from the dielectric constant of the material [18]. The input impedance of the dipole may be determined using the Method of Moment formulation, as described earlier.

*B. Circuit Representation*

The relation between the waveguide aperture port and the terminal port of the dipole may now be described as a two-port impedance network, as shown in Figure 2a. If the material is linear, then the network will be reciprocal, and the mutual impedance between the two antennae will be equal (i.e., $Z_{21} = Z_{12}$) [29]. As this applies to most materials of interest, it is assumed that this will be valid in this derivation.
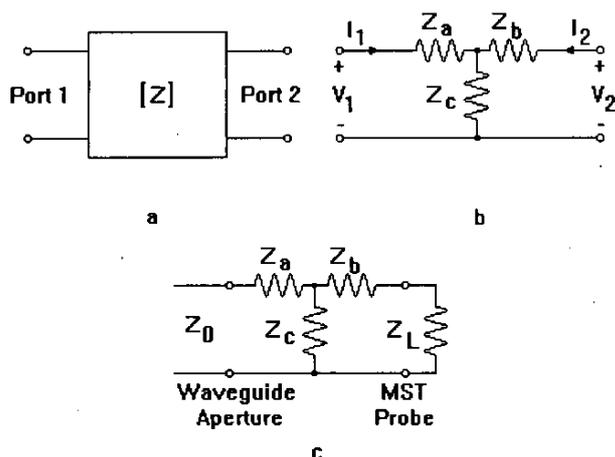


a                     b

c

Fig. 2. Schematic of an embedded MST probe illuminated by an open-ended rectangular waveguide.

The reciprocal two-port impedance network may be easily expressed as a T-network of impedances, as shown in Figure 2b [36]. The impedances associated with these can be calculated from the network impedances $Z_{11}$, $Z_{21}$ and $Z_{22}$ as

$$Z_a = Z_{11} - Z_{21} \tag{5a}$$

$$Z_b = Z_{22} - Z_{21} \tag{5b}$$

$$Z_c = Z_{21} \tag{5c}$$

Finally, when a load impedance is attached to port 2 (i.e., the center terminal of the dipole) as in Figure 2c, an equivalent input impedance at the aperture of the waveguide can be calculated as

$$Z_{in} = Z_a + \frac{Z_c(Z_b + Z_L)}{Z_b + Z_c + Z_L} \tag{6}$$

from which the reflection coefficient of the aperture can be calculated as

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \tag{7}$$

When using a modulated PIN diode loaded dipole, this results in two reflection coefficients, $\Gamma_F$ and $\Gamma_R$, associated with the two states of the diode, due to the two impedances, $Z_F$ and $Z_R$, described above.

### III.    EXPERIMENTAL RESULTS

In order to verify the formulation above, reflection coefficient measurements were made at 10 GHz using an X-Band waveguide as a function of distance from an MST probe. The coherent reflection coefficient was measured using a calibrated HP-8510C vector network analyzer. The MST probe consists of a dipole of length 1.5 cm and a radius of 0.01 cm, centrally loaded with a commercially available PIN diode. The forward- and reverse-biased impedance of the diode at this frequency were approximately $10+j70$ Ω and $20-j20$ Ω, respectively, which match typical values for the diode used [37,38]. Using the network analyzer, the static reflection coefficient at the aperture of the waveguide was measured to be $\Gamma_s = 0.233\angle-75.4°$.

Figures 3a and 3b shows the magnitude and phase of forward- and reverse-biased reflection coefficient as a function of distance between the MST probe and waveguide aperture. As can be seen, there exist strong agreement between the calculated and measured values within 1 cm from the aperture of the waveguide. As the probe approaches the aperture of the waveguide, the variations between the measured and calculated values increase. In the formulation, the effects of the flange as a ground plane on the dipole were not considered. Thus, it is expected that the effects of multiple reflections between the dipole and the waveguide are the cause variation between the calculated and measured values when the two are in close proximity. However, the MST probe will typically be embedded deep enough in the material to ignore the effects of multiple reflections.
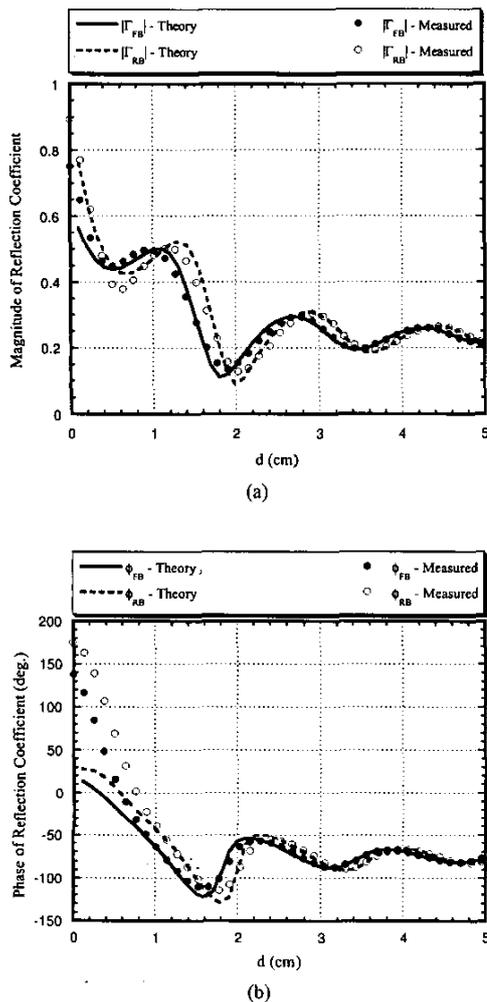
Fig. 3. Comparison of the a) magnitude and b) phase of measured and calculated forward- and reverse-biased reflection coefficient as a function of distance.

## IV.  CONCLUSIONS AND FUTURE WORK

The formulation for the calculation of reflection coefficient at the aperture of an open-ended rectangular waveguide in the presence of an embedded MST probe has been presented. This technique is based upon calculating the mutual coupling between the aperture of the waveguide and the dipole probe, and calculating the effective input impedance of the aperture. The formulation here neglected to account for the effect of the flange of the waveguide as a ground plane on the dipole. However, for the application of embedded MST measurements, this will not be an issue, due to the depth that the MST probe is embedded.

The formulation presented here has been verified in terms of spatial location of the MST probe with relation to the waveguide aperture and load impedance. While no

sensitivity analysis has been performed, the technique requires an accurate static reflection coefficient to be known. Variations in this value may result in inaccurate calculations of the modulated reflection coefficient.

The accuracy of the formulation above would benefit by including the effects of multiple reflections between the dipole and waveguide flange. This can be accomplished by considering the flange as an infinite ground plane, which would affect the current distribution along the dipole. Again, the benefits of this consideration would be greatest when the dipole and waveguide aperture are very near to one another, with the affect decreasing rapidly as the dipole is located farther away.

The choice of PIN diodes used may also improve the sensitivity of this technique. Ideally, a PIN diode whose forward- and reverse-biased states closely matched a short and open, respectively, would be used. PIN diodes with the very low package capacitance and impedance provide this, and are also capable of operating at higher frequencies. However, these types of diodes may not be as robust, and may cost more, than diodes that operate at lower frequencies.

Finally, this formulation may be incorporated into an inversion or root-finding routine, which may be used to provide for an inverse technique of finding the dielectric constant in which the MST probe is embedded from reflection coefficient measurements. Such a technique would need to be efficient in terms of the number of iterations required to converge to a solution, as the calculation of the forward problem requires significant computation time. Also, convergence around a local minimum, rather than the global solution, would need to be considered if such a technique were implemented.

## ACKNOWLEDGMENT

## REFERENCES

[1]  Zoughi, R., *Microwave Nondestructive Testing and Evaluation*, Kluwer Academic Publishers, The Netherlands, 2000.

[2]  Mubarak, K., K.J. Bois and R. Zoughi, "A Simple, Robust and On-Site Microwave Technique for Determining Water-to-Cement (w/c) Ratio of Fresh Portland Cement-Based Material," *IEEE Transactions on Instrumentation and Measurement*, vol. 50, no. 5, pp. 1255-1263, October 2001.

[3]  Bois, K., A. Benally and R. Zoughi, "Microwave Nondestructive Determination of Sand-to-Cement (s/c) Ratio in Mortar," *Research in Nondestructive Evaluation*, vol. 9, no. 4, pp. 227-238, 1997.

[4]  Bois, K., A. Benally and R. Zoughi, "Microwave Near-Field Reflection Property Analysis of Concrete for Material Content Determination," *IEEE Transactions on Instrumentation and Measurement*, vol. 47, no. 3, pp. 628-637, June 1998.

[5]  Bois, K.J., A.D. Benally, P.S. Nowak and R. Zoughi, "Cure State Monitoring and Water-to-Cement Ratio Determination of Fresh Portland Cement Based Materials Using Near-Field Microwave Techniques," *IEEE Transactions on Instrumentation and Measurement*, vol. 47, no. 3, pp. 628-637, June 1998.

[6] Hu, C., T. Case, M. Castle, R. Zoughi and K. Kurtis, "Microwave Evaluation of Accelerated Chloride Ingress in Mortar," *Proceedings of the Twenty-Seventh Annual Review of Progress in Quantitative Nondestructive Evaluation*, vol. 20A, pp. 467-473, Ames, IA, July 17-21, 2000.

[7] Case, J., R. Zoughi, K. Donnell, D. Hughes and K.E. Kurtis, "Microwave Analysis of Mortar Prepared with Type I/II, III and V Cement and Subjected to Cyclical Chloride Exposure," *Proceedings of the Twenty-Eighth Annual Review of Progress in Quantitative Nondestructive Evaluation*, vol. 21B, pp. 498-505, Brunswick, ME, July 29-August 3, 2001.

[8] Peer, S., J. Case, K. Donnell, D. Hughes, R. Zoughi and K.E. Kurtis, "Investigation of Microwave Reflection Properties of Mortar Exposed to Wet-Dry Cycles of Tap Water and Chloride Bath," *Proceedings of the Twenty-Eighth Annual Review of Progress in Quantitative Nondestructive Evaluation*, vol. 21B, pp. 1269-1276, Brunswick, ME, July 29-August 3, 2001.

[9] Qaddoumi, N., S. Ganchev and R. Zoughi, "Microwave Diagnosis of Low Density Glass Fibers with Resin Binder," *Research in Nondestructive Evaluation*, vol. 8, no. 3, pp. 177-188, 1996.

[10] Gray, S. and R. Zoughi, "Dielectric Sheet Thickness Variation and Disbond Detection in Multi-Layered Compsites Using an Extremely Sensitive Microwave Approach," *Materials Evaluation*, vol. 55, no. 1, pp. 42-48, 1997.

[11] Bakhtiari, S., S. Ganchev and R. Zoughi, "Microwave Frequency Optimization for Accurate Thickness or Dielectric Property Monitoring of Conductor Backed Composites," *Materials Evaluation*, vol. 51, no. 6, pp. 740-743, June 1993.

[12] Qaddoumi, N. R. Zoughi and G.W. Carriveau, "Microwave Detection and Depth Determination of Disbonds in Low-Permittivity and Low-Loss Sandwich Composites," *Research in Nondestructive Evaluation*, vol. 8, no. 1, pp. 51-63, 1996.

[13] Qaddoumi, N., S.I. Ganchev, G. Carriveau and R. Zoughi, "Microwave Imaging of Thick Composites with Defects," *Materials Evaluation*, vol. 53, no. 8, pp. 926-929, August, 1995.

[14] Zoughi, R., S. Ganchev, G. Carriveau and N. Qaddoumi, "Microwave Detection of Defects in Thick Multi-Laminate Sandwich Composite Structures," ASNT Spring '95 Conference Paper Summaries, pp. 191-193, Las Vegas, NV, March 20-24, 1995.

[15] Zoughi, R., S. Ganchev and G. Carriveau, "Microwave Nondestructive Testing and Evaluation of Thick Composites," Gordon Research Conference on Nondestructive Evaluation: "Noninvasive Sensing of Materials During Processing," Henniker, New Hampshire, June 26-July 1, 1994.

[16] Qaddoumi, N., L. Handjojo, T. Bigelow, J. Easter, A. Bray and R. Zoughi, "Microwave Corrosion Detection Using Open-Ended Rectangular Waveguide Sensors," *Materials Evaluation*, vol. 58, no. 2, pp. 178-184, February 2000.

[17] Yeh, C. and R. Zoughi, "A Novel Microwave Method for Detection of Long Surface Cracks in Metals," *IEEE Transactions on Instrumentation and Measurement*, vol. 43, no. 5, pp. 719-7215, October 1994.

[18] Bois, K., A. Benally and R. Zoughi, "An Exact Multimode Solution for the Reflection Properties of an Open-Ended Rectangular Waveguide Radiating into a Dielectric Half-Space: Forward and Inverse Problems," *IEEE Transactions on Instrumentation and Measurement*, vol. 48, no. 6, pp. 1131-1140, December 1999.

[19] Bois, K., L. Handjojo, A. Benally, K. Mubarack and R. Zoughi, "Dielectric Plug-Loaded Two-Port Transmission Line Measurement Technique for Dielectric Property Characterization of Granular and Liquid Materials," *IEEE Transactions on Instrumentation and Measurement*, vol. 48, no. 6, pp. 1141-1148, December 1999.

[20] Bakhtiari, S., S. Ganchev and R. Zoughi, "A Generalized Formulation for Admittance of an Open-Ended Rectangular Waveguide Radiating into Stratified Dielectrics," *Research in Nondestructive Evaluation*, vol. 7, no. 2/3, pp. 75-87, 1995.

[21] Bolomey, J.-C. and F.E. Gardiol, *Engineering Applications of the Modulated Scatterer Technique*, Artech House, Inc., Boston, 2001.

[22] Joisel, A., K.J. Bois, A.D. Benally, R. Zoughi and J.C. Bolomey, "Embedded Modulated Dipole Scattering for Near-Field Microwave Inspection of Concrete: Preliminary Investigation," *Proceedings of the SPIE'99 Subsurface Sensors and Applications Conference*, vol. 3752, pp. 208-214, Denver, CO, July 18-23, 1999.

[23] Zoughi, R., N. Bertrand and B. Fosdick, "Potential Application of Embedded Modulated Scatterers in Microwave Near-Field Inspection of Composites," *Proceedings of ICCE/7*, pp. 973-974, Denver, CO, July 2-8, 2000.

[24] Hughes, D. and R. Zoughi, "A Method for Evaluating the Dielectric Properties of Compsites Using a Combined Embedded Modulated Scattering and Near-Field Microwave Nondestructive Testing Technique," *Proceedings of the IEEE Instrumentation and Measurement Technology Conference*, pp. 1882-1886, Budapest, Hungary, May 21-23, 2001.

[25] Hughes, D. and R. Zoughi, "Near-Field Microwave and Embedded Modulated Scattering Technique (MST) for Dielectric Characterization of Materials," Presented at the *Twenty-Ninth Annual Review of Progress in Quantitative Nondestructive Evaluation*, Bellingham, WA, July, 2002.

[26] Bois, K., R. Mirshahi and R. Zoughi, "Dielectric Mixing Models for Cement Based Materials," *Proceedings of the Review of Progress in Quantitative Nondestructive Evaluation*, vol. 16A, pp. 657-663, Plenum Press, NY, 1997.

[27] Richmond, J.H., "A Reaction Theorem and Its Application to Antenna Impedance Calculations," *IRE Transactions on Antennas and Propagation*, vol. AP-9, no. 6, pp. 515-520, November 1961.

[28] King, H.E., "Mutual Impedance of Unequal Length Antennas in Echelon," *IRE Transactions on Antennas and Propagation*, vol. AP-5, pp. 306-313, July 1957.

[29] Carter, P.S., "Circuit Relations in Radiating Systems and Applications to Antenna Problems," *Proceedings of the IRE*, vol. 20, no. 6, pp. 1004-1041, June 1932.

[30] Pistolkors, A.A., "The Radiation Resistance of Beam Antennas," *Proceedings of the IRE*, vol. 17, no. 3, pp. 562-579, March 1929.

[31] Harrington, R.F., *Field Computation by Moment Methods*, Macmillian, New York, 1968.

[32] Moore, J. and R. Pizer, *Moment Methods in Electromagnetics: Techniques and Applications*, John Wiley & Sons, Inc., New York, 1984.

[33] King, R. and C.W. Harrington, Jr., "The Distribution of Current Along a Symmetrical Center-Driven Antenna," *Proceedings of the IRE*, vol. 31, no. 10, pp. 548-567, October 1943.

[34] Harrington, R.F. and J.R. Mautz, "Straight Wires with Arbitrary Excitation and Loading," *IEEE Transactions on Antennas and Propagation*, vol. AP-15, no. 4, pp. 502-515, July 1967.

[35] Landstorfer, F.M. and R.R. Sacher, *Optimization of Wire Antennas*, John Wiley & Sons, Inc., New York, 1985.

[36] Balanis, C.A., *Antenna Theory: Analysis and Design*, John Wiley & Sons, Inc., New York, 1982.

[37] Agilent Technologies, *PIN Diodes for RF Switching and Attenuating*, Technical Data Sheet, 2000.

[38] Agilent Technologies, *Applications of PIN Diodes*, Application Notes, 1999.