Flexibly Connected Thin-walled Space Frame Stability

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FLEXIBLY CONNECTED THIN-WALLED SPACE FRAME STABILITY

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Abstract

The elastic stability of a cubic space frame composed of cold-formed steel members is studied for various bending and warping rigidities. A finite element technique is used to perform a parametric study of the elastic stability response of the flexibly connected frame. The results are compared with previously published observations for hot-rolled steel sections. The criterion used for deciding the significance of warping for hot-rolled sections is not found to be applicable to cold-formed sections.

Introduction

Due to advances in high-strength material technology, lightweight structures are becoming more popular, and structural members are tending to be more slender. Thus, buckling failures are becoming more important, making the calculation of the structure buckling load an important consideration.

With the inclusion of partially restrained and fully restrained joint behavior in the load and resistance factor design (LRFD) it becomes imperative to study the effects of such joints in the stability behavior of frames. Investigations by Ackroyd and Gerstle (1983) and Yu and Shanmugam (1986) studied flexible connection behavior on plane frames while Blandford *et al.* (1988) and Carlberg *et al.* (1990) have investigated the behavior of space frames comprised of hot-rolled sections. However, there has been no published research on the effect of flexible joint behavior on cold-formed steel sections. Flexible joint behavior in cold-formed steel structures is important since these members are generally not connected as rigidly as compared to hot-rolled sections. Further, the sectional properties of hot-rolled sections differ significantly from those of the cold-formed sections, with warping effects considered to be a more significant influence on the behavior of cold-formed sections as compared to hot-rolled sections.

It is the purpose of this paper to consider the stability response of cubic space frames composed of cold-formed steel sections and discuss the differences in the behavior of cold-formed sections with the results of hot-rolled sections reported by Carlberg *et al.* (1990).

Structural Model

A FORTRAN program developed by the third author, Space Frame EIGenvalue program (SFEIG), is used to analyze the portal space frame. The program incorporates the elastic and geometric stiffness matrices, $[K_E]$ and $[K_G]$, as developed by Yang and McGuire (1984, 1986). This finite element formulation also includes the warping degree-of-freedom (d.o.f.) and hence each member has a total of 14 d.o.f. (Fig. 1(a)). This implies that the element stiffness matrix is a 14×14 matrix. The nodal displacement vector for the element shown in Fig. 1(a) will contain a warping d.o.f., *i.e*, $\chi = d\theta_x/dx$, in addition to the six conventional d.o.f.'s $[\ u_x, u_y, u_z, \theta_x, \theta_y, \theta_z\ ]$ at each node. The corresponding force vector shown is $[F_x, F_y, F_z, M_x, M_y, M_z, B]$. Subscripts 1 and 2 indicate the node numbers at which the forces and displacements are considered. Since a discontinuity in the warping displacements of the members will exist at the joints, a zero length connection element (Fig. 1(b)), as developed by Blandford *et al.* (1988), is

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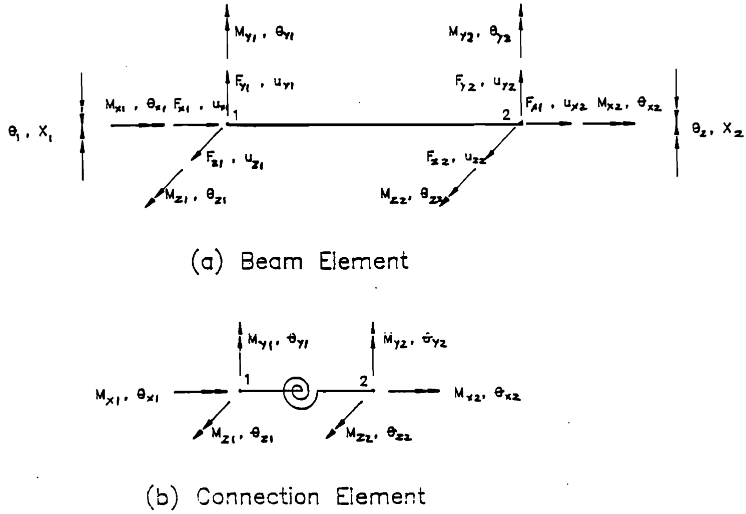


Figure 1: Element Library

used to model the joint behavior. This enables the column and beam members to have different magnitudes of warping at the connection nodes without performing a conventional static condensation on the discontinuous element warping displacements. The characteristic eigen-equation is given by

$$([K_E] + \lambda[K_G])\{\Delta\} = \{0\}$$

where λ is the eigenvalue. The buckling load is the smallest positive eigenvalue, λ_{cr}, and the corresponding eigenvector Φ_{cr} gives the buckling mode. The solution algorithm incorporates secant iteration in a determinant search followed by the application of a "shifted" inverse iteration strategy to obtain the critical eigenvalue, λ_{cr}.

Numerical Results

Figures 2(a) and 2(b) depict the frame with loading, and column orientations for which analyses have been performed. The loading has been specifically chosen to provide a better understanding of the effect of warping on frame stability. The column orientations shown are those which have been considered by Razzaq and Naim (1980) for rigid frame analyses, and Blandford *et al.* (1988) and Carlberg *et al.* (1990) for flexibly connected frame analyses. Based on the eigenvalue convergence studies of Carlberg *et al.* (1990), a four-element discretization per member has been chosen. Such a discretization is found to model the P-Delta and warping effects favorably as compared to a one-element discretization. Two element/member are nearly the same as a four element mesh. Sections chosen for study comprise of I-sections formed by two channels (with unstiffened flanges) connected back to back as specified in the Cold Formed Design Manual (1987). The chosen sections have the properties shown in Table 1.

The material constants E and G are assumed to be 29,000 ksi (213 MPa) and 11,600 ksi (85.2 MPa), respectively. Connection stiffnesses have been taken as

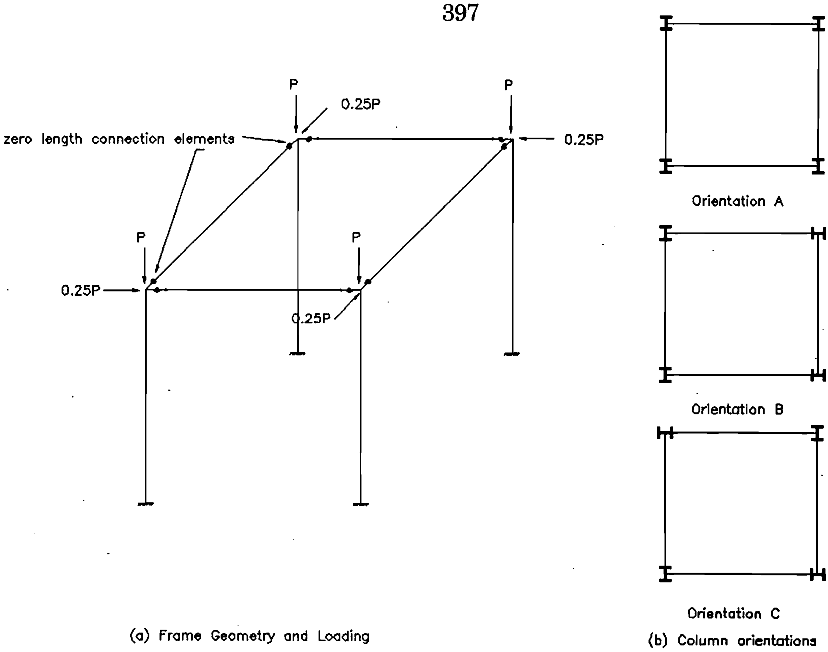


Figure 2: Cubic Space Frame

Table 1: Properties of Sections Used in Analysis

Section ($D \times B \times t$), in.	Area A in^2 (mm^2)	Moment of Inertia in^4 (mm^4), about: Weak Axis, I_y	Moment of Inertia in^4 (mm^4), about: Strong Axis, I_z	Torsional Constant J, in^4 (mm^4)	Warping Constant C_w, in^6 (mm^6)
$7 \times 3 \times 0.105$	2.013 (1298.7)	0.4767 (198,417)	12.436 (5,176,254)	0.0074 (3080)	5.05 (135.6×10^7)
$4 \times 2.25 \times 0.105$	1.225 (790.3)	0.2018 (83,995)	2.572 (1,070,547)	0.0045 (1873)	0.655 (17.6×10^7)
$2 \times 2.25 \times 0.075$	0.598 (385.8)	0.1428 (59,438)	0.375 (156,086)	0.00112 (466)	0.119 (3.2×10^7)

$K_{\theta_x} = 10^{12}$ for a rigid torsion connection.

$(K_{\theta_y}, K_{\theta_z}) = (\frac{3EI_y}{L}, \frac{3EI_z}{L})\frac{\nu}{1-\nu}$ for the bending connection behavior,

where ν is called the bending fixity factor, which varies from 0 to 1. A fixity factor value of 0 indicates a pin connection while that of 1 indicates a rigid connection. From the available experimental results, it is possible to assign values for ν to model different types of connections, *e.g.*, a value of 0.3 for ν models a double seat angle connection. A warping fixity factor a_f is introduced as a measure of flange warping restraint, and also ranges from 0 to 1. This factor is modeled from the warping spring concept of Yang and McGuire (1984). The warping factor a_f takes the value of 0 when the flanges are allowed to warp freely, and 1 when the flanges are fixed against warping. In other words, a_f is a measure of the restraint provided by members at connection nodes. The amount of restraint depends upon the type of connection provided and also by the presence of stiffener plates attached to the flanges of the members at the connection.

A parametric study of the cubic space frame has been performed for the values of $a_f = 0.0, 0.5$ and 1.0 and $\nu = 0.0, 0.25, 0.5, 0.75$ and 1.0 . The influence of these parameters has been studied and is discussed in the following section.

Discussion of Results

Results show that frames with column patterns A and B are not sensitive to the degree of warping restraint. The buckling loads are found to increase steadily with increasing connection fixity. The structure typically exhibits a sidesway buckling mode for these configurations.

Figure 3 shows the normalized critical eigenvalue (critical load) results obtained for the space frame using $7.0 \times 3.0 \times 0.105$ sections, for differing values of a_f and ν. The eigenvalues have been normalized by the critical load for the completely rigid frame with full warping restraint. The critical eigenvectors are obtained for each calculated eigenvalue. From inspection of these eigenvectors (which are scaled deflection values of the buckled frame), it is possible to explain the frame behavior as follows :

It is observed that for a range of values of bending fixity factor ν from 0.0 to 0.3 and for all values of a_f studied, the magnitude of deflections of the column members is larger than that of the beams. Hence, it is evident that in this range of ν, the column members are more prone to buckling than the beams. For ν ranging from 0.4 to 1.0, and for a_f 0.0 and 0.5, the eigenvectors indicate that the magnitude of lateral beam deflection is more than the column deflections, thus decreasing the resisting capacity of the frame considerably. This shows that the connection is now capable of transferring a sufficient amount of the applied lateral load from the columns to the beams, changing the buckling mode of the structure from a predominantly column buckling mode to that of lateral beam buckling. In the curve for $a_f = 1.0$, *i.e.*, the members are completely fixed against warping at the connections, such a transfer of moment is not facilitated, and hence, column failure prevails. Consequently a high value for the critical load is achieved. This behavior can be seen in the typical buckling modes shown in Figs. 4(a) and 4(b).

Similar behavior can be observed for the $4.0 \times 2.25 \times 0.105$ section, whose normalized buckling load curves are shown in Fig. 5. Figure 6 shows the critical buckling load for the $2.0 \times 2.25 \times 0.075$ section, which behaves differently. This can be attributed to the fact that warping is not significant enough for this section to facilitate a transfer of moment from the column to the beam. This behavior is discussed and compared with results obtained for hot-rolled members in the following section.

Comparison with Behavior of Hot-rolled Sections

Based on studies with hot-rolled sections, it is stated (Yang and McGuire, 1984) that warping deformation can be significant for I-section members with $pL < 2.0$ where

$$p = \sqrt{\frac{GJ}{EC_w}}$$

and L = member length. G, J, E, and C_w are as defined earlier. Studies (Carlberg *et al.*, 1990), have confirmed this for a similar frame and loading pattern as presented here, with I-sections W 36X160 (pL = 1.05 for L = 144 in. (3.65m)) and W 12X53 (pL = 2.00 for L = 144 in.). However, results for the cold-formed sections $4.0 \times 2.25 \times 0.105$ (pL = 2.516 for L = 48 in. (1.22m)) presented in Fig. 6 indicate that warping effects are still significant despite the fact that $pL > 2.0$. Only for sections which give higher values of pL, *e.g.*, $2.0 \times 2.25 \times 0.075$ (pL = 2.945 for L = 48 in.), does warping tend to be insignificant as shown in Fig. 6. Another interesting fact to be noticed is that there is a decrease of about 2% in the critical load for $a_f = 1.0$ from the maximum value (occurring in the neighborhood of $\nu = 0.75$) to the value at full bending fixity ($\nu = 1.0$). This behavior is apparently not visible for hot-rolled sections.

Conclusions

Elastic analyses have been performed on the full section of the members of the cubic space frame without local buckling. The alternating strong and weak axis column orientation has been found to provide maximum structure resistance for the loading considered. This is due to the high internal bracing capacity of the column member.

Results indicate that structure buckling resistance is sensitive to low values of connection stiffness. This has also been observed for hot-rolled sections. For partial warping fixity values, as the connection stiffness increases, there is a clear indication of the shifting of the buckling mode from a predominantly column buckling to that of lateral beam buckling, resulting in a decrease of structural resistance to buckling.

The significance of warping in cold-formed steel sections is clearly indicated. The cutoff value of pL below which warping can be considered significant is found to differ from that for hot-rolled sections. It is suggested that pL has to be above 3.0 for cold-formed sections in order to ignore the warping effects. This is in comparison to the value of 2.0 for hot-rolled sections, suggested by Yang and McGuire (1984).

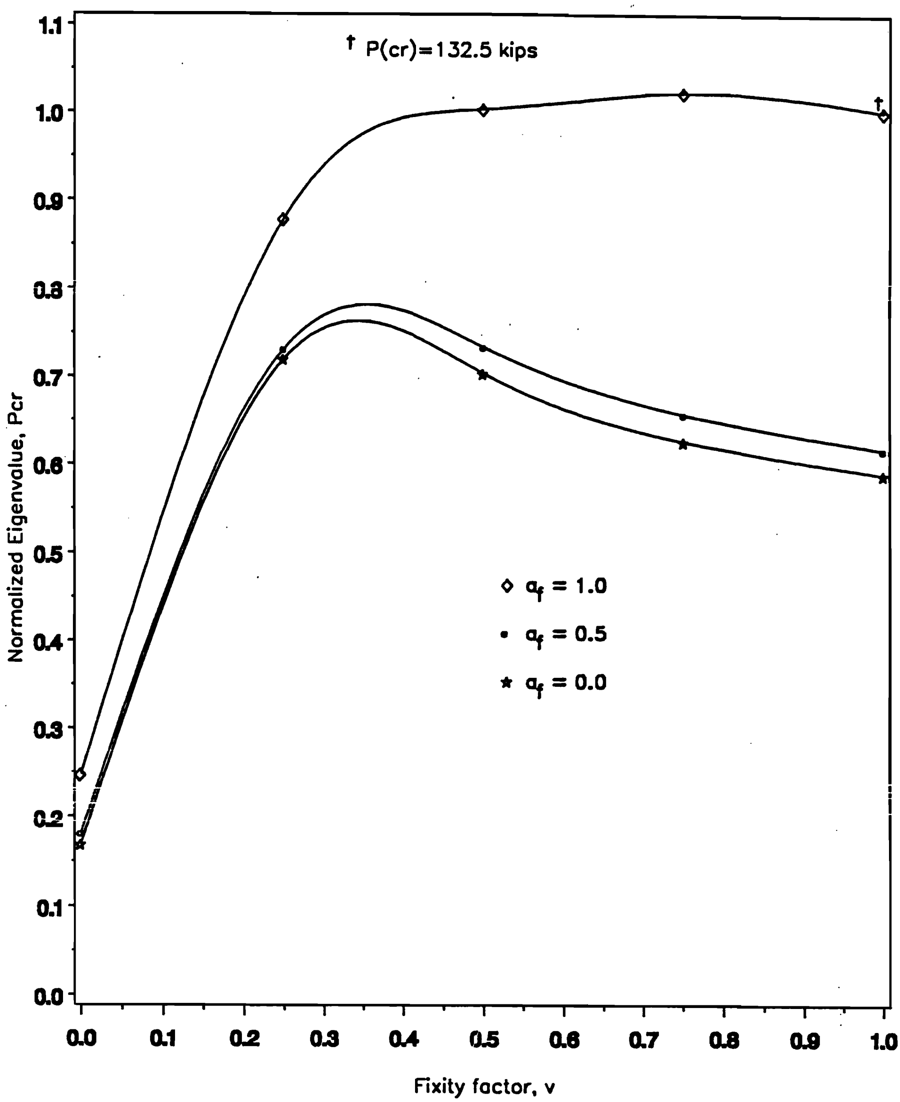
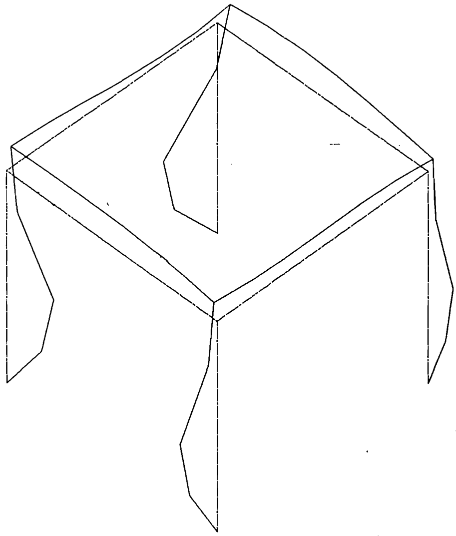
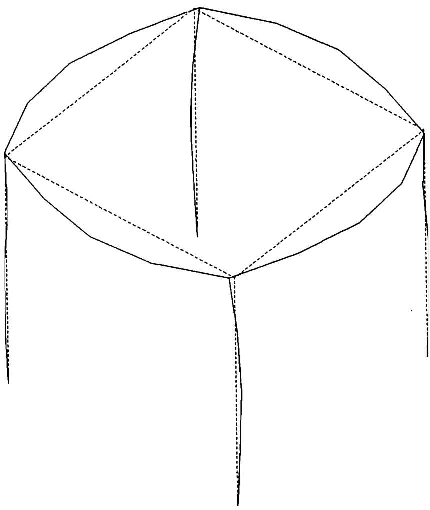


Figure 3: Stability Results for $7 \times 3 \times 0.105$ Section



Buckling Mode for 7 X 3 X 0.105 Section: $a_f = 1$, $v = 1$

(a) Typical Column Buckling Mode



Buckling Mode for 7 X 3 X 0.105 Section: $a_f = 0.5$, $v = 0.5$

(b) Typical Lateral Beam Buckling Mode

Figure 4: Typical Buckling modes for $7 \times 3 \times 0.105$ Section

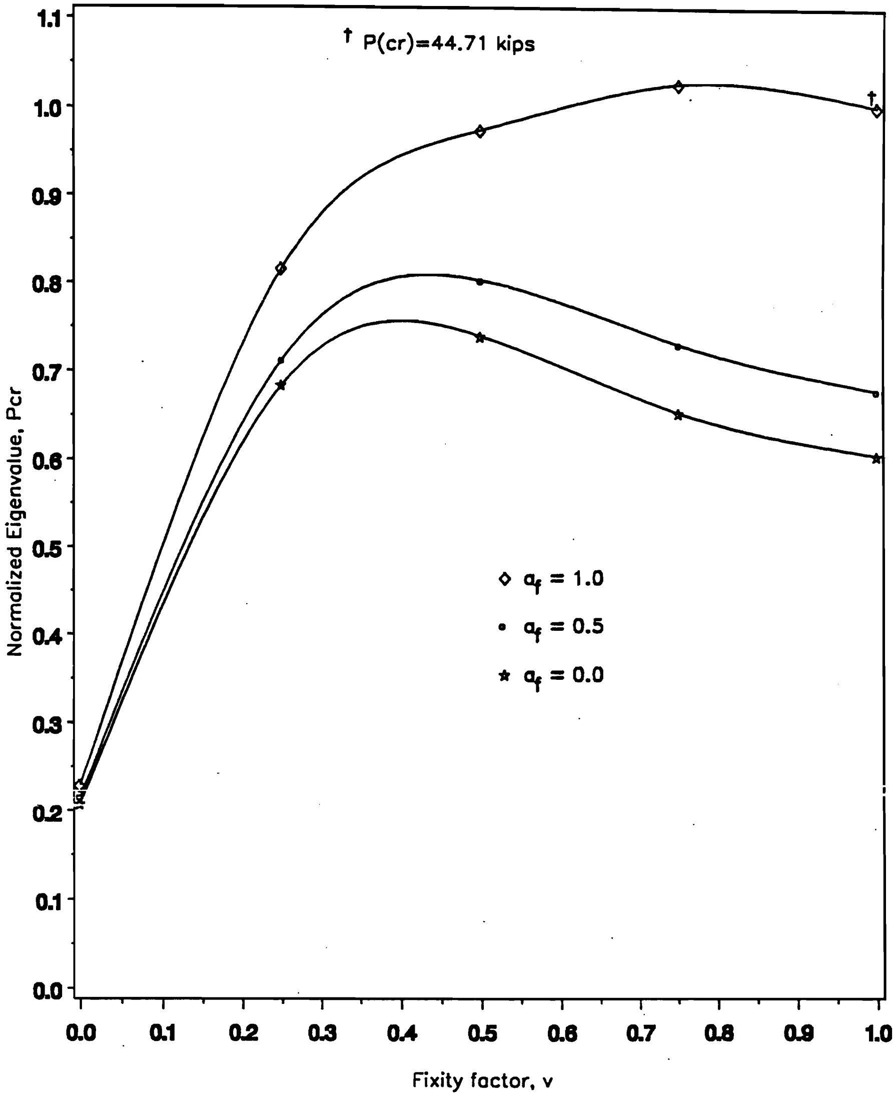


Figure 5: Stability Results for $4 \times 2.25 \times 0.105$ Section

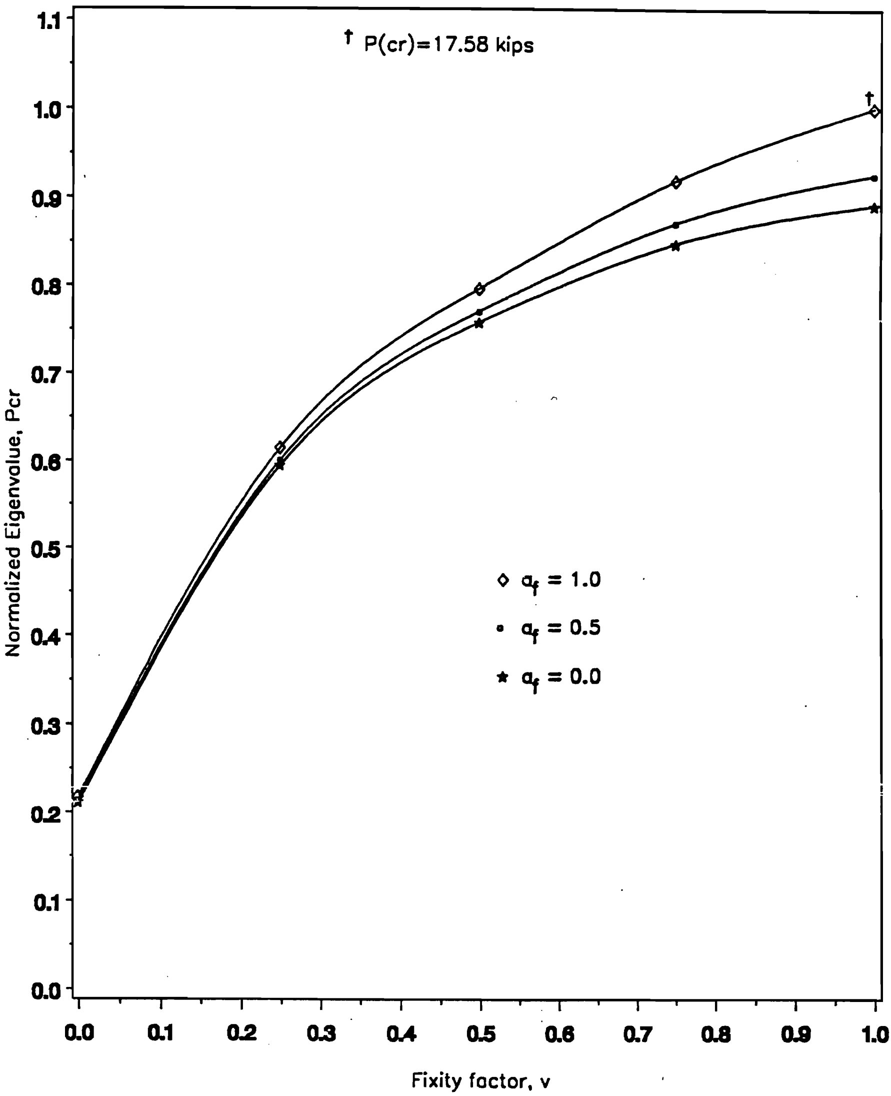


Figure 6: Stability Results for $2 \times 2.25 \times 0.075$ Section

Appendix I - Notation

χ	Warping degree of freedom
λ	Eigenvalue
λ_{cr}	Critical eigenvalue
ν	Bending fixity factor
Φ_{cr}	Critical eigenvector corresponding to the critical eigenvalue
$\theta_x, \theta_y, \theta_z$	Rotational displacement about the axis indicated by the subscript
A	Cross-sectional area
a_f	Warping fixity factor
C_w	Cross-section warping constant
E	Elastic modulus
F_x, F_y, F_z	Force applied in the direction indicated by the subscript
G	Shear modulus
I_y	Weak axis moment of inertia
I_z	Strong axis moment of inertia
J	Torsion constant
K_{θ_x}	Connection torsion stiffness
K_{θ_y}	Weak axis connection rotation stiffness
K_{θ_z}	Strong axis connection rotation stiffness
$[K_E]$	Elastic stiffness matrix
$[K_G]$	Geometric stiffness matrix
L	Member Length
M_x, M_y, M_z	Moment applied about the axis indicated by the subscript
p	A constant which equals $\sqrt{GJ/EC_w}$
u_x, u_y, u_z	Translational displacement in the axis indicated by the subscript

Appendix II - References

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