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LOCAL BUCKLING OF STIFFENED AND UNSTIFFENED ELEMENTS UNDER NONUNIFORM COMPRESSION

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ABSTRACT

Thin plates subjected to linearly varying inplane compression in one direction may undergo local buckling before failure. An analytical procedure is presented for evaluating the local buckling strength based on which equations for the local buckling stress of unstiffened and stiffened elements are presented.

INTRODUCTION

Thin walled members are composed of plate elements which are supported along both edges parallel to the direction of compression (referred to as stiffened elements) and supported along only one edge parallel to the direction of compression with the other completely free (referred to as unstiffened elements). These thin plate elements may experience elastic local buckling and stable postbuckling behaviour when subject to inplane compressive, bending or shear stress. Due to initial imperfections, the bifurcation type of local buckling indicated by small deflection theory is not usually experienced by elements of commercially manufactured thin walled members. However, the postbuckling behaviour expressed in the form of effective width equations is a function of the elastic local buckling stress and hence the theoretical calculation of elastic local buckling stress of thin walled elements is of practical interest. Furthermore, the out of plane deflection of imperfect plates increases drastically at local buckling stress and hence is of interest to designers.

Stowell (1939), Timoshenko (1961), Winter (1959), and Kalyanaraman (1979), have presented methods for evaluating local buckling strength of thin plate elements and members subjected to uniform in plane compression. Rhodes and Harvey (1971) and Walker (1967), have presented analytical procedures for evaluating the local and postbuckling behaviour of thin walled stiffened and unstiffened elements subjected to linearly varying in plane compression. Ramakrishna and Kalyanaraman (1984) have presented closed form equations for local and post buckling strength of thin walled stiffened elements subjected to linearly varying inplane compression based on regression of analytical results.

In this paper Galerkin's procedure has been used to solve the governing differential equation for calculating the local buckling stress of non-uniformly compressed stiffened and unstiffened elements having elastically rotationally restrained longitudinal edges. Through regression, equations developed [Jayabalan (1989)] for the local buckling coefficient are presented as a function of the edge rotational restraint factor, and the non-uniform compression factor. The results of the proposed equations are compared with experimental results.

ANALYTICAL STUDY

Governing Equations

A thin flat rectangular plate compressed by linearly varying displacements in the longitudinal direction as shown in Fig. 1 is considered, where the unloaded longitudinal edges may have one of the following boundary conditions. Both edges are completely restrained against out of plane translation

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and elastically restrained against rotation (stiffened element); only one edge is completely restrained against out of plane translation and other edge is completely free (unstiffened element).

The basic equation governing the elastic behaviour of buckled elastic plate have been derived by Von Karman (1932) and may be written in the non- dimensional form as

$$\frac{1}{\phi^2}\frac{\partial^4\omega}{\partial\xi^4} + 2\frac{\partial^4\omega}{\partial\xi^2\partial\eta^2} + \phi^2\frac{\partial^4\omega}{\partial\eta^4}$$

$$= 12(1-\mu^2)\left[\frac{\partial^2 F}{\partial\xi^2}\frac{\partial^2\omega}{\partial\eta^2} + \frac{\partial^2 F}{\partial\eta^2}\frac{\partial^2\omega}{\partial\xi^2} - 2\frac{\partial^2 F}{\partial\xi\partial\eta}\frac{\partial^2\omega}{\partial\xi\partial\eta}\right] \quad \text{.........(1)}$$

where ξ = x/l; η = y/b; ω is the non-dimensional form of the displacement (ω = w/t) and F is the non-dimensional form of the stress function, defined as :

$$\sigma_\xi = \frac{\sigma_x l^2}{\phi^2 E t^2} = \frac{\partial^2 F}{\partial\eta^2};$$

$$\sigma_\eta = \frac{\sigma_y l^2}{E t^2} = \frac{\partial^2 F}{\partial\xi^2};$$

$$\tau_{\xi\eta} = \frac{\tau_{xy} l^2}{\phi^2 E t^2} = \frac{-\partial^2 F}{\partial\xi\partial\eta} \quad \text{.........(2)}$$

The applied linearly varying displacement may be represented by

$$U = (1 - 0.5\,\alpha + \alpha\eta)\,U_{max} \quad \text{......(3)}$$

where U is the non-dimensional inplane displacement at any point along the width, U_{max} is the non-dimensional inplane displacement at edge η = + 0.5 and α is the non-uniform compression factor defined as

$$\alpha = 1 - \frac{U_{min}}{U_{max}} \quad \text{.........(4)}$$

α = 0, 1, 2 correspond to the uniform compression, triangular compression and pure bending cases, respectively.

The differential equation governing the local buckling stress due to the unidirectional compression may be obtained after disregarding the terms on the right hand side of Eqn.1, except that corresponding to Eqn.3. The value of σ_ξ is set equal to the non-uniform stress as determined by the inplane displacement given by

$$\sigma_\xi = U_{max}(1 - 0.5\,\alpha + \alpha\eta) \quad \text{.........(5)}$$

At the local buckling stress, σ_ξ of stiffened and unstiffened elements can be expressed as

$$\sigma_{\xi,cr} = \frac{K\pi^2 E}{12(1-\mu^2)\,(b/t)^2}$$

where K is the buckling coefficient and μ is the Poission's ratio.

While using Galerkins method for solving Von-Karman's partial differential equation for large deflection of plates, the following deflected shape of the locally buckled plate is obtained.

$$\omega = \sum_{n=0}^{L} q_n Y_n(\eta) \sin \pi\xi \qquad \text{.......(7a)}$$

$$Y_n(\eta) = \sum_{r=0}^{4} C_{nr}\, \eta^{(n+r)} \qquad \text{.......(7b)}$$

The accuracy of the results depend upon the number of terms used in the deflection function. In this study three terms have been used since it was found that three terms are adequate to obtain good results for the standard cases already solved. The value of C_{nr} can be obtained by satisfying the boundary conditions along the unloaded edges of the elements. The elastic edge rotational restraints are defined by a factor ε [$\varepsilon = (M/\theta) \times (b/d)$]. The boundary conditions along the loaded edges are automatically satisfied by the assumed displacement field.

By substituting the deflected shape from equation (7) into the governing differential equation (1) and using Galerkin's method, one gets

$$\int_{-0.5}^{0.5} \left[\frac{d^4Y_n}{d\eta^4} - 2\frac{\pi^2}{\phi^2}\frac{d^2Y_n}{d\eta^2} + \frac{\pi^4 Y_n}{\phi 4} + \frac{\pi^4 K}{\phi^2} U\,(1 - \alpha/2 + \alpha\eta)Y_n\right] Y_j\, d\eta = 0 \qquad \text{.........(8)}$$

where ϕ is the ratio of wave length to width of the plate perpendicular to direction of compression. Performing the integration and varying the values of n and j over the range $0 \le n, j \le 3$, a square matrix of the following form is obtained.

$$[\,[L] - K\,[M]\,]\,\{q_n\} = 0 \qquad \text{.........(9)}$$

Equation (9) is in the form of an eigen value problem. The lowest eigen value and the corresponding eigen vector are of interest in the local buckling analysis, and are obtained using power method of solution. The value of 'K' varies with ϕ and the lowest value of 'K' is of practical interest.

The elastic local buckling coefficient K_e of stiffened and unstiffened elements are functions of the rotational edge restraint factor and non- uniform compression factor. The elastic local buckling coefficient for stiffened and unstiffened elements obtained from numerical solution of Eqn. (9) are plotted in Fig. 2 and 3. Through a regression analysis, the curves shown in Fig. 2 and 3 can be represented by the following equations with a maximum error of 6% and mean error of 2.1% and RMS error of 0.35%

Stiffened Element

$$K_{c,s} = C_1 + C_2 \frac{\varepsilon_1^{0.94} - 7.47}{\varepsilon_1^{0.94} + 7.47}$$

$$C_1 = 5.48 + 0.71 \frac{\varepsilon_2^{1.05} - 8.76}{\varepsilon_2^{1.05} + 8.76} + 3.43\,\alpha - 1.37\,\alpha^2 + 3.13\,\alpha^3$$

$$C_2 = 0.75 + 0.04 \frac{\varepsilon_2^{1.6} - 163.72}{\varepsilon_2^{1.6} + 163.72} + 0.87\,\alpha - 0.17\,\alpha^2 + 0.66\,\alpha^3 \qquad \text{.......(10)}$$

Unstiffened Elements

$$K_{c,u} = A + B \frac{\varepsilon^{0.7} - 1.5}{\varepsilon^{0.7} + 1.5}$$

(a) Maximum compression at free edge:

$$A = 0.853 + 0.195\,\alpha + 0.016\,\alpha^2 + 0.023\,\alpha^3$$

$$B = 0.426 + 0.078\,\alpha + 0.015\,\alpha^2$$

(b) Maximum compression at supported edge :-

$$A = 0.836 + 1.693\,\alpha - 3.914\,\alpha^2 + 5.154\,\alpha^3$$

$$B = 0.415 + 0.967\,\alpha - 2.402\,\alpha^2 + 3.086\,\alpha^3 \qquad \text{............(11)}$$

The values for the elastic local buckling coefficients obtained from Eqn. (10) and (11) are compared with the analysis results in Figs. 2 and 3.

EXPERIMENTAL INVESTIGATION

In order to study the local buckling strength of non-uniformly compressed stiffened and unstiffened elements, thirty four specimens having twenty six unstiffened elements and eight stiffened elements subjected to non-uniform compression were tested. The details of the testing frame, cross section dimensions, instrumentation etc., are given in Jayabalan (1989) and Ramakrishna et.al (1984). The specimens were tested for the cases of non- uniform compression covering the entire range of uniform compression to uniform bending, where the experimental local buckling stress was evaluated using the strain reversal method. The cross section dimensions, non-uniform compression factor and local buckling stress of all the specimens tested are given Table 1 (σ_{cr}, exp, col.5). These experimental values have been used below for comparison with analytical methods.

COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS

For the specimens, the elastic local buckling stress calculated using Eqn. (10) and (11) are presented in Table 1 (σ_{cr}, col.6). It is seen from column 7 in Table 1, that the percentage difference between the theoretical elastic local buckling stress and experimental local buckling stress increases as the ratio of elastic local buckling stress to yield stress increases. It is also well known that both in plate and column buckling the effects of initial imperfection and residual stresses become larger as the elastic local buckling stress reaches the yield stress. In order to account for these effects, a Perry-Robertson type of formula as given below may be used.

$$\sigma_{cr} / \sigma_y = \frac{1}{[1 + \sqrt{(\sigma_y / \sigma_{cr,e})}^{\,2n}\,]^{1/n}} \qquad ....(12)$$

It is found that a value of n' equal to 2.0 gives good correlation with test results. In Table 1, column 9, the theoretical local buckling stress calculated using Eqn. (12) is presented. It is seen that the comparison between the modified local buckling stress and experimental values are much better. The mean difference between the theoretical and experimental values is 7.7% on the conservative side, and the coefficient of variation is 13.2%.

SUMMARY AND CONCLUSIONS

An analytical method for calculating the local buckling coefficient of stiffened and unstiffened elements subjected to linearly varying compression was presented. Closed form equations for calculating the local buckling coefficient of members with stiffened and unstiffened compression elements derived through regression analysis were also given. Results of the experiments conducted on stiffened and unstiffened elements were compared with the analytical values of local buckling stress. It was shown that the closed form regression equations for the local buckling strength of stiffened and unstiffened elements compare well with the test results.

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TABLE 1 - COMPARISON OF LOCAL BUCKLING STRESS

No.	b/t	α	ε	σ_{exp} MPa	$\sigma_{cr,e}$ MPa	% error 5 & 6	σ_Y MPa	σ_{cr} MPa	%error 5 & 9
1	2	3	4	5	6	7	8	9	10
U1	45.3	-0.18	1.68	68.5	80.4	17.5	177.6	73.2	6.9
U2	45.8	0.29	1.29	83.1	94.7	13.9	254.8	88.7	6.7
U3	45.8	-0.31	1.72	81.3	81.3	0.0	155.2	72.0	-11.4
U4	47.1	0.38	1.65	94.6	99.7	5.4	178.7	87.0	- 8.0
U5	46.3	-0.61	1.86	76.1	86.6	13.8	159.1	76.1	0.0
U6	45.6	0.66	1.84	140.6	156.8	11.5	287.3	137.6	- 2.1
U7	65.1	-0.11	1.33	32.7	36.7	12.2	163.1	35.8	9.4
U8	62.1	0.16	0.98	38.4	44.7	16.4	145.3	42.7	11.1
U9	60.8	-0.44	1.54	41.1	46.7	13.6	144.7	44.4	8.0
U10	64.2	0.49	1.77	60.0	60.5	0.8	159.5	55.5	- 7.5
U11	62.3	-0.51	1.58	40.1	45.4	13.2	156.0	43.6	8.7
U12	65.2	0.53	1.78	62.6	61.7	- 1.4	188.3	58.6	- 6.3
U13	30.3	-0.21	1.97	108.2	185.8	71.7	150.1	116.7	7.8
U14	30.2	0.21	1.39	125.4	209.6	67.1	152.0	123.0	- 1.9
U15	29.9	-0.23	2.04	130.0	192.9	48.3	156.5	121.5	- 6.5
U16	30.1	0.29	1.67	126.2	229.7	82.0	169.2	136.2	7.9
U17	84.1	-0.28	1.14	20.1	22.3	10.9	318.4	22.2	10.4
U18	82.8	0.32	1.05	25.7	28.4	10.5	294.2	28.3	10.1
U19	83.2	-0.54	1.34	22.2	24.9	12.1	294.2	24.8	11.7
U20	81.7	0.53	1.39	34.3	37.6	9.6	243.4	37.2	8.4
U21	44.1	-1.56	1.67	109.3	119.7	9.5	284.5	110.3	0.9
U22	59.4	-1.63	1.38	69.1	66.9	- 3.2	249.8	64.3	- 6.9
U23	86.2	-1.74	0.82	28.2	30.3	7.4	305.1	30.0	6.2
U24	44.7	-1.34	1.71	101.3	111.6	10.2	248.8	100.4	- 0.9
U25	59.7	-1.42	1.33	55.7	61.5	10.4	230.0	58.9	5.6
U26	88.9	-1.56	0.92	26.3	27.3	3.8	300.8	27.3	3.6
S1	148.0	0.00	6.20	33.3	45.0	35.1	224.6	44.1	32.4
S2	148.0	0.16	5.70	35.0	50.1	43.1	224.6	48.9	39.7
S3	208.0	0.77	7.40	26.3	37.1	41.0	224.6	36.6	39.2
S4	208.0	0.80	8.90	26.5	38.4	44.9	224.6	37.8	42.6
S5	241.0	2.00	70.40	103.3	125.4	21.4	224.6	109.5	6.0
S6	225.0	1.92	60.30	101.5	131.2	29.3	224.6	113.3	11.6
S7	220.0	1.76	59.50	93.8	114.2	21.7	224.6	101.8	8.5
S8	220.0	1.48	58.40	69.4	82.9	19.5	224.6	77.8	12.1

Unstiffened Element, S - Stiffened Element

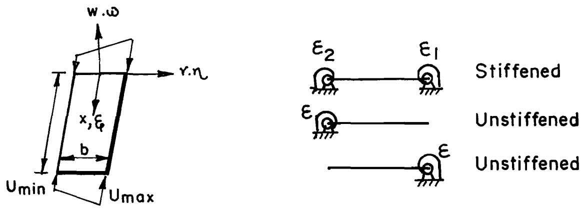

FIG. 1 PLATE ELEMENT MODEL

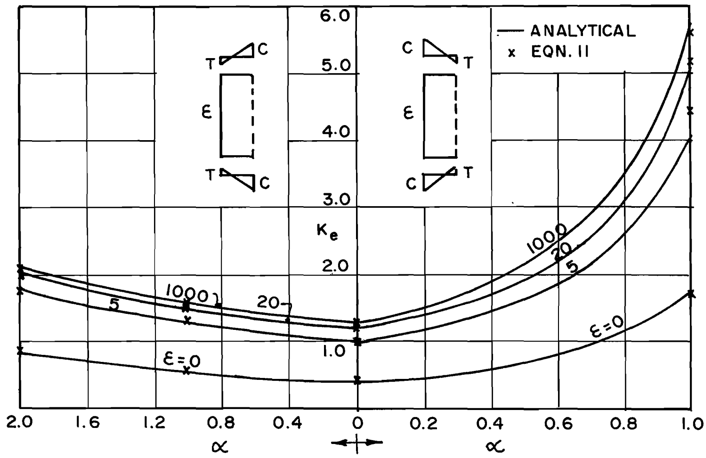

FIG. 2 LOCAL BUCKLING COEFFICIENT OF UNSTIFFENED ELEMENTS

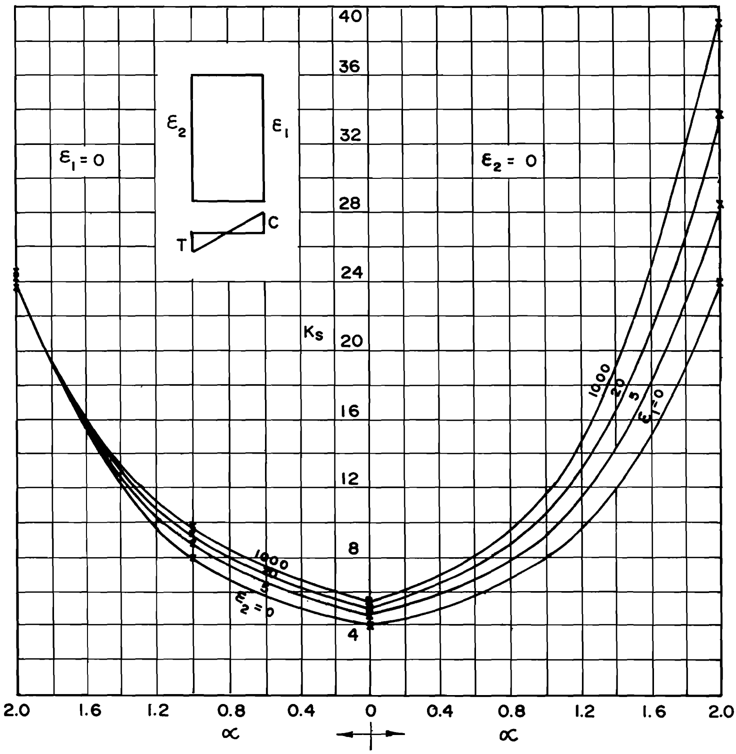

FIG.3 LOCAL BUCKLING COEFFICIENT OF STIFFENED ELEMENTS

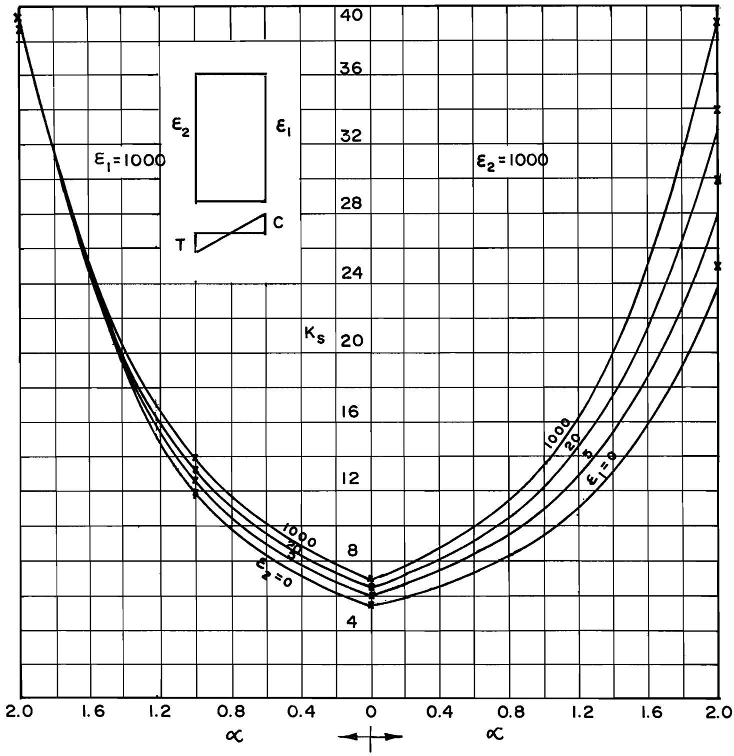

FIG. 4 LOCAL BUCKLING COEFFICIENT OF STIFFENED ELEMENTS