# A Probabilistic Examination of the Ultimate Strength of Cold-formed Steel Elements

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# A PROBABILISTIC EXAMINATION OF THE ULTIMATE STRENGTH OF COLD-FORMED STEEL ELEMENTS

**Benjamin Schafer[1], Mircea Grigoriu[2], & Teoman Peköz[3]**

## ABSTRACT

This paper investigates the ultimate strength of cold-formed steel plates in uniform compression and pure bending with the goal of determining the statistical characteristics (mean and variance) of the ultimate strength. Plate thickness, longitudinal residual stress magnitude, and imperfection magnitude are considered as random variables. Based on existing data appropriate distributions are determined for these three random variables. Using ABAQUS for the strength prediction, two methods: Monte Carlo simulation and Taylor series approximation, are employed to determine statistical characteristics of the plates. The results are compared to the deterministic approach of the AISI Cold-Formed LRFD Specification.

## 1 INTRODUCTION

The sensitivity of some cold-formed steel members to imperfections results in a relatively wide range of scatter when the ultimate strength of cold-formed steel members are evaluated experimentally. As a result of this variability, it is important to perform a probabilistic examination in addition to the usual deterministic approaches. While it is impossible to address all cold-formed steel members for any conceivable loading, it is possible to examine the probabilistic behavior of cold-formed steel elements. Since typical members can be idealized as a composition of different elements, a study of the elements themselves allows an insight into member strength. Comparison of the probabilistic results to existing specifications may highlight the shortcomings of existing approaches and also serve to show the actual variability that exists in the strength prediction of typical cold-formed steel elements and members.

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To lay a foundation for comparison, a deterministic approach is presented first. This approach is typical of the specifications and codes used throughout the world for strength prediction of cold-formed steel. Next, the random variables: thickness, residual stress magnitude, and imperfection magnitude are discussed and appropriate distributions are determined. With that established, the details of the model for an element in pure compression and pure bending are presented. With the problem fully defined, Monte Carlo simulation and Taylor series approximation are used to determine the mean and variance of the ultimate strength of the elements. With the statistical characteristics of the elements known discussion and comparisons are made to the deterministic approach.

## 2 DETERMINISTIC APPROACH

Before beginning a probabilistic examination of cold-formed steel members the deterministic approach of the AISI Specification[1] is presented. In order to examine the capacity of a cold-formed steel member the strength of the component elements must be determined. For instance, a hat-shaped member in flexure (with the top in compression) consists of the component elements shown in Fig. 1. Where the numbering refers to: (1) stiffened element under uniform compression; (2) stiffened elements under stress gradient; (3) unstiffened elements under uniform tension.

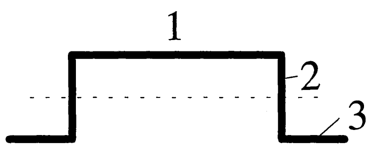



Figure 1: Hat Section and Definition of Cold-Formed Steel Elements

Each component element is typically investigated as a separate plate. Therefore, element 1 can be considered as a simply supported plate under uniform compression. Element 2 is considered as a simply supported plate under a linear stress gradient, tension stress on the bottom, compression stress on the top. The ultimate strength of these plates is determined by using an effective width approach. A plate with an effective width acting at the yield stress of the material is found such that it has the same strength as the actual plate which fails with a nonlinear stress distribution.

### 2.1 Stiffened Element in Pure Compression

The ultimate strength of a stiffened element in pure compression is determined by first calculating the elastic buckling stress and then using Winter's equation to determine the effective width. Winter's equation is an empirical correction to Von Karman's earlier work. The expression accounts for the experimental scatter Winter observed in his and others tests. The equations presented below use the same expressions as in the AISI Specification but are rewritten in a more convenient form for the purposes of this study. The ultimate load a plate may carry in pure compression ($P_{ult}$) can be expressed as:

$$P_{ult} = \lambda P_{cr}$$
$$\text{for } F_{cr} < 2.2F_y$$
$$\lambda = \sqrt{\frac{F_{cr}}{F_y}}\left(1 - 0.22\sqrt{\frac{F_{cr}}{F_y}}\right)\frac{F_y}{F_{cr}}$$
$$P_{cr} = F_{cr}wt$$

where $F_{cr}$ is the linear elastic buckling stress. Using such an approach the width to thickness ratio (*w/t*) completely determines the ultimate strength for a particular set of material properties. Fig. 2 shows how the yield strength, ultimate strength as predicted by AISI, and elastic buckling strength compare for different *w/t* values ($E$=2.03×10$^5$MPa, $F_y$=345MPa). It is clear that for *w/t* values above 100 the ultimate strength increases little.

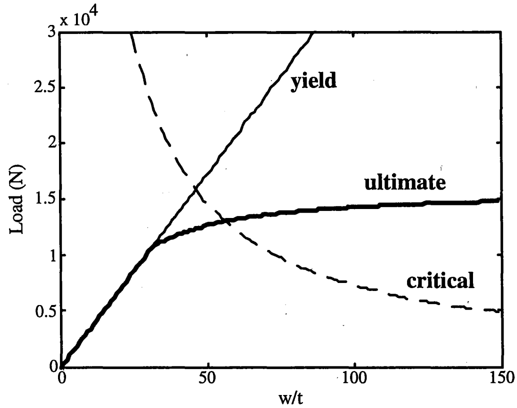



Figure 2: Element in Uniform Compression

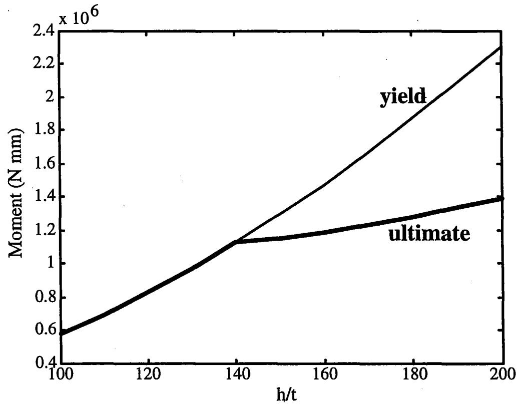



Figure 3: Element in Pure Bending

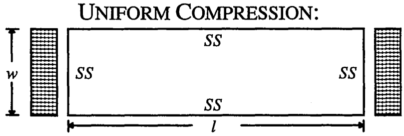
### 2.2 Stiffened Element under a Stress Gradient

For elements under a stress gradient, the AISI Specification uses an effective width procedure similar to that of an element in pure compression. The elastic buckling load is calculated for the entire element, with the plate buckling coefficient suitably modified to account for the loading. Winter's equation is then used to determine an effective width for the entire element. This information is used to determine effective portions for the compression zone of the element. Figure 3 shows how the yield strength, and nominal strength as predicted by AISI, compare for different *h/t* values ($E$=2.03×10$^5$MPa, $F_y$=345MPa).

## 3 PROBABILISTIC MODEL

In order to perform a probabilistic examination of cold-formed steel elements the random and deterministic parts of the problem must be determined. Two component elements selected for further study and the possible parameters investigated are shown in Fig. 4. All the parameters listed could potentially be considered random variables. In some cases the variation would be small or not of interest. In order to focus the investigation, only three quantities: plate thickness ($T$), residual stress magnitude ($R$), and imperfection magnitude ($I$) are considered random. Thickness is selected as a random quantity, primarily because of a provision in the AISI Specification discussed below. Residual stresses are selected for study, because while it is generally agreed that they influence the strength, no agreed upon magnitude exists for typical cold-formed steel members. Imperfections are considered because cold-formed steel members are known to be imperfection sensitive, but the exact mechanisms that cause these imperfections are not known. Hence, a deterministic procedure is not particularly meaningful when examining imperfections. All other quantities are deterministic. Thus, for the purposes of this study, when investigating the ultimate strength of a particular element only the variation from $T$, $R$, and $I$ contribute to the variation present in the ultimate strength.

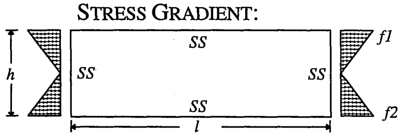
STIFFENED ELEMENTS UNDER UNIFORM COMPRESSION:




*Possible Parameters to be considered:*

- length of plate
- width of plate
- thickness of plate
- modulus of elasticity
- yield stress
- magnitude of residual stresses
- magnitude of imperfections
- distribution of imperfections
- distribution of residual stresses
- plate boundary conditions

STIFFENED ELEMENT UNDER STRESS GRADIENT:




*Possible Parameters to be considered:*

- f1/f2 ~ ratio of applied stress
- length of plate
- height of plate
- thickness of plate
- modulus of elasticity, yield stress
- magnitude of residual stresses
- magnitude of imperfections
- distribution of imperfections
- distribution of residual stresses
- plate boundary conditions

Figure 4: Component Elements (Plates) for Investigation

## 3.1 General Input Parameters

### 3.1.1 Thickness

The statistical characteristics of the thickness of cold-formed steel has been previously investigated[2]. Galambos et al. report that the thickness ($T$) exceeds the design thickness ($t_d$) by 6% and the coefficient of variation (standard deviation/mean) of $T/t_d$ is reported as 0.053. The AISI specification states that if the delivered thickness is 95% of the design thickness then the section is adequate. As a result, for comparison to the AISI Specification, it is appropriate to assume that $T$ is approximately equal to $t_d$. For this study the nominal thickness of interest will be 1mm. Therefore, the random variable $T$ will be assumed normally distributed with mean of 1mm and standard deviation of 0.053mm as shown in Fig. 5. Approximating $T$ as a normal distribution is a reasonable and simplifying assumption. However, it should be remembered that the normal distribution is defined from $[-\infty,+\infty]$. Therefore, negative values of $T$ are possible. Of course, in this case the probability content is negligible for negative realizations of $T$.

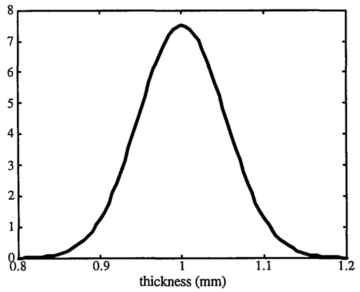


Figure 5: Probability Density Function for $T$

### 3.1.2 Imperfections

The magnitude and distribution of imperfections in cold-formed steel sections has only seen limited study. Imperfections are a function of the thickness of the sheet, the forming process, shipping, handling, installation and other factors. A simplified method for modeling imperfection distributions (often used in analytical solutions) is to assume a distribution equal to one of the buckling (eigen)modes. In this analysis, the first mode is used as the distribution of the imperfection. In order to determine imperfection magnitudes, studies for C-shaped sections and trapezoidal sections are examined[3,4]. The maximum deviation on each measurement line as estimated from graphs is recorded in Appendix Table A1. A histogram of the imperfection data and a lognormal distribution fitted to the data are shown in Fig. 6. Based on this examination the random Variable $I$ is assumed to have a lognormal distribution with a mean of 0.73mm and a standard deviation of 0.0424mm.

### 3.1.3 Residual Stress

The creation of residual stresses is a complex process, storing of the coils, the rolling of the member, final straightening, etc. all contribute to the creation. Nonetheless, efforts have been made to characterize residual stresses. Collected data[4,5,6] on residual stresses are shown in the Appendix Table A2. Typical residual stress distributions have tension on the outside, and compression on the inside. of the plate thickness. The stresses are higher at the corners, and the net stress or membrane stress is generally close to zero. A histogram of the absolute value of the surface residual stress in the flat regions and a uniform, probability density function are plotted in Fig. 7. From this limited data it is concluded that the net residual stress can be considered zero in the flat sections; tension on top, compression on bottom, as observed by Weng[5]. The higher state of residual stress in the small corner areas is neglected in this analysis. For this study, $R$ is assumed to be uniformly distributed over a range of 0 to 55%*Fy*.

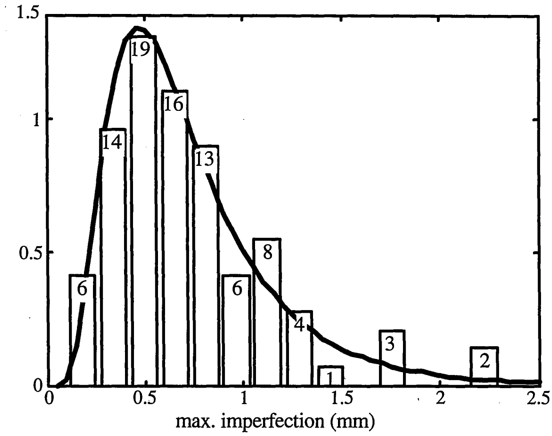



Figure 6: Histogram and Probability Density Function for *I*

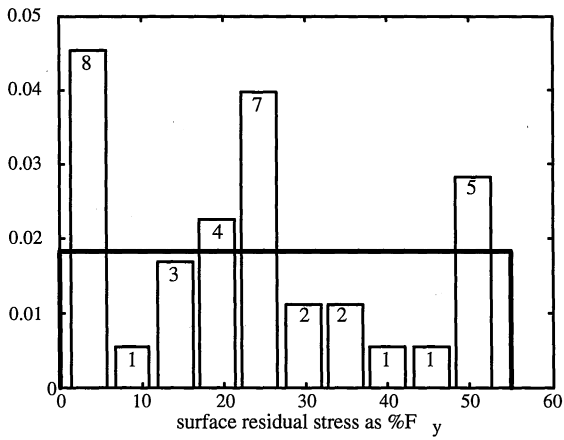



Figure 7: Histogram and Probability Density Function for *R*

### 3.1.4 Material Properties

To model elastic perfectly plastic behavior the modulus of elasticity, Poisson's ratio, and the yield stress are sufficient. For this study all of these parameters are considered deterministic with $E$=2.03×10$^5$MPa, ν=0.3, and $F_y$=345MPa. For virgin steel (no residual stress) a typical tensile coupon is essentially elastic perfectly plastic. However, tensile coupons taken from cold-formed steel elements generally exhibit nonlinear behavior. This nonlinear stress-strain behavior is largely due to the residual stresses existing in the section. Therefore, by including residual stresses in the model the nonlinear portion of the stress-strain diagram is actually captured. In addition, because the residual stresses are random, the variation inherent in the material behavior is also partially captured. One factor not considered in this study, is the variability of the yield stress of the material. In general, the yield stress is expected to be greater than the design yield stress, in this analysis it will be assumed equal and deterministic. This assumption introduces a certain degree of conservatism in to the analysis.

## 3.2 Stiffened Element Under Uniform Compression

### 3.2.1 Length

The length of an element in a cold-formed steel section is generally much larger than the width. For plates with length to width ratio ($l/w$) greater than 4 the elastic buckling stress is reliably estimated by a single buckling coefficient value. Analysis is conducted using a plate with $l/w$ equal to 4 to avoid the effect of length on the solution.

### 3.2.2 Width

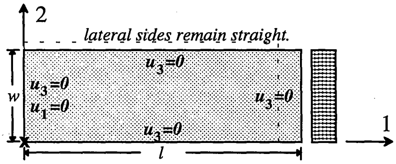
The width to thickness ratio ($w/t$) of the plate is important for determining the ultimate load the plate may carry. In order to fully examine the behavior of stiffened elements under uniform compression, a continuum of different $w/t$ ratios should be investigated. For solutions by simulation, computation of the statistical characteristics at a variety of $w/t$ values is prohibitive because of the number of computer runs required. However, if Fig. 2. captures the overall behavior adequately, then any element with a $w/t$ greater than 50 is significantly effected by local buckling, and thus of interest. As an example, and for simplicity, $w/t$ shall be taken as 100.

### 3.2.3 Boundary Conditions

The boundary conditions of the plate have an important role in determining the ultimate strength. The AISI Specification assumes that a stiffened element under uniform compression can be modeled as a simply supported plate. This assumption is generally conservative, simple, and justified by the fact that at failure significant yielding occurs at the edges of the plates, thus limiting the rotational stiffness in these corner regions. Modeling a simply supported edge leads to ambiguity in the boundary conditions. The moment on all the edges should remain zero, but how does this translate into restrained degrees of freedom? Two possible solutions are: the lateral sides are free to displace in the plane of the plate, but they must remain straight (Case 1), and the lateral sides are completely free to displace in the plane of the plate (Case 2.)

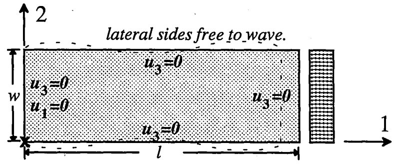
ABAQUS was used to analyze the following problem: What is the ultimate load in uniform compression for the two cases? Given: $l$=400mm, $w$=100mm, $T$=1mm, $E$=2.03×10$^5$MPa, $F_y$=345MPa, $R$=27.5%$F_y$, $I$=0.72mm. Fig. 8 shows the two cases and analysis results. Case 1 corresponds to a plate connected to a "stiff" web, this model is typically used to examine the problem analytically[7,8]. Case 2 corresponds to a plate connected to a weak, or slender web. The behavior and ultimate strength of these two members is quite different. The AISI specification uses the same procedure for analysis of the elements regardless of whether it is connected to a stiff or slender web. Case 2 is selected for this study.

Case 1: Connected to Stiff Web




$P_{ult.} = 2.66 P_{cr}$

Case 2: Connected to Slender Web




$P_{ult.} = 2.19 P_{cr}$

Figure 8: Comparison of Plate Boundary Conditions

## 3.3 Stiffened Element Under Stress Gradient

### 3.3.1 Ratio of Applied Stresses

A stiffened element under a stress gradient covers loading from pure compression to pure tension. However, it is typical that the stiffened element referred to is a web of a member, rather than an element under arbitrary load. For webs, the stress distribution is that of tension on one side, compression on the other. The most basic of this type of loading is pure bending, and therefore for this analysis the pure bending case is investigated.

### 3.3.2 Height

In order to fully examine the behavior of stiffened elements under pure bending, a variety of different *h/t* ratios should be investigated. As Fig. 3 demonstrates the most interesting elements in pure bending are influenced by local buckling in the compression region. In order to investigate a typical element which is susceptible to buckling in bending, *h/t* is selected as 200.

### 3.3.3 Boundary Conditions

The AISI specification assumes that a simply supported plate is adequate for evaluating the element strength. If the lateral (unloaded) sides are forced to stay straight it is analogous to a flexural member with strong flanges. If the lateral sides are free to wave this corresponds to a slender flange. For this analysis, the lateral sides are assumed free to wave in the plane of the plate. The loaded edges are restricted to only move linearly across the edge.

# 4 PROBABILISTIC MODEL: SOLUTION METHODOLOGIES

## 4.1 Monte Carlo Simulation

Thickness (*T*), residual stress (*R*), and imperfection magnitude (*I*) are selected as random. Using the distributions of Figures 5-7, realizations of *T*, *R*, and *I* are generated. Analysis is conducted for a sample of *T*, *R*, and *I*. The results are recorded (in this case the ultimate strength at failure), a new sample of *T*, *R*, and *I* is selected, and another analysis is conducted. If enough samples are used and the process is "stable" then the generated sample mean and variance of the simulation give a reliable approximation of the actual mean and variance of the process.

### 4.2 Taylor Expansion

With ABAQUS as a tool, the ultimate strength becomes a scalar field in T, R, and I that can be sampled pointwise. A Taylor expansion of an unknown function is possible in this situation, because the derivatives necessary for the expansion can be readily approximated by central differences. Thus, it is possible to determine a functional form for the ultimate strength of the element. If the ultimate strength is denoted as *U,* then for a second order expansion this results in an equation of the following form:

$$U = aT + bR + cI + dT^2 + eR^2 + fI^2 + gTR + hRI + jTI + k$$

Where, *a* through *k* are determined by the Taylor expansion. Since the distribution of the random variables T, R, and I is known, the mean and variance of *U* can be readily calculated. (In fact, any moment of *U* can be generated.)

## 5 PROBABILISTIC MODEL: RESULTS

### 5.1 Plate in Uniform Compression

#### Monte Carlo Simulation

The greatest difficulty in using MC simulation is determining the necessary number of samples. Of course more is better, but the analysis time for each sample is on the order of a few hours. (Analysis for this work is conducted on a VAXstation-4000-60 using ABAQUS version 5.3-1.) The mean of the ultimate strength multiplier through 100 samples of *T*, *R*, and *I* is shown in Figure 10. The output from the analysis is λ, the ultimate strength of the plate is $P_{ult} = \lambda P_{cr}$.

After only a few samples it is clear that *T* is the most influential parameter of the problem. The AISI Specification for the ultimate strength is a function of *T*. Simulation of the ultimate strength via the AISI Specification using 500 samples of *T* is shown in Figure 9. After 100 samples the AISI simulation settles down considerably. This is and indication that 100 samples is an adequate amount for the MC simulation.

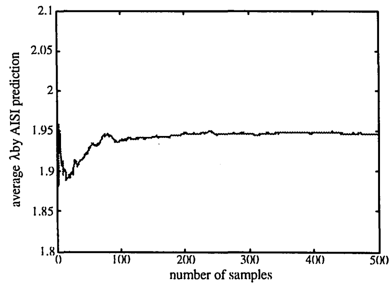



Figure 9: Results for AISI Simulation

Figure 10 visually demonstrates the convergence of the strength multiplier λ. Numerically, it is found that the MC simulation gives a mean for λ of 2.17 and a variance of 0.048. In addition to examining the mean and standard deviation of λ, how the random variables *T*, *R*, and *I* are related to λ is also investigated (Figures 11-13). For the 100 samples, it is clear by looking at Figure 11, that thickness is well correlated with the final answer. Conclusions about the effect of residual stress and imperfection are not as straightforward. However, if a linear regression is performed on the thickness data (fit a straight line to data in Figure 11) the residuals reveal that large imperfections and residual stresses do decrease the strength (as is intuitively expected).

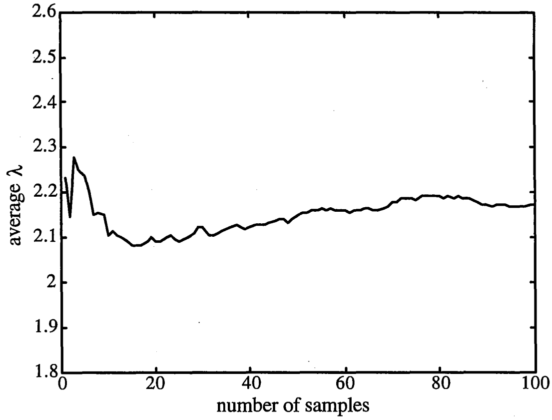



Figure 10: Average of Strength Multiplier λ

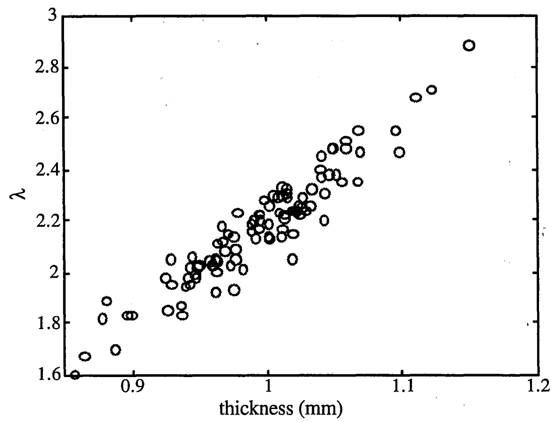



Figure 11: Sample Results for Thickness

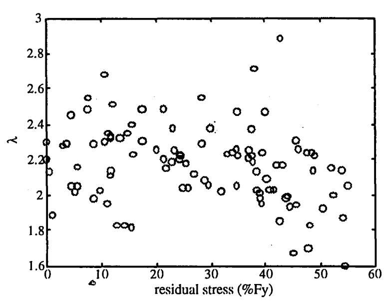



Figure 12: Sample results for Residual Stress

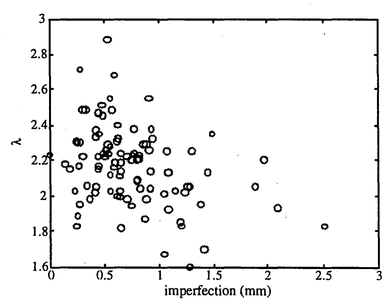



Figure 13: Sample Results for Imperfection

**Taylor Expansion**

To calculate the central differences necessary for the derivatives in the Taylor expansion step sizes for $T$, $R$, and $I$ are needed. The data from the MC simulation is examined in order to determine the step sizes. Figure 11 shows that the strength is approximately linear in $T$. Therefore, a linear regression is performed on $T$ and the residuals for $R$ and $I$ are examined. This reveals that the residuals of $I$ are roughly linear. Therefore $I$ is also regressed and the $R$ residuals are recalculated. By studying the residuals, regions around the mean which are linear with respect to the variables can be determined. Based on this analysis, the following step sizes are selected:

$$\Delta t = 0.053 \text{ (one standard dev.)}, \quad \Delta r = 7.5, \quad \Delta i = 0.4257 \text{ (one standard dev.)}$$

With the step sizes known a functional form can now be readily determined. Analysis is completed on ABAQUS to determine the required points for the central difference calculation. The expansion leads to the following expressions for $\lambda$:

*First Order Expansion:*

$$\lambda = 2.64T - 0.00333R - 0.141I - 0.257$$

*Second Order Expansion:*

$$\lambda = 30.9T - 0.0564R + 1.48I - 14.2T^2 + 8.89 \times 10^{-5} R^2$$

$$+ 0.0552I^2 + 0.0503TR - 0.00157RI - 1.66TI - 14.2$$

With the functional forms known, the mean and variance of $\lambda$ can be readily calculated. For the first order expansion the mean of $\lambda$ is 2.19 and the variance is 0.0484. For the second order expansion the mean of $\lambda$ is 2.18 and the variance is 0.2394.

## 5.2 Plate in Pure Bending

**Taylor Expansion**

Investigation of elements under stress gradient are conducted using Taylor expansion. The step sizes for the random variables are selected the same as in the uniform compression case. The output from the analysis is $\alpha$, the ultimate flexural strength of the element under stress gradient is:

$$M_{ult} = \alpha M_y .$$

The first and second order expansions lead to the following expressions for $\alpha$:

*First Order Expansion:*

$$\alpha = 0.811T - 0.00053R - 0.00941I - 0.15682$$

*Second Order Expansion:*

$$\alpha = 0.946T - 0.00271R + 0.225I + 1.78 \times 10^{-5} R^2$$

$$+ 0.00126TR - 7.83 \times 10^{-5} RI - 0.233TI - 0.28$$

With the functional forms known, the mean and variance of $\alpha$ can be readily calculated. For the first order expansion the mean of $\alpha$ is 0.633 and the variance is 0.0020. For the second order expansion the mean of $\alpha$ is 0.638 and the variance is 0.0105.

# 6 DISCUSSION AND COMPARISON TO AISI SPECIFICATION

## 6.1 Plate in Uniform Compression

For the case studied, the nominal $\lambda$ value determined using AISI is 1.94. (i.e. the ultimate strength is $1.94P_{cr}$). Figure 14 shows a histogram of the MC simulation results and a normal distribution fit to those results. Investigation of the MC simulation data reveals that 13 of the 100 members are predicted to have a strength less than the AISI prediction.

In order to evaluate the Taylor series approximation, and for later use, a distribution for $\lambda$ is needed. Based on Figure 14 it is concluded that a normal distribution adequately captures $\lambda$. Since the MC simulation and the Taylor expansions have different means and variances the percentage of expected understrength members will vary for the different methods. Figure 15 shows the expected percentage of understrength elements based on the MC simulation results. For the MC simulation (now fitted to the normal distribution) the expected percentage of understrength members is 14.5%, for the first order Taylor expansion it is 6.1%, for the second order Taylor expansion it is 31.2%. The large variability in the Taylor expansion results is due to the step sizes selected for calculating the derivatives.

Relying primarily on the MC simulation results for discussion: the nominal AISI prediction lies approximately one standard deviation below the expected average. The percentage of understrength members may at first seem larger than expected, but this is without consideration of resistance factors. For a resistance factor of 0.85, typical for compression, the probability of an understrength member is 0.84%.

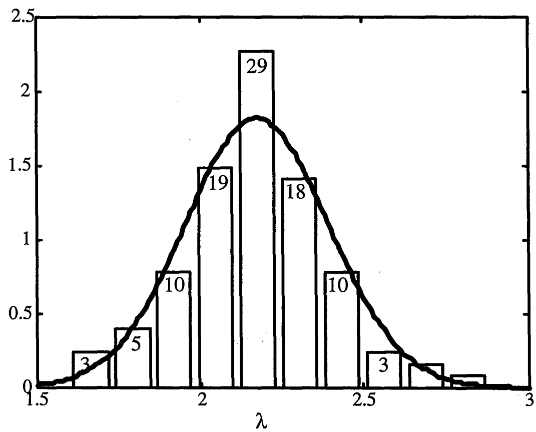



Figure 14: Histogram and Distribution for $\lambda$

## 6.2 Plate in Pure Bending

A similar analysis to that conducted for the uniform compression case is also conducted for the pure bending example. For the example element in pure bending the nominal AISI $\alpha$ value determined is 0.604 (i.e. the ultimate strength is $0.604M_y$). Figure 16 shows the assumed distribution of α and the expected percentage of understrength members using the mean and variance from the first order Taylor expansion. The expected percentage of understrength elements in pure bending is 25.5% for the first order expansion and 37.1% for the second order expansion. For a resistance factor of 0.9, those values become 2.3% and 17.9% respectively. The prediction of elements under a stress gradient is less conservative than for elements in uniform compression alone.

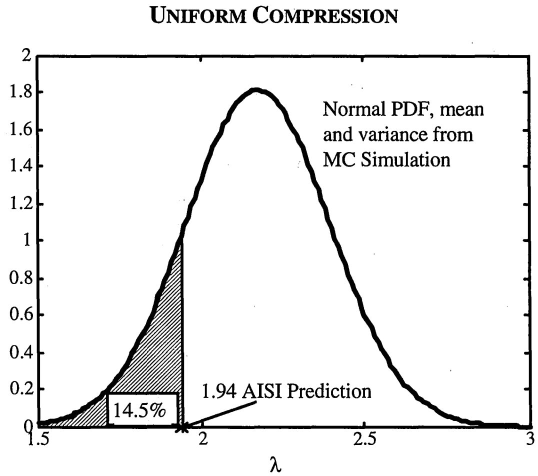



Figure 15: AISI Comparison to λ

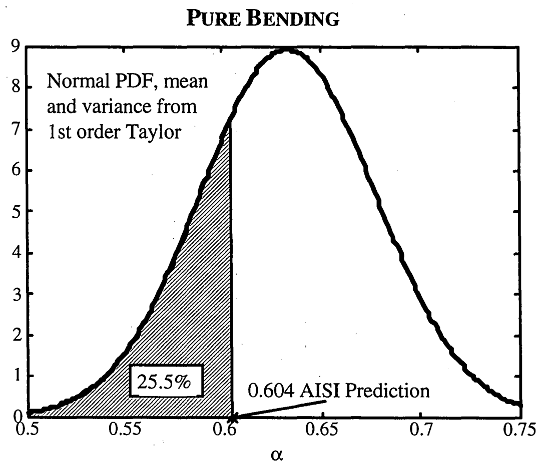



Figure 16: AISI Comparison to α

### 6.3 Example: Flexural Strength of a Hat Section

The analysis of elements in pure compression and bending are of interest primarily because when placed together the elements approximate the behavior of cold-formed steel members. The example mentioned previously is that of a hat section, as in Fig. 1. Consider a section as shown in the inset of Fig. 17. For this hat section in bending, the web and tension flanges are not prone to buckle. If they are considered deterministic, then the variability in the ultimate strength is dependent on the compression flange only. Using the same procedure as the AISI Specification it is found that for this section ($E$=2.03×10$^5$MPa and $F_y$=345MPa) the flexural capacity $M_n$ may be expressed as:

$$M_n = 0.21\lambda^3 - 1.48\lambda^2 + 4.61\lambda + 0.90 \text{ kN m}$$

With the functional form of $M_n$ now known and the distribution of $\lambda$ known (from the MC simulation performed on a plate in uniform compression) $M_n$ can be readily solved by simulation. Samples of $\lambda$ are generated using a normal distribution characterized by the mean and variance found from the MC simulation. Figure 17 shows a histogram of $M_n$ after 1000 samples. The flexural capacity prediction of the AISI Specification is 5.81kNm. The simulation yields a flexural capacity with mean 6.08kNm and standard deviation 0.27kNm. It is found that 147 of the 1000 samples (14.7%) have a flexural strength less than that predicted by the Specification. If a resistance factor of 0.9 is used no understrength members are observed in the sample of 1000. The resistance factor significantly decreases the probability of understrength members, because in the example the variance in the capacity is rather small. For cases with greater uncertainty the resistance factor would not have such a dramatic effect. In addition, the Specification was shown to be less conservative for slender webs, therefore members dominated by these elements could be expected to be less conservative than the example presented.

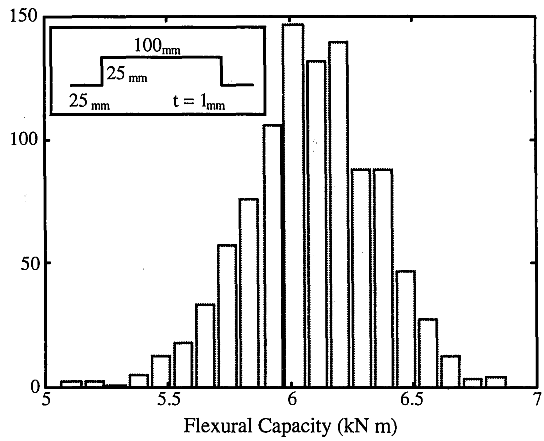



Figure 17: Histogram for Hat Section Example

# 7 CONCLUSIONS

Based on an examination of existing literature in cold-formed steel, typical distributions for thickness, residual stress and imperfection magnitude are determined. Using ABAQUS general models for the strength of a simply supported plate in uniform compression and pure bending are developed. Probabilistic analysis of the ultimate strength is conducted by MC simulation and Taylor expansion for the plate in uniform compression and Taylor expansion for the plate in pure bending. Analysis by MC simulation indicates that the ultimate strength is approximately normally distributed. Analysis by Taylor expansion yields similar mean values but different variances for the members studied. The variance from the Taylor expansion is influenced by the selection of step size for use in the central difference approximation of the derivatives.

A comparison of the statistical results to the AISI Specification indicate that the Specification is less conservative in its prediction of a plate in pure bending than in pure compression. Ignoring resistance factors, the expected probability of an understrength member is ~15% for an element in pure compression and ~25% for an element in pure bending. However, for the flexural member example problem, with the resistance factor included, no understrength members were observed in a sample of 1000. In the future, a Specification procedure that treats the inherent variability of the inputs in cold-formed steel strength prediction directly, and yields a probability based answer to the user may prove useful - for now, Winter's empirical correction factor still appears viable albeit in some cases quite conservative.

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# APPENDIX

Table A1: Imperfection Data

| | Specimen | Line 1 | Line 2 | Line 3 | Line 4 | Line 5 |
|---|---|---|---|---|---|---|
| | | **Magnitude of Maximum Imperfection (mm)** | | | | |
| Kwon | CH1-5-800 | 0.35 | 0.35 | 0.55 | 0.75 | 1.25 |
| C-Section | CH1-6-800 | 0.55 | 0.35 | 0.38 | 0.75 | 0.75 |
| t=1.10mm | CH1-7-400 | 0.90 | 0.75 | 0.60 | 0.45 | 0.55 |
| | CH1-7-600 | 0.50 | 0.30 | 0.38 | 0.50 | 0.70 |
| | CH1-7-800 | 0.55 | 0.70 | 0.90 | 1.20 | 1.30 |
| | CH2-7-800 | 0.35 | 0.25 | 0.45 | 0.80 | 0.95 |
| | CH2-7-1000 | 0.25 | 0.50 | 0.70 | 0.90 | 0.60 |
| | CH2-8-1000 | 0.75 | 0.40 | 0.20 | 0.80 | 1.45 |
| | CH2-10-1000 | 0.45 | 0.45 | 1.30 | 0.80 | 0.50 |
| | CH2-12-1000 | 0.90 | 0.75 | 0.80 | 0.80 | 1.15 |
| | CH2-14-1000 | 0.75 | 0.65 | 0.60 | 0.50 | 0.50 |
| Bernard | ST22 | 0.60 | 0.50 | 0.45 | 0.25 | 0.25 |
| Trap. Sect. | | 0.80 | 0.65 | 0.60 | 0.45 | 0.20 |
| t=0.595mm | IST43 | 1.20 | 1.10 | 0.70 | 0.30 | |
| | | 1.80 | 1.50 | 1.10 | 0.30 | |
| | IST44 | 0.70 | 0.60 | 0.40 | 0.55 | 0.35 |
| | | 0.90 | 0.60 | 0.30 | 0.10 | |
| | IST46 | 2.30 | 1.70 | 1.20 | 0.60 | 0.40 |
| | | 2.30 | 1.70 | 1.20 | 0.60 | 0.30 |
| **Overall Average = 0.72** | | | **Overall Standard Deviation = 0.43** | | | |

Table A2: Residual Stress Data

| | Specimen | Outside | Inside | Net | Outside | Inside | Net |
|---|---|---|---|---|---|---|---|
| | | ***FLATS*** | | | ***CORNER*** | | |
| | | **Residual Stress as %fy** | | | **Residual Stress as %fy** | | |
| Ingvarsson | U1 | 4 | -3 | 1 | 56 | -38 | 18 |
| (C-Section) | U2 | 1 | -2 | -1 | 50 | -31 | 19 |
| Weng | RFC13 | 22 | -24 | -2 | 54 | -41 | 13 |
| (C-Section) | RFC14 | 53 | -53 | 0 | 80 | -56 | 24 |
| | PBC14 | 24 | -24 | 0 | 53 | -47 | 6 |
| | R13 | 53 | -53 | 0 | 70 | -59 | 11 |
| | R14 | 53 | -44 | 9 | 71 | -62 | 9 |
| | P3300 | 21 | -21 | 0 | 42 | -40 | 2 |
| | P4100 | 17 | -20 | -3 | 40 | -40 | 0 |
| | DC12 | 23 | -23 | 0 | 46 | -50 | -4 |
| | DC14 | 26 | -28 | -2 | 53 | -40 | 13 |
| | P11 | 35 | -37 | -2 | 61 | -53 | 8 |
| | P16 | 32 | -41 | -9 | 60 | -54 | 6 |
| Bernard | R410-1 | 13 | -13 | 0 | 3 | 0 | 3 |
| (Trapezoid | R412-1 | -5 | 6 | 1 | -4 | 2 | -2 |
| Section) | R412-2A | -3 | 4 | 1 | -2 | 2 | 0 |
| | R412-2B | 12 | -10 | 2 | 8 | -4 | 4 |
| | *mean* | 22.41 | -22.71 | -0.29 | 43.59 | -35.94 | 7.65 |
| | *st. dev.* | 18.53 | 18.51 | 3.44 | 26.35 | 22.14 | 7.87 |