Design of Thin-walled Beams for Distortional Buckling

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Introduction

The bending behaviour of cold-formed steel beams is far from simple. The double symmetry associated with hot-rolled I-sections is generally uneconomical and the use of thin-walled slender elements means that local buckling of parts of the cross-section in compression may be a significant design factor leading to additional asymmetry of behaviour. Furthermore, the modern tendency to introduce additional folds in order to control local buckling and to produce more favourable global section properties also serves to aggravate the tendency towards distortional buckling.

Generalised Beam Theory (GBT) has wide application to the analysis and design of cold-formed beams and can provide a relatively simple approach to the problems associated with the alternative buckling modes in thin-walled beams and their various interactions. Historically, both the local and global bucking of beams have been intensively researched and the appropriate design methods have been well documented. Recent research shows that distortional buckling may also be critical in beams of practical proportions and the relative lack of research and the weakness of design approaches dealing with distortional buckling in the available codes and standards is a deficiency in the design of cold-formed steel structures. With the aid of GBT, some recently proposed design approaches are evaluated and improvements are proposed. Further investigation shows that the Perry-Robertson equation can be used to introduce a yield criterion in order to empirically modify the elastic distortional buckling moment and thus to predict the buckling strength of a member.

Interaction of buckling modes

GBT is a complex subject with many ramifications and its details, which can be found in references [1]-[6], will not be elaborated here. In contrast to the case of uniformly compressed columns, beams are generally subject to shear forces as well as bending moments. The family of basic second-order equilibrium equations, with consideration of the shear stresses, in the form established by GBT is

$$E\,{}^{k}C\,{}^{k}V'''' - G\,{}^{k}D\,{}^{k}V'' + {}^{k}B\,{}^{k}V + \sum_{i=1}^{n}\sum_{j=1}^{n} {}^{ijk}\kappa_{\sigma}\left({}^{i}W\,{}^{j}V\right)'' - {}^{ijk}\kappa_{\tau}\left(2\,{}^{ij}W'\,{}^{j}V' + {}^{i}W''\,{}^{j}V\right) = {}^{k}q \tag{1}$$

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where a forward superscript k=1,2,...n donates the mode number and

E	is the modulus of elasticity
G	is the shear modulus
${}^{k}C$	is the generalised warping resistance
${}^{k}D$	is the generalised torsional resistance
${}^{k}B$	is the generalised transverse bending resistance
${}^{k}V$	is the generalised deformation resultant
${}^{k}W$	is the generalised stress resultant
${}^{ijk}\kappa_{\sigma}$	is the matrix of second-order terms relating to longitudinal stress
${}^{ijk}\kappa_{\tau}$	is the matrix of additional second-order terms relating to shear stress
${}^{k}q$	is the applied distributed load applicable to mode k

From this equation system it can been seen that, if the stress resultant is not constant along a member, the influence of shear stress is taken into account by the additional second-order terms ${}^{ijk}\kappa_{\tau}$. However, for a long-span beam with an open cross-section these terms have little influence and may be neglected.

An investigation of the interaction of bucking modes or loads can be easily achieved by taking account of different combinations of modes or loads when solving the equation system.

Consider a simply-supported Zed-section beam, whose cross-section is not symmetric, as an example in order to illustrate the interaction of the buckling modes. The dimensions of the cross-section are shown in Figure 1 and the steel material has a modulus of elasticity of 200 kN/mm^2 and Poisson's ratio 0.3. The generalised cross-section properties computed with aid of GBT are given in Table 1 and the corresponding buckling modes are illustrated in Figure 1. The first four modes are the global (rigid body) buckling modes, the fifth is a symmetric distortional buckling mode and the sixth is an asymmetric distortional buckling mode. If pinned restraints at two ends of the beam and a uniformly distributed load applied in the plane of the web are assumed, the solutions of equation system (1), for buckling with consideration of some alternative mode combinations, are shown in Figure 2. For illustrative purposes, in solving the equation system, only the stress resultant of mode 2 (bending moment about major axis x) is taken into account in order to eliminate load coupling which is considered later in this paper.

Figure 2 shows the buckling moments as a function of the span of the simply supported beam. It can be seen that, for spans less than about 2.3 metres, the beam buckles in distortional modes whereas for longer spans, the beam buckles in global modes (lateral-torsional modes). Local buckling of the top flange, which can be investigated by introducing into the analysis an intermediate node in the flange, never occurs. Evidently, in this example, the interaction between the modes and the effect of shear stress are insignificant. The span range over which distortional buckling is critical includes spans used in practice and, if restraints exists, provided for example by the cladding, support braces or anti-sag bars, the range could be much larger. It follows that design for distortional buckling has important practical significance.

Mode k=	C (cm^4)	D (cm^2)	B (kN/cm^2)
1	4.350000	0.000000	0.000000
2	177.5874	0.000000	0.000000
3	12.98406	0.000000	0.000000
4	1214.386	0.032625	0.000000
5	0.113525	0.000756	0.121381
6	0.112084	0.000884	0.054739

Table 1 Cross-section properties

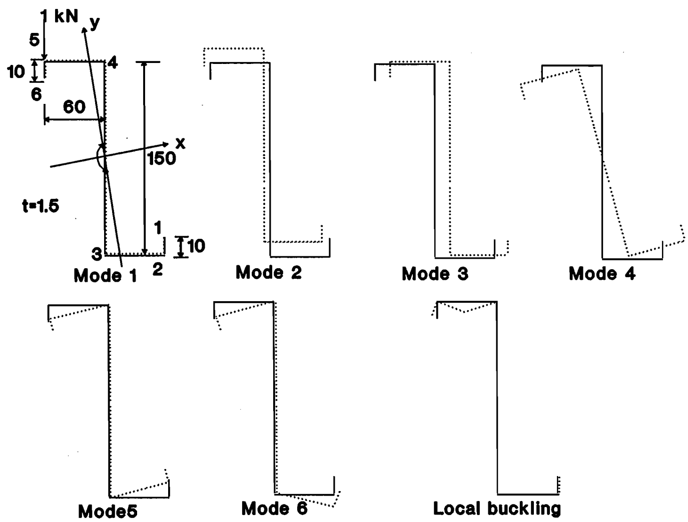

Figure 1 Local, distortional and global buckling modes

For a beam with a symmetrical cross-section, in contrast to the case of column, single mode distortional buckling cannot occur if the bending moment is applied normal to the axis of symmetry. In this case, distortional buckling of the beam takes the form of an interaction of modes 5 and 6. Obviously, if a bending moment is applied parallel to the axis of symmetry, there are single distortional buckling modes but this is often a less important case. Similarly, the Zed section beam has an asymmetric cross-section and cannot buckle in a single distortional mode.

If the applied bending moment is constant along the beam, using GBT, the buckling half wavelength and the buckling stress resultant for single mode buckling can be written in the explicit forms:

$$ {}^{k}\lambda = \pi \left(\frac{E\,{}^{k}C}{{}^{k}B} \right)^{0.25} \tag{2} $$

$$ {}^{i,k}W_{cr} = \frac{1}{{}^{ikk}\kappa} \left(2\sqrt{E\,{}^{k}C\,{}^{k}B} + G\,{}^{k}D \right) \tag{3} $$

Equation (2) leads to the conclusion that the half wavelength for a single mode distortional buckling is independent of the mode of load application which means that compression and bending members have the same buckling half wavelength.

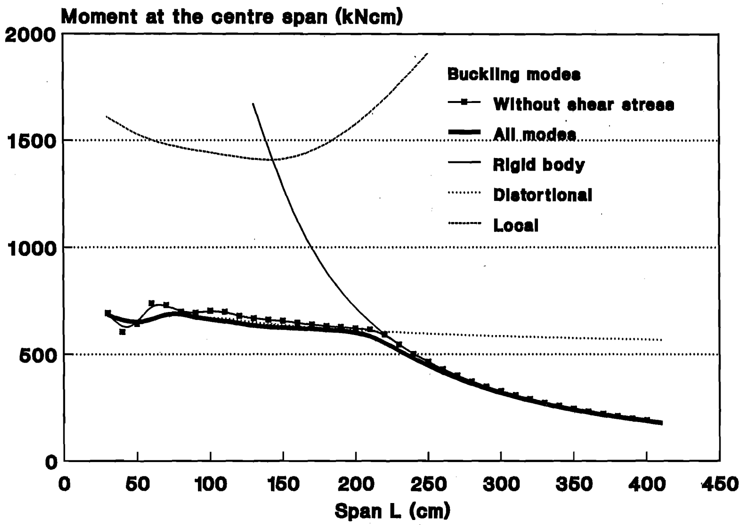

Figure 2 **Buckling mode interaction for a simply-supported Zed-section beam subject to a uniformly distributed load.**

Load coupling

The application of load through the shear centre of the cross-section is difficult to achieve in practice and in most cases a cold-formed beam is subject to an applied torque in addition to the bending moment. This means that twisting of the section is bound to occur unless it is continuously restrained against torsion. In addition, transverse bending moments are caused by the associated cross-section warping and these should also be taken into consideration in the design.

We now return to the cross-section shown in Figure 1 as an example to illustrate the effect of load coupling. It is assumed that a 1 kN concentrated load is applied in the vertical direction at node 5 of the cross-section at the centre of the span. As the load is applied neither at the shear centre of cross-section nor in the direction of the principal axis, in addition to bending moment about major x axis, it will cause a both a torque and a bending moment about the minor y axis. Furthermore, two transverse bending moment distributions ^5W and ^6W will occur due to symmetrical and asymmetrical warping of the cross-section. Figure 3 shows the distribution of these stress resultants along the beam calculated using GBT. It can be seen that the bending moment about the major axis x has a maximum value of $^2W_{max}$ = $-PL\sin\theta/4$ and that the bending moment about the minor axis y has a maximum value of $^3W_{max}$ = $PL\cos\theta/4$ in which θ = 110.089° is the angle between the major axis and the load direction.

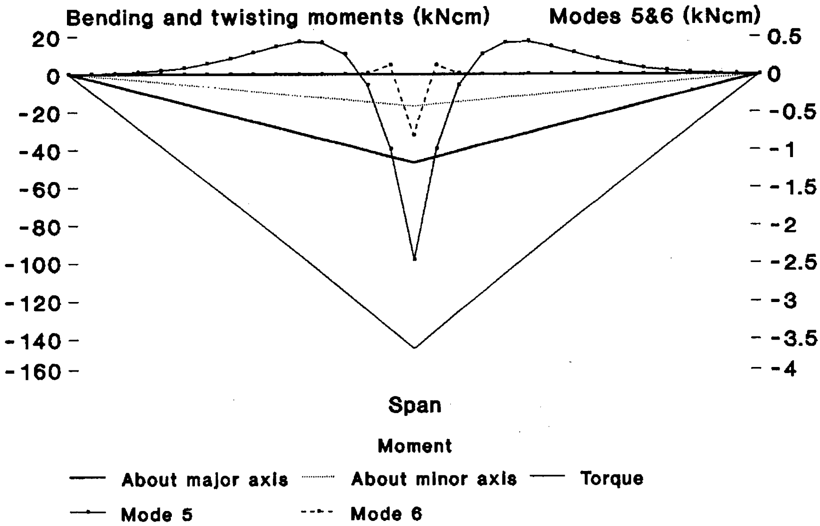

Figure 3 **Stress resultants caused by a 1 kN concentrated load applied at the centre of a simply supported beam of 2 metres span**

Figure 4 shows the effect of eccentricity on the buckling resistance of a simply-supported beam of 2 metres span with the cross-section dimensions shown in Figure 1. It can be seen that, if the load is applied through the shear centre, the effect of the transverse bending moments is trivial. As the load eccentricity increases, so does the torque, and the buckling resistance of the beam decreases rapidly so that the effect of the torque becomes dominant. The stress resultants of modes 5 and 6 have their maximum effect when the load is applied at about 40 mm from the shear centre. However, many specifications do not have detail rules for design against torsion and there is no code which has adequate clauses dealing with the transverse bending stresses which arise as a result of distortion. Both BS 5950 and Eurocode 3 include rules for combined bending and torsion. BS 5950 and the AISI Specification both treat biaxial bending as if the section were loaded in a principal plane and subject to lateral buckling.

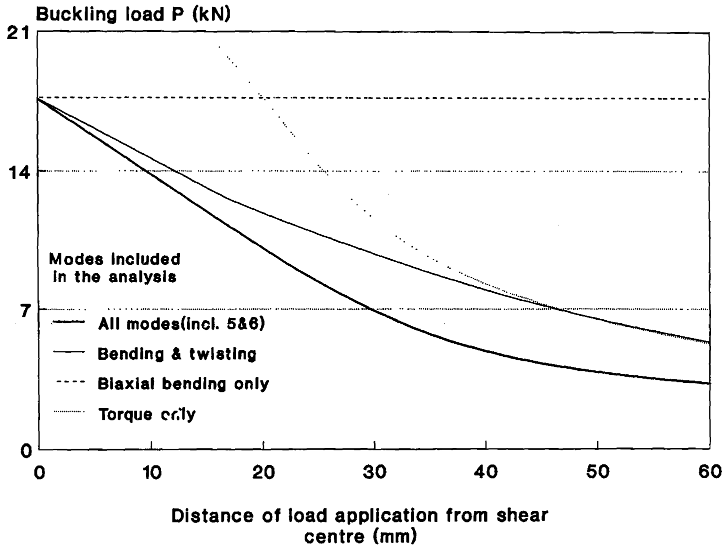

Figure 4 **Effect of load eccentricity on the buckling resistance of a beam**

The eccentricity effect could, of course, be significantly reduced if the top flange of the profile were to be restrained by the cladding. However, modern standing seam and clip-fixed systems offer greatly reduced restraint when compared with traditional cladding systems.

Evaluation of the proposed design approaches for distortional buckling

Evidently, GBT is a yardstick by which other design methods may be assessed but, in the current state of the art, it probably does not provide a practical basis for a design code. Analytical expressions for the distortional buckling of thin-walled beams of general section geometry under a constant bending moment have been developed by Hancock [7], and Serrette and Pekoz [8]. These analytical expressions were based on the simple flange buckling model shown in Figure 5 in which the flange was treated as a compression member with both rotational and translational spring restraints in the longitudinal direction. The rotational spring stiffness k_ϕ and the translational spring stiffness k_x represent the torsional restraint and translational restraint from the web respectively. In their analyses, both of the above of authors chose the translational spring stiffness k_x to be zero.

Hancock's analysis [7] concentrated on bending about the axis of symmetry of the cross-section. Bearing in mind the mode shape shown in Figure 5, the rotational spring stiffness k_ϕ for this case is double that for a column in compression, so that:

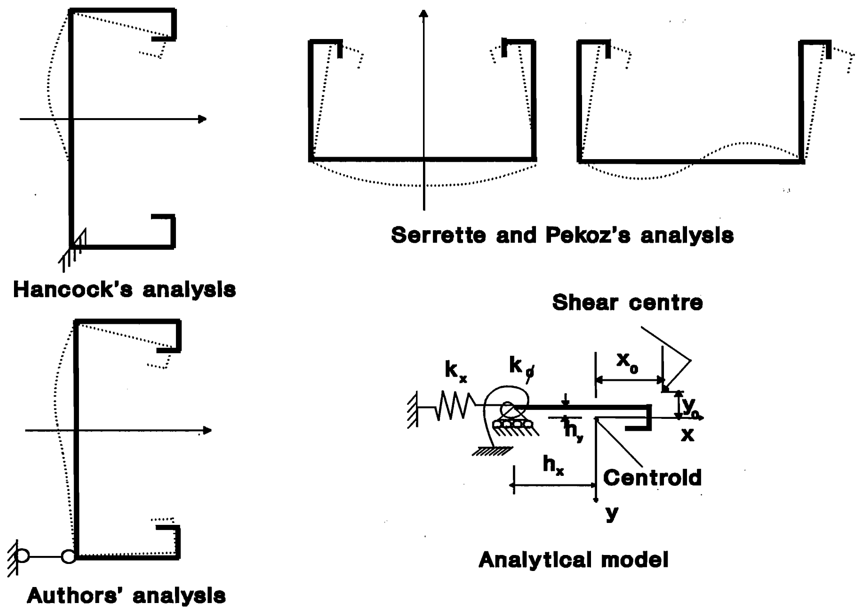

Figure 5 Analytical model for the distortional buckling of bending members

$$k_\phi = \frac{4D}{b_w + 0.06\lambda} \tag{4}$$

and the distortional buckling half wavelength λ is:

$$\lambda = \pi\left(\frac{E\,I_{wc}\,b_w}{4D}\right)^{0.25} \tag{5}$$

where:

b_w is the depth of the web
D $=Et^3/[12(1 - \nu^2)]$ is the plate flexural rigidity
E is the modulus of elasticity
t is the thickness of the profile
ν is Poisson's ratio
I_{wc} $=I_w + I_x(x_0 - h_x)^2 + I_y(y_0 - h_y)^2 - 2I_{xy}(x_0 - h_x)(y_0 - h_y)$
I_w is the warping constant of the compression flange and lip
I_x, I_y are the second moments of area of the compression flange and lip about the x and y axes respectively
I_{xy} is the product second moment of area of the compression flange and lip about the x and y axes

The elastic critical stress for distortional buckling derived by Hancock is:

$$\sigma_{cr} = \frac{W_{cr}}{Z} = \frac{E}{2A_f}\left[\left(\alpha_1 + \alpha_2\right) \pm \sqrt{\left(\alpha_1 + \alpha_2\right)^2 - 4\alpha_3}\,\right] \tag{6}$$

where α_1, α_2, and α_3 are characteristic values related to k_ϕ, λ and the dimensions of the compression flange and lip. A_f is the gross area of the flange and lip and Z is the elastic section modulus of the whole cross-section.

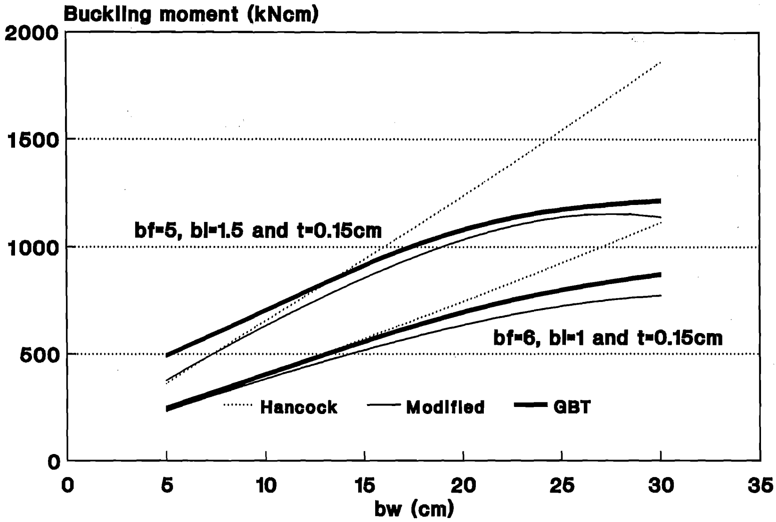

Figure 6 Comparison of design expressions with GBT for channel section beams

Figure 6 compares the results given by Hancock's expressions and GBT for two channel section beams. It can be seen that, as the web depth increases, the values of the distortional buckling moment given by Hancock's expressions become higher due to the neglect of the web local buckling and the assumption of fixity at the bottom (tension) end of the web as shown in Figure 5. In order to improve the accuracy of the design expressions, modifications have been made by authors by introducing a reduction coefficient into equation (4) and by assuming the tension end of the web to be pinned. Hence, we now have:

$$k_\phi = \frac{3D}{b_w}\left(1 - \frac{\sigma'}{\sigma_w}\right) \tag{7}$$

and

$$\lambda = \pi \left(\frac{E I_{wc} b_w}{3D} \right)^{0.25} \qquad \textbf{(8)}$$

where σ' is the buckling stress of the web plate obtained from equation (6) with $k_\phi = 0$ and σ_w is the local buckling stress of the web plate. For a beam of symmetrical cross-section, this can be written as [9]:

$$\sigma_w = \frac{9 \pi^4 D}{32 b_w^2 t} \frac{\left(\frac{b_w^2}{\lambda^2} + 1 \right) \left(1 + 4 \frac{\lambda^2}{b_w^2} \right) \left(1 + 9 \frac{\lambda^2}{b_w^2} \right)}{\sqrt{\left(\frac{27}{25} \right)^2 \left(1 + \frac{\lambda^2}{b_w^2} \right)^2 + \left(1 + 9 \frac{\lambda^2}{b_w^2} \right)^2}} \qquad \textbf{(9)}$$

Figure 6 shows that the modified design expressions give a better estimate of the critical bending moment than Hancock's expressions.

Serrette and Pekoz's analysis was concerned with the bending behaviour of the outstanding legs of the cross-section when a bending moment is applied in a direction parallel to the axis of symmetry. The rotational spring stiffness k_ϕ was determined by:

$$k_{\phi,s} = \frac{D}{\left(\frac{b_f}{2} + \frac{b_w}{3} \right)} \gamma \qquad \textbf{(10)}$$

$$k_{\phi,as} = \frac{D}{\left(\frac{b_f}{6} + \frac{b_w}{2} \right)} \gamma \qquad \textbf{(11)}$$

Where $k_{\phi,s}$ and $k_{\phi,as}$ are the rotational spring stiffnesses for symmetric sections and asymmetric sections respectively and b_f is the width of the flange. $\gamma \leq 1.0$ is an empirical reduction factor to take account of the local buckling of the outstanding leg.

In order to determine the minimum buckling stress, Serrette and Pekoz suggested that the following equation should be solved for a range of buckling wavelength values λ:

$$\sigma_{cr} = \frac{W_{cr}}{Z_g} = \frac{\alpha_1 + \alpha_2}{\alpha_3} \qquad \textbf{(12)}$$

where α_1, α_2, and α_3, which have totally different definitions from Hancock's parameters, are characteristic values related to k_ϕ, λ and the dimensions of the outstanding leg. Z_g is the gross section modulus of the leg.

Although this calculation procedure appears to give good results, the requirement for iteration to find the minimum stress can cause over-elaborate calculations. As this calculation is concerned with buckling in a single mode, and as it has already been shown that the buckling wavelength values λ are independent of the load, the authors suggest that the buckling half wavelength can be assumed to be the same as that of column in pure compression, namely

$$\lambda_s = \pi\left(\frac{E\,I_{wc}\,b_w}{2D}\right)^{0.25} \tag{13}$$

$$\lambda_{as} = \pi\left(\frac{E\,I_{wc}\,b_w}{6D}\right)^{0.25} \tag{14}$$

where λ_s and λ_{as} the are the half wavelengths for symmetric and asymmetric distortional buckling respectively.

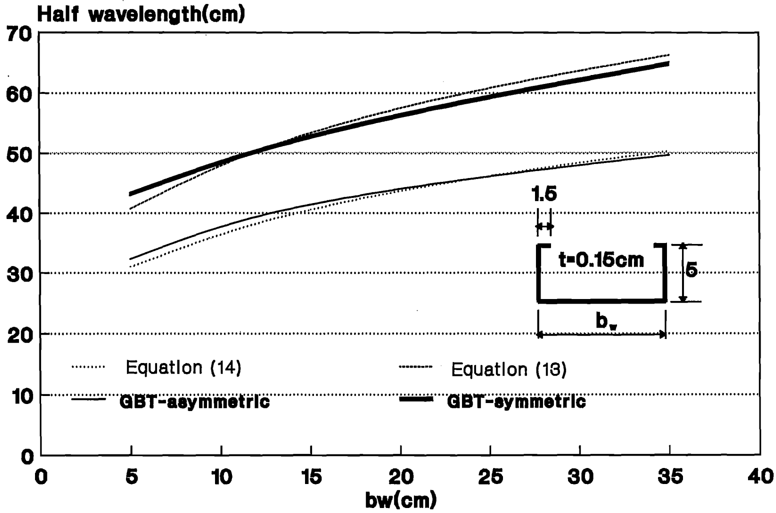

Figure 7 **Comparison of the buckling half wavelength calculated with equations (13) and (14) and with GBT**

Figure 7 shows the half wavelengths from equations (13) and (14) for a channel section beam. It can be seen that, when compared with results given by GBT, the equations produce a good prediction of the half wavelength for distortional buckling. It should be noted here that a symmetric or asymmetric cross-section beam may buckle either symmetrically or asymmetrically. In the design for distortional buckling of the outstanding leg, the buckling stress for both symmetric and asymmetric buckling should be calculated with aid of equations (13) and (14) and the smaller buckling stress chosen.

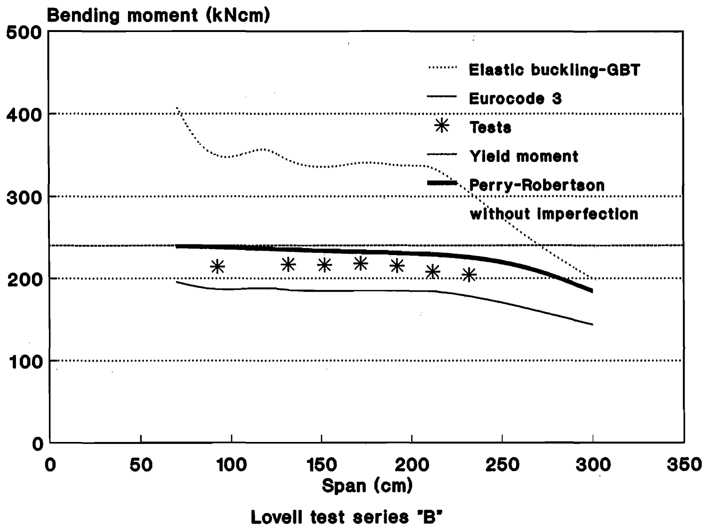

Figure 8 Comparison between test and theoretical results

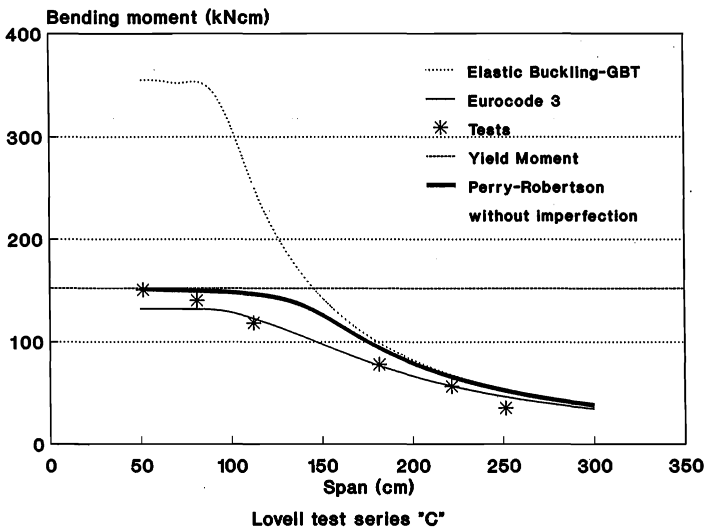

Figure 9 Comparison between test and theoretical results

Comparison with tests

Some typical channel section beams which were tested by Lovell [10] at the University of Salford have been analyzed using GBT. In these tests, a constant bending moment about the major axis was applied to the beam. In the analyses using GBT, based on the test arrangement, the loads were assumed to be applied through the shear centre of the cross-section and the two ends of the beams were assumed to be simply supported with respect to lateral bucking and fixed with respect to torsion.

In order to introduce a yield criterion into GBT, the Perry-Robertson equation and the equations in Eurocode 3 Annex 1.3 (Clause 6.1) [11] with an imperfection factor of 0.21 were adopted. This procedure is in accordance with the trend in current design standards to express all cases of interaction between buckling and yielding in the form of a Perry-Robertson equation. The analytical and test results for some of the specimens are compared in Figures 8 and 9. It can be seen from these results that the use of the Perry-Robertson theory without imperfections gave unsafe results whereas the use of the equations of Eurocode 3 with an imperfection factor of 0.21 gave conservative results.

Conclusions

GBT has a wide application to the analysis and design of cold-formed beams. It can deal with a variety of problems which include buckling mode interaction, alternative loading patterns and load location by means of a simple unified approach.

In the design of flexural members for distortional buckling, using the basic approach suggested by Hancock, modified expressions which take account of local buckling in the web can be used to improve the accuracy of the prediction of the buckling stress. Similarly, equation (13) and (14) can be used to determine the buckling half wavelengths in order to simplify the approach suggested by Serrette and Pekoz.

In the design of a beam undergoing distortional buckling, the elastic buckling moment given by GBT or by one of the design approaches discussed above may be combined with design formulas such as the Perry-Robertson equation or the equations in Eurocode 3 in order to introduce a yield criterion.

References

1. Schardt R, "Verallgemeinerte Technische Biegetheorie" (Generalised Beam Theory), Springer-Verlag, 1989 (in German).

2. Davies J M and Leach P, "First-Order Generalised Beam Theory", J Construct. Steel Research, 31, 1994.

3. Davies J M, Leach P and Heinz D, "Second-Order Generalised Beam Theory", J Construct. Steel Research, 31, 1994.

4. Leach P, "The Calculation of Modal Cross-Section Properties for use in the Generalised Beam Theory, Thin-Walled structures, 19, 1994.

5. Davies J M and Leach P, "Some applications of Generalised Beam Theory", Proc. 11th Int. Speciality Conf. on Cold-Formed Steel Structs., St Louis, Missouri, October 1992.

6. Leach P and Davies J M, "An experimental verification of the Generalized Beam Theory applied to interactive buckling problems", Thin-Walled structures, Vol. 25 No. 1, 1996.

7. Hancock G J, "Design for distortional buckling of flexural members", Proc. Third International Conference on Steel and Aluminium Structures, Istanbul ,1995.

8. Serrette R L and Pekoz T, "Distortional buckling of thin-walled beams/panels. I Theory, ASCE, Journal of Structural Engineering, Vol. 121, No. 4, April 1995.

9. Timoshenko S P and Gere J M, "Theory of Elastic Stability", McGraw-Hill, 1963,

10. Lovell M H, "Lateral buckling of light gauge steel beams", MSc thesis, University of Salford, 1983.

11. Eurocode 3, "Design of Steel Structures", CEN ENV 1993-1-3: Supplementary Rules for Cold Formed Thin Gauge Members and Sheeting, 1996.