Effect of Bracing Stiffness on Buckling Strength of Cold-formed Steel Columns

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Abstract: A three-dimensional second-order analysis is used to study the effect of brace stiffness on the buckling strength of cold-formed steel columns. The finite element formulation uses an iterative updated Lagrangian scheme to include the second-order geometric non-linear effects in the space frame element as well as the connection element used to model a brace. Lateral brace stiffness required to achieve full bracing for a two-dimensional column, and lateral and torsional brace stiffnesses required for a three-dimensional cold-formed steel column are studied. The strength requirement for various brace components, the effects of column initial imperfections, and the impact of varying warping boundary conditions are also investigated.

INTRODUCTION

Light weight structures are becoming increasingly popular resulting in slender structural members. Thus, determination of the buckling behavior of the members is important. It is very common to provide bracing for slender members to increase their buckling capacity. It is essential to provide lateral and/or torsional bracing in order to provide full column bracing and to make sure that the deflections of the column are within permissible tolerances. Full bracing is defined as equivalent in effectiveness to an immovable lateral or torsional support.

The effect of lateral bracing on the buckling strength of columns has been studied extensively by various researchers over the years. Winter (1960) presented a simple elementary method that permits the lower limits of strength and rigidity of lateral support in order to provide full bracing. Wang *et al.* (1980) presented the bracing requirements for locally buckled thin-walled columns. These authors performed bifurcation analysis and load incremental analysis for simple thin-walled columns to determine the restraint stiffness requirements. Wang and Nethercot (1989) investigated the brace stiffness requirements using a three-dimensional analysis and included the effects of initial imperfections and plasticity on the section. Plaut and Yang (1993, 1995) performed extensive parametric studies to determine the lateral brace stiffness requirements for multi-span columns with two or three spans. These authors based all their findings on two-dimensional analyses, i.e., pure flexural analysis about the weak axis for hot-rolled sections. Most of the authors mentioned above presented two-dimensional analyses for pure flexural buckling or bifurcation analysis for flexural-torsional buckling for hot-rolled steel sections.

In this paper, a three-dimensional analysis with the inclusion of second-order geometric non-linear effects is used to determine the buckling loads of cold-formed steel columns with

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various lateral and torsional brace stiffnesses. An initial imperfection is also considered for the various columns considered. A connection element is used to model the brace stiffness at the column midheight. Effects of support warping restraint on the brace stiffness requirements is also investigated.

FINITE ELEMENT FORMULATION OVERVIEW

A computer program GNSFAP (Geometric Nonlinear Space Frame Analysis Program) based on the finite element theory developed by Chen (1990) is used in this study. Some details of the finite element formulation and numerical analysis are explained below, but complete details are provided in Chen (1990), and Chen and Blandford (1991a, b; 1993; 1995).

This finite element formulation for the large-deformation analysis of space frame structures is based on second-order geometric nonlinear theory and Vlasov's theory for thin-walled beams (i.e., large displacement of members with small strains, and includes the warping deformation influence). Rodriguez's modified rotation vector is used to represent angular deformations, which avoids rotational discontinuities at the joints of deformed space frame structures. Two local coordinate systems are used in the finite element development: complete (fixed local) reference which is used to locate the initial beam position, and a cantilever bound reference. A displacement field defined on the cantilever bound reference experiences no displacement or rotation at end a (Fig. 1(a)) except for the warping deformation.

Frame Element : There are eight deformation degrees of freedom for the space frame finite element, $u_{sx}^{b}, u_{sy}^{b}, u_{sz}^{b}, \theta_{x}^{b}, \theta_{y}^{b}, \theta_{z}^{b}, \chi_{x}^{a}, \chi_{x}^{b}$ where subscript 's' refers to the displacements at the section shear center; superscripts 'a' and 'b' refer to beginning and end nodes of the finite element, respectively; translational displacements are represented by u; Euler rotations by θ; and warping deformation by χ. An element stiffness matrix $\mathbf{k}_{\theta}^{f}$ is developed using the usual finite element procedures discussed in Chen and Blandford (1991a) based on a cubic interpolation for torsion and bending about each axis with a linear interpolation of the axial deformation.

For the usual finite element development, the rotational displacement components θ_x ($=\phi_x$), θ_y, and θ_z are chosen, but components θ_y and θ_z do not equal the modified rotation vector components ϕ_y and ϕ_z. A geometric discontinuity results at a corner node where an axial rotation component ϕ_x of one member is assembled to a bending rotation component θ_y or θ_z of another member. For the finite element developed by Chen and Blandford (1991a), components of the modified rotation vector are chosen as the rotational displacement field and the eight deformation degrees of freedom for the cantilever bound reference (Fig. 1(a)) are $u_{sx}^{b}, u_{sy}^{b}, u_{sz}^{b}, \phi_{x}^{b}, \phi_{y}^{b}, \phi_{z}^{b}, \chi_{x}^{a}, \chi_{x}^{b}$. Stiffness matrix $\mathbf{k}_{\theta}^{f}$ is transformed to $\mathbf{k}_{\phi}^{f}$ based on the modified rotation vector components and a cantilever bound reference. Element stiffness matrix $\mathbf{k}_{\phi}^{f}$ is used for element force recovery since rigid body displacements are excluded. The stiffness matrix $\mathbf{k}_{\phi}^{f}$ can be expressed as

$$\mathbf{k}_{\phi}^{f} = \mathbf{T}_{\phi}^{T}\mathbf{k}_{\theta}^{f}\mathbf{T}_{\phi} + \mathbf{k}_{g\phi}^{f} \tag{1}$$

in which transformation matrix $\mathbf{T}_\phi$ and geometric stiffness matrix $\mathbf{k}^f_{g\phi}$ are given in Chen and Blandford (1991a). Superscript 'f' signifies frame element. Furthermore, $\mathbf{k}^f_\phi$ is transformed to matrix $\mathbf{k}^f_{c\phi}$ based on the complete reference. The displacement field (fourteen displacement degrees of freedom) for the complete reference is $U^a_{sx}, U^a_{sy}, U^a_{sz}, \Phi^a_x, \Phi^a_y, \Phi^a_z, U^b_{sx}, U^b_{sy}, U^b_{sz}, \Phi^b_x, \Phi^b_y, \Phi^b_z, \chi^a_x, \chi^b_x$ (Fig. 1(b)) and the stiffness matrix can be written as

$$\mathbf{k}^f_{c\phi} = \mathbf{B}_\phi \mathbf{k}^f_\phi \mathbf{B}^T_\phi + \mathbf{k}^f_{cge} \tag{2}$$

in which the details for the element geometric stiffness matrix $\mathbf{k}^f_{cge}$ and transformation matrix $\mathbf{B}_\phi$ are provided in Chen and Blandford (1991a).

Flexible Connection Element : Development of the flexible connection element follows the same steps shown above for the frame element. Again, two different coordinate systems are used and various assumptions considered in the formulation are explained in Chen and Blandford (1995). Geometric nonlinearity is also considered in the connection element formulation. The connection element also has fourteen degrees of freedom (dof) as in the frame element case including the warping dof. Second-order geometric nonlinear terms are also included as in the frame element. The stiffness matrix on the complete reference $\mathbf{k}^c_{c\phi}$ is represented by

$$\mathbf{k}^c_{c\phi} = \mathbf{B}_\phi \mathbf{k}^c_\phi \mathbf{B}^T_\phi + \mathbf{k}^c_{cg} \tag{3}$$

in which $\mathbf{k}^c_\phi$ is the connection element stiffness matrix defined on the cantilever bound reference; $\mathbf{B}_\phi$ is the connection element displacement - deformation transformation matrix; and $\mathbf{k}^c_{cg}$ is the geometric stiffness matrix on the complete reference (Chen and Blandford 1995).

Transformation details from the shear center to an arbitrary connection point and other transformations involved in the formulation are given in Chen (1990) and Chen and Blandford (1995).

Structure Equations : The beam and connection element stiffness matrices of (2) and (3) are transformed from the local coordinate system into the global coordinate system (Chen and Blandford 1991b) and are then assembled together using the conventional direct stiffness analysis procedure. These global stiffness equations can be represented as

$${}^{k+1}\mathbf{K}_{i-1}\delta\mathbf{q}_i = {}^{k+1}\lambda_i \mathbf{P} - {}^{k+1}\mathbf{F}_{i-1} \tag{4}$$

in which $\mathbf{K}$ is the global structure stiffness matrix; $\delta\mathbf{q}$ is the global iterative change in the displacement vector; λ is a load multiplier; $\mathbf{P}$ is the reference load vector; $\mathbf{F}$ is balanced or equilibrated force vector; pre-superscript k+1 denotes current load step; and subscript i signifies the iteration number. An iterative updated Lagrangian scheme is used to include second-order geometric nonlinear effects, i.e., the structure geometry is updated for each iteration within the load step based on the iterative change in displacements. Work-increment-control and modified Newton Raphson methods are used for the solution of the nonlinear global stiffness equations presented. The details of the work-increment-control method are given in Chen and Blandford (1991b; 1993).

NUMERICAL RESULTS

Extensive verification of the computer program GNSFAP (Geometric Non-Linear Space Frame Analysis Program) has been performed and the results are compared with available analytical solutions. The numerical solutions have been found to be in good agreement with the analytical results.

Two-dimensional analysis results for a thin-walled Euler column are presented for the purposes of studying convergence. Two and three-dimensional analysis results are presented for a cold-formed steel column to study the bracing requirements. The constitutive properties for all the columns considered in this study are: elastic modulus E = 20000 kN/cm^2 and shear modulus G = 7590 kN/cm^2. The boundary conditions for the two-dimensional columns are: at the bottom support displacements u_1 (translation in x-direction), v_1 (translation in y-direction), w_1 (translation in z-direction) and rotations θ_{x1}, θ_{y1}, χ_1 (rotation about strong axis, torsional rotation and warping rotation, respectively) are zero, but rotation about weak axis θ_{z1} is non-zero; at the top support of the column the boundary conditions are that of the bottom support except that displacement along the length of the column v_2 is non-zero. All the displacement degrees of freedom at intermediate nodes are restrained except the weak axis rotation (θ_{z1}), translation in weak direction (u) and translation along the column length (v), so that the column is forced to act as a two-dimensional column. The boundary conditions for three-dimensional column at the bottom support are: restrained against translation in all three directions, free to rotate about weak and strong axes, restrained against torsion but free to warp, unless specified otherwise; and the boundary conditions at the top support of the column are similar to that at the bottom support, except that it is free to translate in the longitudinal direction or along the column length (i.e., roller support). None of the displacement degrees of freedom at intermediate nodes (seven at each node) are restrained. A second-order geometric non-linear analysis is performed for all the columns. An asymmetric initial geometric imperfection Δ_x is specified for both the two- and three-dimensional braced columns studied. The initial deflection (imperfection) is provided by adding together a half sinewave Δ_{x1} with amplitude d_{01} and a full sinewave Δ_{x2} with amplitude d_{02}. These deflections are expressed as

$$\Delta_{x1} = d_{01}\sin\left(\frac{\pi x}{L}\right); \qquad \Delta_{x2} = d_{02}\sin\left(\frac{2\pi x}{L}\right); \qquad \Delta_x = \Delta_{x1} + \Delta_{x2} \tag{5}$$

The magnitudes of the initial displacements in Eqn. (5) are small and each column can be considered to be almost perfect within allowable tolerances. An initial asymmetric imperfection is used so that the second buckling mode will be clearly identifiable. The stability or the buckling load P_{cr} for braced columns is defined as the load corresponding to a midheight lateral deflection of twice the initial imperfection ($2d_{01}$), as for practical design purposes it is undesirable to have deflections greater than $2d_{01}$ at midheight of the column (Winter 1960).

Convergence of Finite Element Solution : A convergence study of the finite element solution is reported in this section and results are compared with the weak axis Euler load P_y of the column. Properties for the thin-walled I-section (Wang *et al.* 1980) are: A_x (area of cross-section) = 11.61 cm^2; I_y (moment of inertia about weak axis) = 33.39 cm^4; I_z (moment of inertia about strong axis) = 665.97 cm^4; J (torsion constant) = 0.421 cm^4; C_w (warping constant) = 3437.26 cm^6; and L (length of column) = 254 cm. The boundary conditions and nodal restraints are that for a two-dimensional column. An initial geometric imperfection Δ_x is given about the weak axis in accordance with Eqn. (5) to initiate bending. An amplitude of d_{01}=0.0005L for the half sinewave and an amplitude of d_{02}=0.0001L for the full sinewave is used in Eqn. (5). Since the column for this case is not braced, the column stability load P_{cr} is defined to be the load at which the tangent stiffness is reduced to 10^{-8} the elastic tangent stiffness.

Fig. 2 shows the buckling load ratio versus an increasing number of elements used to discretize the column. The Euler load of this column is 102.180 kN. In Fig. 2 the stability load P_{cr} is non-dimensionalized with respect to the weak axis Euler load P_y. Convergence is obtained with an eight element discretization. It is seen that even with six elements, the ratio P_{cr}/P_y is 0.963 which is very close to the ratio of 0.959 for an eight element discretization. The converged load is smaller in value as compared to the Euler load due to the inclusion of second-order geometric nonlinear effects in the present analysis. An eight element discretization is used for all the subsequent column analyses.

Lateral Brace Stiffness/Strength Requirement for Two-Dimensional Column : The effect of lateral bracing provided at midheight on the buckling strength of a column is studied. A cold-formed I-section (C4x2.25x0.105) is considered from the Cold-Formed Design Manual (1987) which consists of two channel sections back to back. Properties for this section are: A_x = 3.28 cm^2; I_y = 2.39 cm^4; I_z = 26.47 cm^4; J = 0.078 cm^4; C_w = 23.44 cm^6; and L (length of column) = 64 cm. The boundary conditions and nodal restraints are that of a two-dimensional column. An initial geometric imperfection (Δ_x) is given about the weak axis in accordance with Eqn. (5) to initiate bending. An amplitude of d_{01}=L/1000 for the half sinewave and an amplitude of d_{02}= L/10000 for the full sinewave are used in Eqn. (5). A concentrated force is applied on the top of the column. The lateral brace (with stiffness K_{bx} or non-dimensionalized stiffness S_x) at midheight is modeled using a flexible connection element. The non-dimensionalized brace stiffness is defined as

$$S_x = \frac{K_{bx}\, L}{P_y} \tag{6}$$

Geometric non-linearity in the connection element is also considered. The lateral brace stiffness is adjusted by varying the translational stiffness of the connection element (K_{bx}). All the degrees of freedom (dof) of the connection element except the translational dof are slaved to the midheight node of the column.

Fig. 3 shows the load-displacement curve for the lateral displacements at the midheight of the column at the brace location for a brace stiffness of $S_x = 27.33$ at which the column buckles into the second mode, as shown in Fig. 3.

Fig. 4 shows the stability load ratios, i.e., P_{cr}/P_y, with varying lateral brace stiffness for the two-dimensional column. It is seen that $S_x = 27.33$ yields a stability load of approximately $4P_y$, i.e., the second buckling mode is encountered. Hence, full bracing is achieved for this column. Fig. 5 shows the axial force in the brace with increasing lateral brace stiffness. It is observed that the maximum strength required for the brace is approximately 5.5 % of the weak axis Euler buckling load. Deflected shapes of the column about the weak axis for different values of lateral brace stiffness are shown in Fig. 6. It is observed that the second flexural buckling mode occurs at a lateral brace stiffness of $S_x = 27.33$.

Torsional Brace Stiffness Requirement for a Three-Dimensional Column : The three-dimensional column consists of the same cold-formed I-section (C4x2.25x0.105) as that considered for the two-dimensional case. Boundary conditions and nodal restraints are that of the three-dimensional column. Initial imperfections are provided about both the strong and weak axes of the column in accordance with Eqn. (5) with amplitudes of d_{01}=L/1000 and d_{02}= L/10000. Such imperfections lead to a small initial twist in the column. The full lateral brace stiffness of $S_x = 28$ is assumed for this column which is slightly higher than the stiffness determined from the two-dimensional analysis (i.e., $S_x = 27.33$). Again, a flexible connection element is used to model the torsional stiffness of the brace ($K_{b\theta y}$) to the column at midheight. The non-dimensionalized torsional brace stiffness ($S_{\theta y}$) is expressed as

$$S_{\theta y} = \frac{K_{b\theta y} L}{GJ} \tag{7}$$

where J = torsion constant of the column.

Fig. 7 shows the non-dimensionalized stability loads of the column with increasing torsional stiffness of the brace at midheight. It is observed that if $S_{\theta y} = 0$, torsional buckling with $P_{cr} = 1.6P_y$ occurs, which is very close to the theoretical value of the column torsional buckling load of $1.67P_y$ computed from Timoshenko and Gere (1961). It is seen that a torsional brace stiffness of $S_{\theta y} = 550$ eliminates the torsional buckling mode and forces the column back into the second flexural buckling mode about the weak axis. Fig. 8 shows the bending strength required (23 kN-cm) for the brace provided for the three-dimensional column.

Effects of Warping Restraint at Supports on Torsional Brace Stiffness Requirement : Effects of various warping end conditions of the column are studied. The three-dimensional column considered in the previous section is used. The warping degrees of freedom were not restrained (warping free or warping fixity factor = 0.0; Blandford 1994) in the previous section at the supports. Two different cases are considered in this study. In the first case, the warping fixity factor for the supports is considered to be 0.5 and in the second case, the warping is fully restrained (warping fixed or warping fixity factor = 1.0) at the supports. A flexible connection element is used at each support of the column to model the varying warping stiffnesses. In both cases, a full lateral brace stiffness of $S_x = 28$ is provided. It is seen from Fig. 9 that the torsional brace stiffness requirement reduces with increasing warping fixity. A torsional brace stiffness of $S_{\theta y} = 400$ for the first case leads to the second flexural buckling mode and it is observed that no torsional brace stiffness is required if the column ends are fully restrained against warping, as is considered in the second case.

SUMMARY AND CONCLUSIONS

A finite element formulation for large-deformation analysis of space frame structures based on second-order geometric nonlinear theory (large displacement of members with small strains) and iterative updated Lagrangian geometry corrections has been used to perform analyses of columns with a midheight brace. A flexible connection element included in the program is used to model the lateral and torsional brace of each column.

The lateral brace stiffness required to provide an effective brace for the column to buckle into the second mode has been evaluated when the column is restricted to pure flexural bending about the weak axis. It has been shown by means of space frame analysis capabilities that a very slight initial torque to the column (with warping free at both ends of the column) can lead to torsional buckling rather than the anticipated flexural buckling, even though full lateral brace stiffness is provided to the column. The buckling load becomes very small, i.e., about 1.6 times of the Euler load. In such a case, it is essential to provide torsional and translational brace stiffnesses. It is seen from the results presented that a non-dimensionalized torsional stiffness of about $S_{\theta y}$ = 550 (i.e., 550 times GJ/L of the column) is required to force the column into the second flexural buckling mode. The effect of warping restraint at the ends of the column has a large impact on the torsional brace stiffness requirements. It has been shown that no torsional brace is required when the column supports are fully restrained against warping.

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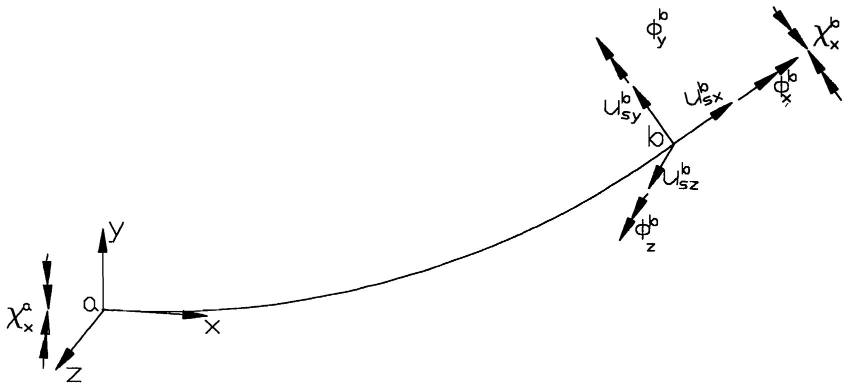

Fig. 1(a). Cantilever Bound Deformation Degrees of Freedom for the Frame Element

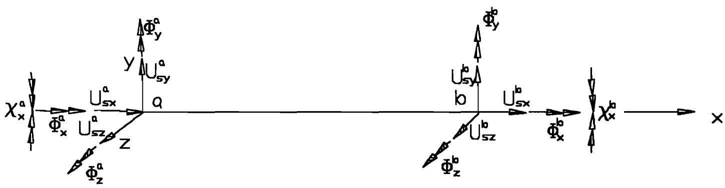

Fig. 1(b). Complete Reference Displacement Degrees of Freedom for the Frame Element

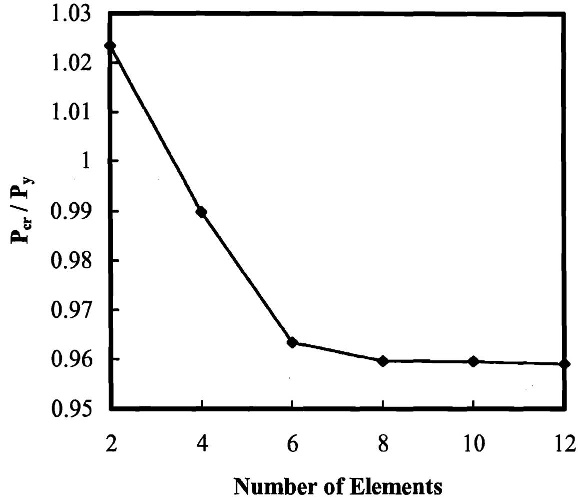

Fig. 2. Convergence of Finite Element Solution

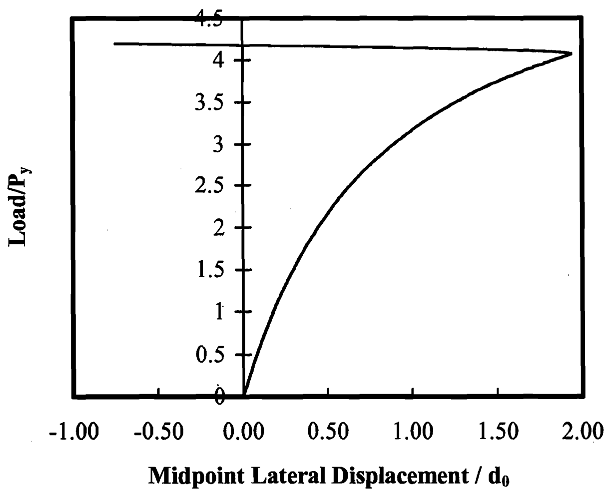

Fig. 3. Load-Displacement Curve for Two-Dimensional Column with Non-dimensionalized Lateral Brace Stiffness of S_x = 27.33

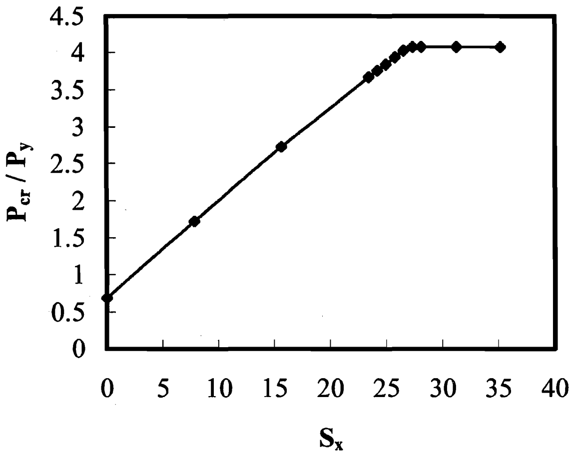

Fig. 4. Variation of Buckling Load With Increasing Lateral Brace Stiffness for Two-Dimensional Column

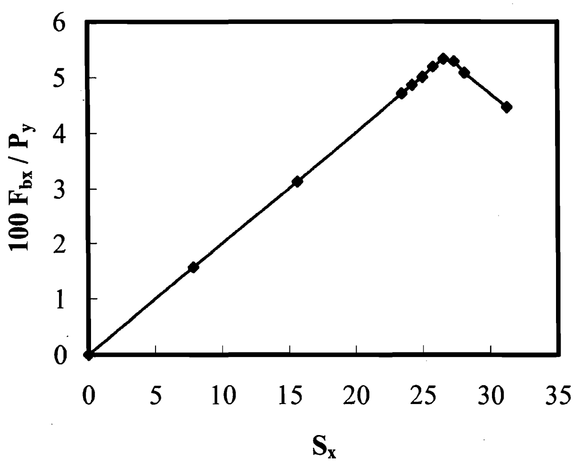

Fig. 5. Variation of Brace Force With Varying Lateral Brace Stiffness for Two-Dimensional Column

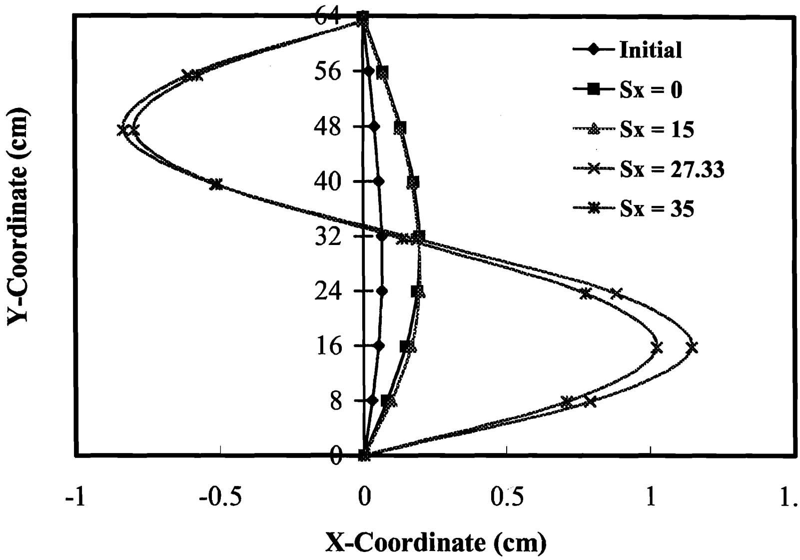

Fig. 6. Deflected Shape of the Two-Dimensional Column About the Weak Axis for Different Values of the Lateral Brace Stiffness

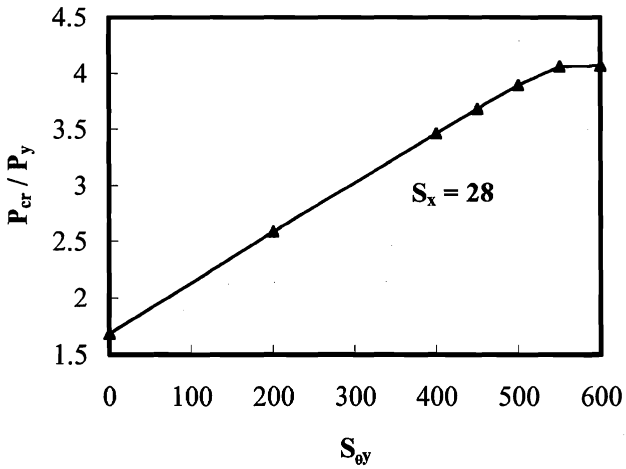

Fig. 7. Variation of Stability Load With Varying Torsional Brace Stiffness for the Three-Dimensional Column

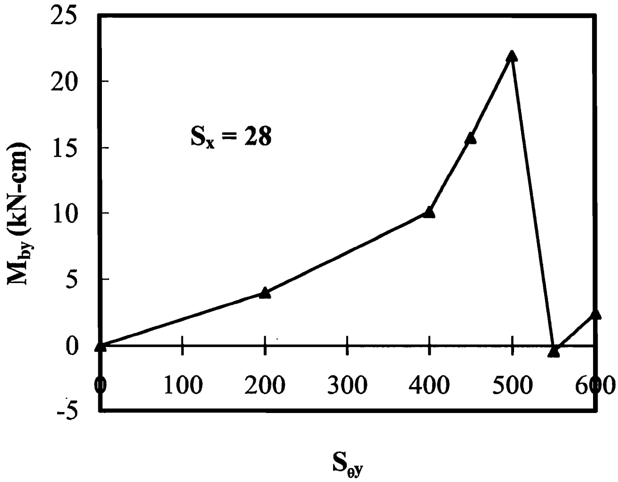

Fig. 8. Variation of Bending Moment in Brace With Varying Torsional Brace Stiffness for the Three-Dimensional Column

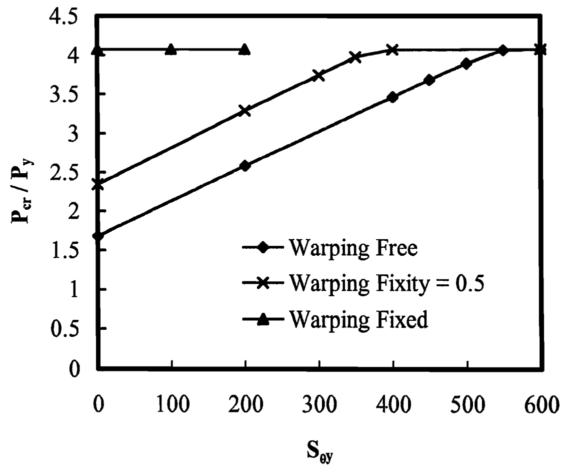

Fig. 9. Variation of Stability Load With Varying Rotational Brace Stiffness for the Three-Dimensional Column with Varying Warping Restraint at Supports