Buckling Mode Interaction in Cold-formed Steel Columns and Beams

C. Jiang

Viorel Ungureanu

J. Michael Davies

Fourteenth International Specialty Conference on Cold-Formed Steel Structures
St. Louis, Missouri U.S.A., October 15-16, 1998

BUCKLING MODE INTERACTION IN COLD-FORMED STEEL COLUMNS AND BEAMS

J. M. Davies(1), C. Jiang(2) and V. Ungureanu(3)

Summary

The buckling behaviour of cold-formed steel columns and beams is far from simple. The three generic forms of buckling, namely local buckling, distortional buckling and overall buckling, generally have different wavelengths and are usually restricted to different span ranges. However, there is also the possibility of interaction among these buckling modes at a certain span length. Thus, local plate buckling and distortional buckling may occur together with lateral-torsional buckling in such a way that they all have an influence on the ultimate load carrying capacity of a member. The influence of local buckling is taken into consideration in design codes by using either an effective width or an effective thickness for the plate element under consideration. However, the consideration of distortional buckling is less-well developed in the codes and the effect of its interaction with other buckling modes is far from clear.

Another factor which influences the ultimate load carrying capacity of a thin-walled section is the interaction of compression force and bending moment. In current design codes, this is usually limited by a certain allowable stress in the extreme fibres of the cross-section. For a stocky member which is not subject to buckling, this is a reasonable assumption. However, for potentially unstable members, the behaviour is complicated and, in most cases, the test results are scattered high above the predictions given by the design codes.

Generalised Beam Theory (GBT) [1-3] can provide explicit analytical expressions for the problems associated with the various interactions of the alternative buckling modes and also the interactions associated with combinations of axial load and bending. This paper, therefore, makes particular use of GBT in assessing the influence of local buckling and distortional buckling on lateral-torsional buckling and the interaction of compression force and bending moment. Based on analyses using GBT together with comparisons with available test results, conclusions are presented which the authors hope will be of benefit for future design codes.

(1) Professor of Structural Engineering, Dept. of Civil Engineering, Manchester School of Engineering, University of Manchester, Manchester M13 9PL, UK
(2) Research Engineer, Dept. of Civil Engineering, Manchester School of Engineering, University of Manchester, Manchester M13 9PL, UK
(3) Research Engineer, Laboratory of Steel Structures, Centre for Advanced and Technical Sciences, Romanian Academy of Sciences, Timisoara Branch, M. Viteazul 24, Timisoara, RO

1. Explicit expressions for buckling interaction using Generalised Beam Theory

The basic differential equations of Generalised Beam Theory (GBT) with the second-order terms included are:

$$E\cdot{}^{k}C\cdot{}^{k}V''''-G\cdot{}^{k}D\cdot{}^{k}V''+{}^{k}B\cdot{}^{k}V+\sum_{i=1}^{m}\sum_{j=2}^{n}{}^{ijk}\kappa\cdot({}^{i}W{}^{j}V')'={}^{k}q \tag{1}$$

where a forward superscript k is used to denote the mode number (k =1, 2, ... n) and

E and G = Young's modulus and shear modulus respectively;
${}^{k}C$ = generalised warping resistance;
${}^{k}D$ = generalised torsional resistance;
${}^{k}B$ = transverse bending resistance;
${}^{ijk}\kappa$ = second order terms representing the deviation forces which are caused by axial stress together with deformation of the member;
${}^{k}W$ = stress resultant (${}^{1}W$ = compression force, ${}^{2}W$ and ${}^{3}W$ = bending moment about the major and minor axes respectively, ${}^{4}W$ = torque);
${}^{k}V$ = generalised deformation resultant.

For bifurcation problems, as there is no load causing deformation prior to buckling, the right hand side term ${}^{k}q$ is zero and we have buckling with a constant stress resultant ${}^{i}W$:

$$E\cdot{}^{k}C\cdot{}^{k}V''''-G\cdot{}^{k}D\cdot{}^{k}V''+{}^{k}B\cdot{}^{k}V+{}^{i}W\sum_{i=1}^{m}{}^{ijk}\kappa\cdot{}^{j}V''=0 \tag{2}$$

If a thin-walled member is assumed to have pinned end conditions and to buckle in a half sine wave, the above equation gives rise to explicit expressions for member buckling with mode interaction. As a special case, if only the single mode "k" is considered, the minimum critical stress resultant and the corresponding half wavelength are [3-4]:

$${}^{i,k}W_{cr} = \frac{1}{{}^{ikk}\kappa}(2\sqrt{E^{k}C^{k}B}+G^{k}D) \tag{3}$$

$$\lambda_{cr}=\pi\sqrt[4]{\frac{E\ {}^{k}C}{{}^{k}B}} \tag{4}$$

From equation (4) it can be seen that the half wavelength depends only on the cross-section properties ${}^{k}C$ and ${}^{k}B$ which are independent of the load.

If two modes "j" and "k" are allowed to buckle interactively in a single half sine wave, there are two cases to consider:

a) for ${}^{iij}\kappa \neq 0$ and ${}^{ikk}\kappa \neq 0$, The minimum critical stress resultant is:

$${}^{i,jk}W_{cr} = {}^{i,j}W\frac{\beta}{2}(1+\omega)[1-\sqrt{1-\frac{4\omega}{\beta(1+\omega)^{2}}}] = \gamma.{}^{i,j}W_{cr} \tag{5}$$

where

$$\beta = \frac{1}{1 - \frac{{}^{ijk}\kappa \; {}^{ikj}\kappa}{{}^{iij}\kappa \; {}^{ikk}\kappa}} \tag{6}$$

and

$$\omega = \frac{{}^{i,k}W_{cr}}{{}^{i,j}W_{cr}} \geq 1 \tag{7}$$

In equation (5), γ is a coefficient which has a value less than or equal to 1 and which reflects the amount of mode interaction. The value of β varies between 1 and ∞. The lowest possible value of γ is 0.5 when β=1 and ω=1, which is the maximum interaction of two modes. These explicit interaction expressions were first obtained by Schardt [4].

b) For ${}^{iij}\kappa = {}^{ikk}\kappa = 0$, equation system (2) can be written as:

$$\begin{cases} {}^{j}P \cdot {}^{j}V_m - {}^{ikj}\kappa \cdot {}^{ijk}W_{cr} \cdot {}^{k}V_m = 0 \\ -{}^{ijk}\kappa \cdot {}^{ijk}W_{cr} \cdot {}^{j}V_m + {}^{k}P \cdot {}^{k}V_m = 0 \end{cases} \tag{8}$$

where

$${}^{k}P = E \cdot {}^{k}C \cdot \left(\frac{\pi}{L}\right)^2 + G \cdot {}^{k}D + {}^{k}B \cdot \left(\frac{L}{\pi}\right)^2 \tag{9}$$

and L is the length of member.

The critical value of the stress resultant ${}^{i,jk}W_{cr}$ can be obtained by solution of equation (8):

$${}^{j}P \cdot {}^{k}P - {}^{ijk}\kappa \cdot {}^{ikj}\kappa \left({}^{i,jk}W_{cr}\right)^2 = 0 \tag{10}$$

Finally, the interaction expression will be

$${}^{i,jk}W_{cr} = \sqrt{\frac{{}^{j}P \cdot {}^{k}P}{{}^{ijk}\kappa \cdot {}^{ikj}\kappa}} \, . \tag{11}$$

The interaction of compression force and bending moment can be obtained by considering combinations of the applied loads ${}^{1}W$, ${}^{2}W$ and ${}^{3}W$ in solutions of equation (2).

2. Mode interaction for a compressed column

Taking a lipped channel section column with cross-section dimensions: lip width b_l = 15 mm, flange width b_f = 50 mm, web depth b_w = 90 mm and material thickness t = 1.5 mm as an example, in which the material of the member has a modulus of elasticity E = 200000 N/mm^2 and Poisson's ratio ν = 0.3, the cross section properties computed using GBT are listed in Tables 1 and 2. The corresponding buckling modes are illustrated in Figure 1 in which the first four modes are global buckling (rigid body) modes, the fifth and sixth modes are the symmetric and asymmetric cross-section distortional buckling modes respectively and the seventh mode is the local web buckling mode. The eighth and ninth modes are the asymmetric and the symmetric lip buckling modes respectively.

Mode interaction only occurs when the relevant off-diagonal terms in the ^{1jk}k matrix are non-zero. It follows from Table 2 that $^{123}\kappa = {}^{132}\kappa = {}^{156}\kappa = {}^{165}\kappa = {}^{178}\kappa = {}^{187}\kappa = {}^{167}\kappa = {}^{176}\kappa = {}^{169}\kappa = {}^{196}\kappa = 0$ which means that, for a symmetric column, there is no mode interaction between the symmetric or asymmetric modes. However, there is potential interaction between the symmetric buckling modes and between the asymmetric buckling modes.

The interaction between lip and flange distortional buckling is trivial because the buckling half wavelengths of the lip and flange modes are in most cases quite different. For the same reason, there is very little interaction between the local or distortional buckling and the global buckling modes. However, the interaction between local and distortional buckling may be significant. For example, the interaction coefficient γ for symmetrical distortional and local web buckling is 0.804 when the column length is 200cm.

k=	C(cm^4)	D(cm^2)	B(kN/cm^2)
1	3.3000	0.0000	0.0000
2	45.903	0.0000	0.0000
3	12.845	0.0000	0.0000
4	252.69	0.0248	0.0000
5	0.1674	0.0006	0.0626
6	0.2018	0.0007	0.2024
7	0.0013	0.0010	0.9123
8	0.0006	0.0021	2.8189
9	0.0005	0.0021	3.0703

Table 1 - Section properties of a lipped channel section

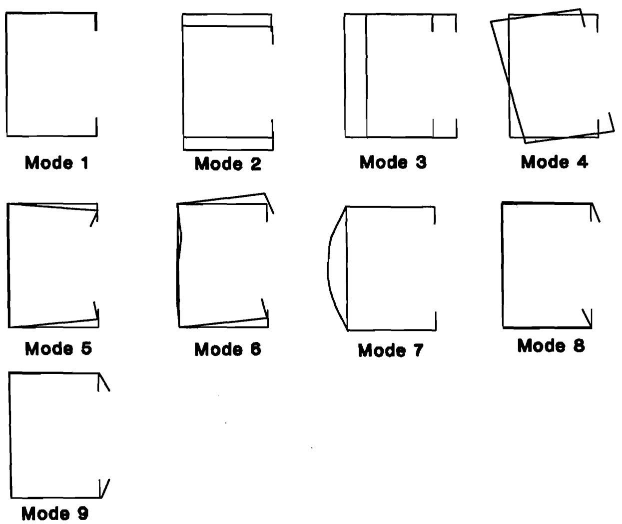

Fig. 1 - Buckling modes for a lipped channel section subject to uniform compression

Figure 2 shows some results obtained for the channel section column with two ends pinned. It shows that the column buckles in the local mode for shorter lengths and in the torsional mode for the longer lengths. There is no interaction between the rigid body mode 4 and the local web mode 7 or the distortional mode 5 because they are asymmetric and symmetric modes respectively. For global (rigid body) buckling, only modes 2 and 4 couple and their coupled critical stress is ${}^{2,4}\sigma_{cr}$ = 10.577 kN/cm^2 for 200 cm span according to equation (5). There is relatively trivial interaction between rigid body mode 4 and asymmetric distortional mode 6 because their interaction parameter γ = 0.953, and the interaction between local mode 7 and distortional mode 5 is more significant with an interaction parameter γ = 0.810 according to equation (5).

k=	j=2	3	4	5	6	7	8	9
				${}^{1jk}K$				
2	1.0000	0.0000	-4.4281	0.0000	0.2854	0.0000	-0.1190	0.0000
3	0.0000	1.0000	0.0000	0.1735	0.0000	-0.2217	0.0000	-0.0918
4	-4.4281	0.0000	37.4086	0.0000	-1.4720	0.0000	0.7891	0.0000
5	0.0000	0.1735	0.0000	0.2567	0.0000	-0.0941	0.0000	0.0226
6	0.2854	0.0000	-1.4720	0.0000	0.2619	0.0000	-0.0299	0.0000
7	0.0000	-0.2217	0.0000	-0.0941	0.0000	0.1913	0.0000	0.0001
8	-0.1190	0.0000	0.7891	0.0000	-0.0299	0.0000	0.0837	0.0000
9	0.0000	-0.0918	0.0000	0.0226	0.0000	0.0001	0.0000	0.0788
				${}^{2jk}K$				
2	0.0000	0.0000	0.0000	0.0931	0.0000	-0.0202	0.0000	0.0347
3	0.0000	0.0000	0.9999	0.0000	0.0396	0.0000	0.0106	0.0000
4	0.0000	0.9999	0.0000	-0.4964	0.0000	0.0552	0.0000	-0.2464
5	0.0931	0.0000	-0.4964	0.0000	0.0717	0.0000	-0.0145	0.0000
6	0.0000	0.0396	0.0000	0.0717	0.0000	-0.0090	0.0000	0.0091
7	-0.0202	0.0000	0.0552	0.0000	-0.0090	0.0000	0.0109	0.0000
8	0.0000	0.0106	0.0000	0.0145	0.0000	0.0109	0.0000	-0.0214
9	0.0347	0.0000	-0.2464	0.0000	0.0091	0.0000	-0.0214	0.0000
				${}^{3jk}K$				
2	0.0000	0.0000	-0.9997	0.0000	0.1952	0.0000	-0.0295	0.0000
3	0.0000	0.0000	0.0000	-0.0202	0.0000	0.1219	0.0000	-0.0388
4	-0.9997	0.0000	11.7961	0.0000	-1.2668	0.0000	0.4106	0.0000
5	0.0000	-0.0202	0.0000	0.1346	0.0000	0.0313	0.0000	0.0093
6	0.1952	0.0000	-1.2668	0.0000	0.1717	0.0000	0.0023	0.0000
7	0.0000	0.1219	0.0000	0.0313	0.0000	-0.0805	0.0000	-0.0236
8	-0.0295	0.0000	0.4106	0.0000	0.0023	0.0000	0.0434	0.0000
9	0.0000	-0.0388	0.0000	0.0093	0.0000	-0.0236	0.0000	0.0414

Table 2 - Second order cross-section properties for a channel section

For a uniformly compressed column of relatively short length with an axis of symmetry, there is a general tendency for the member to buckle in a symmetrical local or distortional mode.

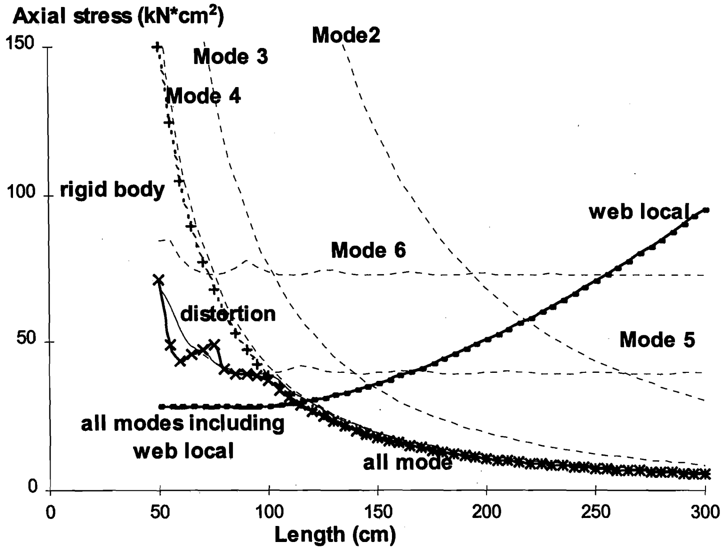

Fig. 2 - Buckling curves for a pin-ended lipped channel column (CH90 x 50 x 15 x 1.5)

3. Mode interaction for a beam

In contrast to the case of column, for a beam with a symmetrical cross-section, single mode buckling cannot occur if the bending moment is applied normal to the axis of symmetry because the relevant diagonal terms in the ${}^{2jk}k$ matrix are all zero, i.e. ${}^{222}\kappa = {}^{233}\kappa = {}^{244}\kappa = {}^{255}\kappa = {}^{266}\kappa = {}^{277}\kappa = {}^{288}\kappa = {}^{299}\kappa = 0$. In this case, the buckling of the beam takes the form of an interaction of two or more modes but the interaction occurs only between the symmetric modes or the asymmetric modes.

Again taking the above lipped channel section as an example, when a constant bending moment is applied about its symmetric axis, rigid body buckling is an interaction between modes 3 and 4 and distortional buckling is the interaction of modes 5 and 6. For a 200cm span, the critical stress ${}^{3,4}\sigma_{cr} = 29.59$ kN/cm^2 can be obtained from equation (11).

The curve of distortional buckling interaction in Figure 3 is obtained using the finite difference method because the distortional buckling curve is not a pure sine curve and equation (11) is not valid. From Figure 3 it can be seen that the buckling interaction between local or distortional and global buckling is not significant in this case.

Using the same method it can be seen that:

1) If a bending moment is applied parallel to the axis of symmetry, there are individual distortional buckling modes and their interaction behaviour is similar to that of a column, but there are no individual global modes 2 and 3 because ${}^{322}\kappa = {}^{333}\kappa = 0$.
2) The buckling interaction of a cold-formed member under an applied torque can be analysed in a similar way based on the value of ${}^{4jk}\kappa$ but this is not included in this paper.
3) For a member with a point-symmetric cross section, for instance a Z-section, there is no single mode buckling and such a section always buckles by the interaction of two or more modes.

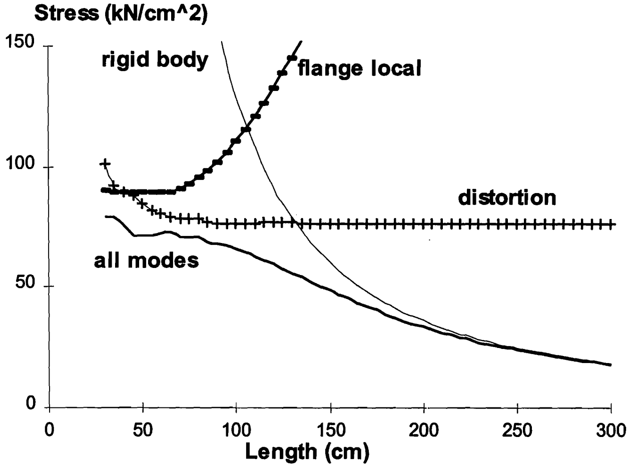

Fig. 3 - Buckling curves for a simply supported lipped channel beam subject to bending about the axis of symmetry (CH90 x 50 x 15 x 1.5)

4. Interaction of bending and compression

Now let us consider whether there is linear relationship for interaction between axial load and bending, as used in typical codes:

$$\frac{P}{P_c} + \frac{M_x}{M_{cx}} + \frac{M_y}{M_{cy}} \leq 1 \tag{12}$$

where P is the axial compressive force, M_x and M_y are bending moments about the x and y axes respectively; P_c is the ultimate resistance in pure compression and M_{cx} and M_{cy} are the ultimate moments of resistance in pure bending.

Assuming that above lipped channel section member has pinned end boundary conditions and that its major and minor axes are denoted by x and y respectively, two representative lengths are chosen for consideration, namely 70cm and 200cm, in which the member buckles firstly in a local or distortional mode and secondly in a flexural-torsional mode. By systematically changing the ratio of the compression force to bending moment, load interaction curves can be obtained by solving equation (2) and these are illustrated in Figures 4 and 5. GBT, of course, gives elastic bifurcation loads and, in order to introduce a yield criterion, the Ayrton-Perry formula is used to convert the GBT results into failure loads:

$$P_{ult} = \frac{1}{2}[(P_s + (1+\eta)P_E) - \{(P_s + (1+\eta)P_E)^2 - 4P_sP_E\}^{1/2}] \tag{13}$$

where:
- P_{ult} = the failure load of the member;
- P_s = the "squash" load evaluated using the effective cross-section;
- P_E = the Euler buckling load evaluated for the full cross-section;
- η = an imperfection parameter.

$$\eta = 0.002 \cdot \left(\frac{L_E}{r} - 40\right) \geq 0 \tag{14}$$

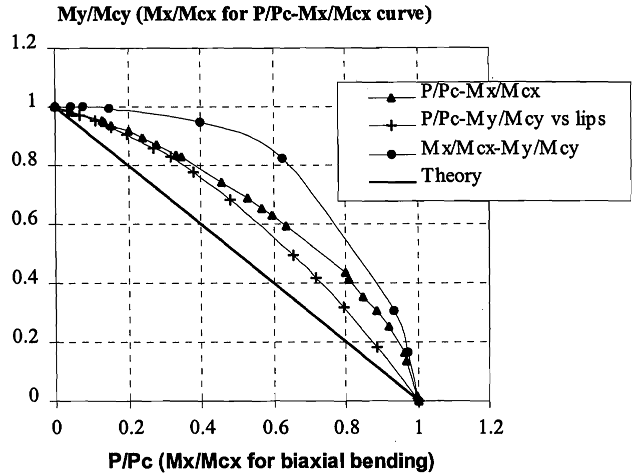

Fig. 4 - Interaction of compression force and bending moment in a 200 cm long lipped channel section (CH90 x 50 x 15 x 1.5)

Three curves are given in Figures 4 and 5, namely:

P/P_c - M_x/M_{cx}	= combination of axial load with bending about the symmetric axis;
P/P_c - M_y/M_{cy} vs. lips	= combination of axial load with bending about the asymmetric axis (lips in compression);
M_x/M_{cx} - M_y/M_{cy}	= combination of bending about the two principal axes.

The line indicated "theory" is, of course, the linear interaction relationship given by typical codes.

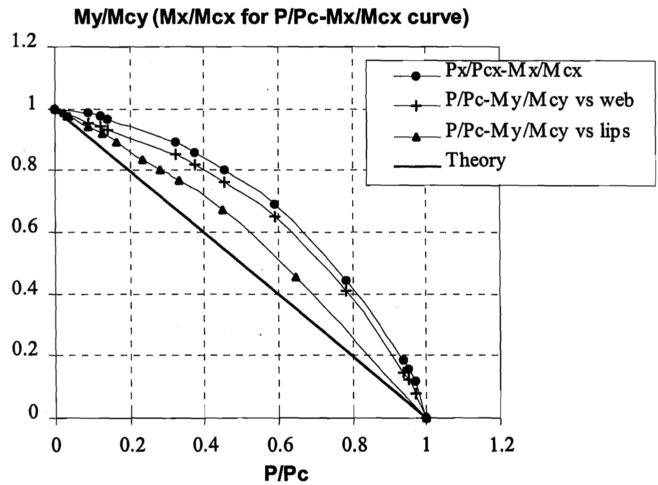

Fig. 5 - Interaction of compression force and bending moment in a 70 cm long lipped channel section (CH90 x 50 x 15 x 1.5)

From Figures 4 and 5, it can be seen that the interaction relationship in each case is a curve rather than the straight line assumption of equation (12). The interaction here is far from simple and depends on the stress distribution in the cross-section rather than the stress value in the extreme fibre in compression. Clearly, a member subject to a combination of modest compressive force and bending moment about the axis of symmetry may have a much greater stability than is predicted by the codes.

5. Comparison of tests and theoretical results

The influence of local buckling is taken into account in cold-formed member design codes or standards, for instance, EC3 (Eurocode 3) [5] and AISI (American Iron and Steel Institute) [6] by using the concept of effective width or thickness. However, the interactions between local and distortional modes and between distortional and global modes of buckling are not included in the codes and standards. Recent research [7] has shown that both approaches could be used for sections which undergo distortional buckling before or at the same time as local buckling.

Figures 6 to 10 show some comparisons between test results and the predictions given by GBT (Ayrton-Perry), EC3 and AISI. In these figures, $N = A \cdot f_y$ is the short column yield load [8,9]. In general, the Ayrton-Perry formula with the elastic buckling forces obtained using GBT provides the best mean fit to the test results. The curves obtained according to EC3 and AISI, which are based on semi-empirical methods, experimentally calibrated, follow the trend of the test results, but tend to be rather conservative [14].

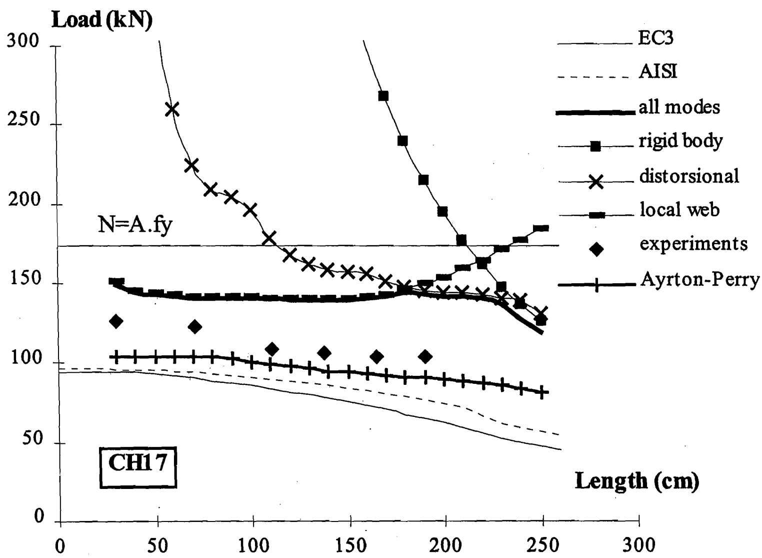

Fig. 6 - Comparison between test results, the GBT approach and available design methods
For the CH17 series of lipped channel columns [8]

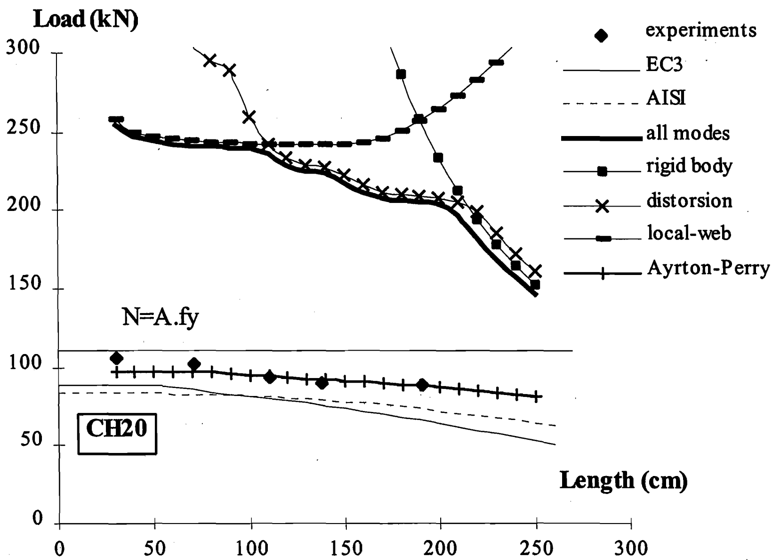

Fig. 7 - Comparison between test results, the GBT approach and available design methods for the CH20 series of lipped channel columns [8]

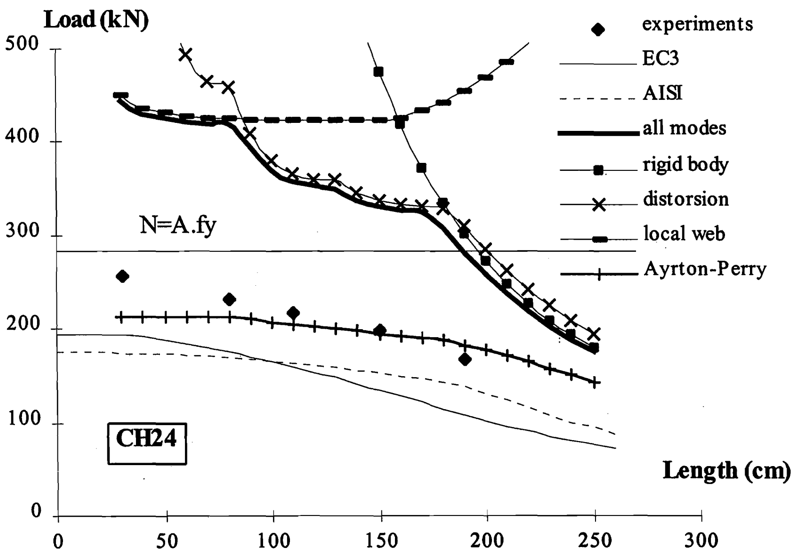

Fig. 8 - Comparison between test results, the GBT approach and available design methods for the CH24 series of lipped channel columns [8]

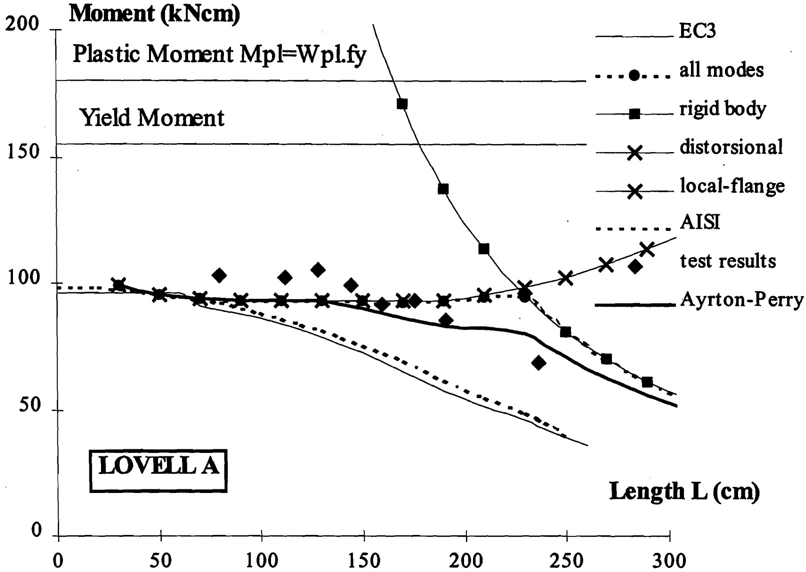

Fig. 9 - Comparison between test results, the GBT approach and available design methods for the Lovell A series of lipped channel beams [9]

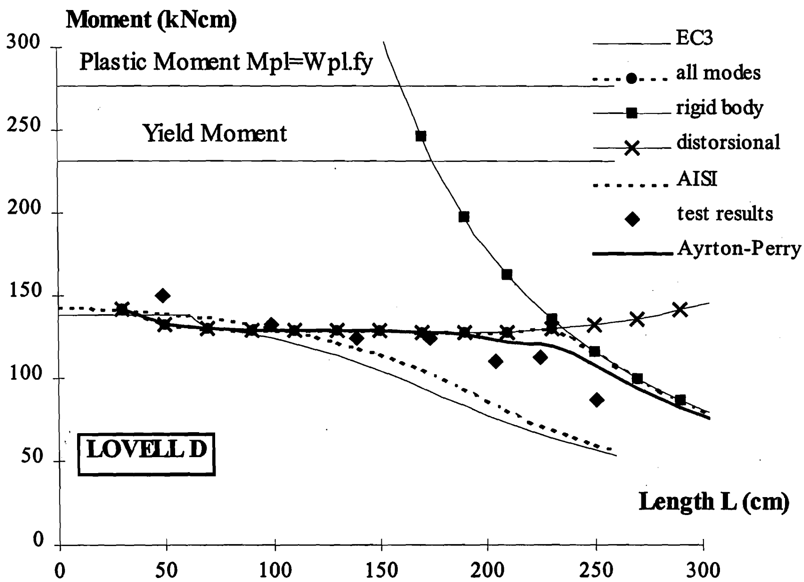

Fig. 10 - Comparison between test results, the GBT approach and available design methods for the Lovell D series of lipped channel beams [9]

In order to consider the interaction between axial load and bending moment, data from 26 eccentric compression tests carried out by Loughlan and Rhodes (1980) were used [10,11]. The test specimens were cold-formed lipped channel sections with both ends pinned. An eccentric axial load was applied so that the specimens were bent about the asymmetric axis with the web in compression. In the tests, the failure of all of the columns was controlled by flexural buckling. Another series of eccentric compression tests carried out by Loh [12,13] on lipped channel sections bent about the axis of symmetry were also used. The buckling lengths used in the determination of the compression and flexural buckling strength are L_{cx}=L, L_{cy}=0.5L and L_{cT}=0.5L.

Two theoretical analyses were carried out using GBT and EC3 [5] in which the bending moment was taken to be the applied load multiplied by the eccentricity. In the analyses using EC3, the approach without lateral-torsional buckling was used and, in the GBT analyses the Ayrton-Perry formula was used to convert the linear buckling results into failure loads. The comparisons between test and prediction are given in Figures 11 and 12 for the Loughlan & Rhodes' tests and in Figures 13 and 14 for Loh's tests. In case of the Loughlan and Rhodes tests, the comparisons show that both theoretical analyses give good correlation with the tests and the mean values of the ratio of test to theoretical results are 1.076 for GBT and 1.188 for EC3. In case of Loh's tests the comparisons again show that the two theoretical analyses give good correlation with the tests and mean failure load ratios of 1.047 for GBT and 1.268 for EC3 were obtained. EC3 can give a satisfactory prediction here because of the relatively small bending moment obtained in tests under eccentric axial load. If the ratio of bending moment to axial load is increased, the design codes generally give a conservative prediction and the advantage of using GBT can be seen to give a better effect.

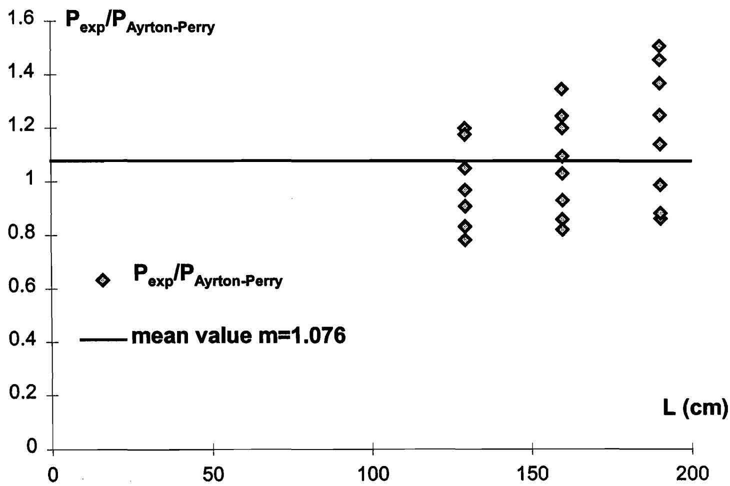

Fig. 11 - Comparison between the Loughlan-Rhodes tests and the results given by GBT

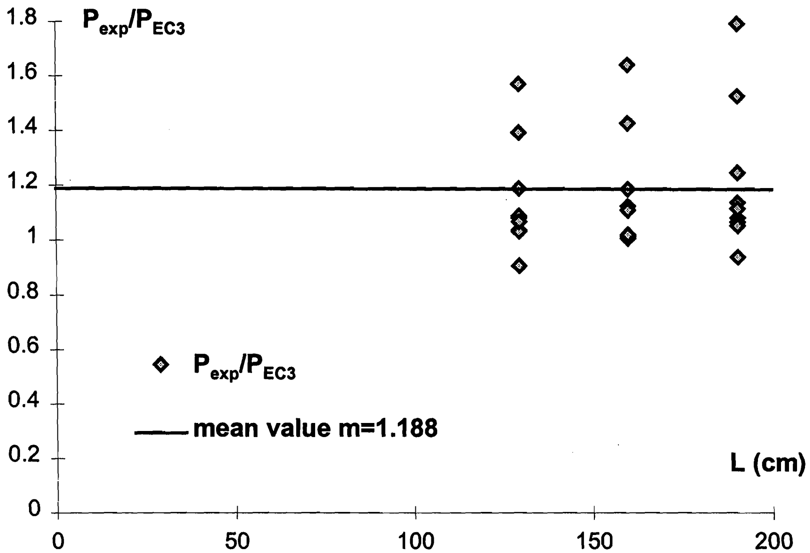

Fig. 12 - Comparison between the Loughlan-Rhodes tests and the predictions of EC3

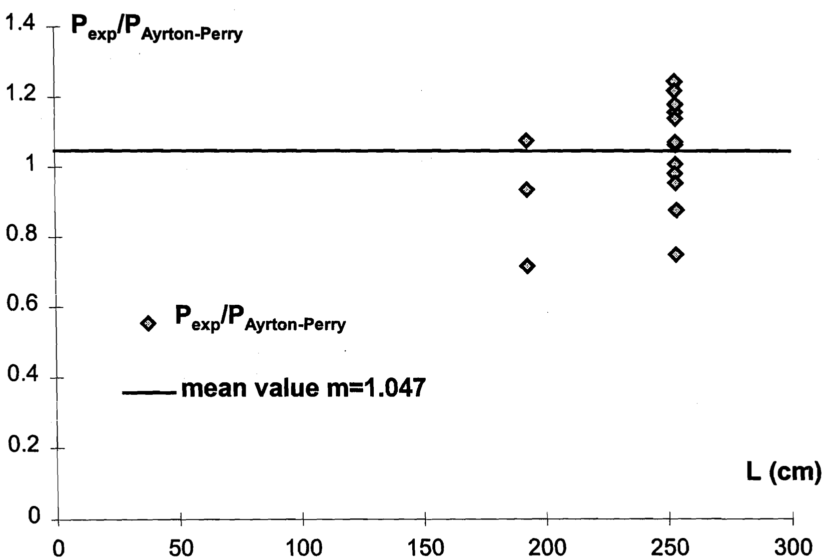

Fig. 13 - Comparison between Loh's tests and the predictions of GBT

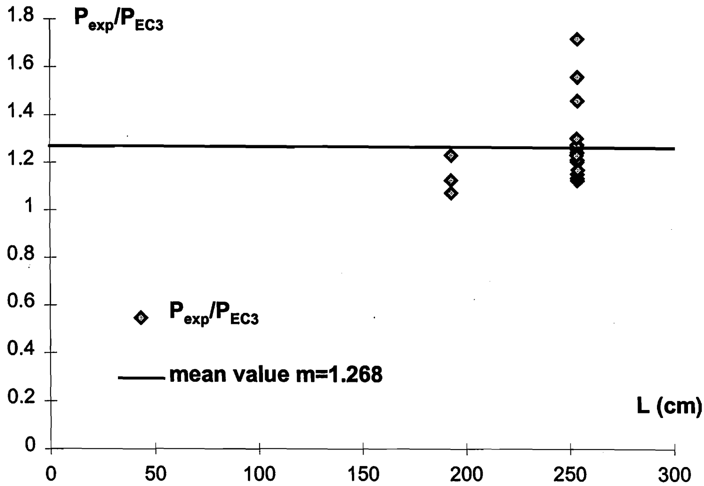

Fig. 14 - Comparison between Loh's tests and the predictions of EC3

6. Conclusions

Mode interaction in both columns and beams has been analysed using GBT and some significant characteristics have been revealed. It can be seen that GBT offers a good tool to with which to analyse the interaction of buckling modes.

The interaction relationship for compressive force and bending moment in thin-walled members is not as simple as is suggested by the assumptions in current design codes because the failure load depends on the stress distribution in the cross-section rather than the stress in the most highly stressed fibre in compression. This is the reason why test results tend to be scattered far from the prediction based on the maximum stress assumption. GBT offers a rational approach to this problem and the codes clauses may be improved as a result of further research.

Appendix—Notation

${}^{k}B$ = transverse bending resistance;
${}^{k}C$ = generalised warping resistance;
${}^{k}D$ = generalised torsional resistance;
${}^{ijk}\kappa$ = second order terms representing the deviation forces which are caused by axial stress together with deformation of the member;
${}^{k}W$ = stress resultant (${}^{1}W$ = compression force, ${}^{2}W$ and ${}^{3}W$ = bending moment about the major and minor axes respectively, ${}^{4}W$ = torque);
${}^{i,jk}W_{cr}$ = critical value of the stress resultant;
${}^{k}V$ = generalised deformation resultant;
E = Young's modulus;

G	= shear modulus;
L	= the length of member;
M_x, M_y	= the bending moments about the x and y axes respectively;
M_{cx}, M_{cy}	= the ultimate moments of resistance in pure bending;
P	= the axial compressive force;
P_c	= the ultimate resistance in pure compression;
P_E	= the Euler buckling load evaluated for the full cross-section;
P_s	= the "squash" load evaluated using the effective cross-section;
P_{ult}	= the failure load of the member;
η	= imperfection parameter.

Appendix—References

1. Schardt R., Verallgemeinerte Technische Biegetheorie (Generalised Beam Theory), *Springer Verlag* 1989.
2. Davies J. M. and Leach P., First-order Generalised Beam Theory", *J Construct. Steel Research,* 31 (1994) 187-220.
3. Davies J. M. and Leach P., Second-order Generalised Beam Theory", *J Construct. Steel Research,* 31 (1994) 221-241.
4. Schardt R., Lateral torsional and distortional buckling of channel- and hat sections, *J Construct. Steel Research,* 31 (1994) 243-265.
5. Eurocode 3, Design of Steel Structures Part 1.3: Cold-formed Thin Gauge Members and Sheeting, *ENV 1993-1-3, February 1996.*
6. AISI- American Iron and Steel Institute Cold-Formed Steel Design Manual, American Iron and Steel Institute Nov. 1986
7. Hancock G. J., Kwon Y. B. and Bernard E. S., Strength design curves for thin walled sections undergoing distortional buckling, *J Construct. Steel Research,* 31 (1994) 169-186.
8. Lau S.C.W. and Hancock G.J., Inelastic buckling of channel columns in the distortional modes *Thin-Walled Structures* 10 (1990), 59-84
9. Lovell H. M., Lateral buckling of light gauge steel beams. MSc Thesis, University of Salford, 1983
10. Loughlan J. and Rhodes J., "The interactive buckling of lipped channel columns under concentric or eccentric loading", *Proc. Int. Conf. on Thin-Walled Structures,* University of Strathclyde, Glasgow, 1980.
11. Weng C. C. and Lin C. P., "Study of the maximum strength of cold-formed steel columns", *J. Struct. Engrg.,* ASCE, 118(1), 1992, 128-146
12. Loh T.S., "Combined axial load and bending in cold-formed steel members", PhD Thesis, Cornell University, Ithaca, New York, 1985, 392p.
13. Salmi and Talja A, "Design of cold-formed HSS channels for bending and eccentric compression (Bending about the axis of symmetry)", Technical Research Centre of Finland, VTT-TIED-1505, September 1993.
14. Dubina et al, "Recent interactive buckling approaches for cold-formed thin-walled members", *Proc. of the 2nd Conf. on "Coupled Instabilities in Metal Structures – CIMS'96",* Liege, Belgium, 5-7 September 1996 (ed. by J. Rondal, D. Dubina, V. Gioncu).