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DYNAMIC INTERACTION BETWEEN RAIL TRACK SYSTEMS AND THE SUBSOIL SOLUTIONS IN THE FREQUENCY- AND TIME DOMAIN

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ABSTRACT

For the numerical simulation of dynamic soil-structure interaction problems both a frequency and a time domain formulation are presented. In order to be capable to consider more sophisticated models of the structure, the frequency domain algorithms for homogeneous and layered halfspaces have been coupled to the Finite Element Program ANSYS. Flexibility functions are presented for a concrete slab track system. Furthermore the stress distribution in the subsoil is calculated and visualized. The time domain formulation is applied for demonstrating the basic phenomena of a moving load passing by with sub- and supercritical speed. Besides that, a nonlinear, tension-free condition of contact between the track and the subsoil is mentioned briefly.

INTRODUCTION

The steady increase of the travelling speed of modern high-speed trains accompanied by higher dynamic interactions with the underlying soil led to an increasing effort for the maintenance of ballast supported railroad tracks. As a consequence new slab track systems have been designed and constructed. These slab track systems exhibit a distinct dynamic behavior compared with ballast supported railroad tracks, whereby in all cases the interaction with the semi-infinite subsoil has to be taken into account.

In this paper a procedure for the simulation of the dynamic behavior of the track (ballasted track or slab track systems) and the subsoil is presented. Based on the substructure method, the system under consideration has been divided into two substructures which have completely different properties: a) the track (finite structure) and b) the subsoil (unbounded continuum). First, the basic equations describing the dynamic behavior of each substructure have been derived independently. Then, the influence of the unbounded soil on the dynamic behavior of the structure is introduced at the interface between both substructures as a boundary condition for the structure. In the present paper this boundary condition is based on a displacement-force relationship (flexibility formulation) calculated by using the influence functions (Green's functions) for a layered or homogeneous halfspace.

SUBSTRUCTURE 'TRACK'

The track will be modeled by using the Finite Element Method. Hence, the governing equations of motion will be transformed to a linear set of equations written in the nodal degrees of freedom (DOF) of the structure. Eq. (1a) gives the resulting dynamic equilibrium of the structure in the frequency domain, whereas Equation (1b) expresses the equilibrium in the time domain for the time step t^{i+1}

$$(-\omega^2\mathbf{M} + i\omega\mathbf{D} + \mathbf{C})\mathbf{u} = \mathbf{P} - \mathbf{Q} \tag{1a}$$

$$\mathbf{M}\ddot{\mathbf{u}}^{i+1} + \mathbf{D}\dot{\mathbf{u}}^{i+1} + \mathbf{C}\mathbf{u}^{i+1} = \mathbf{P}^{i+1} - \mathbf{Q}^{i+1} \tag{1b}$$

M: Mass matrix of the structure
D: Damping matrix of the structure
C: Stiffness matrix of the structure
u: Vector of the nodal displacements
P: Vector of the applied external loads
Q: Vector of the interaction loads between soil and structure
ω : Angular frequency
i : Imaginary unit
i: Index of time stepping

The vector of the interaction loads **Q** represents the influence of the subsoil on the dynamic behavior of the structure and enforces the coupling of both substructures in the sense of the soil-structure interaction.

SUBSTRUCTURE 'UNBOUNDED SOIL'

Frequency domain formulation

The description of the unbounded soil starts from the so called influence functions for harmonic point loads acting on the surface of the halfspace. They can be calculated analytically in the case of a homogeneous halfspace [Lamb, 1904] or semi-analytically by means of the Thin-Layer-Method [Kausel, 1986] in the case of a layered one. In Eq. (2) the relationship between the soil displacements $w_\alpha(\omega,r)$ $(\alpha = x,y,z)$ at an arbitrary point due to a point load $P_\beta(\omega)$ $(\beta = x,y,z)$ acting at the surface of the halfspace is given exemplarily for the homogeneous halfspace by means of the Green's function $f_{\alpha\beta}(\omega,r)$

$$w_\alpha(\omega,r) = \frac{1}{Gr} f_{\alpha\beta}(\omega,r) P_\beta(\omega) . \qquad (2)$$

The distance between the source (point load) and the receiver is given by r, whereas G represents the shear modulus of the soil.

In order to obtain the resultant interaction loads **Q** it is necessary to solve a mixed boundary-value problem in which zero stresses are imposed on the soil surface outside the interface, while at the interface displacements according to the motion of the upper structure have to be imposed. To overcome this problem, the interface is discretized into N sub-areas (contact elements) of uniform rectangular shape [Savidis & Richter, 1979]. Within each sub-area the contact stresses are assumed to be constant (Fig. 1 left). Since influence functions for point loads are used, quite arbitrary shaped geometries as well as arbitrary distributed contact stresses are basically possible. The soil displacements are represented by the corresponding quantities at the midpoints of the sub-areas (interaction points, see Fig. 1 right). These assumptions necessitate the calculation of the soil displacement $w^{(j)}$ at the j^{th} interaction point caused by the uniform contact stresses $q^{(k)}$ acting on the k^{th} sub-area. To accomplish this, Eq. (2) must be integrated over the loaded area ΔA

$$w_\alpha^{(j)} = \frac{1}{G} \iint_{\Delta A} \frac{1}{r} f_{\alpha\beta}^{(jk)} dA \; q_\beta^{(k)} = F_{\alpha\beta}^{(jk)} q_\beta^{(k)} \qquad (3)$$

Using a compact vector-matrix notation, the combined effect of all contact stresses acting on the soil-structure interface (expressed by the vector **q**) on the soil displacements at all interaction points (captured by the vector **w**) can be written as

$$\mathbf{w}(\omega) = \mathbf{F}(\omega) \cdot \mathbf{q} \quad \Rightarrow \quad \mathbf{q} = \mathbf{F}^{-1}(\omega) \cdot \mathbf{w}(\omega) \qquad (4a,b)$$

Thus, $\mathbf{F}(\omega)$ can be interpreted as the flexibility matrix of the soil.

Next, it is necessary to relate kinematically the structural displacements **u** with the displacements **w** at the interaction points. This is necessary because the structural degrees of freedom at the soil-structure interface generally do not coincide with those introduced for the subsoil (displacements at the interaction points). This can be done by means of a simple linear transformation matrix $\mathbf{T}^u$ involving the element shape functions

$$\boldsymbol{w} = \boldsymbol{T}^u \cdot \boldsymbol{u} \qquad (5)$$

A similar transformation matrix $\boldsymbol{T}^Q$ holds for the transformation of the contact stresses to their resultants, the so-called interaction loads as,

$$\boldsymbol{Q} = \boldsymbol{T}^Q \cdot \boldsymbol{q} \qquad (6)$$

by taking into account the applied external work. Introducing the vector **q** (Eq. 4b) *in* Eq. (6) *and performing the transformation* given in Eq. (5), the interaction loads can be written in terms of the nodal DOF of the structure as

$$\boldsymbol{Q} = \boldsymbol{T}^Q \cdot \boldsymbol{q} = \boldsymbol{T}^Q \cdot \boldsymbol{F}^{-1} \cdot \boldsymbol{T}^u \cdot \boldsymbol{u} = \boldsymbol{C}_B \cdot \boldsymbol{u} \qquad (7)$$

The interaction loads **Q** in Eq. (1a) can be replaced by Eq. (7), where the matrix $\boldsymbol{C}_B$ appears to be the dynamic stiffness matrix of the soil in terms of the nodal DOF of the finite structure.

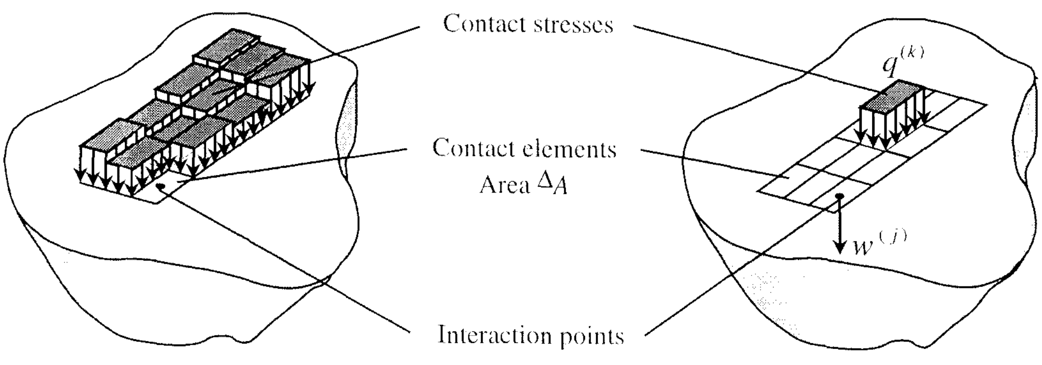

Fig. 1. Spatial discretization of the contact area between the finite structure and the unbounded soil (contact mesh) with the assumption of uniform contact stresses within each sub-area (contact element)

Time domain formulation

The relationship between the vector of the soil displacements **w** at all interaction points and the vector of the uniform contact stresses **q** of all sub-areas, both arranged in $3N\,x\,1$-vectors, can be expressed by means of a convolution integral:

$$\mathbf{w}(t) = \int_0^t \mathbf{F}(t-\tau)\cdot\mathbf{q}(\tau)d\tau \qquad (8)$$

with $$\mathbf{F}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathbf{F}(\omega)e^{i\omega t}d\omega \qquad (9)$$

being the unit impulse (Dirac impulse) response matrix. Its element $F^{(jk)}$ denotes the displacement (flexibility) at the j^{th} interaction point due to uniform contact stresses acting on the k^{th} contact element. It can be derived by applying a numerical Fast Fourier Transformation on the flexibility of the subsoil $\mathbf{F}(\omega)$. Writing Eq. (8) in a discretized form yields the absolute soil displacements at the time t^{i+1}:

$$\mathbf{w}^{i+1} = \underbrace{\Delta t\mathbf{F}^{i}\cdot\mathbf{q}^{1} + \ldots + \Delta t\mathbf{F}^{1}\cdot\mathbf{q}^{i}}_{\text{History part}} + \underbrace{\Delta t\mathbf{F}^{0}\cdot\mathbf{q}^{i+1}}_{\text{Actual part}} \qquad (10)$$

$\mathbf{F}^{(i)}$ stands for $\mathbf{F}(k\Delta t)$ while the superscript at **q** indicates the corresponding time. For the following considerations it proves to be appropriate to split the expression containing all the contact stresses into one part caused by the known contact stresses up to the time t^{i} (history part) and a second part caused by the unknown contact stresses $\mathbf{q}^{i+1}$ at the time t^{i+1}. Remember that the latter have to be determined. Denoting the history part of the soil displacements with $\mathbf{w}^{hist}$ and interpreting $\Delta t\mathbf{F}^{0}$ as the actual flexibility matrix $\mathbf{F}^{act}$, Eq. (10) can be rewritten as follows:

$$w^{i+1} = w^{hist} + F^{act}\cdot q^{i+1} \qquad (11)$$

Based on Eq. (11) the coupling of the subsoil to the finite structure is straight forward. After simple transformations similar to those being described in the frequency domain, the relationship between the interaction loads and the actual displacements can be determined as

$$Q^{i+1} = C^{akt}\cdot u^{i+1} - Q^{hist} \qquad (12)$$

with $$\mathbf{C}^{akt} = \mathbf{T}^{Q}\cdot\left[\mathbf{F}^{akt}\right]^{-1}\cdot\mathbf{T}^{u}$$

$$\mathbf{Q}^{hist} = \mathbf{T}^{Q}\cdot\left[\mathbf{F}^{akt}\right]^{-1}\cdot\mathbf{w}^{hist}$$

Given initial values for both displacements and velocities, the equation of motion (1b) can be solved by a time step integration scheme, such as the implicit Newmark's β method described in [Bathe & Wilson, 1976]. The displacements and velocities at time t^{i+1} resulting from a predictor-corrector-formulae will be used to evaluate the current interaction loads from Eq. (12). After evaluating the equation of motion Eq. (1b) and correcting the displacements as well as the velocities by means of the corrector-step, an iteration process is carried out until a desired tolerance is satisfied. Finally, after the interaction problem has been solved, it is also possible to calculate, the soil displacements at any point in the subsoil.

With the procedure described above the interaction problem is solved in the time domain, although the Green's function representing the (linear) subsoil have been determined in the frequency domain. Hence, this procedure is called a 'hybrid domain procedure'. Beyond that, a 'pure time domain procedure' has been developed, in which the Green's functions are determined directly in the time domain [Bode, 2000]. This procedure is substantially more efficient. However, by now it is limited to a homogeneous, linearly elastic and isotropic halfspace.

Since the equations of motion are solved in the time domain, any nonlinearities in the track model can be considered. To do this the developed soil algorithms have been coupled to the Finite Element Program ANSYS, [Hirschauer et. al., 2000]. To include a nonlinear, non-cohesive contact condition between the structure and the subsoil, the described algorithm has to be modified slightly [Savidis et. al., 2000].

NUMERICAL RESULTS

Frequency dependent flexibilities of the slab track

First, the dynamic interaction between a slab track system and a homogeneous subsoil is investigated. Fig. 2 shows the Finite Element mesh used for modeling the slab track. It consists of the rails supported by pads and 17 ties, a layered elastic plate and the subsoil, which is numerically described with Green's functions, however it is not included in the illustration in Fig. 2. All components have been modeled as linearly elastic or linearly viscoelastic. The parameters for the investigations have been taken from a benchmark test carried out within the research project "System dynamics and long-term behavior of the vehicle, track and subsoil" funded by the German Research Association (DFG). The system is loaded symmetrically by two vertical and time harmonic forces at the rail above the middle tie.

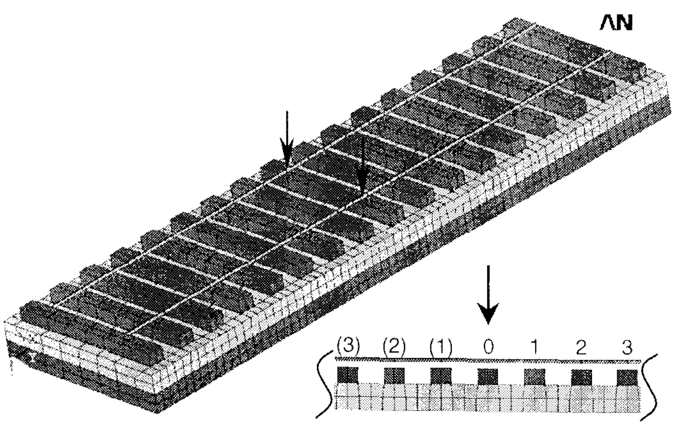

Fig. 2. Analyzed model of a slab track system consisting of rails, pads, ties and the slab. The subsoil is not illustrated.

Fig. 3 shows the absolute values of the vertical flexibilities at the upper side of the loaded tie (no. 0) and the 3 adjacent ties (no. 1-3) as a function of the frequency. In Fig. 4 the corresponding values at the upper side of the rail can be seen. Starting from the static value the flexibilities at the ties (Fig. 3)decrease strongly within the frequency range between 20Hz and approximately 50Hz, whereas the values at higher frequencies nearly remain at a constant value. On a qualitative level this behavior holds for a wide range of elastic plates interacting with the halfspace

In contrast to this, a quite different dynamic behavior of the flexibilities with respect to the upper side of the rail can be observed. Starting from the static values, the flexibilities decrease again up to 30Hz, but with further increasing frequency also the flexibilities are increasing. At about 110Hz a maximum of the flexibilities, slightly higher than the static value, can be seen in Fig. 4. This behavior can be related to the very soft pads between the rails and the ties, used in slab track systems.

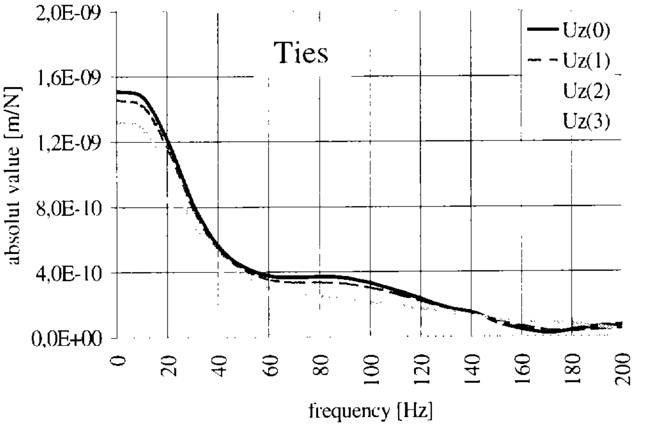

Fig. 3. Absolut value of the flexibilies at the upper side of the ties no. 0 –3 as a function of frequency

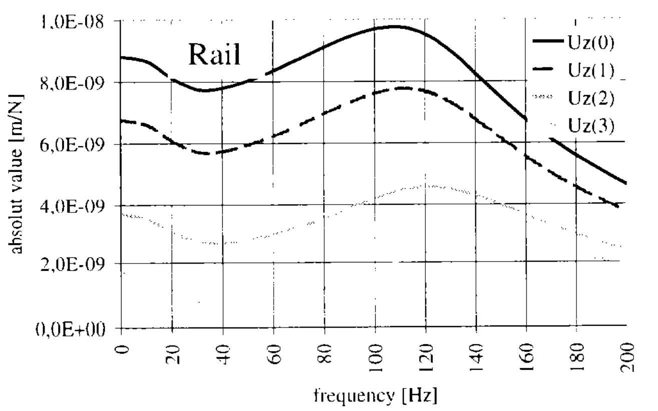

Fig. 4. Absolut value of the flexibilies at the upper side of the rail at the ties no. 0 –3 as a function of frequency

Stress distribution under the slab track

A simplified model of the track has been used to visualize its motion and to calculate the contact stresses as well as the stresses in the interior of the soil due to a harmonic excitation with a frequency of 50Hz. The concrete slab track has a length of 15.0m, a width of 3.0m, a height of 0.3m, a Young's modulus of 30000MN/m² and a Poisson's ratio $\nu = 0.25$. The subsoil is assumed to be a homogeneous halfspace with a shear modulus of 80MN/m², a shear wave velocity of 200m/s and a Poisson's ratio $\nu = 0.33$.

The snapshot in Fig. 5 shows the displacements of the track and the surrounding free field, whereas the corresponding contact stresses are given in Fig. 6. The stress distribution shows extreme values in the centre and at the edges beneath the plate.

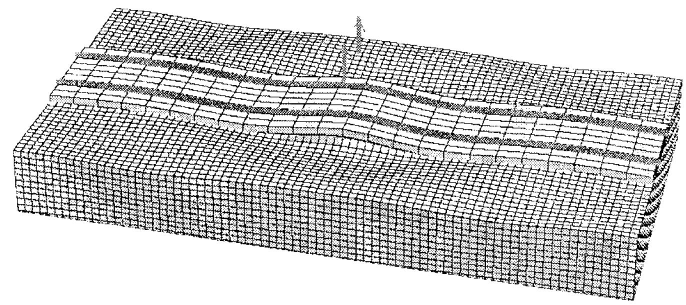

Fig. 5. Snapshot of the displacements of a slab track system and the free field due to a harmonic excitation with a frequency of 50Hz (exaggerated for visualization)

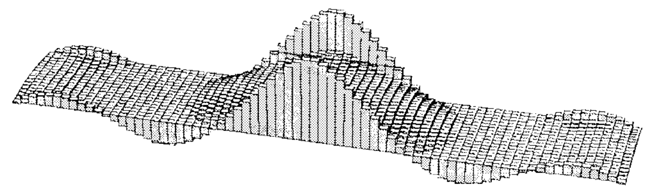

Fig. 6. Snapshot of the contact stresses underneath a slab track due to a harmonic excitation with a frequency of 50Hz

Fig. 7. Snapshot of the vertical normal stresses in the interior of the halfspace under a slab track due to a harmonic excitation with a frequency of 50Hz

The stress wave propagation into the interior of the halfspace is influenced by the slab. The point loads acting on the rails is widely distributed due to the stiffness of the track and therefore acts as a distributed load on the halfspace (Fig. 7).

Simulation of a moving wheel-set

To simulate a moving wheel-set a system of 11 rigid ties resting on a homogeneous soft subsoil has been investigated, see Fig. 8. The wheel-set has been modeled by two concentrated loads moving from tie to tie with a constant speed. Thereby the number of ties was sufficient to simulate the substantial phenomena of moving loads.

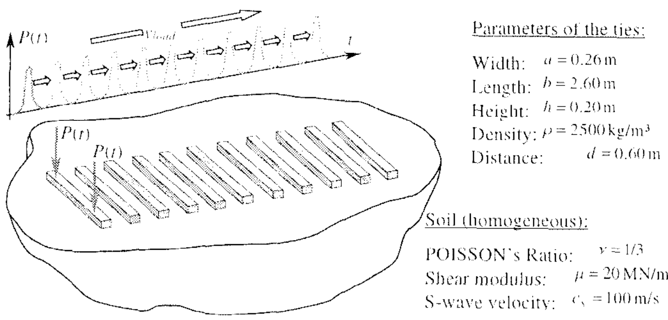

Fig. 8. Simulation of a moving wheel-set along a system of 11 rigid ties

With v_{load} = 225km/h and v_{load} = 450km/h two different velocities have been chosen in order to investigate the effects of travelling with subcritical and supercritical speed, compared to the speed of the most dominant Rayleigh wave at the surface of the soil ($v_{Rayleigh}$ = 335km/h).

In Fig. 9 and Fig. 10 snapshots of the displacement field are shown for the passage with subcritical and supercritical speed, respectively. Regarding the subcritical case (Fig. 9), surface waves can be detected in front of the load. This is indicated by the displacements of the soil surface as well as by the ties in front of the load being in motion before the arriving of the load.

However, in the case of a passage with supercritical speed (Fig. 10) no surface waves can be detected in front of the moving load. The ties being approached by the moving load are still almost at rest (the leading, but not particularly distinctive longitudinal waves disturb the status of the perfect rest). As expected, a wedge-shaped wave front in the free field shows up behind the load, which is known from acoustics as being a Mach' cone.

CONCLUSIONS

The preceding examples have shown the efficiency of the implemented procedures for the frequency and the time domain formulation. This opens a broad spectrum of practice-relevant applications to be calculated.

In particular the coupling of the developed algorithms to the Finite Element Program ANSYS allows the investigation of the interaction of quite arbitrary rail track systems with the layered subsoil. Nonlinear constitutive relations can be considered as well. To design the track system inner forces of any component of the track may be extracted. Moreover, the dynamic stresses in the subsoil – to be used for an estimation of the track settlement – can be obtained in a subsequent procedure. The included non-cohesive condition of contact allows a more realistic modeling of the coupling between the ties to the subsequent layers.

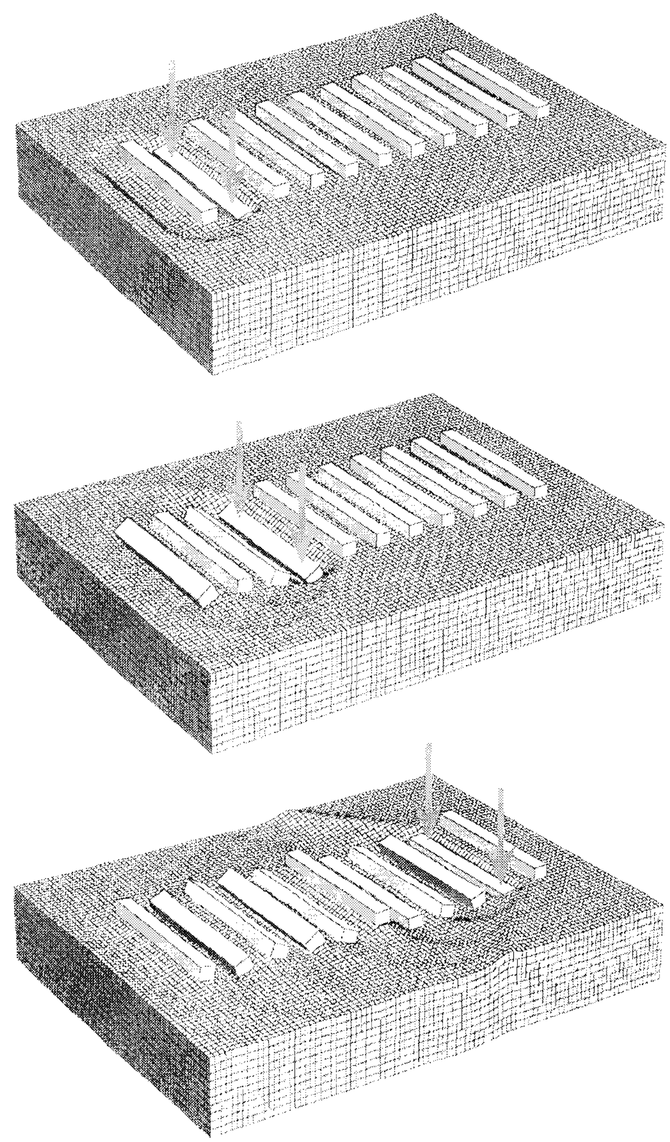

Fig. 9. Snapshots of the motion of a 11-tie system; subcritical case (v_{load}=225km/h < $v_{Rayleigh}$=335km/h)

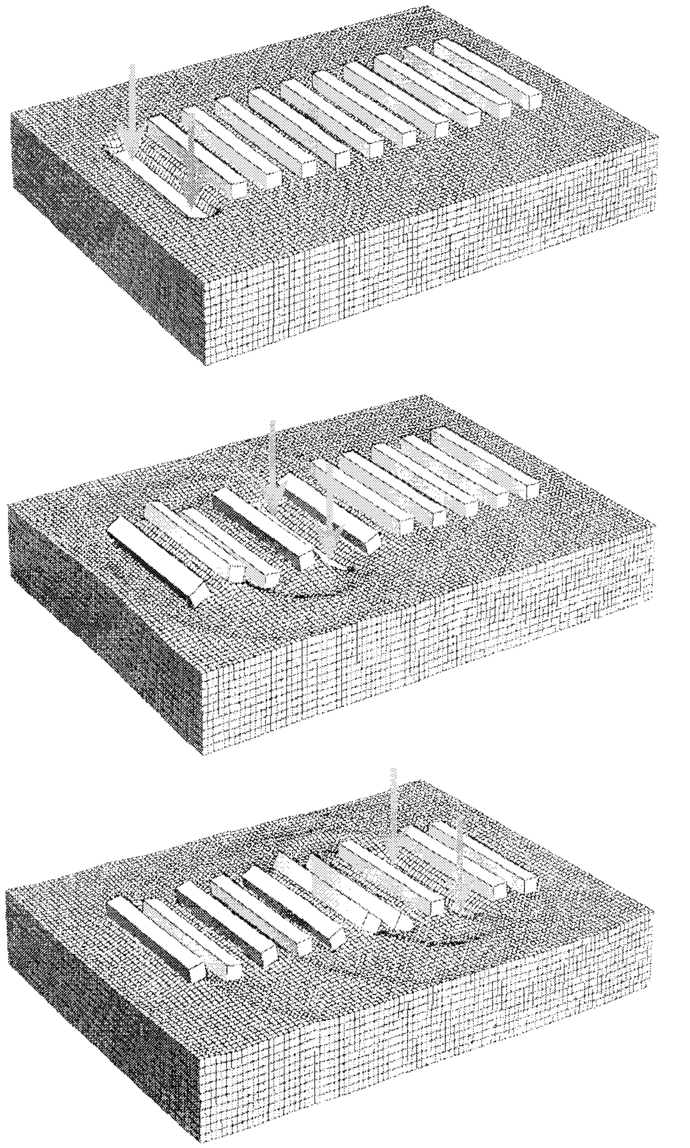

Fig. 10. Snapshots of the motion of a 11-tie system; supercritical case ($v_{Load}=450km/h > v_{Rayleigh}=335km/h$)

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