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Intensional reasoning about knowledge

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ABSTRACT

As demands and ambitions increase in Artificial Intelligence, the need for formal systems that facilitate a study and a simulation of a machine cognition has become an inevitability. This paper explores and developes the foundations of a formal system for propositional reasoning about knowledge. The semantics of every meaningful expression in the system is fully determined by its intension, the set of complexes in which the expression is confirmed. The knowledge system is based on three zeroth-order theories of epistemic reasoning for consciousness, knowledge and entailed knowledge. The results presented in the paper determine the soundness and the completeness of the knowledge system. The modes of reasoning and the relations among the various epistemic notions emphasize the expressive power of the intensional paradigm.

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I. INTRODUCTION

A. PROLEGOMENA TO A THEORY

"All men by nature
desire to know".
Metaphysics, Aristotle

1. Genesis. The contemporary echo of the idea of an intelligent machine originates with the invention of the stored program computer and the reflections by Alan Turing and Claude Shannon. The idea is certainly behind the emergence of Artificial Intelligence as a valid scientific discourse in the second half of the 20th century.

The possibility that an artifact could perform cognitive functions, ones exclusively in the domain of human competence, is intriguing in its implications. It appears that human nature has been deprived of the last marvels of existential uniqueness: the ability to reason and to create knowledge which are intrinsic components of intelligence.

Indeed, a respectable number of researchers in Artificial Intelligence have explicitely equated the cogitative potential of humans with the one of artifacts. There are two plausible sources for the equivalence drawn between artifacts and humans: the first one is the complexity of artifacts and the second is the rationalist tradition in the analysis of human cognitive behavior.

Complexity is reflected in the essential attributes of each artifact, its relative independence, flexibility and multi-usefulness.

All three of the attributes are present both on a structural (or hardware) level and on a functional (or software) level.

As a direct consequence of complexity one has the effect of unpredictability. Regardless of the level of determinism that one wants to simulate in artifacts, the unpredictability is always present. This might be a consequence of the empirical notion that results of a process are always at least one level higher in their complexity than the process that has produced them. Hence, it appears that unpredictability is unequivocally opposed to the rationalist tradition in the analysis of human or artificial cogitative competence.

Intellectual disciplines usually evolve over many centuries. The 'success' of any intellectual discipline is often measured in how well the fundamental principles of the discipline have been encapsulated with in a formal system. The mathematical method, which in essence coincides with pure formality and rationality, has been accepted as a metric of how serious and 'hard' a certain scientific endeavor is. Rationality requires clearly defined problems, and the problems in AI defy stable formulations which is a fact often ignored in the analysis of knowledge.

The rationalist method has also been adopted in the identification of the phenomenon that creates knowledge, experience and ratiocination. The notion of experience is reduced to that of a modification of a knowledge base (KB). The initial set of facts represents an apriori knowledge built into the system which upon the application of valid inference, becomes an experience for a reasoning agent. Informally, an argument, as a result of inference, is valid if and only if its premises could not be true and the conclusion false.

Ratiocination is to be understood as a form of analysis entirely dependent on the inherent power of the 'mind' to reason. By identifying the process of reasoning with that of computing, then the brute force capabilities of the artifact have generally (although not universally) been accepted as a form of ratiocination.

This characterization of experience and ratiocination is consistent with the axiomatic acceptance (in Artificial Intelligence) of the dictum "cognition is a resolute computation". From a pragmatic view this is quite necessary. After all, there will always be differences between a form and a context and one must recognize that form is a prerequisite to computation. By adopting the principles of mathematical logic, the present work closely follows the rationalist tradition in Artificial Intelligence.

However, the work differs from the rationalist tradition in the requirement that the reasoning about knowledge be within the system. Cognition requires inner presence, or as Heidegger argues "being-in-the world". The intention of the epistemic system is to capture part of the presence in the world by empowering the artifact with its own mechanism for reasoning about its knowledge. Thus, it is important to distinguish between the method which is rationalistic and the intention which is mentalistic. The mentalism that has been induced by the intention, should not be confused with the phenomenon of anthropomorphic fallacy, the overtones of which are addressed in the next section.

2. <u>The Limits of Formality</u>. As used throughout the work and commonly understood, the word formal is synonymous with mechanical. The seminal works of Godel and Church and the definition of Hilbert's Programme put to rest the argument of unlimited expressive power of a formal system [9, 16], hence on mechanization.

The results of Godel and Church are deus ex machina, they liberate instead of confine. These results are a vindication in the resolution of mentalistic problems with formal methods. It is reasonable to assume that formality is just the first step due to the present structural and functional constraints of the artifact and the incomplete knowledge of ourselves.

To an extent, the source of the problem, an adequate simulation of of bona fide human cognitive properties, is the nature of the discipline where Man is the object of and the model for the study or "a measure of all things". The inevitable human interference has mislead some researchers to look for and recognize purely anthropomorphic features in artifacts.

For example, the aim of Searle (in Torance [51]) is to distinguish between weak AI "we can build powerful tools to mimick and study the mind", and strong AI "something can think in virtue of instantiating a computer program". According to Searle, the former problem is solvable, while the answer to the later is a definitive no. Others have asserted that on an abstract level the mind and the computer are the same. The argument is too strong on the grounds that there is no a definitive characterization of human cognitive properties and functions.

A prima facie value is the symbiotic relationship between AI and Man, for although we can "still learn more about machines from the study of man" as Von Neumann put it, there is a lot to be learned about man from the way some artifacts work also. An extension is never a replacement.

These arguments urge us to propose the principle of 'rational commitment'. It is simply a recognition of the fact that the question of intellectual equality between Man and Machine is an exercise in impuissance. The rational commitment does not imply commitment to the rationalist tradition only or exclusion of the mentalism. As stated earlier, the formalization is based on an abstraction. Abstractions occasionally create deviant images of reality, a fact well-recognized in the effort to formalize common sense reasoning. The commitment is a statement against the reduction of intelligence and its versatility to a single coordinate.

By necessity the epistemic model starts ad hominem, by desire it ends ad machinum. If the necessity and the desire coincide in extension, the results will be more than gratifying. Both should be acknowledged whenever the question is raised of how comprehensive, compelling, and complete a certain theory is.

3. Desiderata. Any serious intent by an artifact to encompass a level of cognitive generality postulates an integral ability on its part to reason with deductive, inductive, and evidential rules [12]. The evaluation of evidence is also subject to inductive and deductive criteria. A distinction is made in order to stress the meta-system significance of evidence. In essence, the rules of evidence govern

the epistemic environment of an intelligent agent. They determine the course of proper reasoning since in extension the true evidence coincides with knowledge.

The assessment of knowledge within a reasoning system is usually a result of an internal or an external query. The nature of the request might be either to revise the current knowledge of the system (in view of some new situation) or to answer whether some proposition is known or not. It is the later type of response that the epistemic model is trying to capture. Inter alia, the solution of reasoning about what is known may also prove necessary to address the problem of knowledge revision.

The AI community shares the controversy concerning the ontological differences between declarative and procedural knowledge with so many paradigms in epistemology. Declarative knowledge is expressible through the mode 'knowing that', while procedural knowledge through the mode 'knowing how'. The argument is easily extended by introducing another mode, often used in the process of explanation, expressed through 'knowing why'.

The position taken in this research is as follows: both procedural and explanatory modes are reducible to a declarative mode. It is true that explanatory and procedural knowledge do require some form of causal ordering (an instance of which is the notion of plan), but I believe that the ordering also can be expressed through a set of declarations (or propositions).

Given this point of view, namely, that all modes of knowledge are reducible to a declarative or propositional mode, propositions are treated as objects of knowledge and the notion of knowing as an empirical binary relation between a reasoning agent and an object (proposition). However, the argument will be put forward that there is a more primitive notion of knowledge, the one such as consciousness which in a sense is a prerequisite for knowing. Again, consciousness will be treated as an empirical binary relation between an agent and a minimal concept expressing proposition. Thus, it is implicitly posited that concepts are the ontological constituents of propositions.

Three objectives motivate the present research. The first one is to explore the ontology of knowledge. The second objective is to develope a zero-order theory for reasoning about concepts. Finally, the third objective is to modify the zero-order theory for reasoning about concepts to a zero-order theory for reasoning about knowledge.

Doxastic and epistemic inclinations such as knowing and believing are expressed through the verbs of propositional attitudes, know and believe [50]. Propositional attitudes usually define or appear in an intensional context. A context is intensional if its co-referential expressions such as singular terms, predicates or sentences which have the same denotation are not substitutable without changing the truth value of the context as a whole. Otherwise the context is termed as extensional. Since the distinction between extension and intension is essential in modelling knowledge and belief, a complete section is devoted to this issue later in the work.

Although, an occasional reference is made to the doxastic inclinations such as belief and awareness, this research addresses mainly their epistemic counterparts, knowledge and consciousness. However, one can find the metaphysical difference between knowledge and belief rather interesting in understanding the nature of knowledge.

Knowledge is a correct interpretation of reality. Imposing correctness on the interpretation is to distinguish knowledge from mere ideas, opinions, perceptions, and beliefs. To know excludes being wrong.

As defined, the notion of knowledge is identical with the notion of truth. De facto one of these notions would be redundant in the presence of the other. The relation between truth and knowledge has a distinct locus in the cognitive behavior of man.

Knowledge is intrinsically committed to representation and identification. The representation deals with the ontological properties of knowledge. The identification addresses the metaphysical properties.

It is commonly understood that "an agent a knows Π" if and only if:

(1) The Truth condition is satisfied: Π is true

(2) The Belief condition is satisfied: a believes Π

The term 'agent' denotes an individual capable of reasoning, while Π stands for an arbitrary proposition. The first condition is the least controversial [4]. It is a matter of intuitive sanction and linguistic necessity.

Contrasting views exist with respect to the Belief condition. One view accepts the condition either in a strong form 'knowledge entails certainty' or in a weak form 'it is not the case that knowledge entails certainty'. The other view denies the Belief condition either in a weak form 'it is not the case that knowledge entails not belief' or in a strong form 'knowledge entails not belief'.

In summary, the study of an epistemic context requires that the two conditions be satisfied. On the other hand, in a doxastic context the Truth condition is neither necessary nor adequate.

4. Method. Various paradigms for knowledge representation in Artificial Intelligence such as semantic networks, frames and units, reach for mathematical logic whenever a need for justification and rigor arises. This is done by transliteration of the object formal system into a modified system of logic.

It is not surprising that the need for formalization has been plagued with empirical adventures under the name of heuristic adequacy. For example, some have proposed [23] a twelve-valued logic as a vehicle for studying the progressive growth of plants. A closer look into this work reveals a reduction of the twelve-valued to a two-valued logic. One might agree with Ryle who posits that formal logic is a regimentation of the relevant sectors of the ordinary discourse [18]. However, extreme care should be taken in order not to defeat the aim and intent of logic: to discriminate invalid from valid arguments and to provide rigorous and simple standards for their expression and evaluation.

The research is based on the seminal work of Richard Montague [44] in linguistics and philosophy. The standard of rigor Montague imposed on the study of properties of natural languages has been that of mathematical logic. He recognized that the formal clarity of set-theoretic semantics or model theory is undoubtedly one of the most important factors in developing a whole family of logical theories such as pragmatics, intensional and deterministic systems that would enable him to explore and formalize the subtleties of natural language.

The study of language (both formal and natural) is commonly partitioned into three branches-syntax, semantics and pragmatics. Syntax is concerned with relations between linguistic expressions, while semantics with relations between expressions and the objects to which they refer. Pragmatics, on the other hand, is concerned with relations between expressions, the objects to which they refer, and the contexts of use of the expressions. Thus, pragmatics is in a sense generalization of semantics [44].

The work of Montague on both semantics and pragmatics, with the exception of the linguistic and philosophical communities, has received very little attention in Artificial Intelligence and the cognitive sciences in general. Recently, Vardi [55] has explored the intensional approach in a doxastic context using the methodology of constructive worlds of various depth.

The reason for this unintended 'ignorance' is two-fold. First, the aforementioned insistence of Montague on mathematical rigor has been, at least in the opinion of this author, misunderstood to be irreconcilable with the mentalistic school in cognitive sciences. Again, the misconception is a result of not distinguishing between intentions and methods. Otherwise, Montague would not have introduced the notions of possible worlds and contexts of use if he had thought that classical semantics was sufficient to deal with problems of natural languages. Secondly, due to his untimely death, many prophetic articles written by Montague were left in a cryptographic form which certainly causes difficulties in identifying possible areas of application.

The initial work in developing an axiomatic theory of the logic of intensions was done by Church [10]. To provide an interpretation, Carnap [7] introduced the idea of state of affairs, a sort of plausible qualifier for the intension of an expression. He posited that in essence the extension is a function of two arguments, the expression and the state of affairs. The semantic considerations for the intensional logic were elaborated by Kaplan in his dissertation, where the possible state of affairs were models of the corresponding language [25]. A general axiomatized system for the Montague type of intensional theory was developed by Gallen [15].

The zero-order epistemic system proposed in this research should be classified as a generalized extension of ordinary propositional calculus to accommodate reasoning about knowledge. The nature of the extension is to be understood as follows: the basic system and the extended system share the same vocabulary, and have the same theorems and rules of inference a propo the shared vocabulary. The extension is done in order to enrich the expressive power of the underlying system via some extended vocabulary, additional axioms and rules of inference.

It is implicit that the arguments are subject to the criteria of rationality, material adequateness, and intuitive admissability. In the absence of consensus with respect to the standards for inclusion of an arbitrary formal system to the family of logics, the question would be ignored because it leads to a metaphysical circus [23].

The use of a 'logical system' follows from the intention to put forward an explanatory model for understanding the use of epistemic notions in an ordinary discourse and within the framework of intensional logic. Clearly, the laws of logic and the laws of thought are not isomorphic [19]. The former are considered to be an expression of ability rather than obligation.

Thus, given an intellect and sufficient time (effort), logical and material information, inferences could be made about some state of affairs. In essence that is the parthenogenetic nature of logic or the potential to induce new knowledge in an environment where the information content is restricted to the initial one. Hence, the intuitive understanding of logic that we have is probably best expressed by Witgenstain who wrote " ... logic is not a theory of the world, but its reflection". It is again a question of pragmatics.

B. PRINCIPIA FUNDAMENTA

> "If modern modal logic was conceived in sin, then it has been redeemed through Godliness."
> The Unprovability of Consistency, Boolos

1. Propositional Calculus. By a propositional language P is understood a language of which the symbols are drawn from the following categories:

(1) Improper symbols

1.1. Logical constants

(a) ¬ read 'it is not the case that'

(b) & read 'and'

(c) v read 'or'

(d) -> read 'if . . . then'

(e) <-> read 'if and only if'

1.2. Parenthesis, brackets, and commas;

(2) Proper symbols

A denumerable set of propositional variables:

{p, q, r, s, . . . }.

Both, the improper and proper symbols denote the set of primitive symbols of P. A formula is a finite sequence of symbols. If f and g are formulas, then the concatenation of f and g, denoted by fg, is a formula of P.

It is important to distinguish between the language P, or for that matter for any language which is the object of study , and the English language extended with mathematical vocabulary and symbols in which the discussion about P takes place. The language one studies is called the object language, and the extended English is called meta-language. The theorems about the propositional system are proved in both the object language and the meta-language.

One of the objectives of a formal language is to be able to generate in a unique and precise manner the set of "meaningful expressions" which belong to the language. The set of meaningful expressions of P is actually the set of the well-formed formulas of P. They are inductively defined by the following set of formation rules:

(R1) A propositional variable standing alone is a wff.

(R2) If f is a wff, then $\neg f$ is a wff.

(R3) If f and g are wffs, then [f & g] is a wff.

The set of wffs of P is the intersection of all sets F of formulas such that:

(1) $p \in F$ for each propositional variable p.

(2) For each formula f, if $f \in F$, then $\neg f \in F$.

(3) For all formulas f and g, such that $f \in F$ and $g \in F$. then $[f \ \& \ g] \in F$.

The axiom system of P consists of all wffs which have the following forms:

(A1) $\neg[f \vee f] \vee f$

(A2) $\neg f \vee \bullet g \vee f$

(A3) $\neg[\neg f \vee g] \vee \bullet \neg[h \vee f] \vee \bullet g \vee h$

The number of axioms of P is infinite. Any further reference to the axioms of P, (A1) through (A3), will be in the form of "all propositional tautologies". There is only one rule of inference in P which is:

(MP) From f and f -> g, to infer g.

Let H denote a set of wffs of P. A proof of wff f from the set H of hypothesis is a finite sequence f_1 , . . ., f_m of wffs such that f_m is f and for each i ($1 \leq i \leq m$) at least one of the following conditions is satisfied:

(C1) f_i is an axiom.

(C2) f_i is a member of H.

(C3) f_i is inferred by (MP) from wffs f_j and f_k where $j < i$, and $k < i$.

Any logical system that contains the axioms of P will be termed as a standard logical system. A proof of wff f in P is a proof of f from the empty set of hypothesis. A wff f is a theorem of P if and only if f has a proof in P. Given set H of hypothesis, the derivability of a wff f from H is denoted by $H \vdash f$. To indicate that wff f is a theorem of P one writes $\vdash f$. The decision problem for a logical systems is the problem of finding an effective procedure for determining of any wff f whether or not f is a theorem of the logical system.

So far all the logical notions that were introduced for P, are essentially syntactic. In order to appreciate and understand the full expressive power of a logical system, one also needs semantics. For example, the solution for the decision problem for P, might be rather difficult in syntactic terms. Hence, the definition of salient logical and metalogical attributes such as consistency, satisfiability, validity, soundness, and completeness which apply to logical systems is necessary.

An assignment of truth values to propositional variables is a function from the set of variables to the set {F, T} or {0, 1} of truth values where F (0) and T(1) denote Falsehood and Truth respectively. Following the notation that was introduced by Von Neumann, 2 stands for the set of truth values {0, 1}. The value V(f) of wff f with respect to the assignment A, is defined by induction on the complexity of f in P. A wff f is tautology if and only if V(f) = 1 for all assignments A. When a wff f is tautology it is denoted by $\models$ f. A wff f is contradiction if and only if V(f) = 0 for all assignments A.

A wff f is valid with respect to interpretation if and only if V(f) = 1 for all assignments of values to its variables. Again, the validity of wff f is denoted by $\models$ f. An interpretation of a logical system is sound if and only if all the axioms are valid and the rules of inference preserve validity.

A logical system is consistent with respect to negation if and only if there is no wff f such that $\vdash$ f and $\vdash \neg$f are in the system. A logical system is complete if every tautology is a theorem. When a logical system is both sound and complete then one may say that the logical system is determined. The logical system of propositional calculus P is consistent and determined. For proofs of these results the reader is referred to Mendelson [42]. The account of the propositional system presented here is based on the exceptional text by Andrews "An Introduction to Mathematical Logic and Type Theory: The Truth through Proof" [1].

The modal systems that follow are extensions of classical propositional calculus. The logical and meta-logical notions described here are applicable to modal systems too, However, some modifications are necessary to accommodate the 'interpretations' of modal operators.

2. Systems of Modal Logic. A formal system conceived to study the nature of logical necessity, impossibility, and contingency is known as modal logic. The distinction between a necessity and a contingency is a metaphysical one and is not to be confused with the epistemic difference between a priori and a posteriori truths. The question whether these notions coincide in extension is still an open one.

The difference between necessary and contingent truths is sometimes taken to be analogous to the difference between 'analytic' and 'synthetic' truths. An analytic truth is defined as 'true solely in virtue of its meaning' and a synthetic truth as 'true in virtue of facts'.

Given a proposition Π, another proposition can be formed asserting that Π is necessary, with the expression "it is necessary that Π". The new proposition will be true when Π is necessary, and false when Π is not necessary. In this case, 'it is necessary that' is a monadic proposition forming operator (or a sentenial operator) on propositions.

The operator used to form the new proposition is not a truth-functional one; given that p is false it follows that p is not necessary, while if p is true it certainly does not follow that p is necessary. Operators not subject to truth-functional interpretation are termed modal; hence the attribute modal to the systems of logic that use them. Another monadic sentenial operator, expressible in the form 'it is possible that' represents the possibility operator. In view of the many different notations in modal logic, L shall be used for the operator of necessity and M for the operator of possibility. The next step is to present the basic modal systems.

The well formed formulas of a modal system are built out of atomic formulas in the same manner as they were built in the system P of propositional calculus. Also, if f is a wff so are Lf and Mf. The weakest system of modal logic K has the following axiom schemes and rules of inference:

(A0) Truth-functional tautologies

(A1) L(f -> g) -> Lf -> Lg

(MP) From f, f -> g infer g

(NC) From Lf infer f

If we add the axiom scheme:

(A2) Lf -> f

we get the system T. The Brouwershe system is obtained with the addition of:

(A3) f -> LMf

and the system S4 with the addition of (A4) to the system T:

(A4) Lf -> LLf

Finally, S5 is obtained from T with the addition of:

(A5) Mf -> LMf

Systems that include all the theorems of K and are closed under rules of inference (MP) and (NC) are called normal.

Kripke's seminal work in providing semantics for the inclinations of modality [27, 28, 30] is closely followed in representing the interpretation for modal logic. A Kripke structure is an ordered triple <W, W0, R>, where W is the set of possible worlds; W0 is an element of W and stands for the actual or 'real' world; and R is a reflexive relation on W, named a relation of accessibility. Given two arbitrary worlds W1 and W2, W1 R W2 means that every proposition true in W2 is possible in the world W1. If additional requirements are imposed on the relation R, such as transitivity or symmetry the different modal systems are obtained as before.

Assume we have a Kripke structure <W, W0, R>, which will be denoted with KS. A model V on KS is a binary function V(P, G), where P varies over the set of atomic formulae and G over the set of possible worlds W. The range of V is naturally the set 2. The assignment of the truth-values to the non-atomic formulae is done by induction. For example, if V(f, G) = 1 and V(g, G) = 1 and assuming V(f, G) and V(g, G) are already defined for all G ϵ W, one can define V(f & g, G) = 1; otherwise V(f & g, G) = 0.

The definition of V(Lf, G) is interesting. Let V(Lf, G) = 1 for every G' ϵ W such that GRG' holds, otherwise let V(f, G) = 0. Then f is necessary in G if and only if f is true in all worlds G' which are possible relative to G. The completeness theorem, first proved by Kripke [30], equates the syntactical notion of provability in modal logic with the semantical notion of validity.

The standard language of predicate calculus, given in Andrews [1], can be augmented in the same manner as the propositional calculus was. With the addition of the monadic operator L and the appropriate modal interpretation the modal predicate calculus is obtained. There are many more fundamental (and possibly open) problems concerning the modal predicate system than the propositional system.

The objections put forward by Quine concerning modal logic are: there is neither a clear reason nor motivation for formalization, and the interpretation presents insuperable difficulties [23]. For scientific and mathematical reasoning, argues Quine, there is no need to extend classical logic. The argument is short sighted. A sufficiency for mathematical and scientific discourse might not be appropriate for ethical, legal, or for that matter a common sense context. The denial of any formalization is equal to the assertion that these concepts are empty which is at least a disputable argument.

The introduction of quantification to modality for Quine is a nightmare. This is due to his criteria of: (1) ontological commitment-a test of what kinds of things a theory says there are; (2) ontological admissability- admit only those entities for which there is an adequate criteria of identity. The quantifiers carry the ontological commitment. They are the device by means of which people talk about things. Modalities do not, in contrast, directly talk about things; they express the ways of talking about things [22, 23].

Different problems arise from the behavior of singular terms appearing in the scope of modal operators. Modal operators are referentially opaque or intensional and the substitution (or the law of Leibnitz) fails in modal context. This means that in the scope of a modal operator, substitution of one singular term for another one with the denotation of the same object can change the truth value of the sentence. The failure of the Leibnitz's law is regarded by some to be conditio sine qua non that a satisfactory interpretation of identity exists in modal systems. Take for example (Quine in Linsky [38]):

(1) $9 \equiv$ the number of planets

which is certainly a true utterance. If one substitutes the identity (1) into the sentence

(2) $L\ (9 > 7)$

then he gets a false sentence

(3) L (the number of planets > 7)

The replies to Quine's criticism have been numerous and vivid. For the curious reader, Linsky [38] contains an exceptional collection of articles and Haack [23] is a scholarly account of all the difficulties the area bristles with.

Another problem is that of transworld identity: which individuals in different possible worlds are to count as the same. So far, the most appealing solution has been proposed by Kripke [29], where a proper name is is equated with a rigid designator which denotes the same individual in all possible worlds.

The formal semantics, based on the possible worlds approach developed by Hintikka [18] and Kripke [27], dispersed some of Quine's criticism and to an extent settled the question of the interpretability of modal systems.

Currently, there are three different approaches concerning the nature of the possible worlds. Kripke's approach is a conceptualist one, for him possible worlds are the ways people express their conception of the world to be different. The linguistic approach, due principally to Hintikka [18], identifies the possible worlds with a maximally consistent sets of sentences. Finally, Lewis takes the view that possible worlds are abstract entities independent of a language or a thought [23].

The intent of the section was to give an overview of the important problems raised by the development of modal logic. A confinement to issues relevant to the epistemic concepts has closely been observed. In all fairness to Quine, it must be stated that he never did question the mathematical soundness of the modal system. Another sign of vindication, at least with respect to knowledge and belief, comes again from Quine who as quoted by Linsky in [38] says:

> "What makes me take the propositional attitudes more seriously than logical modality is a different reason: not that they are clearer, but they are less clearly dispensible. We cannot easily forswear daily reference to belief, pending some substitute idiom as yet unforeseen. We can much more easily do without reference to necessity."

3. The Classic Theory of Knowledge. The formal conception of a system for logical assessment of knowledge dates back to the efforts of Von Wright in his 1951 work "An Essay on Modal Logic". The ideas developed by Von Wright in passim motivated Hintikka's work on epistemic logic, which resulted in the lengthy treatment of the subject [19]. Before proceeding with a detailed account of the theory developed by Hintikka, which is referred to as 'the classical theory of knowledge', the reader should bear in mind that 'the theory' is based on modal logic.

Hintikka's intention was to formalize the basic epistemic inclinations such as "a knows that", "a believes that", "it is possible for all that a knows", and "it is compatible with everything a knows that". The four epistemic inclinations are referred to by four monadic operators, 'K', 'B', 'P', and 'C' respectively. 'a' is a free individual symbol denoting an agent. Since our interest is focused on knowledge, only the axiom system and rules of inference relative to it are presented:

(A1) All propositional tautologies

(A2) Kaf & Ka(f -> g) -> Kag

(A3) Kaf -> f

(A4) Kaf -> KaKaf

(A5)¬Kaf -> Ka¬Kaf

(MP) From f and f -> g, infer g

(NC) From f infer Kaf

where f, and g are well-formed formulas, and the formula 'Kaf' is the formal counterpart of 'the agent a knows that f'.

The axiom (A5) was rejected by Hintikka as unintuitive. The axiom system, excluding (A5), is determined since the system is isomorphic to the modal system S4. When A5 is included, the system is determined again whence the system is isomorphic to the modal system S5.

The first axiom and the rule of inference (MP) are credit from the propositional calculus. A source of controversy and critique in the AI community are the second axiom and the necessitation rule of inference. It appears that they define the agent as a 'perfect knower'. Its knowledge is closed under implication and everything true, by necessitation, is known to the agent 'a'.

The axiom in question is of a particular interest. It expresses the notion that whenever someone knows anything, he knows all its logical consequences. The phenomenon has appeared under different names in the literature on epistemology such as the consequential closure, the conservation law, and the paradox of logical omniscience.

The acceptance of this axiom, it has been argued, eliminates the need for search and is not admissable in a resource-bounded environment. The axiom is considered to be a sufficient reason to reject any model-theoretic analysis of epistemic inclinations.

Hintikka in [20] suggests two directions in resolving the paradox of omniscience, that is, to delineate the set of logical consequences for which the paradox will hold. The first avenue is to put a syntactical restriction on the deductive argument that leads from f to g. The idea is that the number of free individual symbols and the number of layers of quantifiers determines the number of individuals in a sentence. Hence, the parameter of an argument from f to g should never be larger than the respective parameters in f and g. The other avenue is a probabilistic one and reflects the urn theory. The nested quantifiers can be thought of as successive draws of individuals from an urn. The later interpretation may have only one disadvantage: a draw is too much like a search attempt.

An interesting approach has been proposed by several authors such as Cresswell [11], Kripke [31], and Hintikka [21]. The paradigm goes under different names such as non-standard worlds, impossible worlds, and non-normal worlds, but the respective notions are similar. Thus, in these 'unworldly' worlds not all valid formulas need to be true and inconsistent formulas may be true. However, as it is shown later while reviewing the work of Levesque [36] who calls these non-standard worlds "incoherent situations", an agent still has an opportunity for a perfect knowledge. The mode of reasoning has only been changed, but a strong intuitive sanction is lacking.

A3 is the axiom of veridicality; it states that an agent knows only those things that are true. The grounds for its inclusion were already explained in the discussion of the nature of knowledge. In a doxastic context this axiom is not valid.

The last two of the axioms are known as axioms of introspection. A4 is the axiom of positive introspection, if an agent knows something then he knows that he knows it. A5 is the axiom of negative introspection, if an agent does not know something then he knows that he does not know. Both axioms involve meta-level inference about knowledge, or what is commonly termed as iteration of knowledge. The theoretical ground for their inclusion or exclusion are still not settled, but both axioms are proven to be useful in a multiagent environment and distributed environment.

Hintikka's idea of consistency, in his own words, "is immunity to certain kinds of criticism". Instead of speaking about consistency and inconsistency, he defines respectively the counterterms defensibility and indefensibility. A valid set of sentences is termed as a set of self-sustaining sentences.

Originally, Hintikka [19] formulated his semantics in terms of sets of sentences called model sets. When Kripke [27] introduced the possible world as a primitive in the context of modal logic, his theory supplanted all earlier theories of states of affairs or sets of sentences as models. As a consequence, Hintikka [18] reformulated his semantics in terms of possible worlds. Thus, a sentence is self-sustaining if it is true in all possible worlds, and defensible if it is true in one such world.

Hintikka's answer to the questions raised by Quine is the concept of multiple referentiality. It simply states the fact that identities hold in some possible world but not in others is due to the reference of the singular terms to the objects in different ways in different possible worlds. Provided that this is true, a combination of quantifiers and epistemic operators should not cause difficulties as long as it is done for individuals which exist in a particular world.

The quantification across epistemic operators presents another problem. The position of Hintikka on the issue is to be prohibitive: the principle of substitution is not to be applied unless additional premises are introduced. It is possible that the restriction deviates from the intended generality of the principle, however, additional premises are quite intuitive in epistemological context.

The persistence Professor Hintikka exhibited in his work on the foundations of logic of knowledge has prevented the reduction of the theory to an intellectual accident. His efforts have been rewarded by the proliferation of research in computer science, theory of games, economics, communications, cryptography, and Artificial Intelligence. It is to the last that we focus our attention.

C. EPISTEMOLOGY AND ARTIFICIAL INTELLIGENCE

"An expert is one who does not
have to think. He knows."
F. L. Wright

1. The Initial Stage. The renewed interest in a general machine intelligence was sparked by an article co-authored jointly by McCarthy and Hayes [39] in 1969. In the paper they examine the problem of creating a program capable of inferring in a formal language a strategy that achieves a predetermined goal. According to McCarthy and Hayes, the task requires formalization of the following concepts: causality, ability, and knowledge. With this in mind they proceed by specifying an intelligent artifact, termed Reasoning Program.

The artifact has external and internal levels of communication and various sorts of representation for pictures, scenes, situations, and inference. The program involves both heuristic and epistemological structures. Once the knowledge is adequate, the system should be able to find a strategy and prove it is correct. The authors are only interested in the epistemic aspect of the Reasoning Program. Incidentally, the program that deals with the epistemic aspect is called the Missouri Program.

McCarthy and Hayes attempted to define criteria for an adequate representation of the world. Accordingly, they are: metaphysical-the form of representation does not contradict the facts of reality it intends to represent; heuristic-the reasoning process is expressible in the representation; epistemological-it can be used to express the facts as they really are. In addition, the article presents the thoughts behind possible formalizations of situations, actions, and strategies. Their first-order situation calculus, extended later by McCarthy [41] has had an extraordinary influence. Recently, an analogous formalism termed mental calculus has also been elaborated by McCarthy [40].

The primary concern of McCarthy is to specify the necessary knowledge conditions. If these conditions are met, the reasoning program is able to execute a plan correctly. The research advocates explicit reasoning on the part of an agent concerning his ability to carry out actions. This amounts to writing down all the necessary conditions and verifying their truth-value prior to undertaking any action.

The disadvantages to the proposed method are: the possibility of knowledge explosion due to a large number of proper axioms needed to describe each individual action, and consequently the proof procedures tend to be extremely long.

The theory developed by Hayes and McCarthy adheres to one of the two predominant schools of thought relative to a formal treatment of knowledge, the so-called sentenial or syntactic paradigm where the knowledge is a relation between an agent and a sentence. In the other school, known as the semantic paradigm, knowledge is a relation between an agent and a proposition. The merits of both schools are the subject of examination in the next pages.

2. The Semantic Paradigm. The intuitive appeal of propositional approach springs from the possible world semantics which has an elegant mathematical formulation and is subject to logical analysis.

The intention of Moore [43] is to explain the relation between knowledge and action. In his system, an agent who knows the combination of a safe should be able to reason and conclude that he also knows how to open the safe. The system is motivated by the theory of Hintikka and is isomorphic to the modal system S4. The standard tense logic [48] along with the situation calculus by McCarthy [40] are interpreted with possible world semantics to accommodate reasoning about actions.

The paradox of logical omniscience is briefly examined and made unquestionable on the grounds of a two-fold analogy: the general default rule which states that something is to be assumed true unless otherwise stated, and the frictionless case in physics.

To analyze statements of the form KNOW (a, Π) read as "an agent a knows that Π", Moore defines a relation of compatibility denoted by K. Now, K(a, W1, W2) means that the possible world W2 is compatible with what A knows in a possible world W1, i. e., as far as 'a' is concerned he might just as well be in W2. The relation of compatibility is essentially a reinterpretation of the accessibility relation as defined by Kripke [27].

With respect to actions, an agent 'a' knows how to perform an action if it knows the arguments of the action. In this case, an executable description of an action is to count as its rigid designator, as it is a computer program an executable description of a computation to an interpreter.

For Levesque [36, 37], both the paradox of logical omniscience and the syntactic paradigm are not admissable. The former denies reality, while the later leads to semantic horrors. His solution is to delineate the class of all beliefs into two subclasses, a class of implicit beliefs and a class of explicit beliefs. The class of explicit beliefs is properly included in the class of implicit beliefs for which the omniscience holds. Thus, an agent is still a perfect knower when it reasons about his implicit beliefs.

Each possible world is a situation, where a given proposition p might be: undefined, false, true, or both true and false making a situation inconsistent. Although, Levesque denies using a non-standard world semantics calling his impossible worlds incoherent situations, the result is the same as in the models of Cresswell [11] and Hintikka [21]. What is different is that his agents reason in relevance logic based on the four-valued system proposed by Belnap.

An encouraging result is that, at least for a propositional case, it is easier to calculate what is believed than what is implied by the belief. Thus, given a knowledge base (a set of sentences) KB and a sentence p in a conjuctive normal form, to determine if KB logically implies p is NP-complete. However, to determine if KB entails p has O(mn) algorithm, where $m = |KB|$ and $n = |p|$. Some recent work has been done in supplementing full semantics for the believe system of Levesque that has a decidable segment.

3. The Syntactic Paradigm. Quine and Carnap have recommended that modalities such as 'necessary', 'possible', and similarly 'know' and 'believe', could be treated as predicates of expressions rather than propositional operators [44]. For instance, the sentence "It is necessary that the ice is cold" is to be replaced by the sentence "'The ice is cold'is necessary". Notice that the operator of modality is not being prefixed to a sentence, but to a name of a sentence. The advantages are the elimination of nonextensional context and the possibility for an application of predicate calculus with identity. A few researchers in AI have found these advantages attractive enough to develop first-order syntactical theories of belief and knowledge. An account of the most prominent ones follows.

A first-order formalism for multi-agent planning environment is the locus of the Konolige's interest [33]. It is essentially the syntactic counterpart of Moore's theory. The work gives a careful axiomatization of the language levels and the nesting of beliefs. The proof method is a complex deduction process based on the semantic attachment technique originated by Weyhrauch in 1980 [56]. The reasoning process performed by an agent is modelled as an inference procedure in the object language. Essentially, Konolige simulates the behavior of agents performing cooperative tasks that require that each agent know the future states of its knowledge and the plans and actions of other agents.

The deduction model of belief, yet another theory of Konolige [34], is an attempt to address the question of logical omniscience in a more sophisticated manner. An agent, provided with a sound logic, is placed in a resource bounded environment. It is a symbol-processing system where beliefs are represented in some internal mental language. An agent reasons about his beliefs by manipulating these syntactic objects.

Since the process of inference is recognized as being resource-dependent, the derivation of all the consequences of beliefs is logically incomplete. The semantics for the system has an 'outside' role, i. e., the semantics is needed by an external observer to analyze the beliefs of an agent.

A common problem with the syntactic approach is its impure relation with semantics. For example, any two sets of sentences are to be accepted as distinct semantic entities, such as (f v g) and (g v f) which are for the agent different belief sets. Konolige 'solves' this problem by imposing a deduction rule which demands the presence of both sentences in the base set of beliefs. It is easy to notice the ad hoc character of the syntactic approach; a collection of sometimes hardly intelligible rules which are to replace at least part of the semantics.

The syntactic theory of belief and action, as examined by Haas [17], treats beliefs as sentences in a first order logic. The application of quotation marks, Quine [46], is done to individual terms rather than whole expressions. The method of proof is based on the so-called Reflection Schema which is an infinite set of axioms. Preference is given to Kripke's theory of truth as a method to deal with paradoxes. Consequently, most of the sentences are expected to be grounded which means that the determination of their truth value is reached without resorting to an infinite recursion [32].

A robust theory of knowledge, communication, action, and planning, is behind yet another attempt in syntactics by Morgenstern [45]. As usual, the theory starts with the definition of a syntactic predicate 'know' that ranges over names of sentences. Thus the model remains a first order theory inspite of the quantification over sentences. The theory of truth by Kripke is again embraced to deal with paradoxes.

The axiom system is similar to the one formulated by Moore, a fortiori Hintikka. In the first axiom the system of propositional calculus is replaced by the system of predicate calculus. An agent does not know all the axioms in the system. It knows only the axioms of predicate logic and the axioms that constitute the system of knowledge.

Most of the major objections to the syntactic paradigm came from Montague [44]. He proved with reference to the "Knower Paradox" the following result: if 'KNOW' is a syntactic predicate, one can construct a sentence ϕ, such that ϕ if and only if KNOW(a, " $\neg\phi$"). Assuming logical omniscience, veridicality, and necessitation hold, ϕ is inconsistent.

A recent article by Riviers and Levesque [37] vindicates to an extent the syntactic theory by showing that any modal logic can be translated to a first-order logic with a predicate ranging over sentences. The authors assert that any intensional operator governed by a reasonable modal theory, i. e., a theory containing all extended theorems and closed under modus ponens, could be treated syntactically. Unfortunately, some of their claims appear to be obscure (at least to this author) which might be an indication that an additional work is needed concerning the problem of which paradigm, the syntactic or semantic, is more powerful and adequate.

4. The Hybrid Revision. Building on the work of Levesque [36], Faigin and Vardi [13], and Halpern and Moses [24] developed a logic of general awareness with the intention of preserving the elegance of possible world semantics while resolving the paradox of omniscience. An agent might implicitly have all the logical consequences of his beliefs, but the explicit beliefs are the ones he has and acts upon.

At each possible world a syntactic filter (or an awareness operator) is introduced to delineate the explicit from implicit beliefs. Thus the theory has now three unary modal operators: B (explicit belief), L (implicit belief, and A (awareness). In their model, an explicit belief is semantically defined as implicit belief restricted to the sentences permitted by the awareness. If an agent is aware of all the sentences in the system, then the classes of explicit and implicit beliefs are identical. The model enables reasoning in a resource-bounded environment, although its complexity is increased. However, it might be necessary to induce small constraints into the semantics in order to treat modalities in a more efficient way.

In the overview, the focus was on general systems rather than on specific applications. The intention was to emphasize the relevance of the research to the issues of computational reasoning and knowledge representation. Hence, a few areas of application are briefly enumerated.

It appears that it is almost impossible to delineate the boundaries of the research areas which are not influenced by reasoning about knowledge. To attempt a definition may prove to be imprudent and counterproductive. After all, no science has benefited more from confluence of ideas than AI.

II. THE SYSTEM OF KNOWLEDGE

A. THE ONTOLOGY OF KNOWLEDGE

"The totality of meaning is
never fully rendered . . ."
Merleau-Ponty

1. Extension and Intension. Since the publication of Frege's work [14], the functionality principle according to which the meaning of an expression is a function of the meanings of its constituents has generally been accepted in linguistics. However, due to the ambiguities present in the context of natural languages Frege recognized the need for a distinction between an extension or denotation of an expression ϕ in some language L and its intension or sense. To reduce the number of possible constituents let us use the term 'entity' to represent things, items, objects or individuals. The few examples below illustrate why the distinction between an intension and an extension is necessary.

For instance, take a particular word such as 'book'. The intension of 'book' is a definition, such as "a lengthy treatment of a subject, printed and binded, and comes on the market as a commodity". The extension of 'book' is the set of all books in the world. Simpliciter, if the word 'book' denotes a concept of a book then the intension is all entities that define the concept, while the extension is all those entities that 'fall' under the concept.

Certain authors consider the distinction between an intension and extension in the semantics of natural languages to be analogous with the distinction between an episodic and a semantic memory [52]. The episodic memory holds particular facts which correspond to extensions, while the semantic memory holds general principles which are intensions. For Umberto Eco, the intension of an expression corresponds to the theory of signification and the extension of an expression corresponds to the theory of communication. Our interpretation is the traditional one. The intension of an expression is a function over the set of possible worlds.

Assume an arbitrary language L and given an expression ϕ in L, denote the extension of ϕ with EXT (ϕ) and its intension with INT (ϕ). When the expression ϕ is a sentence, then the EXT (ϕ) is the set 2.

Does the definition of extension of the expressions presented above help in any way to extract the meaning that each expression carries within itself? Consider the sentences $\phi \equiv$ "Two plus two are four" and $\psi \equiv$ "John and Ann are step-siblings". The principle of functionality is satisfied. If both sentences are true (or false) then the sentences have the same extension. But ϕ and ψ have entirely different meanings and thus represent different informational sources for a reasoning agent.

The distinction between an extension and an intension is even more apparent in the examples where the principle of functionality fails, such as oblique or intensional contexts. The famous example is the 'morning star-evening star' paradox. Recalling the previous exposition on modalities, the sentence "Necessarily the morning star is identical with the morning star" is true with respect to all possible worlds. So, if one is to follow the functionality principle and replace the second occurrence of 'morning star' with 'evening star' the resulting sentence "Necessarily the morning star is equal to the evening star" is true. On the contrary, the second sentence is false. Why? It is quite possible that there is a world in which the stars are not identical, although both constituents "morning star" and "evening star" have the same extension.

The 'morning star-evening star' paradox could also be examined in an epistemic context. The following sentence "If the Morning Star ≡ the Evening Star, then Ann knows that the Morning Star appears in the morning if and only if Ann knows that the Evening Star appears in the morning" is not generally considered to be a logical truth. By logical truth of a sentence ϕ in a language L one understands that ϕ is true under all possible interpretations of L. But this is consistent with our understanding of epistemic notions. In order for ϕ to be a logical truth, ϕ should have an additional premise "Ann knows that the Morning Star ≡ the Evening Star". Indeed, the additional premise can be regarded as a deviation from the standard logic. But the deviation is acceptable as long as one tries to capture different epistemic and doxastic inclinations. These propositional inclinations are entirely different enterprise.

One possible solution to the problem is to avoid possible world semantics and modalities. It is simple enough and wrong. Assume that Mr. Spock was a student in the Computer Science Department at UMR. While pursuing his degree he supported himself by working at nights in the library, where one his coworkers used to be Mrs. Robinson. After graduation, Mr. Spock is hired by Bell Labs. Since Mr. Spock currently works for Bell Labs, the expression "coworker of Spock" is coextensive with the expression "member of Bell Labs". Now consider the expression "former coworker of Mr. Spock" which is coextensive with Mrs. Robinson. Replacing the expression "coworker of Mr. Spock " with a "member of Bell Labs" yields the expression "former member of Bell Labs" which is not coextensive with Mrs. Robinson. The principle has failed again without mentioning any possible worlds. Tense modalities are vivid examples for demonstrating an intensional context [15].

Clearly, the extension of an expression must somehow depend on the syntactical context in which it occurs. An expression ϕ used in an ordinary context has an extension EXT (ϕ) while used in a referentially oblique context its extension becomes INT (ϕ). To preserve the principle of functionality one might introduce for each expression ϕ a new one denoted by ϕ' or the concept of ϕ, such that the extension of ϕ' is the intension of ϕ. As a consequence the decomposition of any expression could be done in terms of both extensions and intensions [15], which depends on the nature of context in which each individual construct appears.

None of the terms concerning the context of the use of an expression in some language L, such as possible state of affairs, possible worlds, points of reference or 'indexes' satisfies our intuition. The idea of worlds at least with respect to reasoning by artifacts is too ambitious. As Hintikka quotes Savage's suggestion in [20], it is better to think in terms of some 'small worlds'. Extensive world theories are implausible creations at the current level of AI. The term 'points of reference' is more appropriate in reasoning when tense modalities are involved. Hence, a new term 'complex' is introduced which supplants all aforementioned terms with a similar meaning.

Complex is simply a set of propositions which are cognitively accessible to a reasoning agent and sets of complexes which stand for those propositions. The non-empty denumerable set of complexes shall be denoted by K. Thus, given a sentence ϕ its intension, INT (ϕ) is exactly the proposition expressed by the sentence ϕ. The idea is to regard the object of knowledge to be a proposition. Hence, knowledge is an empirical relation between an entity (reasoning agent) and a proposition. The important role that propositions and concepts play in an intensional context is addressed in the next sections.

Is there a need for the notion of complexes in the treatment of knowledge? Assume that there are two physically identical complexes, yet what different agents know with respect to those complexes is not necessarily identical. The position taken in the research is a conceptualist one. Complexes are not only necessary as primitives for analysis of epistemic and doxastic notions, but they are indispensible if one is to capture what is vacuously called a multitude of conceptions or imagination. Both complexes and intensions are possibly the initial steps toward a comprehensive theory of meaning and understanding.

The next question is concerned with the relation of the logic of intensions and the theory of knowledge. The logic of intensions comprehends modal logic, whence knowledge is a modality. Both, the intensional and higher order modal logic are alternative formulations to each other. Moreover, the monadic modalities coincide with the properties of propositions.

2. Knowledge-carriers. The condition imposed on the propositions that constitute the epistemic environment of an agent was that they be true. The question is whether propositions are the entities capable of truth or falsity, and in general what kind of entities are likely candidates to be 'truth-bearers'? The choice is limited to sentences and propositions.

Sentences are examined first. A sentence is any complete and correct string of expressions in a given language according to its grammatical rules. For example, "The sun is hot" is a sentence while "The nice morning" is not. It is common to distinguish between sentence types and sentence tokens. Thus, by writing "The sun is hot", "The sun is hot", one might say that there is one sentence type inscribed twice or that there are two sentence tokens.

The distinction between sentence-tokens and sentence-types is one of the reasons why sentences should be rejected as possible candidates for 'truth-bearers'. It is not clear to which of those two entities the truth values should be attributed. Another argument against their candidature for truth-bearers is that sentences are dependent on the underlying language and physical existence.

Consider the analogy with the numerals and numbers, where the former represent sentences and the later propositions. To numbers mathematical properties are easily assigned, to numerals it is meaningless to do so. Sentences can be inscribed and erased, and thus cease to exist. The propositions expressed by those sentences will still be present and true. The sentence "The sun is hot" could be written in Serbo-Croatian such as "Sunce je toplo" and express the same proposition as the former sentence does; an indication that there is no one-to-one correspodence between sentences and propositions. The same proposition can be expressed by different sentences. The extra-linguistic character makes the propositions suitable for knowledge-carriers, a fortiori truth-bearers.

The independence of knowledge from any particular language being used to express it is what was termed in the PROLEGOMENA as an extra-system significance of knowledge. There are arguments (Church [9], Bradley and Swartz [5]) that no qualification is to be assigned to propositions other than defining them as abstract entities capable of truth and falsity. This is far too 'abstract'.

Therefore, each proposition is identified with a set of possible complexes in which the proposition is true. In other words, a proposition is a function from the set of possible complexes to the set 2. Hence, a plausible epistemic context of a reasoning agent is a set of propositions in which every proposition is a set of complexes in which the proposition is 'true'.

An identification of the object of knowledge as a set of possible complexes is not without its problems. Consider the following two propositions:

Π_1: "Two plus two is four."

Π_2: "The earth is round."

Denote with ϕ_1 and ϕ_2, the sentences expressing the two propositions Π_1 and Π_2 respectively. If the sentence $\phi \equiv \phi_1 <-> \phi_2$ is logically true then the sentences ϕ_1 and ϕ_2 are logically equivalent. This entails that the propositions Π_1 and Π_2 are logically equivalent. The logical equivalence of Π_1 and Π_2 implies that they are true in the same set of possible complexes. Why is it so ? The relation between propositions and complexes equates the notions of logical equivalence and semantic equivalence. Thus is not possible to distinguish between a propositional equivalence and an propositional identity. The phenomenon is a consequence of set theory, where each set is uniquely determined by its members. If S_1 and S_2 are two sets and they have the same members then $S_1 \equiv S_2$.

But the propositions Π_1 and Π_2 are far from being identical. Although both true, they have entirely different contexts of information and convey different meanings. To better understand the difficult problem of propositional identity it is necessary to explore the ontological status of the propositions.

3. <u>Concepts</u>. To assert that an entity has an ontological status is to recognize the existence of its intrinsic structure. The structure then can be contrasted with other entities of the same or different types. An example of the later was the analysis of sentences and propositions. So the focus here is on entities of the same type, i. e., propositions.

The existence of an inner structure entails two different characteristics of an entity with reference to the complexity of its structure. When the structure of an entity is reducible to more primitive entities than the original entity is a compound entity or molecular one. When the original structure is irreducible to more primitive entities then one speaks about atomic entities.

For propositions, the argument put forward is the following: propositions are compound entities and their essential constituents are concepts. Regarding the concepts as primitive constructs of propositions does not imply that concepts themselves do not possess inner complexity on their own. A concept may or may not be the subject of further logical analysis. Certain concepts may also entail the existence of other concepts which fall under the the original concepts.

Chronologically, the notion of concept is among the primal entities in the vocabulary of reasoning [5]. Concepts have always been suitable as a reference but quite elusive for a rigorous definition. It is quite possible there is no definition. McCarthy [41] openly admits of using the term 'concept' in the study of first-order theories without any attempt to discuss it. However, the acceptance of concepts as essential entities does not prevent us from giving at least some explanatory account of their 'existence'.

Certains authors [5, 14] think that it is important to disassociate concepts from conceptions and ideas. They apply to concepts the same type of 'abstract detachment' as they do to propositions. It is true that conceptions and ideas could basically be treated as psychological entities which mirror some internal 'mental state' of an agent. But to deny them any role in the formulation of concepts amounts to the denial of possibility in the process of reasoning. What is important to realize is that concepts have the same kind of extra-linguistic property as propositions do.

Just what kind of entities are used to express concepts? The general agreement is that there are two [5, 46]. Expressions of reference, which can be undetermined such as "something" or "everyone", or determined such as "John" or "morning star". Also, propositional functions (or open sentences) which contain a locus which is later substituted with a referring expression. Consider the following proposition:

Π: "The color of blood is red."

In this case the referring expression is "the color of blood" and the propositional function is "... is red". The set of concepts that appear in the proposition Π is {being a color, being a blood, being red}. No truth truth assignment could be given to propositional functions. A distinction, therefore, should be made between open sentences which are concept-expressing entities and closed sentences used to express propositions and are subject to an assignment of truth-value.

As Quine [46] pointed out, any closed sentence could be transformed to an open sentence. For the proposition P the transformation yields:

TΠ: "x is the color of the blood and x is red."

Identifying propositions with a set of complexes in which the propositions were true has a two-fold implication with respect to concepts. First, concepts are related to the complexes through the propositions of which they are essential constituents. Second, in view of the fact that propositions were knowledge-carriers, concepts must be considered as constituents of knowledge. In essence, concepts are the primary building blocks of of knowledge.

With reference to the inner complexity of concepts a distinction is made between linear and non-linear concepts. Simpliciter, a concept is linear if it is not subject to logical analysis; a concept is non-linear if it is. By a logical analysis understand the process of reduction of a concept to another concept which is a synonym or an epistemic alternative of the original concept.

A necessary or analytic truth is one that holds in all complexes. For example, consider the concepts 'mother' and 'green'. The claim is that the concept of a mother is a non-linear one and the concept of green is a linear one. A possible logical analysis of the concept of a mother is a 'female-parent', but no parallel analysis is applicable to the concept of being green. Someone may argue that analysis of 'green' could produce another concepts like 'wavelength', 'reflection', and 'absorption'. These are synthetic or contingent truths because these truths are dependent on the existence of 'greeness' in the resonant complex. A resonant complex is a complex in which the logical and physical truth are coextensive. It will be correct to accept that the resonant complex represents reality.

The question of synonymity is very important and extremely difficult. Intensions are not sufficient for a satisfactory treatment of synonymity. Therefore, no distinction is going to be made between identical and synonymous propositions.

Other than synonymity, there are many modal-like relations that could be defined among concepts. Thus given two concepts Δ_1 and Δ_2 the relations of agreeability, disagreeability, universal agreeability, relevance, and universal relevance can be defined.

Concept Δ_1 is agreeable with concept Δ_2 if and only if there is at least one complex in which both concepts Δ_1 and Δ_2 apply to the same entity. For example, the concepts of being a professor and him knowing the subject that he is teaching.

Concepts Δ_1 and Δ_2 are disagreeable if and only if there is no a complex in which both concepts apply to the same entity. The concept of bachelor and the concept of being married is an example of disagreeable concepts.

Concepts Δ_1 and Δ_2 are universally agreeable if and only if there is no complex in which the application of Δ_1 to an entity does not entail the application of Δ_2 to the same entity. The propositions

Π_1: "Paul and John have one parent in common."

Π_2: "Paul and John are step-siblings."

contain respectively the concepts of "having one parent in common" and "being step-siblings". One can observe that if the first concept applies to some entities (for example Paul and John) the second one applies to the same entities by necessity.

A concept Δ is relevant if and only if there is an entity in at least one complex that falls under the concept Δ. The example of a relevant concept is the concept of being divisible by two.

A concept Δ is universally relevant if there is at least one entity in each possible complex that falls under the concept Δ. The concept of self-identity is an example of universally relevant concept, assuming of course that each complex contains at least one entity.

Concepts are potentials for knowledge depending on the existence of an entity that might possibly fall under the concept. A concept is a selector. It selects entities from the universe of all possible entities and those entities represent its extension. If the concept is empty then the extension is the empty set. An example of an empty concept is the concept of "a round square". The concept of a round square is empty because the concept is logically impossible.

The argument put forward by Bradley and Swartz is that the decomposition of a proposition to concepts is order-sensitive [5]. For instance, take the proposition

Π: "John was on the top of Jim."

and assume that the proposition Π is true. Some of the possible combinations for the concepts that appear in the proposition Π are represented by the following sets of concepts {being John, being on the top, being Jim}, {being Jim, being on the top, being John}, and {being John, being Jim, being on the top}. The first set of concepts reflects the proposition P, while the second set of concepts yields another proposition:

Σ: "Jim was on the top of John."

The third set of concepts is not determined at all with respect to the original meaning of the proposition Π.

An analogy underlining the distinction between ordered and unordered sets of concepts can be found in geography. An agent is given the assignment to visit some cities. To know the ordering is like having a complete map of the tour with all the cities and the roads that interconnect. Here concepts represent cities. To have concepts without an ordering amounts to being a conscious only that the cities might exist on the map. It may even be the case that a road between some cities does not exist. What is missing is the map (or the appropriate mappings between the concepts).

To conclude, a proposition may be defined as an ordered sequence of concepts subject to a truth-value assignment.

The prima facie denial of the attribution of truth values to concepts should not be understood as too categorical. The role of the concepts as the building blocks of knowledge must essentially be recognized through a formal system that can deal with concepts.

B. A ZEROTH-ORDER THEORY OF CONSCIOUSNESS

> ". . . but we need notions,
> not notations."
> A. Tarski

1. Criteria for Conceptual Satisfaction. Consciousness about concepts is a first level of reasoning leading towards knowledge. But open sentences and propositional functions, which are the principle devices to express concepts, can not be attributed truth-value. Hence, we stipulate the existence of minimal concept expressing propositions. For every possibly non-empty concept Δ denote by Π_O its corresponding 'minimal proposition of concept expression ' which has the form "The concept of . . . exists" or "There is a concept of . . .". The gap is to be substituted with any concept. For instance, if the particular concept is a car then the corresponding minimal proposition of concept expression is "There is a concept of a car". The predication of existence in the minimal proposition of concept expression is to be understood to vary over possible entities. The set of actual entities is a subset of the set of possible entities.

By a language of consciousness L_C is understood a language of which the symbols are drawn from the following categories:

[1] Improper symbols as defined for the system P;

[2] A denumerable set of minimal concept expressing propositions $\Pi_C = \{p, q, r, \ldots\}$

[3] A monadic operator of consciousness 'CON' (read 'conscious of');

[4] Individual variables ranging over a denumerable set of

all reasoning agents $PA = \{a_1, \ldots, a_i, \ldots\}$

The formulas of L_C are denoted by p,..., f, g, h, ... Let $\Gamma(L_C)$ be the set of all formulas of L_C which is the smallest set that contains Π_C, is closed under the logical connectives and contains ${}_a\text{CON}(f)$ whenever $f \in \Gamma(L_C)$ and $a \in PA$.

The standard notation for expressing epistemic notions as an empirical relation between an agent and a proposition is slightly modified. This is done to prevent any ambiguity relative to whether or not an agent is in the scope of an epistemic operator. Therefore, the term ' ${}_a\text{CON}(p)$ 'is a formal counterpart of "an agent 'a' is conscious of p".

A conceptual structure for L_C is a tuple $M = \langle K, \Phi, \Theta, V\rangle$ where:

[1] K is a nonempty set of complexes;

[2] Let $\rho(K)$ be the power set of K, i. e., the set of all sets of possible complexes. Then Φ is a function from the set Π_C to the set $\rho(K)$, that is

$$\Phi\colon \Pi_C \to \rho(K)$$

The function Φ provides each minimal concept expressing proposition with an intension;

[3] Θ is a function from {a} x K to the set $\rho(\rho(K))$, viz., the function Θ assigns to an agent 'a' who is in the complex k the set of all minimal concept expressing propositions that an agent is conscious of in the complex k. In a multi-agent case the function Θ is defined as:

$$\Theta\colon PA \times K \to \rho(\rho(K))$$

instead of

Θ: $\{a\} \times K \rightarrow \rho(\rho(K))$

[4] V is a function of confirmation (or valuation)

V: $\Gamma(L_C) \times K \rightarrow 2$

such that:

(a) $V(p, k) = 1$ if and only if $p \in \Pi_C$ and $k \in \Phi(p)$

(b) For any wff f and any complex k, $V(\neg f, k) = 1$ if and only if $V(f, k) = 0$;

(c) For any two wffs f and g and any complex k, $V(f \;\&\; g, k) = 1$ if and only if $V(f, k) = 1$ and $V(g, k) = 1$;

(d) For any wff f and any complex k, $V({}_a\mathrm{CON}(f), k) = 1$ if and only if the $\mathrm{INT}(f) \in \Theta(a, k)$.

Intuitively, the INT(f) of any wff $f \in \Gamma(L_C)$ determines a set of complexes which is a subset of K, where the wff f is 'inevitable'. The intension of f is interpreted as a set of complexes that confirm f. So for an arbitrary wff f $\mathrm{INT}(f) = \{k: M, k \models f\}$. Given a conceptual structure M and a complex $k \in K$, the expressions $V(f, k) = 1$ under the interpretation M and $M, k \models f$ are identical. The role of the functions Φ and Θ is explained in the Example 1.

Example 1. Let $K = \{i, j\}$ be a set of complexes and let $\Pi_C = \{p, q, r\}$ be a set of concept expressing propositions. The range of Φ is the set $\rho(K) = \{\Lambda, \{i\}, \{j\}, \{i, j\}\}$, where Λ denotes the empty set. The intensions to the elements of Π_C are arbitrarily assigned such that $\mathrm{INT}(p) = \{i\}$, $\mathrm{INT}(q) = \{j\}$, $\mathrm{INT}(r) = \{i, j\}$. The range of Θ is the set $\rho(\rho(K)) = \{\Lambda, \{\Lambda\}, \{\{i\}\}, \{\{j\}\}, \{\{i, j\}\}, \{\Lambda, \{i\}\}, \{\Lambda, \{j\}\}, \{\Lambda, \{i, j\}\}, \{\{i\}, \{j\}\}, \{\{i\}, \{i, j\}\}, \{\{j\}, \{i, j\}\}, \{\{i\}, \{j\}, \{i, j\}\}, \{\Lambda, \{i\}, \{j\}\}, \{\Lambda, \{i\}, \{i, j\}\}, \{\Lambda, \{j\}, \{i, j\}\}, \{\Lambda, \{i\}, \{j\}, \{i, j\}\}\}$. To realize the unique relation between propositions and complexes let us express $\rho(\rho(K))$ in terms of the propositions p, q, and r. Now, the range of Θ becomes the set $\rho(\rho(K)) = \{\Lambda, \{\Lambda\}, \{p\}, \{q\}, \{r\}, \{\Lambda, p\}, \{\Lambda, q\}, \{\Lambda, r\}, \{p, q\}, \{p, r\}, \{q, r\}, \{p, q, r\}, \{\Lambda, p, q\}, \{\Lambda, p, r\}, \{\Lambda, q, r\}, \{\Lambda, p, q, r\}\}$. Let the value of Θ at the complex i for an agent 'a' be $\Theta(a, i) = \{p, q, r\}$. This means that an agent 'a' is conscious of three concepts in a complex i. For each complex i, the value $\Theta(a, i)$ determines a unique set of consciousness that is assigned to a reasoning agent. By imposing certain restrictions to the sets of consciousness one can capture different modes for reasoning about consciousness. For instance, if $\Lambda \in \Theta(a, i)$ then an agent 'a' can be conscious of the impossible. When $\Theta(a, i) \equiv \Lambda$ then an agent 'a' residing in a complex i is in a state of complete ignorance.

Assume that K is a non empty set of complexes and that each complex k has at least one entity. Let p, q, and r are concept expressing propositions associated with the concepts Δ_p, Δ_q, Δ_r respectively. Thus, the modal relations among concepts in terms of intensions are:

•Two concepts Δ_p and Δ_q are agreeable if and only if $INT(p) \cap INT(q) \not\equiv \Lambda$.

•Two concepts Δ_p and Δ_q are disagreeable if and only if $INT(p) \cap INT(q) \equiv \Lambda$.

•Two concepts Δ_p and Δ_q are universally agreeable if and only if $INT(p) \equiv INT(q)$.

•A concept Δ_r is relevant if and only if $INT(r) \not\equiv \Lambda$.

•A concept Δ_r is universally relevant if and only if $INT(r) \equiv K$.

Similarly, the definition of a proposition as an ordered sequence of concepts is defined in terms of intensions:

•An intension of a proposition is the intersection of intensions of all the concept expressing propositions which constitute the proposition.

How does a conceptual structure M differ from a standard Kripke structure KS for reasoning about knowledge? Recall that Kripke structure is a tuple KS = <K, KR, R, V> where K is the nonempty set of possible complexes; KR is the 'actual' world or the resonant complex . R is a relation of accessibility and V a valuation function. The relation of accessibility R is not an element of the conceptual structure M. Therefore, epistemic notions such as knowledge and consciousness or doxastic notions such as belief and awareness are not modelled as a relation between two complexes.

Is there any penalty in the case of intensional approach for excluding the relation of accessibility? It appears that the reasoning agent has been deprived from his 'interworld' intuition. The relation of accessibility was the one that provided an agent with a set of epistemic alternatives. For consider a wff f which represents some fact. An agent knows the fact f, if and only if f is true in all complexes which are conceivable for the agent from a complex k.

But a reasoning agent still has its 'intensional' intuition. Consider two wff f and g such that INT(f) ≡ INT(g). Provided $_{a}$CON(f) is true then $_{a}$CON(g) is also true. As expected, in the intensional paradigm a reasoning agent has lost the ability to distinguish between logically equivalent formulas.

A modal type of omniscience has been replaced with an intensional 'omniscience'. While it is desirable to be conscious of concepts which are epistemic alternatives to each other, the side-effect of the intensional approach is counterintuitive and largely idealized. Moreover, there is no satisfactory way to deal with the phenomenon of omniscience with a single primitive qualifier such as a complex and to remain faithful to the pure intensional paradigm.

2. Axiomatization and Determinism. To say that a system is axiomatizable is to state that there is a set of wff T (ordinary those are the axioms of the system) such that members of the system are those wffs derived from T by an acceptable rules of inference. If the principal of wffs is finite then the system is finitely axiomatizable.

The fundamental system S_c for reasoning about consciousness is finitely axiomatizable. The system S_c is defined by the following axiom schemas and rules of inference:

(PC) All propositional tautologies

(RE) From f <-> g infer $_{a}$CON(f) <-> $_{a}$CON(g)

The axioms (PC) are credit from propositional reasoning. The rule of inference (RE) is actually the rule of substitution of equivalents. The Rule (RE) is a consequence of the intensional approach in the formulation of the system S_C. The attribution of fundamental to the system is clear: the reasoning power of an agent 'a' is a minimal one due to the absence of the modes of reasoning introduced in the classical theory of knowledge.

It is interesting that validity in the semantics of possible complexes is a three-fold phenomenon. First, one can speak about a local validity of a wff formula f, i. e., when a formula f is valid in a complex k. A wff f is structurally valid if and only if f is valid in all complexes belonging to a structure or a model. Finally, a global validity of a wff f is defined with respect to a class of conceptual structures or models $\Omega(M)$. The global validity corresponds to the idea of universal validity in the propositional system P.

A system S_C for LC (or the Logic of Consciousness) is said to be sound with respect to a class of conceptual structures $\Omega(M)$ if and only if every theorem of S_C is valid in $\Omega(M)$. The system S_C is complete in LC if and only if every valid formula f in $\Omega(M)$ is a theorem of the system. To prove that the system S_C is determined in LC, i. e., sound and complete, we need a few definitions and two lemmas which have become a standard [25, 31, 68] in all the proofs pertaining to determinism.

A wff f is S_C inconsistent if $\vdash \neg f$ is a theorem of S_C. A wff f is S_C consistent if $\dashv \neg f$ in S_C. The idea of S_C consistency can be extended to a set of formulas. Let Ψ denote a set of well formed formulas. If Ψ is finite, i. e., $\Psi = \{f_1, \ldots, f_k\}$ then $C(\Psi)$ stands for a conjuction of the formulas of Ψ. The set of formulas

Ψ is S_c inconsistent if and only if $\dashv \neg C(\Psi)$ in S_c. When Ψ is an infinite set, define the S_c consistency in the following manner: the set Ψ is S_c consistent if and only if every finite subset of Ψ is S_c consistent. A set of formulas Ψ is called maximal if and only if for every formula $f \in \Psi$, either $f \in \Psi$ or $\neg f \in \Psi$. The set of formulas Ψ is called maximal S_c consistent, denoted by MAX (Ψ), if and only if Ψ is both maximal and S_c consistent. In other words, a set of formulas is maximal S_c consistent if and only if any other conceivable extension will make it inconsistent.

Given a formula $f \in \Gamma(L_c)$, relative to the system S_c, a proof set of $|f|$ is the set of all MAX(Ψ) sets of formulas containing f [8].

The next two lemmas are given without a proof.

Lemma 1: Suppose that Ψ is MAX(Ψ) set of formulas with respect to

the system S_c. Then

(a) $f \in \Psi$ if and only if $\neg f \notin \Psi$

(b) $f \;\&\; g \in \Psi$ if and only if $f \in \Psi$ and $g \in \Psi$

(c) f <-> $g \in \Psi$ if and only if $f \in \Psi$ if and only if $g \in \Psi$

(d) Every theorem of S_c is in Ψ

Lemma 2: Every S_c consistent set of formulas Ψ has a maximal S_c consistent extension.

Proposition 1: The system S_c is determined with respect to a class of conceptual structures $\Omega(M)$.

Proof:

(i) (soundness)

The soundness of the system follows from the fact that it contains all the axioms of (PC) which are tautologies. Also, the rule of inference (RE) preserves validity. For consider a class of conceptual structures $\Omega(M)$ such that $f \leftrightarrow g$ is valid, i. e., $\models f \leftrightarrow g$. The last assertion holds if and only if $INT(f) \equiv INT(g)$. Then for any complex k and any set of consciousness $\Theta(a, k)$ in an arbitrary conceptual structure M, $INT(f) \in \Theta(a, k)$ if and only if $INT(g) \in \Theta(a, k)$. Thus for all complexes k, $V({}_aCON(f), k) = 1$ if and only if $V({}_aCON(g), k) = 1$ which implies that $\models {}_aCON(f) \leftrightarrow {}_aCON(f)$ for all $M \in \Omega(M)$.

(ii) (completeness)

Consider the following tuple $M = \langle K, \Phi, \Theta, V\rangle$ where:

[1] K is a nonempty set of complexes and each complex is maximal S_C consistent set of formulas;

[2] $\Phi: \Pi_C \to \rho(K)$

[3] $\Theta: \{a\} \times K \to \rho(\rho(K))$

[4] V is a function of confirmation

$V: \Gamma(L_C) \times K \to 2$

such that:

(a) $V(p, k) = 1$ if and only if $p \in \Pi_C$ and $k \in \Phi(p)$

(b) For any wff f and any complex k, $V(\neg f, k) = 1$ if and only if $V(f, k) = 0$;

(c) For any two wffs f and g and any complex k, $V(f \,\&\, g, k) = 1$ if and only if $V(f, k) = 1$ and $V(g, k) = 1$;

(d) For any wff f and any complex k, $V({}_aCON(f), k) = 1$ if and only if the $INT(f) \in \Theta(a, k)$.

The proof rests on the fact that the LC is based on on the intensional logic. If so, then the following assumption is correct: for any complex k and any wff f, $f \in k$ if and only if $|f| \in \Theta(a, k)$. The intension of any wff f consists of all those complexes k that confirm f. If each complex k is MAX it follows that INT(f) = $|f|$. Otherwise, INT(f) will not include all the complexes that confirm f. The case when $f = {}_a\text{CON}(g)$ such that $\text{CON}(g)_a \in k$ if and only if $|g| \in \Theta(a, k)$ follows from the definition of a conceptual structure M part ([4], (d)) and the validity of Rule (RE).

The proof proceeds by induction on the structure of a wff $f \in \Gamma(L_C)$. One has to show that for every $k \in K$ and every $f \in \Gamma(L_C)$ the relation (**) holds:

(**) $V(f, k) = 1$ if and only if $f \in k$.

Case 1: Let f = p. $V(p, k) = 1$ iff $k \in \Phi(p)$ and so $k \in$ INT(p). Since INT(p) = $|p|$, it follows that $p \in k$.

Case 2: Let $f = \neg f$. Then (**) follows from Lemma 1(a).

Case 3: Let f = g & h. Then (**) is a consequence of Lemma 1(b).

Case 4: Consider that $f = {}_a\text{CON}(g)$. Let the relation (**) hold for f = g. By the inductive hypothesis V(g, k) =1 and INT(g) = $|g|$, so for every k whenever INT(g) $\in \Theta(a, k)$ so is $|g| \in \Theta(a, k)$. From the definition of conceptual structure, part ([4](d)), INT(g) $\in \Theta(a, k)$ if and only if $V({}_a\text{CON}(g), k) = 1$ if and only if ${}_a\text{CON}(g) \in k$.

Case 5: Suppose that f, g $\in \Gamma(L_C)$ and V(f <-> g, k) = 1 for every k $\in$ K. From (**) follows that f <-> g $\in$ k for every k $\in$ K. So f <-> g belongs to every k where k is MAX(k, whence f <-> g is a theorem of S_C. By Lemma 1(c) then for every k $\in$ K, f $\in$ k if and only if g $\in$ k. Using the rule (RE) if f <-> g is a theorem of S_C so is $_a$CON(f) <-> $_a$CON(f) a theorem of S_C. Therefore, $_a$CON(f) <-> $_a$CON(f) $\in$ k for all k $\in$ K.

Hence M is a conceptual structure for S_C. If $\in \Gamma(L_C)$ and f is S_C valid formula, then V(f, k) = 1 for all k $\in$ K. From (**) follows that f $\in$ k for all k $\in$ K. But each k $\in$ K is MAX set of wff. Applying Lemma 1(d) yields that f is a theorem of S_C , i. e., provable in S_C.▪

Corollary 1: A set of formulas Ψ of LC is S_C consistent if and only if Ψ is $S_C$satisfiable.

Proof:

(only if)

Let M be a conceptual structure as defined in Proposition 1. Since Ψ is $S_C$consistent then Ψ is a subset of some k $\in$ K. Therefore, V(f, k) = 1 for every f $\in \Psi$. Thus Ψ is satisfiable.

(if)

Let Ψ be S_C satisfiable. Then for any formula f $\in \Psi$, V(f, k) = 1 for some k $\in$ K. The last assertion states that if f is satisfiable, $\neg$ f is certainly not. So Ψ is consistent with respect to negation in the system S_C.▪

The system S_c in LC is decidable if there is effective procedure for deciding whether or not a wff f in LC is a theorem in S_c. The methodology is similar to the one introduced by Chellas [8] with respect to classical modal systems. To prove decidability one requires that the notions of axiomatizability and conceptual structure finiteness are recognized by the system.

The system S_c in LC is axiomatizable and all its axioms are decidable (follows from the decidability of PC). The rule (RE) is a reasonable rule of inference. It always determines the relation between premises and the conclusion. The axiomatizability S_c provides for a positive test of provability (or a theoremhood) in the system S_c. Thus, the existence of a positive test for theoremhood qualifies the system as semi- decidable.

In order to be fully decidable, simpliciter decidable, the system S_c along with its complete axiomatization must have the property of having a conceptual structure which is finite. The finiteness of the structure provides a negative test for provability in the system.

A conceptual structure is finite if and only if K has a finite number of complexes; otherwise the conceptual structure is infinite. Let $\Omega(M)$ be a class of conceptual structures for S_c. Imposing a standard enumeration on the elements of $\Omega(M)$ yields $M_1, \ldots, M_i, \ldots$ The class is enumerable, since each M_i is finite. The test for negative theoremhood is now reduced to finding an M_i which is a conceptual structure and showing that some wff f is falsified in M_i. That the both tasks are finite is due to the fact that the axiomatization is finite and the models are finite. If the number of complexes is n (where n is a finite number) then the corresponding conceptual structure has at most 2^n complexes. Since the decidability problem is co-NP-complete the tractability of the problem should be understood in principle and not in practice.

Proposition 2: The system S_c of LC is decidable.

3. Modes for Reasoning. The ability to reason for an agent in the system S_c, other than the standard axioms of (PC) and the Rule (RE) is quite limited. It is a compromise between the flexibility and generality of the intensional approach and the rigid power of modal logic systems such as S4 or S5. We find this to be consistent with the heuristic approach in AI.

A restriction of the class of conceptual structures $\Omega(M)$ with respect to which the system S_c is determined will increase the number of logical 'devices' (axioms or theorems) in the system. The restrictions are accomplished by imposing plastic constraints on the sets of of consciousness $\{\Theta\ (a, k)\}$. The constraints are termed plastic since as long as they hold for a particular system the system is determined. The class of conceptual structures for which the system is determined with respect to a plastic constraint is denoted by $\Omega(M_r)$ where $\Omega(M_r)$ is a subset of $\Omega(M)$. The axioms of the restricted system are actually metatheorems of the generalized system $\Omega(M)$. Some of the acceptable modes of reasoning about consciousness are presented below.

Mode 1:(Consciousness about impossible) This mode was introduced in Example 1. The plastic constraint imposed on an arbitrary set of consciousness Θ (a, k) is:

PC1: $\Lambda \in \Theta(a, k)$

For an agent to be conscious of the impossible means that it is conscious of a wff formula g such that g -> f & ¬f. Let the intension of f to be INT(f) and define the intension of ¬f to be INT(¬f) = K − INT(f). Denote the intension of ¬f with CINT(f), whence CINT(f) is a complement of INT(f) with respect to the set of all complexes K. The formula g is inconsistent since (f & ¬f) is canonically unsatisfiable where INT(f) $\cap$ CINT(f) $= \Lambda$.

There are situations in a reasoning space of an agent that is encompassed by a KB which may be declared to be inaccessible or impossible to be encountered. These situations, which are described by formulas of type g, then will eventually lead to either a destruction of the reasoning mechanism (such as in a robot) or to violation of the entire KB. Prima facie there is a utility in the requirement that a reasoning agent be conscious of the impossible.

The mode of reasoning about the impossible, however, is inadmissable in a doxastic context. One cannot believe "Unicorn exists" and "Unicorn does not exist" in the same complex k. A distinction should also be made between the formulas ($_{a}$CON(f) and $_{a}$CON(¬f)) and $_{a}$CON(f & ¬f) which are entirely different modes of reasoning. The later mode requires a time resonant interval for the wffs f and ¬f, while the former mode refers to two different time intervals, regardless of how infinitesimal that interval difference is. The argument against being conscious of the impossible comes from the behavior of canonically unsatisfiable formula (or contradiction) in standard propositional calculus. Anything is derivable from a contradiction which is the reason why propositional logic is so concerned about consistency. So the implication is clear. If an agent is conscious of the impossible it is conscious of everything possible. This makes the (MT1) inappropriate for its inclusion in the system of knowledge.

Mode 2:(Distribution of consciousness) The process of reasoning from universal facts toward particular facts is accomplished by the metatheorem:

MT2: ${}_{a}CON(f \& g) \rightarrow {}_{a}CON(f) \& {}_{a}CON(g)$

which is true if

PC2: If $INT(f) \cap INT(g) \in \Theta(a, k)$ then $INT(f) \in \Theta(a, k)$ and $INT(g) \in \Theta(a, k)$

holds. The distribution of consciousness is unproblematic. One can modify the 'then' part of PC2 by dropping either the INT(f) or the INT(g) to get a single importation.

Mode 3:(Collection of consciousness) Evidently, the Mode 3 is the converse of the Mode 2. In this case one has captured the process of reasoning from particular facts toward universal facts.

MT3: ${}_{a}CON(f) \& {}_{a}CON(g) \rightarrow {}_{a}CON(f \& g)$

which is true if

PC3: If $INT(f) \in \Theta(a, k)$ and $INT(g) \in \Theta(a, k)$

then $INT(f) \cap INT(g) \in \Theta(a, k)$

holds. It is an unproblematic mode from the intuitive point of view. A substitution of the "if ... then" condition in either one of the plastic constraints PC2 and PC3 with an "if and only if" condition will yield both modes of reasoning (MT2 and MT3) in a single mode.

Mode 4:(The Omniconsciousness of Truth) If a reasoning agent is to recognize canonically true things, denoted by T, then a fairly simple constraint is to be met:

MT4: ${}_{a}CON(T)$

if

PC4: $K \in \Theta(a, k)$

A canonically true formula is true in every complex. Therefore, the intension of T must be the set of all possible complexes, i. e., $INT(T) \equiv K$.

Mode 5:(Conscious about Consciousness) This mode involves iteration of consciousness operators. The metatheorem of positive introspection has the form:

MT5: ${}_{a}CON(f) \rightarrow {}_{a}CON(CON(f))$

and is valid

PC5: if $INT(f) \in \Theta(a, k)$ then there exist a set of complexes $\hat{K}$ such that $\hat{K} \equiv \{i | INT(f) \in \Theta(a, i)\}$ and $\hat{K} \in \Theta(a, k)$.

The arguments against the acceptance of the axiom of positive introspection in an epistemic context are few. They are usually due to the psychological phenomena of subconsciousness and repression. The existence of these phenomena is rather speculative even with respect to human intellect. To discuss the subconsciousness and the repression in connection with the artifact reasoning is probably unacceptable at the moment.

There may be some technical difficulties, however, with the realization of either positive or negative introspection. Let the number of epistemic operators on the left side of (MT5) be denoted by δ which stands for a degree of reasoning about consciousness or knowledge. One can observe that degree of reasoning δ always induces a higher degree of reasoning $\delta + 1$. The higher degree of reasoning is reflected in the existence of the set $\hat{K}$ which requires that the cardinality of the set of consciousness $\Theta(a, k)$ to be increased. So, in general, if one wants to have an unlimited degree of reasoning then the sets of consciousness must be infinite.

Mode 6:(Conscious about the Unconsciousness) The mode for negative introspection, i. e. , to be unconscious implies to be conscious about something that you are unconscious off is somewhat of a linguistic circus and caveat. But the difficult admission of this peculiar linguistic expression is not a sufficient reason for the mode of negative introspection to be rejected in reasoning by artifacts. The negative introspection is formalized as follows:

MT6: $_a\neg CON(f) \rightarrow {}_aCON(\neg CON(f))$

and is valid

PC6: if $INT(f) \notin \Theta(a, k)$ then there exist a set of complexes $\hat{K}$ such that $\hat{K} \equiv \{i \mid INT(f) \notin \Theta(a, i)\}$ and $\hat{K} \in \Theta(a, k)$.

Consider that $\Gamma(L_C)$ is augmented with another monadic operator 'AWE' so that if a wff $f \in \Gamma(L_C)$ then $_aAWE(f) \in \Gamma(L_C)$. The dual of the consciousness operator 'CON' is the operator of awareness 'AWE' such that:

$V({}_aAWE(f), k) = 1$ if and only if $CINT(f) \notin \Theta(a, k)$.

The difference between the two inclinations, consciousness and awareness, is a difference between definitive and possible information. Given a complex k, a reasoning agent 'a', and a wff f, by the definition of consciousness for 'a' to be conscious of f means that the $INT(f) \in \Theta(a, k)$.

The epistemic operator 'CON' is a sort of existential quantifier over the set of possible concepts and hence complexes for an agent. Various concepts may exist within a KB. However, they do exist for an agent 'a' if and only if the concepts are members of an agent's set of consciousness for some $k \in K$. In the case of awareness, an agent is aware of a formula f if the complement of the intension of f is not element of its set of awareness. There is no guarantee that the intension of f is in its set of awareness. Let 'a' be conscious that "the grass is green" in a complex k. The agent 'a' is aware that "the grass is green" in a complex k if and only if 'a' is not conscious that "the grass is not green".

The problem with notions such as negative introspection and awareness is beyond the intuitive admissability. The theoretical basis for both negative introspection and awareness is somehow superfluous and troublesome. The reason is that with modalities there is no 'a clear cut negation' as there is one in classical logic. A comparative study of the plastic constraint (PC6) and the condition for awareness reveals the effect of negating an epistemic operator. Thus, the negative introspection can alternatively be formulated as:

MT6: $_a$AWE(f) -> $_a$CON(AWE(f))

The definition is consistent with the idea of duality in modal and intensional theories. Absence of consciousness does not entail total ignorance about certain concept. It does entail that there is a possibility, no matter how small, that a concept is accessible.

C. A ZEROTH ORDER THEORY OF KNOWLEDGE

> "The most efficient way to solve a
> a problem is to already know how to
> solve it. Then one can eliminate search
> entirely."
> The Society of Mind, Marvin Minsky

1. Criteria for Epistemic Satisfaction. By an epistemic language L_e is understood a language of which the symbols are drawn from the following categories:

[1] Improper symbols as defined for the system P;

[2] A denumerable set of atomic propositions Π_e.

The elements are: p, q, r, . . .

[3] A monadic operator of knowledge 'KNOW'

(read 'know');

A monadic operator of consciousness 'CON'

(read 'conscious of');

[4] Individual variables ranging over a denumerable set of reasoning agents $PA = \{a_1, \ldots, a_i, \ldots\}$

The formulas of L_e are denoted by p,..., f, g, h, ... Let $\Gamma(L_e)$ be the set of all formulas of L_e which is the smallest set that contains Π_e, is closed under the logical connectives and contains $_a$KNOW(f) and $_a$CON(f) whenever $f \in \Gamma(L_e)$ and $a \in PA$. The set of minimal concept propositions Π_c is a subset of the set of all atomic propositions Π_e.

An epistemic structure for L_e is a tuple $E = \langle K, \Phi, \Theta, V \rangle$ where:

[1] K is a nonempty set of complexes;

[2] The function Φ provides each atomic proposition with its intension:

$$\Phi: \Pi_a \rightarrow \rho(K)$$

[3] The function Θ assigns to a reasoning agent its knowledge set:

$$\Theta: \{a\} \times K \rightarrow \rho(\rho(K))$$

[4] V is a function of confirmation

$$V: \Gamma(L_e) \times K \rightarrow 2$$

such that:

(a) $V(p, k) = 1$ if and only if $p \in \Pi_e$ and $k \in \Phi(p)$

(b) For any wff f and any complex k, $V(\neg f, k) = 1$ if and only if $V(f, k) = 0$;

(c) For any two wffs f and g and any complex k, $V(f \& g, k) = 1$

if and only if V(f, k) = 1 and V(g, k) = 1;

(d) For any wff f and any complex k, V($_a$KNOW(f), k) = 1 if and only if the INT(f) $\in \Theta$(a, k) and k $\in$ INT(f). The condition for knowing can be expressed through consciousness as: for any wff f and any complex k, V($_a$KNOW(f), k) = 1 if and only if V($_a$CON(f), k) = 1 and k $\in$ INT(f).

The difference between epistemic structures and conceptual structures is reflected in the criteria ([4](d)) for confirmation. The modification is necessary if the Axiom of Truth (AT)

(AT) $_a$KNOW(f) -> f

is to be included in a formal system for the logic of knowledge (LK). The Axiom of Truth underlines the metaphysical distinction between consciousness and knowledge. In the case of consciousness, where Θ(a, k) is a set of propositions that an agent is conscious about in a complex k, the complex is just an observation (or a reference) point for an agent. Consequently, it is not necessary for a complex k to be included in the intension of any proposition which is a member of Θ(a, k). The complex k is detached from the consequences of the consciousness. When knowledge is involved, due to the distinction that was made between knowledge and other cognitive phenomena such as perceptions, beliefs, conceptions, the reference point such as the complex k is affected by what an agent knows. The propositions that an agent knows must be 'true' in the complex in which the propositions are known.

A formal system S_e for the LK is axiomatized as:

(PC) All propositional tautologies

(AT) $_a$KNOW(f) -> f

(RE) From f <-> g infer $_a$KNOW(f) <-> $_a$KNOW(g)

Let $\Omega(E)$ denote a class of epistemic structures. Then the following result holds for all epistemic structures $E \in \Omega(E)$.

Proposition 3: The system S_e is determined with respect to $\Omega(E)$.

Proof: The proof of this proposition is quite similar to the one in the Proposition 1. Assume that every complex k is a MAX(f), i. e., is maximal consistent set of formulas and f is wff.

(i) (soundness)

The case that matters is the Axiom of Truth. Suppose that every epistemic structure E satisfies the condition for knowledge ([4](d)) and that $_a$KNOW(f) is a theorem at a complex k. The last assertion holds if INT(f) $\in \Theta$(a, k). From the condition ([4](b)) it follows that k $\in$ INT(f). This posits that f is true at k.

(ii) (completeness)

The important case to be proved is when a wff f = $_a$KNOW(f). All other cases are reducible to the results from Lemma 1. Let INT(f) $\in \Theta$(a, k) from which follows that since every k is maximal consistent set of formulas then INT(f) = |f|. The last assertion is true if and only if $_a$KNOW(f) $\in$ k. Applying the Axiom of Truth yields that f $\in$ k.∎

The metatheorems (MT2) to (MT6) that were discussed concerning the LC may accordingly be defined for the epistemic system and used as a metatheorems in LK. The metatheorems for knowledge are:

ME2: $_{a}$KNOW(f & g) -> $_{a}$KNOW(f) & $_{a}$KNOW(g)

ME3: $_{a}$KNOW(f) & $_{a}$KNOW(g) -> $_{a}$KNOW(f & g)

ME4: $_{a}$KNOW(T)

ME5: $_{a}$KNOW(f) -> $_{a}$KNOW(KNOW(f))

ME6: $_{a}\neg$KNOW(f) -> $_{a}$KNOW($\neg$KNOW(f))

The (MT1) is not even arguable in the case of knowledge. It contradicts the very basic assumption: every knowledge set $\Theta(a, k)$ must be consistent. The acceptance of (MT1) is a violation of the Truth condition.

The Principle of Existence in LC asserts that: A concept Δ exists if and only if there is an agent who is conscious of the concept. Since the present research is primarily concerned with modeling propositional or declarative knowledge, another important principle may hold in general, the so-called Principle of Invariance which says that: a knowledge set of a reasoning agent is invariant with respect to any finite permutation of its constituent propositions.

Assume that the Principle of Invariance does not hold for LC. So, order-sensitivity is preserved in LC and one can give an alternative formulation of knowing an atomic proposition from its uniquely associated concept expressing propositions. This formulation of knowing depends on the restrictive Principle of Linguistic Competence: all of the reasoning agents know the language they communicate in. Then given a valid formula f, the following result holds in LC + LK:

(C & K) CON(p) & CON(q) & ... CON(r) -> KNOW(f)

provided that: the order of p, q, ..., r is preserved and p, q, ..., r are the only propositions appearing in f. The INT(p) $\cap \ldots \cap$ INT(r) is a subset of the INT(f), k $\in$ INT(f) and k $\in \bigcap$ INT(p) where p is any of the constituent concepts of f. The result, due to the restrictions, is not a particularly useful one. Also, from a computational standpoint, one may envision an infinite number of concept expression propositions appearing in the valid formula f.

What about the interplay between the epistemic operators 'CON' and 'KNOW' within the scope of one another? According to the definition of knowledge, consciousness is implied by knowledge. Thus, if an agent 'a' knows a fact represented by wff f, then the agent 'a' is conscious of f. The converse is not true. The metatheorem (KC) can be accepted as a global axiom of the knowledge system:

(KC) $_a$KNOW(f) -> $_a$CON(f)

One point has to be made clear. The interplay of epistemic operators, although interesting because of various possible notions to be depicted, can be quite problematic. For instance, consider

(CK) $_a$CON(KNOW(f))

By setting KNOW(f) equal to g, g = KNOW(f), then g is treated like any other wff. The original intent, to consider an epistemic inclination as an empirical relation between an agent and a proposition is preserved. A plausible importation is obtained by applying (AT) to (CK) in LC+LK which yields:

(CKC) $_a$CON(KNOW(f)) -> $_a$CON(f)

The relation (CKC) is valid if the following plastic constraint is satisfied: there exist a set of complexes $\hat{K}$ such that $\hat{K} = \{j : INT(f) \in \Theta(a, j)\ \&\ j \in INT(f)\} \in \Theta(a, k)$ then $INT(f) \in \Theta(a, k)$. Similarily the importation:

(KCC) $_a$KNOW(CON(f)) -> $_a$CON(f)

is valid if the (AT) is an axiom of LC+LK and the following constraint is satisfied : if there exist a set of complexes $\hat{K}$ such that $\hat{K} = \{j : INT(f) \in \Theta(a, j)\}$ then $INT(f) \in \Theta(a, k)$, and $k \in INT(f)$.

A somewhat weaker principle (KCK) can be obtained from (ME2) which states that if an agent knows something then an agent is conscious that it knows.

(KCK) $_a$KNOW(f) -> $_a$CON(KNOW(f))

The principle (KCK) requires that if $INT(f) \in \Theta(a, k)$ and $k \in INT(f)$ then there exists a set of complexes $\hat{K}$ such that $\hat{K} = \{j : INT(f) \in \Theta(a, j)\}$ and $\hat{K} \in \Theta(a, k)$. No other combination of the epistemic operators 'CON' and 'KNOW' is allowed in the system of knowledge.

2. Entailed Knowledge. The epistemic language L_e is augmented with another monadic epistemic operator 'EKNOW' (read 'explicitely knows'). Thus, for every wff f whenever $f \in \Gamma(L_e)$ so is $EKNOW(f) \in \Gamma(L_e)$. The criteria for confirmation of entailed knowledge is:

(EK) $V({_a}EKNOW(f), k) = 1$ if and only if there exist a formula g such that $INT(g) \in \Theta(a, k)$ and $k \in INT(g)$, and INT(g) is a subset of INT(f).

A wff f is known by an agent at a complex k if and only if there exist some other wff g at the complex k which entails the formula f. The relation of entailment, viz., 'p entails q', is a converse of the relation 'q logically follows from p'. The entailment plays a crucial role in modal and relevance logic. It was introduced in logic to avoid the paradoxes of material implication (denoted in this research with p -> q) which is read as "if it is not the case that p and not q". The strict implication in modal logic is read as "if it is impossible that p and not q".

In relevance logic (like modal logics there is a whole family of relevance logics [23]) it is recognized that entailment is a converse of deducibility. Moreover, relevance logicians argue that the notion of deducibility in the classical logic is defective, because the question of relevance is ignored. So for instance, in terms of the logic of relevance, q is deducible from p if and only if the derivation of q uses p. The digression in in relevance logic was necessary to underline the importance of entailment in any kind of reasoning and particularly about knowledge.

Assume that there is a knowledge base (KB) which is queried with the question: does the grass have any color? If the reasoning is based on entailment then the answer is a result of the existence of a proposition in KB asserting that "the grass is green". The reasoning is based on the idea that a fact is known which necessarily entails another fact. An entailed fact is usually more general one than a fact that entails. For instance, being green entails being colored, but being colored does not entail being green. Intuitively, one may conclude that it is implicit knowledge that is modelled not an explicit one. But recall the answer given by the KB in the above example. That the grass is colored was entailed by an explicit fact in KB that the grass is green.

Let $\Omega(E_x)$ represent a class of entailed (or explicit) epistemic structures. Suppose that each $\Omega(E_x)$ is defined precisely in the same manner as the ordinary epistemic structures with the exception of the condition ([4](d)). Then the following propositions are true.

Proposition 5: If $|= f \rightarrow g$ (f entails g) is a valid formula in $\Omega(E_x)$ then $|= {}_a\text{EKNOW}(f) \rightarrow {}_a\text{EKNOW}(g)$ is a valid formula in $\Omega(E_x)$.

Proof: Assume that $|= f \rightarrow g$ is valid in $\Omega(E_x)$ which implies that INT(f) is a subset of INT(g). Let h be a wff formula such that INT(h) $\in \Theta(a, k)$ and $k \in$ INT(h). If INT(h) is a subset of INT(f), then by the euclidian property of inclusion, INT(h) is a subset of INT(g). Hence, for every $k \in K$ if $V({}_a\text{EKNOW}(f), k) = 1$ then $V({}_a\text{EKNOW}(g), k) = 1$.(••). The Proposition 5 states: the relation (••) represents a reasonable rule of inference in the logic of entailed knowledge (LEK). The rule of inference which is valid in the class $\Omega(E_x)$ is referred to as (RX).▪

The distribution of knowledge (MT2) is a valid formula in $\Omega(E_x)$. When an explicit knowledge is considered then there is no need for any plastic constraints. The MT2 is not anymore a metatheorem; it assumes the full power of being an axiom in the system of explicit knowledge.

Proposition 6: The formula

(MT2) $_a$EKNOW(f & g) -> $_a$EKNOW(f) & $_a$EKNOW(g)

is valid in the class of epistemic structures $\Omega(E_x)$.

Proof: Assume that V($_a$EKNOW(f & g), k) = 1 for every k $\in$ K. Then there is a wff h so that INT(h) $\in \Theta$(a, k), k $\in$ INT(h), and INT(h) is a subset of INT(f & g). But INT(f & g) = INT(f) $\cap$ INT(g). The last assertion implies that INT(h) is a subset of INT(f) and INT(h) is a subset of INT(g). Hence V($_a$EKNOW(f), k) = 1 and V($_a$EKNOW(g), k) = 1.▪

The relation between entailed knowledge and ordinary knowledge is interesting. As one can expect the entailed knowledge implies knowledge simpliciter. The converse is not true in general. The next two results elaborate on the relation between the two types of knowledge.

Proposition 7: The rule of inference

(RE) From f <-> g infer $_a$EKNOW(f) <-> $_a$EKNOW(g) is valid in the class $\Omega(E_X)$ of epistemic structures.

Proof: Assume that $\models f \leftrightarrow g$ for each $k \in K$ and each $K \in \Omega(E_X)$. If $f \leftrightarrow g$ then $INT(f) \equiv INT(g)$. Suppose there exist a wff h such that $INT(h) \in \Theta(a, k)$ and $k \in INT(h)$. The INT(h) is a subset of INT(f) if and only if INT(h) is a subset of INT(g). Hence, $V({}_aEKNOW(f), k) = 1$ if and only if $V({}_aEKNOW(g), k) = 1$.▪

The consequences of the Proposition 7 is that (1) the rule of inference (RX) implies the rule of inference (RE), and (2) the logic of entailed knowledge is included in the logic of knowledge. Naturally, one may ask when they are equivalent, i. e., when is $\Omega(E) \equiv \Omega(E_X)$.

Proposition 8: The LEK and LK are equivalent, $\Omega(E_X) \equiv \Omega(E)$, if MT2 is valid in $\Omega(E)$.

Proof: The proposition is true if (RX) is a reasonable rule of inference in LK. Assume that (MT2) is valid in $\Omega(E)$. To prove that (RX) is valid only one of the importations is necessary. Let the plastic constraint (PC2)

if $INT(f) \cap INT(g) \in \Theta(a, k)$ then $INT(g) \in \Theta(a, k)$

hold in the class $\Omega(E)$. Suppose $\models f \leftrightarrow g$, $\models f \rightarrow g$, and ${}_aKNOW(f) \leftrightarrow {}_aKNOW(g)$ are true for each $k \in K$ and each $K \in \Omega(E)$. By (PC) if $f \rightarrow g$ is true so is $f \leftrightarrow (f \,\&\, g)$. The application of (RE) to $f \leftrightarrow (f \,\&\, g)$ yields ${}_aKNOW(f) \leftrightarrow {}_aKNOW(f \,\&\, g)$. Using the (MT2) on ${}_aKNOW(f \,\&\, g)$ one obtains ${}_aKNOW(f \,\&\, g) \rightarrow {}_aKNOW(g)$ which finally gives ${}_aKNOW(f) \rightarrow {}_aKNOW(g)$.▪

Two theories are equivalent if and only if they have equivalent axioms and rules of inference. Therefore, LEK and LK are equivalent theories about knowledge.

3. <u>Action Structures</u>. In a rational environment there is a symbiotic relationship between actions and knowledge. Actions should be based on knowledge and knowledge should be enlarged by actions. The later type of actions, i. e., 'knowledge extension actions, is the single type of actions admissable in the system.

An action is a sequence of one or more events, where events are properties of time as continuum. A distinction is to be made between instant events which are mapped to moments of time and duration events which are mapped to intervals of time. There is also a metaphysical difference between actions and events which reflects their causality. Actions are internally caused changes in properties of time, while events are externally caused changes in properties of time. For us both phenomena are identical and the common term 'action' is used as a reference.

The time continuum is not a consideration in the present study of actions which means that actions are to be understood as discrete in nature. Thus, an action is change in a property of a complex. What is the property that is changed? It is the knowledge set $\Theta(a, k)$ which is uniquely assigned to an agent at a complex k. Any change in a knowledge set is an action. But the change in an arbitrary knowledge set is a result from a move of an agent 'a' from one complex to another complex. So an alternative interpretation of an action is a translation between complexes.

According to the extent of knowledge and the nature of actions, the class of all reasoning agents may be devided to three subclasses. The three subclasses are properly included in each other in the following order a class of self-conscious agents, a class of agents that has epistemic integrity, and a class of autonomous agents. A reasoning agent is said:

•to be self-conscious if it knows its name.

•to have an epistemic integrity if it is self-conscious and it knows its axioms and rules of inference.

•to be autonomous if it has an epistemic integrity and all its actions are internally caused or self-controlled.

The system of knowledge considers agents with epistemic integrity only. An action structure for a reasoning agent 'a' is a tuple $A\Sigma_a = \langle\Theta, T\rangle$ where:

[1] Θ is a family of knowledge sets for an an agent 'a' and each element of the family is indexed by the complex it is assigned to.

[2] T is a translation relation defined on the family of knowledge sets, a fortiori on the set of complexes K, and having the following properties:

(a) T is reflexive

$\Theta(i)\ T\ \Theta(i)$

(b) T is antisymmetric

If $\Theta(i)\ T\ \Theta(j)$ and $\Theta(j)\ T\ \Theta(i)$ then $\Theta(i) \equiv \Theta(j)$

(c) T is euclidian

If $\Theta(i)\ T\ \Theta(j)$ and $\Theta(j)\ T\ \Theta(k)$ then $\Theta(i)\ T\ \Theta(k)$

and i,j,k $\in$ K and $\Theta(i)$, $\Theta(j)$, and $\Theta(k) \in \Theta$. An epistemic potential of an agent is the cardinality of its knowledge set. Three different types of actions are superimposed by the action structures on a reasoning agent. The first type of actions are called affirmative or positive actions. The affirmative actions increase the epistemic potential of a reasoning agent. The second type of actions are the negative ones which decrease the epistemic potential of a reasoning agent. The last ones are the neutral actions which do not change the epistemic potential.

Positive actions are characteristic of monotonic reasoning or the inter-complex conservative extension of knowledge. Negative actions are descriptive of non-monotonic reasoning and relevant to knowledge revision. At the present level the system does not allow non-monotonic reasoning. The enforcement of monotonicity is the reason why the translation T has to be antisymmetric.

An alternative formulations of active and neutral actions could be given in terms of the information content of a knowledge set.

•For any two complexes i, j $\in$ K a translation T from a complex i to a complex j is called informative if and only if $\Theta(i)$ is a proper subset of $\Theta(j)$. The set difference between the knowledge sets $\Theta(j)$ and $\Theta(i)$ is termed as information gain and is denoted with IG(i↑j).

•A translation T is called non-informative or if and only if for any two complexes i, j $\in$ K, the following identity $\Theta(i) \equiv \Theta(j)$ holds. When there is a sequence of complexes i, j, k, ... such that all translations from one complex to another complex are non-informative, the epistemic potential of a reasoning agent is said to be in an equilibrium.

The total number of action structures in a system of knowledge is mn, where $m = ||PA||$ and $n = ||K||$, and $||A||$ denotes the cardinality of an arbitrary set A. The analysis of the action structures could be done in terms of latices (or equivalently posets) where the two basic operations are set union and intersection and the relation of partial order (translation) T is a set inclusion.

D. OMNISCIENCE REVISITED

> "The disadvantage of exclusive attention
> to a group of abstractions,
> however well founded, is that,
> by nature of the case, you have
> abstracted from the remainder of things."
> Science and the Modern World, N. Whitehead

1. Intensional Omniscience. Ignoring the problem of omniscience in reasoning about knowledge is considered to be a sinful act against this consecrated issue in epistemology. Thus, once again we face the most vivid discourse in the logical analysis of epistemic and doxastic notions, the question whether or not knowledge, consciousness, awareness and belief are invariant with respect to logical equivalence. It is essential to realize, however, that the invariance occurring in the analysis of propositional attitudes does not induce the question of logical validity. The problem with omniscience is the admission of its intuitive plausibility and computational feasibility.

Is the issue of intuitive admissability of logical omniscience reducible to ad hominem arguments? The epistemological literature is flooded with stories that are used as a basis for rejecting the deductive closure [35]. The arguments are always the same: the acceptance of omniscience produces highly idealized reasoning environments. In these environments, and the notorious and absurd examples persist, a reasoning agent as a perfect knower must know the answer to the problems of Goldbach conjecture, the last theorem of Fermat, and whether or not P = NP.

A simple way to dismiss the arguments against omniscience is to acknowledge that any formal system is based on abstraction, a fortiori on some form of idealization of reality. Most of the logical systems, both classic and extended, are certainly not intended to be used by 'idiots' or 'savants', unless in the case of the later it is their privileged area of excellence. But even the most able and ingenious minds working on a certain problem could easily fail to foresee all the consequences of their results.

If one accepts that all logical inferences are purely analytical, or as it was stated in PROLEGOMENA parthenogenetic, then any increase in the information content of the reasoning space is entirely psychological. As a consequence, Hintikka claims that the omniscience is not "only admissable but inevitable". But as one could observe from the extensive survey of methodologies dealing with the problem of omniscience, Hintikka is not definite on the inevitability of omniscience. The question of satisfactory treatment from the standpoint of semantics is still an open problem.

It is important to delineate, at least partially, the problem of logical omniscience with respect to reasoning by artifacts and reasoning by humans. When artifacts are involved in reasoning about knowledge, the insistence on intuitive admissability is an outside issue. So one may designate a certain number of complexes according to priorities which are context-dependent. These complexes will constitute a small KB where an unrestricted interchange on the basis of logical equivalence may be allowed. Thus, although in a syntactic manner, the omniscience will be localized. An interchange on the global level among various knowledge bases could be unrestricted only for propositions that are synonymous.

The last discourse has implicitly opened the Sisyphean problem of relevance in reasoning about knowledge. Relevance is a ubiquitous research problem in its own right and I strongly suspect that a solution in general terms is ever possible. The problem is that relevance and its intuitive interpretation are inherently context-dependent notions.

In our case relevance is introduced for a denotational purpose. It stands for that generic part (or a set of formulas) of the knowledge base (KB) for which unrestricted interchange on the basis of logical equivalence is permitted.

2. Restricting The Omniscience. The omniscience occurring in the intensional context is due to the definition of intensional confirmation and is reflected in the rule of inference RE. How does one proceed in in dealing with omniscience while attempting to preserve the three fundamental principles of any formal system: the

material and the logical adequateness, and the intuitive admissability.

First of all one needs to indicate the omnipresent character of the propositional attitude that that holds in all situations or times with respect to certain complexes. Secondly, one must determine the set of complexes that are not subject the usual recursive truth conditions. The effect of leaving out the truth conditions for a number of complexes is a division of the original system into two subsystems, a strong and a weak subsystem. The strong system has the usual truth conditions and the logical and metalogical attributes apply to it. The weak system has only the rudimentary soundness principles.

The methodology followed, to restrict the locally omniscient system S_r in LK, is basically the one proposed by Hintikka [21], Kripke [31], and Rantala [51]. In addition to the nonempty set of possible and stable complexes K, a new set K′ is introduced which is possible empty and contains unstable complexes. The unstable complexes are elements of the weak system. The Rule (RE) should be restricted in such a way that is to apply only to some wffs in $\Gamma(L_e)$.

Let $\Gamma(L_e)$ be a set of all wffs in LK and let $\Gamma(L_r)$ be an arbitrary recursive subset of $\Gamma(L_e)$. Then the restricted system for reasoning about knowledge S_r is axiomatized in the following way:

(PC) All propositional tautologies

(AT) ${}_a\text{KNOW}(f) \rightarrow f$

(RE) If f, g and $(f \leftrightarrow g) \in \Gamma(L_r)$ then infer

$${}_a\text{KNOW}(f) \leftrightarrow {}_a\text{KNOW}(g)$$

The system S_r is different from the system S_e in that the Rule (RE) is restricted to the formulas that belong to S_r. The restriction imposed on the rule (RE) is syntactic certainly is not justified by the intensional semantics. Hence, any semantics for the restriction is going to be context-dependent. The restriction, in a way, is an admission that either the propositional attitudes are not purely intensional or the understanding and the definition of an intension should be revised. For the present, one has to be content with the small doses of syntax in order to prevent, to an extent, the occurrence of any form of omniscience.

The interpretation for the system S_r is a revised epistemic structure $E_r = \langle K, K', \Phi, \Theta, V \rangle$ where:

[1] K is a nonempty set of complexes;

[2] K′ is a set of unstable complexes;

[3] The function Φ provides each atomic proposition with its intension:

$$\Phi: \Pi_a \rightarrow \rho(K \cup K')$$

[4] The function Θ assigns to a reasoning agent its knowledge set:

$$\Theta: \{a\} \times (K \cup K') \rightarrow \rho(\rho(K \cup K'))$$

[5] V is a function of confirmation

$$V: \Gamma(L_e) \times (K \cup K') \rightarrow 2$$

such that:

(a) $V(p, k) = 1$ if and only if $p \in \Pi_e$ and $k \in \Phi(p)$

(b) For any wff f and any complex k, $V(\neg f, k) = 1$ if and only if $V(f, k) = 0$;

(c) For any two wffs f and g and any complex k, $V(f \,\&\, g, k) = 1$

if and only if $V(f, k) = 1$ and $V(g, k) = 1$;

(d) For any wff f and any complex k, $V({}_a\text{KNOW}(f), k) = 1$ if and only if the $\text{INT}(f) \in \Theta(a, k)$ and $k \in \text{INT}(f)$, such that $\Theta(a, k) = \{|f|: {}_a\text{KNOW}(f) \in k\} \cup \Xi$ where $\Xi = \{\text{INT}(g): k \in \text{INT}(g)\}$.

[6] For every stable complex k and every unstable complex k' if $f, g \in \Gamma(L_r)$ and $V(f <-> g, k) = 1$ then $V(f <-> g, k') = 1$;

Proposition 9: The system S_r is determined with respect to a class of epistemic structures $\Omega(E_r)$.

Proof:

(i) (soundness)

From the definition of a restricted epistemic structure follows that all of the axioms are valid. Also the condition [6] imposed on the Rule (RE) shows that it also preserves validity with respect to any complex. The system S_r is sound with respect to an interpretation E_r.

(ii) completeness

Each stable complex k is a maximal E_r consistent set of formulas . The nature of unstable complexes is determined by the following construction:

For each $k \in K$ let $k' = \{f: {}_a\text{KNOW}(f) \in k\}$.

The unstable complexes, as one can observe, are not subject to the usual truth-recursive conditions. The confirmation function V for the unstable complexes is defined so that for every $k' \in K'$

$V(f, k') = 1$ if and only if $f \in k'$ The condition for knowing says that for any wff f and any complex k, $V({}_aKNOW(f), k) = 1$ if and only if $V(f, k) = 1$ and $V(f, k') = 1$ iff that $k' \in INT(f)$.

As before, one has to prove that for every wff f complex k the (**) $V(f, k) = 1$ if and only if $f \in k$ is true with respect to all restricted epistemic structures $\Omega(E_r)$.

Let $f = {}_aKNOW(g)$ and assume that (**) holds for $f = g$. (only if) $V({}_aKNOW(g), k) = 1$ then $k' \in Int(g)$ which implies that $V(g, k') = 1$ and so $g \in k'$. By the construction of k' then ${}_aKNOW(g) \in k$. (if) If ${}_aKNOW(g) \in k$ then $g \in k'$, and hence $V(g, k') = 1$. Also, by (AT) $g \in k$. From the hypothesis $V(g, k) = 1$ so it follows that $V({}_aKNOW(g), k) = 1$.

Suppose f, g, and $f \lt\!\!-\!\!\gt g \in \Gamma(L_r)$. Then $V(f \lt\!\!-\!\!\gt g, k) = 1$ for every $k \in K$. Applying Lemma 1 then $f \lt\!\!-\!\!\gt g \in k$ for every $k \in K$. By the Rule (RE) if $f \lt\!\!-\!\!\gt g$ is theorem of S_r so is ${}_aKNOW(f) \lt\!\!-\!\!\gt {}_aKNOW(g)$ a theorem of S_r. The last assertion is true if ${}_aKNOW(f) \in k$ if and only if ${}_aKNOW(f) \in k$ for every $k \in K$. By the construction of k', $f \in k'$ if and only if $f \in k'$. Therefore, $V(f \lt\!\!-\!\!\gt g, k') = 1$ for every $k' \in K$.

Assume the wff f is a valid formula such that $E_r \models f$. Then $V(f, k) =1$ for every $k \in K$, and by (**) $f \in k$. Since each complex k is a maximal S_r consistent set of formulas, then by Lemma 1 each complex contains every provable formula of S_r.∎

The result, although, technical in nature is due to quite serious semantic considerations. The Proposition 9 shows that a knowledge depth of a reasoning agent can be restricted within a formal system.

A knowledge depth is the number of logically equivalent formulas. When an agent knows one of them, it knows them all. Applying the restriction to a small but crucial set of formulas has certainly useful computational properties. Also, the number of inference steps used in the process of reasoning can be limited for this KB.

III. CONCLUSIONS AND FUTURE DIRECTIONS

A. THE RELEVANCE OF THE KNOWLEDGE SYSTEM

> "It really is a nice theory.
> The only defect I think it has
> is probably common to all philosophical
> theories. It is wrong."
> Naming and Necessity, S. Kripke

> "That theory is worthless.
> It is not even wrong."
> W. Pauli

The present work provides only the necessary foundation for a comprehensive intensional theory for reasoning about knowledge. However, the advantages over the classic modal theory are noticeable. The SK (or the System of Knowledge) is more general and flexible than the theories based on classic modalities. The flexibility is due to the presence of plastic constraints which can capture practically any mode of reasoning. Also the problem of omniscience is differently formulated in the intensional paradigm than in the modal paradigm. Since, the Rule of Necessitation and the relation of accessibility are not present the issue of resource-boundness is not as acute as with the modal systems. All these arguments make the propositional case for intensional reasoning about knowledge a rather complete one.

An extensive consideration should be given to the further development of the action structures. At the present level their role is a marginal one. But in order to be autonomous the system of knowledge must have full control over the actions. This can be achieved if an agent had access to various knowledge sets. What is needed is an action relation similar to the one of accessibility but without the possible side-effects of modal omniscience.

The intention was to formalize the fundamental notions of knowledge in an intensional setting. Therefore, a number of technical problems were left out. One of these technical issues is a common knowledge. In his landmark paper, Aumann showed that when two agents have the identical priors and their posteriors is a common knowledge, then these posteriors must be identical [3]. The idea of a common knowledge and the notions of "everybody knows" and "all agents know" are closely related.

Assume that there are two agents 'a' and 'b', and a proposition p which is declared to be a common knowledge. This is achieved if 'a' knows p, and 'b' knows p, and 'a' knows that 'b' knows, and 'b' knows that 'a' knows, and 'a' knows that 'b' knows that 'a' knows, and so on.

In the intensional environment it is fairly straightforward to formalize a common knowledge. Let f be a wff that represents a fact which is declared to be known to every reasoning agent in the system. The formula f is a common knowledge if and only if for all $k \in K$ and all $a_i \in PK$ the intension of f , $INT(f) \in \Theta(a, k)$. Therefore, $V(f, k) = 1$ if and only if $V({}_aKNOW(f) \& \ldots \&_n KNOW(f), k) = 1$ for all $k \in K$. Let CM(f) denote that the wff f is a common knowledge and augment the epistemic language L_e with a monadic epistemic operator 'AKNOW' (read 'all know' or 'everybody knows'). Then $V(CM(f), k) = 1$ if and only if $V(AKNOW^n(f), k) = 1$ for all $k \in K$.

However, if a complex is the only primitive notion in the system then the problem of propositional identity and equivalence remains unsolved. A possible avenue for resolving the omniscience is based on some archaic ideas in epistemology [15, 25, 44] where a new primitive such as a context is introduced in the semantics of the system. Then a meaning of a wff f is a function from the set of ordered pairs {<i, a>, <j, b>, <k, c>, ...} to the set 2 and is denoted by MEAN(f). The first element of each order pair is drawn from the set of possible complexes K, while the second element from the set of possible contexts Υ. Now, the identity between two propositions, p and q, can be defined as an identity between the respective meanings of the propositions, i. e., MEAN(p) ≡ MEAN(q). The notion of meaning of an expression allows an unrestricted interchange on the basis of identity at every complex and in every context.

Similarily, the issue of non-monotonic reasoning can be addressed by introducing time as a primitive notion. Now, the extended meaning of a wff f can be defined as a set of ordered triples {<i, a, t>, <j, b, u>, ...}. The idea is to have monotonic reasoning on a micro-level within small time intervals. The effect of non-monotonic reasoning will be on a macro level, where a knowledge set of a reasoning agent does not necessarily have conservative extensions. The use of context and time represent interesting possibilities for an extension of the propositional system for reasoning about knowledge.

B. QUANTIFICATION THEORY

"The future enter into us,
in order to transform itself in us,
long before it happens."
Letters to a Young Poet, R.M. Rilke

Suppose that the epistemic language L_e is augmented with individual variables, predicate variables, as well as with individual and predicate constants. The language L_e also admits improper symbols such as ∃ (read 'exists'), ∀ (read 'for all'), ∇ (read 'for some'), and Å (read 'for most'). The operator of necessity L is to be read as 'it is understandably so'. Another descriptive symbol which can also be classified as an improper symbol denoted by 'U' and stands for 'there is a unique ... such that'[44].

Knowledge is again modelled as an empirical relation between an agent and a proposition. This empirical relation is represented by a two-place predicate constant which admits individual constants ranging over the set PA and zero-place predicate variables which stand for the propositions. The standard set of quantifiers is augmented to accommodate, as it has been proposed by Zadeh [58], reasoning about a common sense.

Assume u is the proposition "There exist an object x which is blue and Jim knows that Ann knows that Tom knows". There are four separate propositions p, q, r, and s nested in the proposition u. These are:

(s) There is an object x which is blue.

(r) Tom knows the proposition s.

(q) Ann knows the proposition r.

(p) Jim knows the proposition q.

The propositions p, q, r, and s appear in an indirect context. Let p", q", r", and s" denote the concepts expressed by the propositions p, q, r, and s respectively. For instance p" stands for the expression UPL (P <-> f) which is read as 'there is a unique zero-place predicate variable P such that the zero-place predicate variable P is understandably equal to the formula f'[44]. The whole expression stands for the proposition p expressed by the formula f. Let BLUE(x) be a one-place predicate constant, and the names of the agents Jim, Ann and Tom are abbreviated to the first character respectively. Then the proposition u in an epistemic context has the form:

∀x((∃x) & jKNOW(a"KNOW("tKNOW("BLUE(X)))))

The introduction of the quantification and the predicate variables reveals the true higher-order nature of intensional logic. A sufficient extension for modeling epistemic and doxastic inclinations is a second-order system. In general any theory, first-order, second order up to the transfinite ordinals can be regarded as an instance of the type theory. Higher-order theories have commonly stronger semantics than any first-order theory. The distinction between different types of objects which is the basis of type theory is already found in most of the mathematical and logical reasoning. All this, nevertheless, has not contributed yet to the acceptance of type theory in Artificial Intelligence [1].

A strong argument against the higher-order theories is that these theories are not complete. Secondly, from a technical point of view such as resolution and unification, the higher-order theories are more complicated. However, today there is a number of generalized completeness theorems for higher-order systems. Also for the last ten years many researchers have done excellent work on the unification problem for higher-order theories.

In essence, some of the technical difficulties encountered in type theory are a small price to pay for the power and naturalness of the expression that can be found in the higher order theories. Therefore, the present propositional system for intensional reasoning about knowledge has two reasonable extensions, quantification theory and resolution-based procedures for logical systems that involve epistemic operators. And the difficult task of developing a full and a comprehensive theory of reasoning about knowledge promises no lack of interesting arguments.

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