29 Mar 2001, 4:00 pm - 6:00 pm

Adjustment Method of the Hysteresis Damping for Multiple Shear Spring Model

Osamu Ozutsumi
Institute of Japanese Union of Scientists @ Engineers, Japan

Susumu Iai
Ministry of Transport, Japan

ADJUSTMENT METHOD OF THE HYSTERESIS DAMPING FOR MULTIPLE SHEAR SPRING MODEL

Osamu Ozutsumi
Inst. of Japanese Union of Scientists & Engineers
Sendagaya 5-10-9, Shibuya-ku, Tokyo 151-0051, Japan

Susumu Iai
Port and Harbour Research Institute, Ministry of Transport, Japan
Nagase 3-1-1, Yokosuka 239-0826, Japan

ABSTRACT

In simulating the behavior of sandy soil under the cyclic loading condition using the multiple shear spring model, it is necessary to adjust the damping constant of the model. We describe a method for adjusting the constant in this paper. If you adopt the conventional Masing rule to decide the unloading curve of each spring, the entire damping constant of this model, which is superposition of those of all springs, would become larger than that measured in the laboratory for large strain level. Though the damping constant of each spring is controllable by amending the Masing rule, there is no obvious way to decide the constant of each spring. In order to reproduce the actual damping constant, we expressed the damping constant of each spring as a function of displacement at which unloading of the spring has started, and determined the coefficients of the expression from the actual damping constant. So we can amend the Masing rule for each spring so as to realize the damping constant for each spring. The method is adopted to the effective stress analysis program FLIP developed by the authors. In this paper we explain the method and show the results of the computer simulations by FLIP program.

INTRODUCTION

Characterization of cyclic behavior of sandy soil is important for evaluating deformation in soil structures during earthquakes. Towhata et al. [1985] proposed a multiple shear spring model to represent the relationships between shear stress and shear strain for sandy soil under the plain strain condition. As shown in Fig.1, this model is represented by a movable point located within the circular fixed boundary defined in shear stress/strain space and connected to the boundary with an infinite number of virtual springs. Each spring corresponds to a virtual simple shear mechanism having a various orientation. The relationship between force and displacement of each spring follows the hyperbolic type load displacement relationship. The displacement of the movable point from the center represents the mobilized shear strain and the resultant of forces acting on the point represents the shear stress induced in the soil. This model automatically takes into account anisotropic behavior of soil caused by principal stress axis rotation.

During earthquakes, soil undergoes cyclic loading, and the shear stress - shear strain relationship of soil traces a hysteresis loop. The area enclosed with the loop corresponds to the hysteresis damping. In order to simulate soil behavior, we must set the shear stress - strain relationship for unloading or reloading process. For idealizing the soil behavior under drained condition, we often idealize these hysteretic behavior based on the Masing rule. The unloading and reloading curves with the Masing rule, if combined with hyperbolic stress strain relationship, reproduce larger damping constant than that typically measured in the laboratory in large strain level. For a commonly adopted one dimensional simple shear type model, Ishihara et al.[1985] proposed a method for adjusting the damping constant by amending the Masing rule.

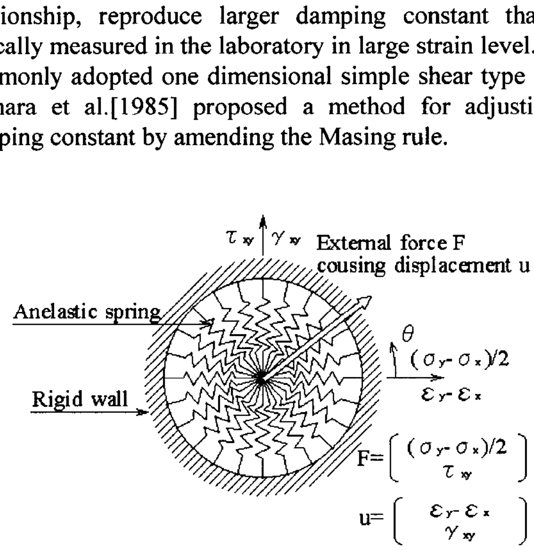

Fig.1 Multiple shear spring model

A problem remains for the multiple shear spring model. If we apply the Masing rule to each spring composing the multiple shear spring model, the model would also overestimate the damping constant of the soil in large strain level. Iai et al. [1990,a] proposed a method for determining the damping constant for each spring in order to adjust the damping constant of the multiple shear spring model as a whole. The damping constant of each spring was defined as a function of its displacement, which corresponds to the reverse point on the backbone curve. They used the method proposed by Ishihara et al.[1985] for adjusting the damping constant of each spring. Iai et al.[1990,b] had installed this procedure in the computer program called FLIP which estimates damage to the soil structures caused by liquefaction based on the effective stress method.

In this paper, we will explain the method for determining the damping constant of each spring from the damping constant defined for overall stress-strain relationships of soils. We will show the shear stress – shear strain relationships of sandy soil under the drained condition obtained from the simulations performed using FLIP program to demonstrate the validity of the proposed method.

MODELLING OF SHEAR MECHANISM

We trace the formulation described by Iai et al.[1990,a] for shear mechanism of sandy soil under the plain strain condition using the multiple shear spring model.

Multiple Shear Spring Model

$\gamma(\theta)$ stands for the displacement of the spring located in the θ-direction (see Fig.1). $F(\gamma)$ stands for the force per unit angle of the spring. The relationship between the spring displacement and the strain components in the soil is given by Eq.(1) and the relationships between the spring forces and the shear stress components in the soil are given by Eq.(2) and Eq.(3).

$$\gamma(\theta) = (\varepsilon_y-\varepsilon_x)\cos\theta + \gamma_{xy} \sin\theta \quad (1)$$

$$(\sigma_y-\sigma_x)/2 = \int F(\gamma(\theta))\cos\theta \, d\theta \quad \text{(the integral range : } 0\rightarrow 2\pi) \quad (2)$$

$$\tau_{xy} = \int F(\gamma(\theta))\sin\theta \, d\theta \quad \text{(the integral range : } 0\rightarrow 2\pi) \quad (3)$$

$(\varepsilon_y-\varepsilon_x)$ and γ_{xy} in Eq.(1) are strain difference and shear strain respectively. $(\sigma_y-\sigma_x)/2$ in Eq.(2) and τ_{xy} in Eq.(3) are stress difference and shear stress respectively.

Loading

We postulate that the relationship between the force $F(\gamma)$ and the displacement γ of the spring conforms to the hyperbolic model under the loading condition. That is, the relationship between the spring force v (= $F(\gamma)/F_m$) which is normalized by the strength of the spring (F_m) and the spring displacement u (= γ/γ_m) which is normalized by the reference strain of the spring (γ_m) is given by Eq.(4).

$$v = u / (1+|u|) \quad (4)$$

This function is shown as a backbone curve in Fig.2. The strength (F_m) and the reference strain (γ_m) are given by Eq.(5) and Eq.(6) respectively.

$$F_m = (1/4)\tau_{max} \quad (5)$$

$$\gamma_m = (F_m/G_0)\pi \quad (6)$$

τ_{max} and G_0 in these equations stand for the shear strength and the initial shear modulus of the soil respectively.

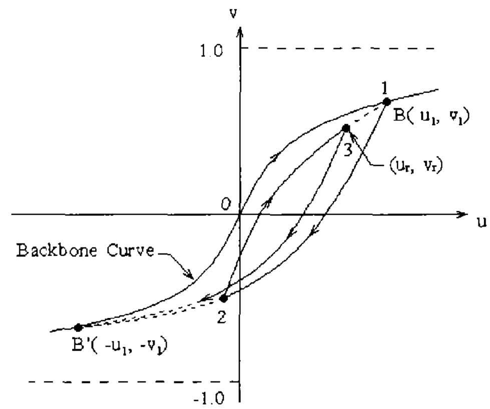

Fig.2 Schematic figure of loading/unloading in the normalized plane

Unloading and Reloading

We suppose that a spring is unloaded at a point (u_l,v_l) on the backbone curve as shown in Fig.2. The coordinate (u_l,v_l) denotes the reversal point from the backbone curve in the normalized plane.

It is assumed that, until getting back to the backbone curve, the relationship between the force and the displacement of the spring is described as a hyperbolic curve, the neutral point of which is located on the last reversal point (u_r,v_r) (these points are designated as 1, 2 or 3 in Fig.2) and which cross the point (u_l,v_l) or $(-u_l,-v_l)$ in case of reloading or unloading respectively. These hyperbolic curves are given by Eq.(7):

$$(v'-v_r')/2\delta = \{(u'-u_r')/2\delta\} / \{1+|(u'-u_r')/2\delta|\} \quad (7)$$

in which u', v', etc. are transformed variables from u, v, etc. using Eq.(8) or Eq.(9) in order to adjust the magnitude of the hysteresis damping.

$$u' = u / \xi(u_l) \quad (8)$$

$$v' = v / \eta(u_l) \quad (9)$$

δ in Eq.(7) is given by Eq.(10) which is derived from a condition that the hyperbolic curve given by Eq.(7) crosses the point $(-u_l',-v_l')$ or (u_l',v_l').

$$\delta = (1/2)\, |(\pm u_l'-u_r')|\cdot|(\pm v_l'-v_r')| \,/\, |(\pm u_l'-u_r')-(\pm v_l'-v_r')| \quad (10)$$

In the above equation, the double signs become minus in case of unloading, and plus in case of reloading. In the first unloading process from the backbone curve, we want to make the Eq.(7) have the same form as the unloading curve based on the Masing rule. This condition derives the expression of $\eta(u_1)$ as follows.

$$\eta(u_1) = (\xi(u_1)+|u_1|) / (1+|u_1|) \quad (11)$$

We can decide the value of $\xi(u_1)$ solving the next equation numerically.

$$D(u_1/\xi) = h(u_1) \quad (12)$$

The unloading curve from the backbone curve conforms to the Masing rule on the u'-v' plane as stated previously, so its damping constant on this plane is given by the function $D(u_1')$, the expression of which is shown in Eq.(13) (Ishihara et al.[1985]).

$$D(u) = (4/\pi)(1+1/|u|)\{1-(1/|u|)\ln(1+|u|)\} - (2/\pi) \quad (13)$$

The function $h(u_1)$ in Eq.(12) stands for a damping constant of the spring. We could determine the function $h(u_1)$ in order that the multiple shear spring model as a whole shows the given damping constant. We describe the method for determining $h(u_1)$ in the next chapter.

DAMPING CONSTANT OF EACH SPRING

Damping Constant of Multiple Shear Spring Model

As stated previously, the damping constant of each spring of the multiple shear spring model is designated as $h(u_1)$.

We think a hypothetical stress path of the soil corresponding to a simple shear test under the drained condition. That is, first, we consolidate the soil isotropically, second, load simple shear stress τ_{xy} on the soil giving shear strain γ_{xy} (= $U\gamma_m$), third, unload the stress until getting to the reverse point on the backbone curve, and finally reload until getting back to the former state. The variable U is equal to u_1 of the spring placed in the direction of angle 90 degree.

The displacement of the spring located in the θ-direction at the beginnings of the unloading is given by Eq.(14).

$$U\sin\theta\, \gamma_m \quad (14)$$

At this moment, the force per angle dθ of the spring is given by Eq(15):

$$\{U\sin\theta / (1+|U\sin\theta|)\}F_m d\theta \quad (15)$$

because each spring stays on the backbone curve. The strain energy per angle dθ is designated by Eq.(16).

$$w(U,\theta)d\theta = e(U,\theta)\, \gamma_m F_m\, d\theta \quad (16)$$

The function e(U,θ) in the above equation is defined as follows.

$$e(U,\theta) = ½(U\sin\theta)^2 / (1+|U\sin\theta|) \quad (17)$$

The damping energy Δwdθ of a hysteresis loop is given by Eq.(18) based on the definition of the damping constant h(U).

$$\Delta w(U,\theta)d\theta = 4\pi w(U,\theta)h(U\sin\theta)d\theta \quad (18)$$

Therefore the damping constant of the multiple shear spring model is given by Eq.(19) or (20). The range of the integrations in these equations is from 0 to 2π.

$$H(U) = (1/4\pi) \int\Delta w(U,\theta)d\theta / \int w(U,\theta)d\theta \quad (19)$$

$$= \int e(U,\theta)\, h(U\sin\theta)d\theta / \int e(U,\theta)d\theta \quad (20)$$

This damping constant is compatible with the damping constant calculated on the corresponding stress-strain space (Iai et al.[1990,a]).

Damping Constant of Each Spring

We assume that the damping constant of each spring could be expressed by Eq.(21).

$$h(U) = \sum_k E_k(|U|/\tau_k) / (1+|U|/\tau_k) \quad (21)$$

Each term of the right side of the above equation is a function whose value increases approximately from zero to E_k as the normalized displacement |U| varies from $\tau_k/10$ to $10\tau_k$. Therefore, if we arrange the values of τ_k in the suitable interval, the unknown function h(U) could be expressed by Eq.(21), even if it has any form. We should determine the values of E_k.

We substitute Eq.(21) for h(U) in the Eq.(20), then the damping constant of the multiple shear spring model is given by Eq.(22).

$$H(U) = \sum_k \varphi_k(U)\, E_k \quad (22)$$

The function $\varphi_k(U)$ in the above equation is given by Eq.(23).

$$\varphi_k(U) = \int e(U,\theta)(|U\sin\theta|/\tau_k)/(1+|U\sin\theta|/\tau_k)d\theta / \int e(U,\theta)d\theta \quad (23)$$

On the other hand, we let H'(U) denote the actual damping constant measured in the laboratory test. For example, H'(U) represents the hyperbolic relation as proposed by Hardin et al.[1972], and this damping constant only depends on the shear strain at the unloading point as shown in Eq.(24).

$$H'(U) = H_{max}(\gamma/\gamma_{ref}) / \{1+(\gamma/\gamma_{ref})\}$$

$$= H_{max}|\pi U/4| / (1+|\pi U/4|) \quad (24)$$

The H_{max} is the maximum value of the damping constant, and $\gamma_{ref} \equiv \tau_{max}/G_0$.

We decide, finally, the values of E_k using the method of least square. That is, we could find the values of E_k which make the sum of square I shown in Eq.(25) the smallest.

$$I = \sum_m (H(U_m) - H'(U_m))^2 \quad (25)$$

We can obtain Eq.(26) from Eq.(22), because $\lim \varphi_k(U) = 1$ ($U\rightarrow\pm\infty$).

$$\lim H(U)\ (U\rightarrow\pm\infty) = \sum_k E_k \quad (26)$$

On the other hand, we can obtain Eq.(27) from Eq.(24).

$$\lim H'(U)\ (U\rightarrow\pm\infty) = H_{max} \quad (27)$$

Comparing Eq.(26) with Eq.(27), we get a following condition about parameter E_k.

$\sum_k E_k = H_{max}$ (28)

This is a constraint condition to decide the values of E_k with the method of least square.

When the function h(U) given in Eq.(21) has only one term, namely :

$h(U) = E_1(|U|/\tau_1) / (1+|U|/\tau_1)$ (29)

we can set $E_1 = H_{max}$ by Eq.(28). So, we should adjust τ_1 so that the sum of square I of Eq.(25) becomes the smallest.

Approximation of H' (U) by This Method

We will show the validity of the above-mentioned method using a model of sand whose initial shear modulus (G_0) is 85000kPa, shear strength (τ_f) is 63.0kPa, and reference strain (γ_{ref}) is 7.4×10^{-4}.

We expressed the damping characteristics of sand, giving damping constant as shown in Eq.(21) or Eq.(29), which is a special case of Eq.(21), to each spring. Although the springs of the multiple shear spring model are continuously distributed in the θ-direction, we replaced an infinite number of springs with 30 springs per a quarter of circle.

Typical damping characteristics of sandy soil obtained experimentally (Zen et al.[1987]) is shown with dashed line in Fig.3a. We consider this curve to be a target function H', and show the approximated curves in the same figure. In case of polynomial approximation (Eq.(21)), we use 10 terms and choose the values of τ_k as follows.

$\tau_k = 0.001\times10^{0.5(k-1)}$ (k= 1, ···, 10) (30)

Though test value is reproduced well as for the polynomial approximation, the monomial approximation misfits the damping constants at lower strain level by Fig.3a.

On the other hand, if we pick the hyperbolic relation by Hardin et al.[1972] shown in Eq.(24) for the target damping function H', both the polynomial and the monomial approximation reproduce the H' well (see Fig.3b).

A hyperbolic type function shown in Eq.(24) is adopted as a target function H' in the FLIP program. Though the actual sand displays a few percent of damping constant at lower strain level as shown in Fig.3a, the hyperbolic curve can't reproduce these damping constant. We think that this characteristics in such a low strain level can be reproduced by Rayleigh damping given separately.

RESULTS OF SIMULATIONS

We carried out computer simulation using the FLIP program to examine the adequacy for the proposed method. We used the same characteristics of soil for the simulation as mentioned above, i.e., G_0 = 85000kPa, τ_f = 63.0kPa, and $\gamma_{ref} = 7.4\times10^{-4}$.

First, a finite element for sand was consolidated isotropically or anisotropically under the drained condition. Then we examined the stress-strain relation of the soil element during monotonic or cyclic simple shear under the drained condition. At this stage, we loaded shear stress τ_{xy} onto the element under the restraint condition of constant vertical normal stress (σ_y) and constant horizontal strain (ε_x) (see Fig.4). We also disregarded any effect of dilatancy. And we supposed, for sake of simplicity, that τ_{max} and G_0 did not depend on the effective confining pressure. We adopted the Eq.(24) as a target function H', and used the polynomial approximation (Eq.(21)), but we could have the same result using the monomial approximation (Eq.(29)). We replaced an infinite number of springs with 6 springs per a quarter of circle for this simulation..

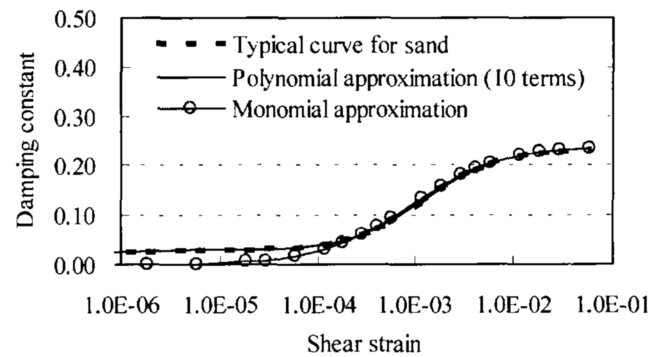

Fig.3a Typical damping characteristics of sand obtained empirically and its approximated curves

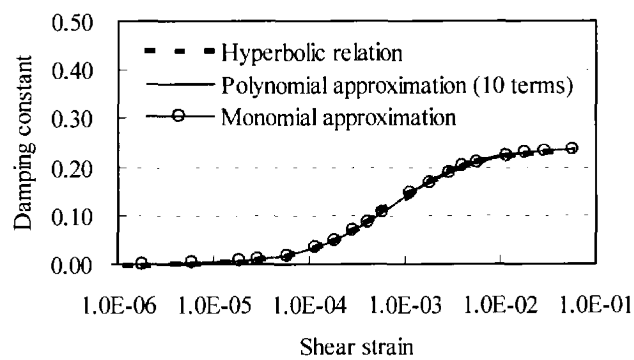

Fig.3b Hyperbolic type damping characteristics by Hardin et al.[1972] and its approximated curves

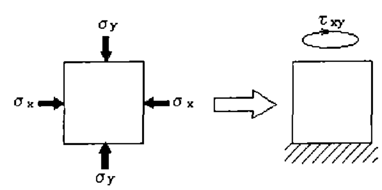

Fig.4 Conceptual diagram of simulation for simple shear test

Cyclic Simple Shear after Isotropic Consolidation

After isotropic consolidation with confining pressure 98kPa, we applied shear stress oscillating sinusoidally with amplitude 55kPa. The relationships between shear stress (τ_{xy}) and shear strain (γ_{xy}) are shown in Fig.5. The figure has four graphs. The first three of them correspond to the case of H_{max} = 0.35, 0.25, 0.15 respectively. The last figure corresponds to the case where the unloading curve of each spring conforms to the Masing rule.

We confirmed that each area surrounded by the hysteresis loop is just equal to the prescribed values, and the smaller the H_{max}, the smaller the tangential modulus ($\equiv \Delta\tau_{xy}/\Delta\gamma_{xy}$) at the unloading point. The area corresponds to the last figure based on the Masing rule is fairly large.

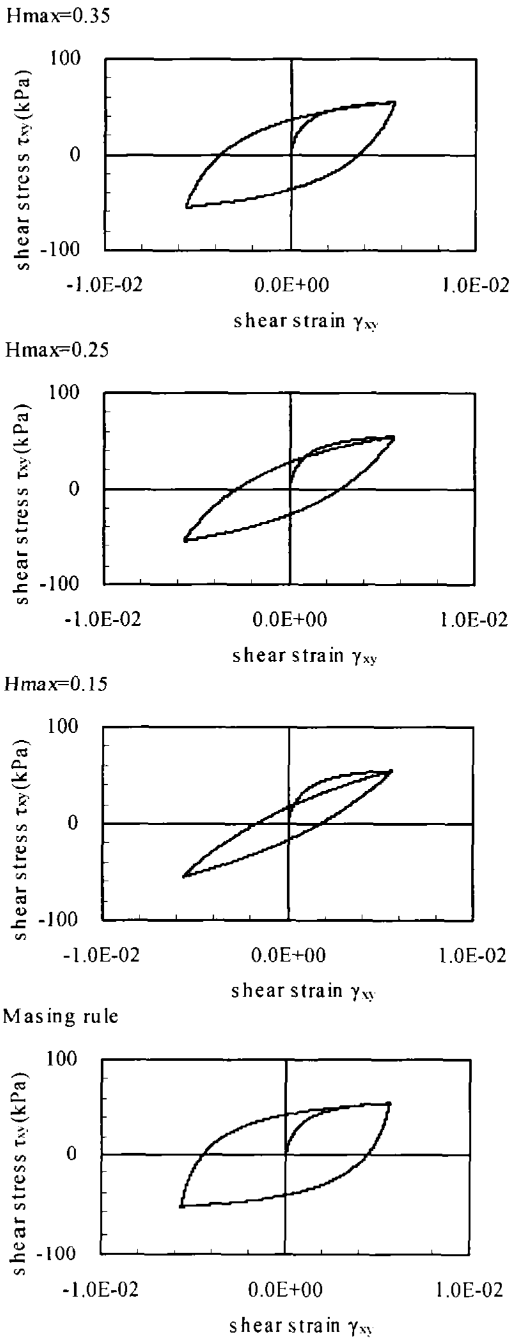

Fig.5 Shear stress – shear strain relationships (computed): Cyclic simple shear after isotropic consolidation

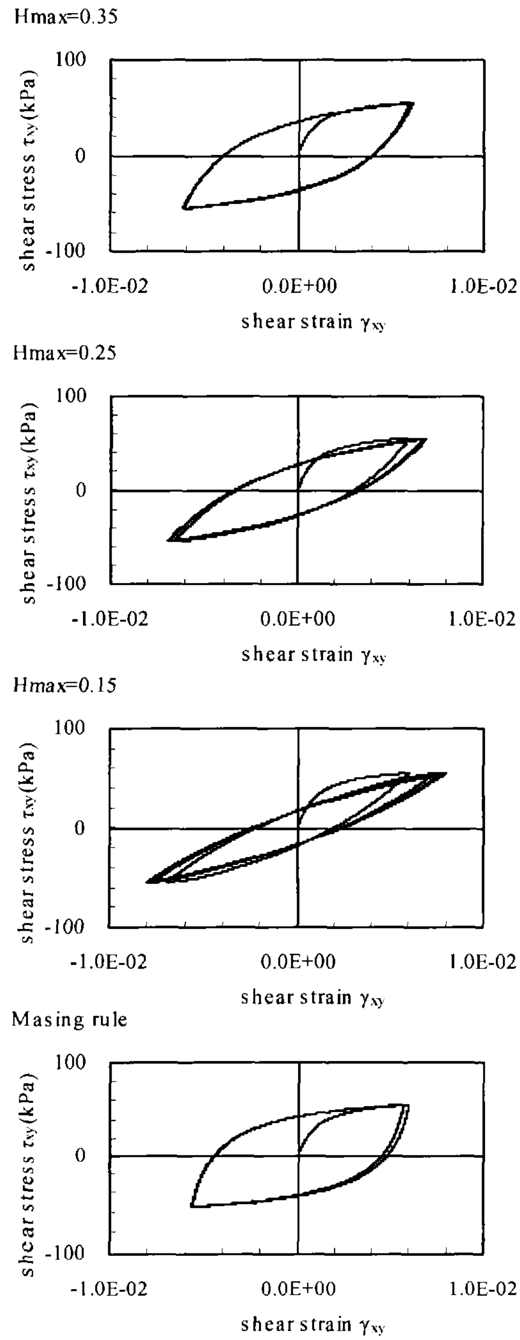

Fig.6 Shear stress – shear strain relationships (computed): Cyclic simple shear after anisotropic consolidation

Cyclic Simple Shear after Anisotropic Consolidation

The conditions for these simulations are the same as the above cases unless stress state after consolidation. We anisotropically consolidated the soil element with K_0 ($\equiv \sigma_{x0}/\sigma_{y0}$) = 0.5. Four graphs in Fig.6 are shown in the same way as Fig.5.

Each graph of Fig.6 shows the same tendency as those in Fig.5, and it is understood that the damping constant could be controlled in the same way as the case of isotropic consolidation. According to Fig.6, shear strain becomes a little large during cyclic loading and this tendency is as remarkable as H_{max} is small. The damping constant corresponds to the loop after softening is equal to the prescribed value. We suppose that this softening depends on increase in the normal horizontal stress (σ_x) due to the decrease in the shear modulus.

Initial Shear Modulus after Consolidation

After the consolidation with various K_0 values, we gave the shear strain (γ_{xy}) of 10^{-6} to the soil element by the strain control method, and took the shear stress (τ_{xy}) at that time. Then, we got the initial shear modulus (G_0) as ratio of the stress to the strain. These modulus are shown in table 1. As shown in table 1, the initial shear modulus decreases in case of the anisotropic consolidation, and the stronger anisotropy, the lager decrease in G_0. And, the degree of the decrease in G_0 is as large as H_{max} is small, among the same K_0.

After the anisotropic consolidation, some displacement occurs to each spring of the multiple shear spring model unless springs located in the direction of ±90 degree. If we apply shear stress τ_{xy} to the soil element, which is in that condition, the tangential stiffness of each spring is smaller than that of undeformed spring. Specially, as for the spring getting unloaded at that time, tangential stiffness becomes small, as H_{max} is small. So the initial shear modulus becomes small, as H_{max} is small.

In case of analyzing a horizontally layered system, we first apply gravity to the system, and K_0 of the sandy soil usually becomes about 0.5. In such a case, especially with small H_{max}, we should pay attention to the decrease in the initial shear modulus. If we consider that the initial shear stress or the shear strength depends on the confining pressure, we have to take account of another effects by this dependency, though it isn't dealt with here.

Table 1 Initial shear modulus G_0 (unit:kPa)

H_{max} =	0.15	0.25	Masing rule
K_0=2.0	54600	59700	61400
1.5	67600	70600	70700
1.0	84900	84900	84900
0.5	54600	59700	61400
0.3	30900	37000	49400

CONCLUSION

If we expressed the relationship between shear stress and shear strain of sandy soil based on the multiple shear spring model, it is necessary to adjust properly the damping constant of each spring of the model in order to simulate soil behavior during earthquakes. We proposed the method for adjusting the damping constant of each spring. In this method, we supposed that the damping constant of each spring is a function of the displacement at the unloading point of the spring, and could be expressed by a polynomial composed of functions, which is similar to the hyperbolic type function. The method of least square was adopted to determine unknown coefficient of the functions.

Then, we showed that the damping constant of the multiple shear spring model could be controlled by this method, through numerical simulations by the FLIP program, which is equipped with this method. We also demonstrated decrease in initial shear modulus after anisotropically consolidation, and the decrease in shear modulus is as remarkable as H_{max} is small. So, we should pay attention to this phenomenon when we adopt this method.

REFERENCES

Hardin, B.O. and V. P. Drnevich. [1977]. "Shear modulus and dampimg of soils : design equation and curves", Journal of Soil Mechanics and Foundation Division, ASCE, Vol.98, No.SM7, pp.667-692.

Iai, S., Y. Matsunaga and T. Kameoka. [1990,a] "Parameter Identification for a cyclic mobility model", Report of Port and Harbour Research Institute, Vol.29, No.4, pp. 57-83

Iai, S., Y. Matsunaga and T. Kameoka. [1990,b] "Strain space plasticity model for cyclic mobility", Report of Port and Harbour Research Institute, Vol.29, No.4, pp. 27-56

Ishihara, K., N. Yoshida and S. Tsujino. [1985]. "Modelling of stress-strain relations of soils in cyclic loading", Proceedings of 5th International Conference on Numerical Methods in Geomechanics, Nagoya, Vol.1, pp. 373-380.

Towhata, I. and K. Ishihara. [1985]. "Modelling soil behavior under principal stress axes rotation", Proceedings of 5th International Conference on Numerical Methods in Geomechanics, Nagoya, Vol.1, pp. 523-530.

Zen, K., H. Yamazaki and Y. Umehara. [1987]. "Experimental study on shear modulus and damping ratio of natural deposits for seismic response analysis", Report of Port and Harbour Research Institute, Vol.26, No.1, pp. 71-113.