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A Review of the Influence of Initial State Shear (K_α) and Confining Stress (K_σ) On Failure Mechanisms and Earthquake Liquefaction of Soils

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A REVIEW OF THE INFLUENCE OF INITIAL STATIC SHEAR (K_α) AND CONFINING STRESS (K_σ) ON FAILURE MECHANISMS AND EARTHQUAKE LIQUEFACTION OF SOILS

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Abstract:

Geotechnical engineering profession has continued to use the in situ measurement based empirical procedure to perform soil liquefaction analysis. This method was initially developed based on past earthquake performance of level ground sites. For sloping ground and larger depths or confining stress, the method requires appropriate corrections such as K_α for initial static shear and K_σ for confining stress. The recommended correction factors in the current state of practice have been presented by NCEER (National Center for Earthquake Engineering and Research). These recommendations, however, do not distinguish between the distinct shear behavior of soils on the opposite sides of the critical state line. These factors can significantly influence the results and alter the final conclusion and outcome of an analysis. This paper reviews the existing soil liquefaction research data focusing on these factors based on the critical state soil mechanics framework. It is shown that the undrained response of soils with stable yielding during both monotonic and cyclic loading can be correlated with the distance of the initial state from the critical state line. K_α and K_σ are shown to correlate well with this distance. Constant cyclic resistance ratio lines are found to lie parallel to lines of constant initial state. Furthermore, K_σ variation is found to be material specific.

INTRODUCTION

Liquefaction and post-liquefaction behavior of soils is of considerable importance to practicing geotechnical engineers and researchers. A number of different methods have been developed in the past few years for the analysis of liquefaction and its consequences. These include methods based on the empirical correlation of in situ measurements (Seed et al. 1984), the steady state methodology (Poulos et al. 1985), and numerical analyses (Finn 1998). While significant progress is being made on each of these methods, the engineering profession will most likely continue to use the simplified methods based on empirical correlations.

The correlation based liquefaction analysis procedure was initially developed for simple level ground conditions to estimate potential for liquefaction based on past earthquake performance (Seed et al. 1984). It does not, however, account for many key aspects such as the effects of initial state of stress and stress history of the soil on liquefaction performance.

Current liquefaction analysis procedures account for these effects through the use of a number of correction factors. The correction factors for the effect of overburden pressure, K_σ, and initial static shear stress, K_α, have been widely researched in recent years in this regard (Seed and Harder 1990). The recommended correction factors in the current state-of-practice have been presented in a report by the National Center for Earthquake Engineering and Research (NCEER 1996/97). Existing recommendations, however, do not distinguish between the distinct shear behavior of soils on opposite sides of the critical state line (Schofield and Wroth 1968). Such opposing shear behaviors have a fundamental bearing on the liquefaction potential of soils. Use of correlations that do not account for the distinguishing behaviors on either side of the critical state line would therefore lead to erroneous conclusions. This would result either in failures or unwarranted expenditures to the public.

This paper reviews the underlying mechanisms of the influence of confining stress and initial static shear on soil liquefaction based on critical state soil mechanics framework. Its objective is to highlight the mechanisms governing the correction factors K_σ and K_α based on critical state soil mechanics principles.

BACKGROUND

Many research workshops (NCEER 1996-97; Lade and Yamamuro 1999) and individual researchers (e.g. Ishihara 1993, Marcuson 1996) have presented comprehensive discussions of the recent advances in liquefaction research. These studies have highlighted the advances in numerical as well as empirical approaches towards the solution of the liquefaction problem.

Significant progress has been made on the development of numerical methods for analysis of liquefaction and its consequences (Finn 1998; Manzari and Dafalias 1997). However, pore pressure generation and shear behavior of soils is complex and the validity of such effective stress analysis has remained difficult to be demonstrated in actual field conditions. In view of this, the engineering profession has and will most likely continue to use the empirical correlations based on in situ measurements to assess liquefaction potential and seismic response.

In the penetration-based methods liquefaction is defined either as 100% increase in pore pressure or as a predetermined level of plastic strain accumulation. A factor of safety against liquefaction, FSL, is defined as:

$$FSL = CRR/CSR \qquad (1)$$

where, CSR is the cyclic stress ratio caused by the design earthquake, and CRR = τ/σ'_0 is the cyclic resistance ratio in which τ is the applied shear stress and σ'_0 is the effective overburden stress. CSR is routinely estimated reasonably accurately using computer programs such as "SHAKE". The cyclic resistance ratio at depth, CRR_σ, is estimated indirectly from:

$$CRR_\sigma = CRR_1 \times K_\sigma \times K_\alpha \times K_m \qquad (2)$$

where CRR_1 is the reference cyclic resistance ratio at confining stress ratio of 100kPa; K_σ and K_α are the correction factors defined earlier and K_m is a correction factor for the Richter magnitude of the design earthquake.

CRR_1 is determined based on $(N_1)_{60}$ from the Seed liquefaction chart (NCEER 1996/1997) developed using past historical data from level ground and shallow conditions up to an effecting confining stress of 100kPa for a reference Richter magnitude of 7.5.

Based on field and laboratory test data on "undisturbed" sand samples obtained from frozen ground under a dam, Pillai and Byrne (1994) developed a K_σ correction curve for fine sands (Fig. 1). The curve recommended by the NCEER (1996/97) for fine sands shown on the same figure is consistent with that developed by Pillai and Byrne (1994). NCEER further recommends that the same curve to be valid for a broad spectrum of materials including gravels and silts. This, however, may not be true given their varying mechanical properties.

Some researchers (Vaid and Thomas 1995 and Vaid and Sivathayalan 1999) have suggested that sands with lower densities (loose sands) generally produced higher K_σ values and different curves exist for different densities. We believe that this is not consistent with the definition of K_σ. By definition, K_σ is a comparison of the CRR_σ at a particular confining stress to the reference CRR_1 at 100kPa at a constant density. K_σ therefore, inherently includes density in its definition and should not vary again with this parameter.

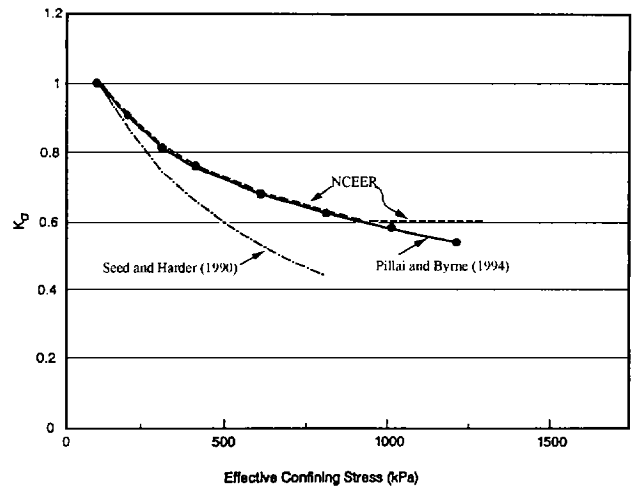

Fig. 1. K_σ variation with confining stress.

Pillai (1991) has suggested that K_α depends on the initial state parameter, which defines the distance of initial state of the soil to that of critical state in an e-ln p′ plot. Based on field and laboratory tests on undisturbed sand samples obtained after freezing the ground under a dam, Pillai et al. (1995), observed that K_α was affected by stress rotation; and that the induced pore pressure ratio decreased with increase in static shear (Fig. 2). This in turn resulted in increased stiffness of the liquefied soil and inhibited the potential for large deformation (Byrne et al. 1994). These mechanisms are interdependent and they could not only influence the triggering potential but also the post-liquefaction deformation.

K_σ and K_α factors are dependent on the stress-deformation characteristics, the material type including "fines content" and mechanical properties of the soil. Despite their strong interdependency on these factors, the existing correlations are derived in isolation and related to physical properties such as "relative density" or "percentage fines" of the soil. Furthermore, existing empirical correlations do not make any distinction between the behavior of soils on the loose side of critical state where pore pressures increase upon shearing and those on the dense side of critical where pore pressures tend to decrease. Correlations that do not distinguish between these opposite behaviors would lead to erroneous conclusions.

THE CONCEPT

Yielding of Sand and State Boundary Surface

Roscoe, Schofield and Wroth (1958) quote experimental evidence that the ultimate state of any soil specimen during a continuous remolding and shear flow will lie on a critical state line (Fig. 3) with equation:

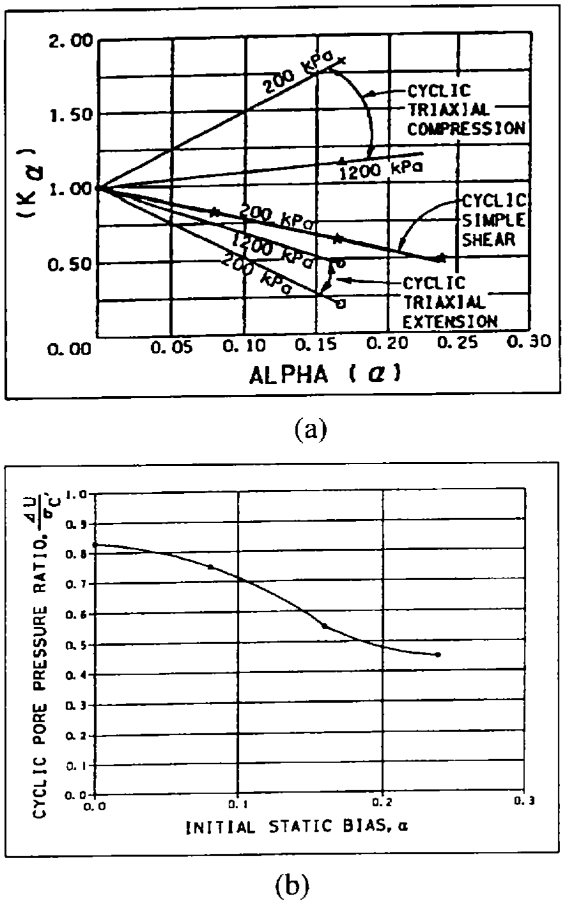

Fig. 2. Initial state, K_α, and pore pressure ratio (Pillai et al.1995).

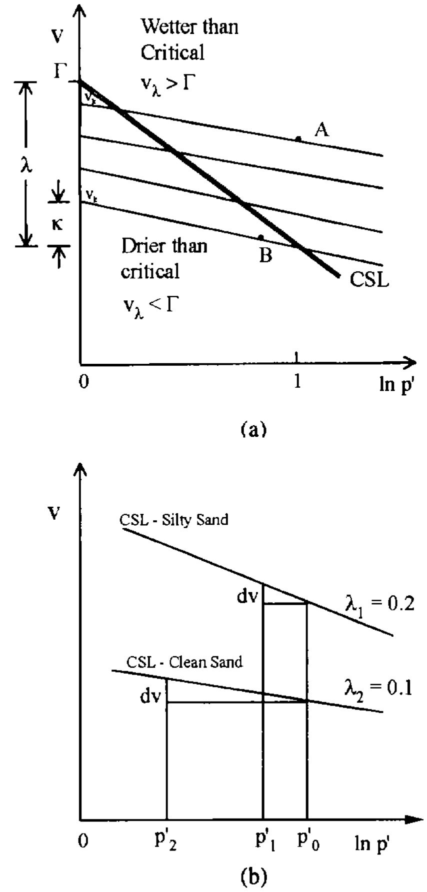

Fig. 3. State diagrams (a) Critical state parameters; (b) Critical state lines for sand.

$$\Gamma = v + \lambda \ln p' = v_\kappa + (\lambda - \kappa)\ln p' \quad (3)$$

where v = 1 + e is the specific mean volume, $p' = (\sigma'_1+\sigma'_2+\sigma'_3)/3$ is the mean effective compressive stress and $q = (\sigma'_1-\sigma'_2)$ is the deviatoric stress. λ is the slope of the critical state line and κ is the slope of the elastic swelling line in the v – ln p′ diagram. The critical state line can be seen as one of a family of parallel lines with equation $(v + \lambda \ln p') = v_\lambda$. Soils with $v_\lambda > \Gamma$ are said to be looser than critical and those with $v_\lambda < \Gamma$ are denser than critical. Loose soils tend to contract upon shearing whereas dense soils tend to dilate. The existence of the critical state line for sands has been confirmed by subsequent studies (Thurairajah 1961; Wroth and Bassett 1965). They further showed that ultimate critical state plane is inclined with a slope M in the q - p′ space with equation q = Mp'.

The critical state line and the concept of plastic stable yielding are central in the development of critical state soil mechanics framework (Schofield and Wroth 1968). The framework has become useful for understanding and modeling the shear and volumetric behavior of soils. In the critical state framework, the state of soils is defined in a 3-D, mean effective normal stress (p'), shear stress (q), and void ratio (e) or specific volume (v) space. Limits to stable states of soil behavior are defined by a state boundary surface in the 3-D, p'-q- v space. Plastic yielding (irrecoverable strains) or large strain accumulation occurs when soil states remain on the state boundary surface. The 2-D representations of the state boundary surface in the q-p′ space and v-ln p′ space are shown in Fig. 4. The shape of the state boundary surface depends on the soil type and its mechanical properties, M, λ, and κ. The original surface consisted of the Hvorslev Surface on the dilative domain and the Cam clay yield surface on the contractive domain (Schofield and Wroth 1968). Schofield (1980) has proposed "a tension crack surface" as the limiting surface on the dilative side at low effective mean stresses (Fig. 4).

Critical state soil mechanics therefore divides the soil behavior at limiting states into three distinct classes of failure. The limiting lines OA and OG (Fig. 4) indicate states limited by *fractures or fissures*; AB and GE indicate that Hvorslev's Coulomb *faults* on rupture planes will limit behavior; BD and ED indicate Cam-clay yield and sediment layer *folds*. All classes of observed mechanisms of large displacements in soils could be characterized as regimes belonging to fractures, faults, or folds.

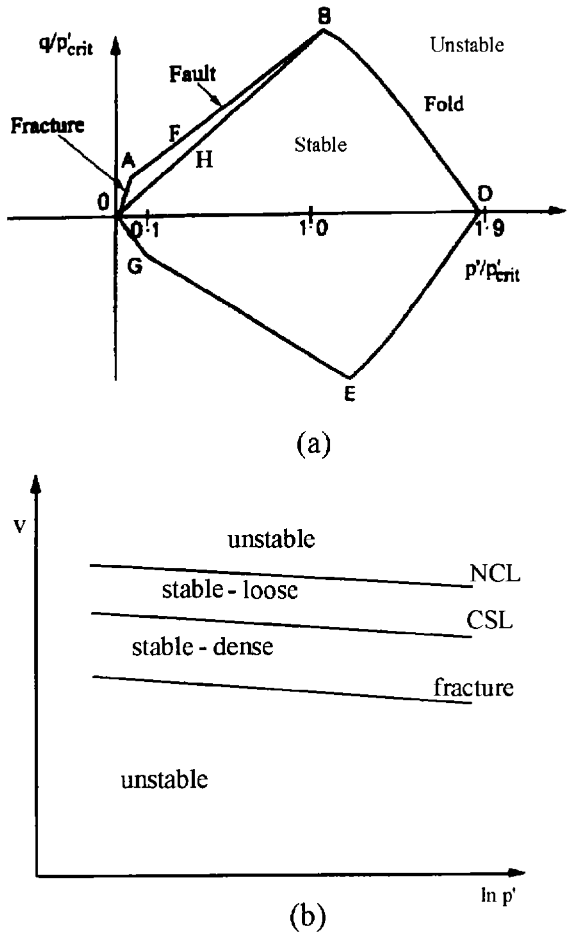

Fig. 4. Limiting states of soil behavior.

Soil states on the fracture surface result in the development of unstable fissures and cracks openings. Heavily overconsolidated clays and overcompacted sands at low confining stresses could reach this limiting state. The presence of high hydraulic gradients in the fracture region would result in the rapid disintegration of the soil mass into a clastic debris. Accordingly, Muhunthan and Schofield (2000) defined liquefaction as a class of failure that occurs when soil is on the dense side of critical state, near zero effective stress, and in the presence of high hydraulic gradients. This definition is closer to that used by Seed but with the additional requirement of the presence of high hydraulic gradient for liquefaction initiation.

Conditions similar to fracture on the dilative side can exist on the contractive domain but outside the normal consolidation line (Fig. 4). Soils in these states such as wind deposited loose sands, air pluviated or moist-tamped sands are susceptible to abrupt collapse upon shearing. Such collapse has been found to occur on the "collapse surface" at a stress ratio q/p' less than M (Sladen et al. 1985).

For sands, stable states of yielding occur only within a narrow band on both the looser and denser sides of critical state line (Fig. 4b). Very often to characterize the stable behavior of such sands under different confining stress or initial static shear researchers resort to moist-tamped and air-pluviated methods of specimen preparation. These samples, however, fall outside the stable yielding region. Their resulting structure may not have the same mechanical properties and very often result in large volumetric strain accumulation and liquefaction due to structural collapse. Such results may not be indicative of the behavior of soils in the stable region. Recent results on the shear behavior of in situ and laboratory moist-tamped specimens has clearly highlighted this phenomenon (Hoeg et al. 2000).

Significance of λ in Liquefaction

The factors that influence the undrained shear deformation response of sand in the field including liquefaction potential can be modeled and accurately predicted based on the knowledge of the initial state, mechanical properties (M, λ, κ) and the loading mode. The initial state would include the "initial static shear" (K_α), "initial confining stress" (K_σ), and void ratio or density. The mechanical properties, M, λ, κ will reflect the frictional, compressibility/swelling, physical gradation and "fines content". Even a relatively small variation in these mechanical properties is significant for sand shear behavior compared with clays. Fig. 3(b) illustrates a typical critical state line (CSL) of clean sand and silty sand. For clean sand, the slope of the CSL, λ, is generally less than that for silty sand. Therefore, the state boundary surface for the clean sand would be much smaller and flatter compared with that of the silty sand. It can be seen that for the same amount of plastic volumetric strain clean sand could generate pore pressure 100 times that for the silty sand which has a λ twice that of sand (Fig. 3b). Therefore, clean sand will have a much higher potential for liquefaction than silty sand.

The parameter λ alone plays the most significant role in the case of strength-deformation and liquefaction of soils. For sands λ is small and therefore they are susceptible for large volumetric strain accumulation and liquefaction. An increase in fines content in sands will increase λ and consequently decrease the potential for liquefaction. Past research has not appreciated the use of this key parameter and has preferred to use physical properties such as "relative density" and "fines content" in liquefaction studies. These physical properties are empirical and are no substitutes for the mechanical parameters to characterize strength-deformation and liquefaction of soils.

Since sand behavior is different on either side of the critical state line the distance of the initial state from critical state line can be a suitable measure to characterize it. For soils with stable yielding, this measure can be correlated with undrained shear deformation response during both monotonic and cyclic loading.

Initial State Parameter

The initial state parameter ψ_p is defined as the ratio of p_a'/p_c' where p_a' and p_c' are the initial mean effective stress, at critical state (Fig. 5). For ψ_p values larger than 1.0, the soil would exhibit contractive behavior, and for ψ_p values less than 1.0, the soil would exhibit dilative behavior. Soil states with ψ_p <0.1 (Schofield 1980) or >2.0 (outside NCL), would be unstable or quasi-stable and could collapse under small perturbation.

The difficulty of accurately establishing the CSL in the e-ln p′ space has been a principal roadblock for advancing the actual mechanisms of soil liquefaction and deformation based on stress history. A new method has been developed by the authors to establish the critical state line in the e-p space based on the energy principles that were used in the original cam clay model (Roscoe et al., 1963). This has been verified for undrained triaxial compression tests on isotropically consolidated sands (Raveendra 2000). Once the critical state line (CSL) in the e-ln p′ space is established for the loading mode, the stress history can be defined.

Strength Ratio and Initial State of Sand

During undrained shear deformation of sand in the stable yielding region, the strength ratio at any given strain level is given by the Cam clay model (Roscoe et al. 1963):

$$\frac{q}{p} = \frac{M}{\left(1 - \frac{\kappa}{\lambda}\right)} \ln \frac{p_e'}{p'} \qquad (8)$$

Therefore, a constant ln (p_e'/p') means that spacing lines in the e-ln p′ diagram are parallel to each other and to the critical state line. For constant ln (p_e'/p'), ψ_p values remain the same, and:

$$\frac{q}{p} = \frac{M}{\left(1 - \frac{\kappa}{\lambda}\right)} \text{ x } C = \text{constant} \qquad (9)$$

or q/p' ($= 2s_u/p'$) = constant for a given ψ_p and strain level along the state boundary surface. Therefore, constant strength ratio contours will be parallel in the e – ln p′ diagram within the stable states of yielding.

Similar relationships can be postulated to exist under cyclic loading. In other words, the normalized undrained shear strength q/p', cyclic resistance ratio q_{cyc}/p', and residual strength ratio q_r/p' are all direct functions of ψ_p, loading mode, and the shape of the state boundary surface.

Cyclic Resistance ratio with Confining Stress

Tumi (1983) has performed extensive cyclic simple shear tests on Tailing and Ottawa sands. The CRR data obtained can be mapped on e - log σ'_v space as shown in Fig. 6 for Tailing sand and Fig. 7 for Ottawa sand. From these figures it can be seen

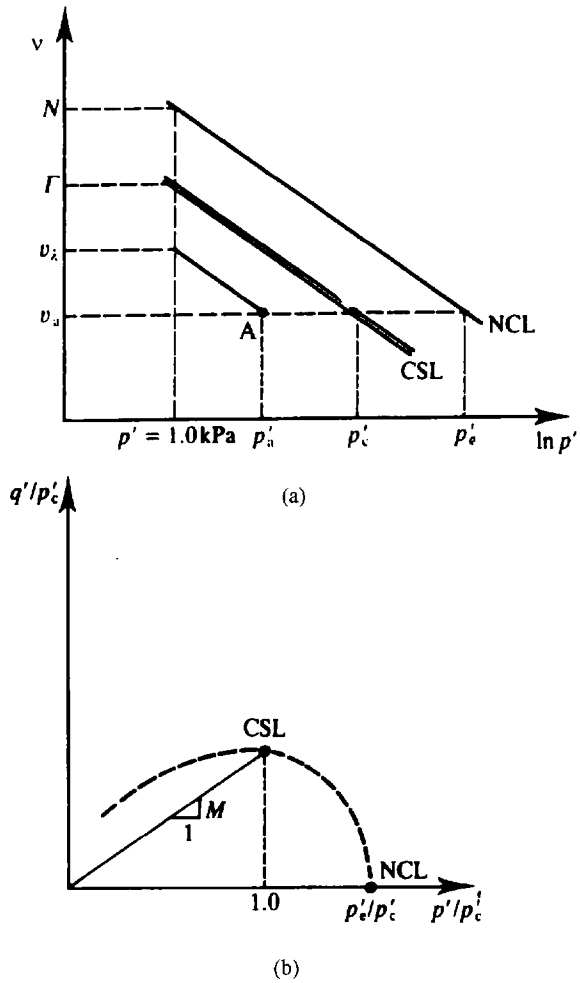

Fig. 5. Normalizing parameters for shear tests (Atkinson 1993).

that constant CRR values lie on parallel lines of constant initial state ψ_p. This is consistent with the previous observations and the postulation by Pillai (1991) that cyclic resistance ratios would be constant for constant ψ_p values and contours of constant CRR would be parallel to the critical state line.

The data of Tumi (1983) can also be used to derive K_σ values by comparing the CRR for a given state to that at 100 kPa and at constant void ratio. A unique curve was derived for various confining stresses (depths). The K_σ curves derived for Ottawa sand and Tailing sand are shown on Figs. 6 and 7 respectively. It can be seen that K_σ variation with confining stress (depth) is different despite the fact that they are both fine sands confirming our previous observation that it is governed by the mechanical property λ of each soil.

Cyclic Resistance Ratio with Initial Static Shear

Early studies on sands (Seed 1983) indicated that the CRR could increase with increase in initial static bias. But later studies on loose contractive sands (Rollins and Seed 1990) indicated that CRR could reduce significantly with increase in

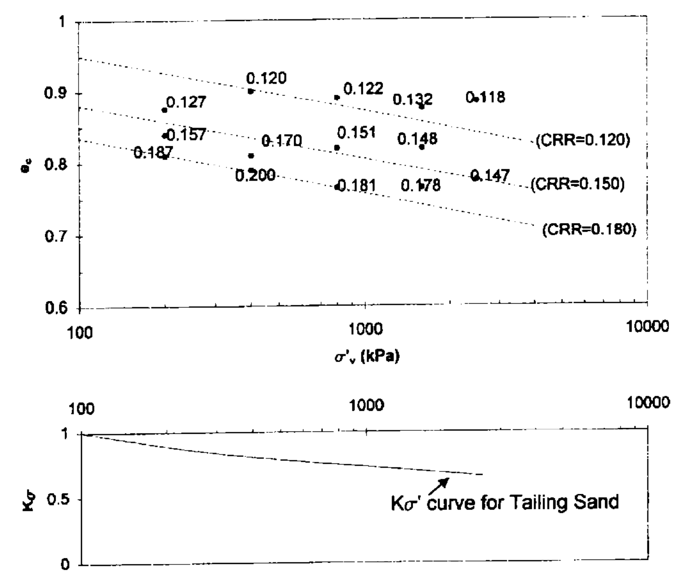

Fig. 6. Mapping of CRR of Tailing sand (data from Tumi 1983).

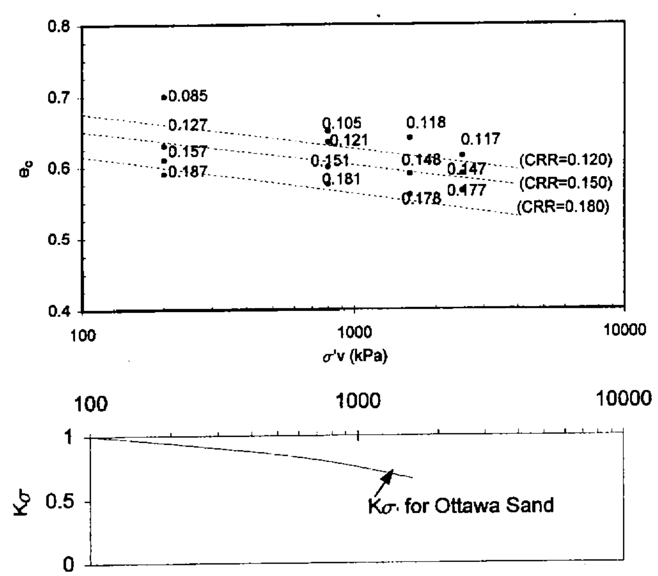

Fig 7. Mapping of CRR of Ottawa sand (data from Tumi 1983).

α. Pillai (1991) postulated that the influence of α on CRR depended on a more fundamental factor defined by the initial state ψ_p of the soil. In general, the more contractive (the larger the ψ_p), the larger the decrease of CRR with increase in α. On the opposite side, the more dilative (smaller the ψ_p) , the larger the increase of CRR with increase in α. Pillai (1991) presented general mechanisms of the dependency of CRR on α. These mechanisms are valid for stable states on either side of the CSL (Fig. 4).

Therefore, sands looser than critical should be divided into two categories; stable and quasi - stable yielding based on their depositional origin. Separate correlations may be needed for these categories. Similarly, on the dilative side very dense soils at low confining stress become unstable.

A majority of the existing research studies on liquefaction potential correlations has been based on relative density (D_r). Relative density is not sufficient to characterize the initial state of the soil and the resulting correlations are empirical. Therefore, engineers will have great difficulties in reaching realistic conclusions on liquefaction.

CONCLUSIONS

There has been considerable research effort put into understanding the influence of confining stress and initial static shear on cyclic strength and the potential for liquefaction of sands. Sand has a chamelonic shear behavior. When viewed from different windows, it tends to give different conflicting pictures. As a result different views and correlations relating to liquefaction have been put forward in the literature. Very often these contributions fail to capture the actual behavior of sand under varying conditions. Part of the reason has been the use of empirical physical properties such as relative density and fines content as compared to more fundamental mechanical properties of the soil to describe soil behavior. The use of test results of soils in "unstable state" (e.g. moist-tamped) has often led to a misrepresentation of the behavior of "stable state" soils (alluvial deposits/ pluviated samples). This is particularly true in the study of effects of initial static shear on liquefaction. The correction factors that are used for confining stress (K_σ) and initial static shear (K_α) in the current state-of-practice are at best empirical. Their use without an understanding of the mechanisms governing shear deformation may mislead the final outcome of liquefaction analysis.

The importance of the initial state parameter in correlationg to liquefaction behavior has been highlighted. It has been shown that constant cyclic resistance ratio lines follow contours of parallel lines of constant initial states. K_σ - curve of a sand is unique. Experimental data presented (Figs 6 and 7), however, show that K_σ variation with confinement is different for different materials. More fundamental research based on mechanical properties of soils such as λ, κ, and M are needed to study the influence of confining stress and initial static shear on liquefaction of soils. Such focused research would lead to more realistic correlations that would enhance the reliability of the simplified procedures used in the current state-of-practice.

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