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DYNAMIC YOUNG'S MODULUS AND AXIAL STRAIN RELATIONSHIPS

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ABSTRACT

Equations of Young's moduli of sands as a function of the axial strain amplitude for different types of dynamic loading were developed from a series of resonant column tests. Ottawa 20-30 sand specimens were excited longitudinally with one of three types of loading at three different confining pressures. In the sinusoidal tests, excitation signals were generated by a variable frequency sine-wave oscillator. In the random tests, input signals were generated by a white-noise generator and a pulse signal generator was used in the impulse tests. Input and output signals were analyzed by an FFT analyzer in the random and impulse loading tests. Under each type of loading it was found that the Young's modulus normalized with the initial maximum Young's modulus for the different confining pressures could be unified using a normalized axial strain with a reference axial strain for each loading type. Relationships that determine the variation of the Young's modulus with the axial strain were developed for each type of loading.

INTRODUCTION

Shear modulus, Young's modulus, and damping ratio are considered the primary dynamic properties of soils. Research on the shear modulus and damping ratio of soils are extensive. Data on Young's modulus of soils, however, are very limited. Sinusoidal loading has been almost the only excitation used in soil dynamics research using resonant column techniques. Earthquakes, wind, ocean waves, and certain man-made forces, however, do not provide a sinusoidal pattern of excitation. In order to establish meaningful results that represent field conditions, a nonperiodic loading should be used in laboratory testing. Random vibration testing as a new technique was introduced into soil dynamics research by Young et al. [1977]. Since then, research has been carried out on the determination of the properties of soils under dynamic random torsional loadings, Aggour et al [1982]; Al-Sanad et al [1983] and Amini et al [1988].

In this research, a series of tests was conducted to determine the Young's modulus of sands under various types of dynamic compression loading using the resonant column device. Soil specimens constructed of air-dry Ottawa 20-30 sand were excited longitudinally with one of three types of loading: sinusoidal, random and impulse, under confining pressures of 34.5, 69 and 276 kPa. At each amplitude of excitation, the resonant frequency and response were determined. From the test results, an equation that determines Young's modulus as a function of the axial strain amplitude was developed for each type of loading, and good agreement between the computed values and measured values of Young's modulus of sand specimens were achieved with each equation.

TEST APPARATUS AND PROCEDURES

The resonant column device used in this research was the Drnevich "fixed-free" type with solid cylindrical specimens. The specimens were fixed at the base with the excitation forces applied to the top. The resonant column device has the capability of applying both longitudinal and torsional excitations. The dimensions of the solid cylindrical specimens were 7.5 cm in length and 3.6 cm in diameter. Water was used as the pressure media for the confining pressure on the soil specimens.

To study the Young's modulus under different types of loading, sinusoidal, random and impulse excitations were employed. The sinusoidal signals were generated by a frequency variable sine-wave oscillator and amplified by a power amplifier. The amplified sinusoidal signals were then sent to a coil that provided longitudinal excitation. The acceleration responses of the soil-mass system in the longitudinal direction were picked up by a transducer mounted in the top platen mounted on the soil column. The response signals were amplified by a charge amplifier. The excitation and response signals were connected to an X-Y oscilloscope, and the amplitudes were read from a voltmeter in root-mean-square (rms) values. The random signals used in the random loading tests were generated by a white-noise generator, then filtered through a 2-channel variable cut-off frequency filter. The random excitation signals could be changed according to the required cut-off frequency. The random excitation and response signals were recorded on a B&K 4-channel FM analog tape recorder. The recorded signals were replayed and analyzed on a Rockland digital signal analyzer (FFT). The impulse signals in the impulse loading tests were generated by a pulse signal generator, in which the width and frequency of pulses could be adjusted. The impulse excitation and response signals were also analyzed on the FFT analyzer.

All the specimens were prepared to a relative density of approximately 75-80% using a dry tapping method. After the assembly of each specimen, the specimen was confined under a pressure of 34.5 kPa for about 30 minutes to ensure the vacuum pressure used during the assembly (around 103.5 kPa) was completely released. The predetermined test sequence, composed of 6 to 8 test stages from low to high excitation was then started. At each stage, the soil specimen was excited longitudinally with either sinusoidal, random, or impulse excitation for one minute, and the excitation, response and resonant frequency were measured. Right after a test stage was finished, the next test stage started at a higher strain level. The test sequence continued until the test stage at the predetermined highest strain level was finished. The confining pressure was then increased to a higher value of 69 kPa, then 276 kPa. The same testing procedures were repeated under each confining pressure.

Under sinusoidal excitation, the resonant frequency and response were obtained by adjusting the frequency of the signal generator to a condition at which a vertical ellipse of both the excitation and response was observed on an X-Y oscilloscope.

In the random excitation tests, the excitation and response signals were recorded on a tape recorder first and then analyzed on an FFT analyzer. Since random signals are nondeterministic, a large amount of data was necessary to establish the statistical characteristics of the random signals. In a stationary ergodic random process, the statistical properties of the random signals do not change with time and must be obtained by time averaging. The more averaging, the more reliable the statistical results. It was found that to obtain a smooth transfer function more averaging times were needed for a soil-mass system with higher damping than one with low damping. The averaging times used in this research ranged from 32 to 128. It was found that no further change to the transfer function was found with any further averaging. The resonant frequency and response of the soil-mass system under random excitation could be obtained from either the power spectral density function (PSD) of the response when the PSD of the excitation was relatively flat around the resonant peak, or the magnitude of the transfer function (MTF). Normally, the MTF method is preferred over the PSD method since the MTF is independent of excitation, and is an inherent function of the vibration system. It was found that with an increase of cut-off frequency used with the random signal, a higher strain amplitude was induced. However, when the cut-off frequency was increased to a certain level, say 3 or 4 times the resonant frequency, further increase in the cut-off frequency did not induce further increase in the strain amplitude. Thus a cut-off frequency of 1,000 Hz was used in this research.

In the testing with impulse excitations, the excitation and response signals were directly connected to the FFT analyzer. Impulse signals are similar to random signals in that the PSD functions of impulse signals are continuous functions as are the PSD functions of random signals. The frequency analysis method, therefore, can also be used for impulse excitation testing. Similar to the random loading testing, the MTF was used for the calculation of the resonant frequency and response in the impulse loading tests.

With the measured resonant frequency, the Young's modulus was determined from the wave propagation equation. Axial strain amplitude induced in a soil specimen under sinusoidal loading was calculated from the measured acceleration response and resonant frequency. Under random and impulse loadings, the root-mean-square (rms) value of the strain amplitude was evaluated using the following equation; which was derived from the random vibration theory, Zhang [1994].

$$\varepsilon = \frac{1}{mL}\sqrt{\frac{S_{f_n}}{(4\pi f_n)^3 D}} \qquad (1)$$

in which ε is the strain amplitude, S_{f_n} is the value of the PSD function of excitation at resonant frequency f_n, D and m are the damping ratio and total mass of the soil-mass system, respectively, and L is the length of the soil specimen.

TEST RESULTS

The measured Young's modulus E of the tested sand specimens for different confining pressures under sinusoidal loadings are shown in Fig. 1 as a function of axial strain ε. By dividing the Young's modulus E at each axial strain by the maximum Young's modulus E_{max} of the specimen at the initial stage of each test sequence, the Young's modulus E was normalized as E/E_{max}. The initial maximum Young's modulus E_{max} of each test sequence was calculated from:

$$E_{\max} = E'\left(1 + \frac{\varepsilon'}{\varepsilon_r}\right) \qquad (2)$$

in which E' is the Young's modulus of the specimen at the first test stage of each test sequence; ε' is the axial strain induced in the specimen at the first test stage of each test sequence where the strain levels were very low, below 2×10^{-4} %; ε_r is the reference axial strain at the first test stage of each test sequence, which was calculated from:

$$\varepsilon_r = \frac{\sigma_0}{E_{\max}} \tag{3}$$

where σ_0 is the confining pressure in kPa; and $E_{\max}$ is the maximum Young's modulus of the first test stage of each test sequence.

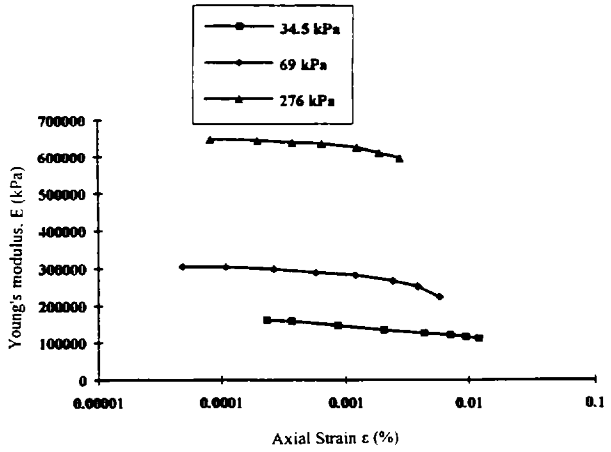

Fig. 1. Young's Modulus, E, vs. Axial Strain, ε, under Sinusoidal Loading for Different

To unify the relationship of the normalized Young's modulus $E/E_{\max}$ with axial strain ε under different confining pressures, it was found that by normalizing the axial strain with the initial reference axial strain as $\varepsilon/\varepsilon_r$ the normalized Young's modulus, $E/E_{\max}$, of the specimens under different confining pressures were unified, Zhang [1994]. The same procedure unified the data for the other types of loading.

EQUATIONS OF YOUNG'S MODULI AS A FUNCTION OF STRAIN

With unified results of $E/E_{\max}$ it is possible to develop equations that relate Young's modulus with the strain. By referring to the equation proposed by Hardin and Drnevich [1972] and the equation of Ramberg-Osgood's model for shear modulus with shear strain, the following equation was suggested for the relationship of Young's modulus with axial strain:

$$\frac{E}{E_{\max}} = \frac{1}{1 + a\left(\dfrac{\varepsilon}{\varepsilon_r}\right)^b} \tag{4}$$

in which a and b are constants. This type of equation has several advantages over other types of equations. First, this equation is dimensionless. Secondly, this equation has meaning for all ranges of strain from zero to infinity; for zero axial strain, the Young's modulus, E, will be the maximum value; when the strain is very large, the modulus reduces to zero. Thirdly, the form of this equation is simple and the curve of the equation can be easily adjusted with the constants a and b. To find the constants a and b, it is necessary to transform Eq. 4 to:

$$\frac{E_{\max}}{E} - 1 = a\left(\frac{\varepsilon}{\varepsilon_r}\right)^b \tag{5}$$

then, plot $(E_{\max}/E - 1)$ against $(\varepsilon/\varepsilon_r)^b$. By adjusting the constant b, a linear relationship through the origin between $(E_{\max}/E - 1)$ and $(\varepsilon/\varepsilon_r)^b$ can be obtained, then the slope of the line is the constant a.

For the results of Young's modulus, $E/E_{\max}$, with $\varepsilon/\varepsilon_r$ under sinusoidal loading, the constants a and b in Eq. 4 were obtained as 0.96 and 0.8 from Fig. 2 and the equation becomes:

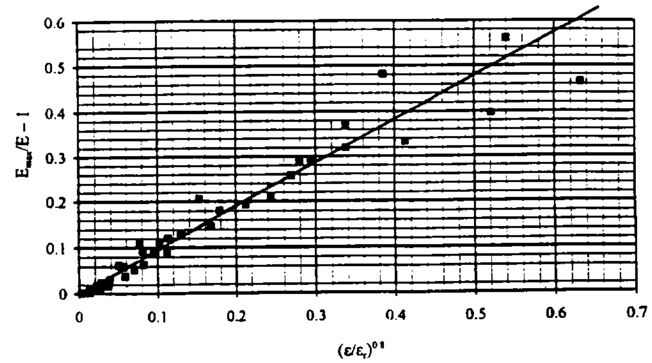

Fig. 2. $(E_{\max}/E - 1)$ vs. $(\varepsilon/\varepsilon_r)^{0.8}$ for Regression under Sinusoidal Loading

$$\frac{E}{E_{\max}} = \frac{1}{1 + 0.96\left(\dfrac{\varepsilon}{\varepsilon_r}\right)^{0.8}} \tag{6}$$

The computed values of $E/E_{\max}$ from the above equation are compared with the measured values in Fig. 3. It can be seen that there is a good agreement between the computed values and measured values. With this equation, we can determine the Young's modulus, E, of sands at any axial strain when the maximum Young's modulus, $E_{\max}$, is known. The maximum Young's modulus can be measured in the field or laboratory, or computed from the equation proposed by Hardin [1978]. His equation was also found to be suitable for the maximum Young's modulus of soils under random and impulse loadings.

The same procedures as described for sinusoidal loadings were used for the equations of Young's modulus under random and impulse loadings, and the constants a and b were obtained. The equations of Young's modulus for random and impulse loadings are as follows.

Random Loading

$$\frac{E}{E_{\max}} = \frac{1}{1+1.5\left(\frac{\varepsilon}{\varepsilon_r}\right)^{0.8}} \qquad (7)$$

Impulse Loading

$$\frac{E}{E_{\max}} = \frac{1}{1+1.32\left(\frac{\varepsilon}{\varepsilon_r}\right)^{0.75}} \qquad (8)$$

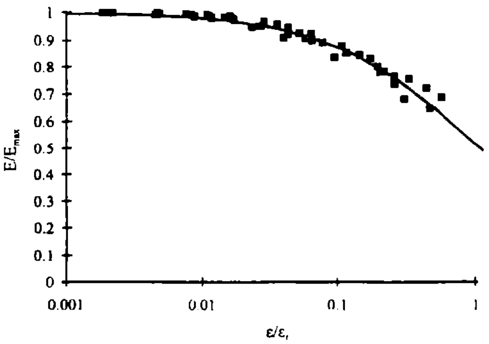

Fig. 3. Comparison of Computed and Measured Values of E/E_{max} under Sinusoidal Loading

The computed values of E/E_{max} from the above equations are compared with the measured values from tests of random and impulse loadings and were in good agreement with the measured values. The unified results of Young's modulus, E/E_{max}, under the three different types of loading are compared in Fig. 4. The computed values from each equation are indicated in the figure. As can be seen, random and impulse loadings had almost the same effect on the Young's modulus E/E_{max}. At an axial strain $\varepsilon/\varepsilon_r$, the Young's modulus, E/E_{max}, reduced more under random and impulse loadings than under sinusoidal loading. Overall, however, the difference on the effect of different types of loading on the Young's modulus was not very significant.

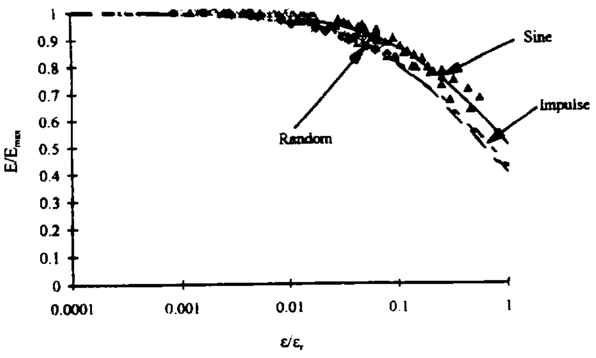

Fig. 4. Comparison of E/E_{max} under Sinusoidal, Random and Impulse Loading

CONCLUSIONS

Young's modulus is a very important dynamic property of soils. Previous research on the Young's modulus of soils were very limited. This research has been conducted for the purpose of studying the Young's modulus of soils under different types of loading and developing constitutive equations of Young's modulus with axial strain amplitude. A series of tests was performed on air dry Ottawa 20-30 sand in a resonant column device. Soil specimens were multi-stage tested with sinusoidal, random, or impulse excitation under confining pressures of 34.5, 69 and 276 k-Pa. From the test results, it can be concluded that:

1. Under all three types of loading, the Young's modulus E decreased with the increase of axial strain. The rate of decrease of Young's modulus with axial strain varied with confining pressure. Under lower confining pressures, the normalized E/E_{max} decreased faster with axial strain.
2. A reference axial strain, ε_r, is a very helpful parameter for unifying the effect of confining pressure on Young's modulus, E. When the normalized axial strain, $\varepsilon/\varepsilon_r$, is used, the normalized Young's modulus, E/E_{max}, under different confining pressures could be unified for each type of loading.
3. With the unified results of E/E_{max} under each type of loading, the following equation was found to be very representative:

   $$\frac{E}{E_{\max}} = \frac{1}{1+a\left(\frac{\varepsilon}{\varepsilon_r}\right)^{b}}$$

   This type of equation has the advantages of being nondimensional, whole-strain-range valid, and easy to fit. It can also be used for a wide variety of soils.
4. Comparing the effect of the three types of loading, random and impulse loadings have almost the same effect on the normalized Young's modulus, E/Emax. Under an axial strain of $\varepsilon/\varepsilon_r$ the reduction of E/E_{max} under sinusoidal loading was slightly smaller than under random or impulse loadings. However, the difference of loading type effect was not very significant.

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