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ABSTRACT

Since the introduction of the SASW method the use of surface waves for soil characterization has gained great popularity in geotechnical engineering. The typical two-receiver testing configuration shows some inherent drawbacks, for this reason a multistation approach is proposed in this paper.
The dispersion curve evaluation with a multistation scheme has several advantages. First of all, the estimate is much more stable and less sensitive to noise and internal phases of the instruments. Moreover signal interpretation is much clearer with respect to SASW approach and there is no need for subjective choices in the construction of the experimental dispersion curve. For this reason the procedure is easily automated, with a great saving of time and the possibility of having a rapid estimate directly on site.
Some experimental results are presented to compare the multistation *fk* analysis of surface waves with the classical two-station procedure of SASW tests. The experimental dispersion curve is finally used for an inversion process based on the simulation of wave propagation in a layered elastic medium. The obtained shear wave profile is then compared with the results of a Cross Hole test.

INTRODUCTION

Geophysical in situ tests are very important tools for the evaluation of dynamic soil properties, especially in hard-to-sample soils. In particular seismic tests supply good quality data regarding soil behavior at very small strain level, suitable for the modeling of the seismic response of soil deposits.
The analysis of surface waves propagation in soils for characterization purposes has gained great popularity in the past decades because of the inherent advantages of such testing techniques. In particular the non-invasive nature of these methods makes them cost and time effective if compared to bore-hole methods, such has cross-hole or down-hole tests. On the other hand, surface wave based methods require a heavy processing of the data and the interpretation is not always straightforward, with the consequent need for specialized personnel.
The use of surface waves for characterization purposes has a long history in seismology, but only recently it has been widely applied in shallow geophysics and in geotechnical engineering. A procedure for the evaluation of soil and pavement moduli from surface wave testing was proposed at the end of the Fifties [Jones, 1958], but it was not widely adopted because of the time consuming acquisition procedure and the inaccuracy of data interpretation.
After the introduction of the SASW (Spectral Analysis of Surface Waves) method [Nazarian and Stokoe, 1984] surface waves based techniques had a strong impulse and they are nowadays widely adopted for dynamic soil characterization.
The SASW method basically uses a two-station experimental procedure for the acquisition of field data. The adoption of a multistation scheme can lead to several improvements in the execution and in the interpretation of field measurements of surface waves propagation. In this paper a comparison between the two approaches is presented, together with some experimental results.

SOIL TESTING USING SURFACE WAVES

Since seismic methods always imply very low strain levels, the behavior of soils is considered linear elastic for the interpretation of the relative field data.
The basic idea of testing procedures based on surface waves propagation is to use their dispersive nature in heterogeneous media. In a linear elastic homogeneous medium Rayleigh waves velocity of propagation is independent on frequency and its value is close to the shear waves velocity one [Richart et al. 1970]. In a vertically heterogeneous medium, phase velocity of surface waves is dependent on frequency and this dependence is implicitly related to the variation of elastic parameters with depth.
If the experimental dispersion curve (i.e. the relationship between surface waves phase velocity and frequency) can be obtained from field data, it is possible to use such information for an inversion process, to estimate the stiffness profile at a site.
It is important to remark that for this purpose the soil is considered as a stack of linear elastic homogenous layers (Fig. 1). Since this model is the basis for the whole characterization process, the application of surface wave based methods is limited

to cases in which such a model is a valid approximation of the real geometry of the soil deposit. Usually the inversion process is performed assuming a reasonable value for density and Poisson ratio of the layers and varying the values of thickness and shear modulus. This is justified by a parametric analysis based on a series of numerical simulations [Nazarian, 1984]. Considering the strong relationship existing between shear modulus G and shear wave velocity, the results are usually presented in terms of the latter.

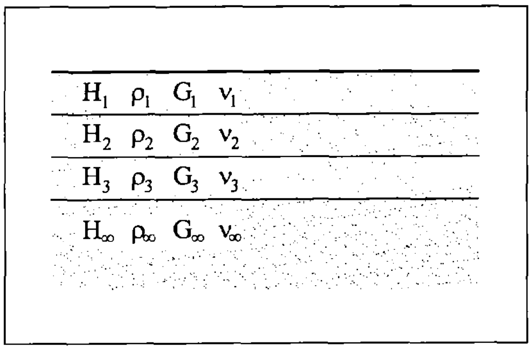

Fig. 1 Layered linear-elastic medium

The dispersive behavior of surface waves can be analyzed using several testing configurations. In the following two different approaches will be considered: the typical two-receiver scheme of the SASW test and the multistation procedure.

The SASW method

The field data acquisition is based on the testing setup reported in Fig. 2. A couple of receivers is moved along a straight line starting from the point source with an inter-receiver spacing typically equal to the spacing between the source and the first receiver (D=X with reference to Fig. 2). The source can be either impulsive (as for example a weight drop) or controlled, acting in a sweep-sine mode. From the frequency domain analysis of the relative signals, information over a broad frequency range can be obtained at once [Nazarian and Stokoe, 1984].

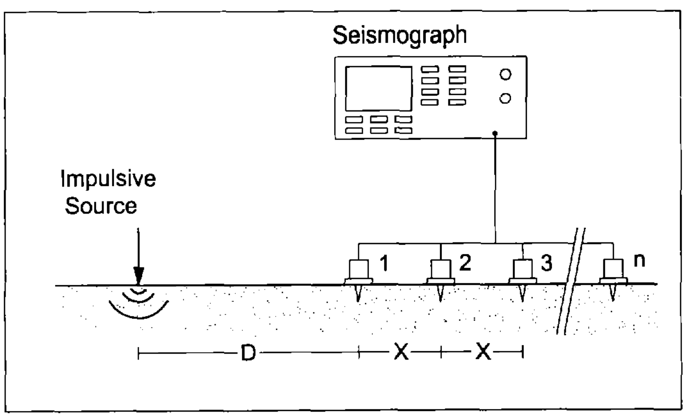

Fig. 2 Field setup for the SASW test

The signal analysis procedure is based on the cross-power spectrum of the two signals, which phase $\Theta_{12}(f)$ can give the information relative to the phase velocity V_R of surface waves, according to the expression:

$$V_R(f) = \frac{2\pi \cdot f}{\Theta_{12}(f)} \cdot X \qquad (1)$$

where f is the frequency and X is the inter-receiver distance.
Different array spacing supply information about different frequency bands and allow to reconstruct a portion of the dispersion curve. Repeating the test with a sufficient number of different array spacing, information in a wide enough frequency range is obtained. Small receiver spacing and light sources are used for the high frequencies, while larger receiver spacing and heavier sources are used to get information related to the low frequency range. The necessity of having different configurations arises from the attenuation of high frequency component that makes it impossible to get useful information with wide receiver spacing and from the near-field effects that prevent the use of receivers close to the source for inferring the information about low frequencies. For this reason a filtering criterion is commonly applied to the information extracted from a single testing setup [Ganji et al., 1998].
One of the main problems associated to the interpretation of SASW data is due to the unwrapping of the cross-power spectrum phase. Indeed, being it a complex quantity, it is defined in a modulo-2π representation that is unsuitable for further processing and needs to be unwrapped to get a full-phase representation, that is necessary for the determination of the phase delay between the two receivers. Such process is a very ticklish one and can be hardly automated. In particular, in the case of the SASW test, the presence of low frequency noise can easily prevent any useful information to be extracted from the signals. This is one of the reasons why a very high signal-to-noise ratio is required in SASW data and, consequently, many repetition of the test are required in any given array configuration to get suitable data trough a stacking process in the frequency domain. Usually the coherence function is used as an indicator of data quality in a given frequency range and only information corresponding to high values of such function are considered.

The *fk* analysis of surface waves

The use of a multistation procedure can strongly improve the reconstruction of the surface wave dispersion curve. The data are collected along a straight line starting from the source location, at n receiver points with constant spacing (Fig. 3). Since the data are collected simultaneously at all the receivers, the wave identification is very robust and it is not sensible to single receiver problems.
The dispersion curve can be obtained transforming the data collected in space and time domain in a different domain where the surface wave dispersion is easily recognized as the location of energy peaks. Procedures based either on the frequency-wavenumber (*fk*) domain [Gabriels et al., 1987] or on the

frequency-slowness *(fp)* domain [McMechan and Yedlin, 1981] are appropriate for this purpose.

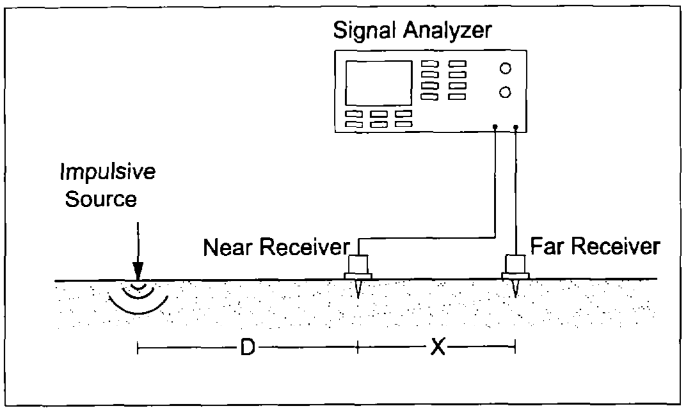

Fig. 3 Multistation testing setup

The analysis in the frequency-wavenumber domain is based on the application of a 2D Fourier Transform to the field data. Starting from the expression of a wave-field as the superposition of surface wave modes [Aki and Richards, 1980], it is possible to show that the peaks in the frequency-wavenumber spectrum are associated to the propagation of surface waves [Tselentis and Delis, 1998]. To get an accurate estimate of the spectrum, it is important to correct the traces to account for geometrical attenuation of surface waves, which can be roughly estimated as inversely proportional to the square root of the distance of the single receiver from the source.

Considering at each frequency *f*, the energy peak location in the wavenumber domain *k*, the phase velocity $V_R(f)$ can be written as:

$$V_R(f) = \frac{2\pi \cdot f}{k} \quad (2)$$

The frequency range over which information related to the dispersion curve can be obtained depends on receiver spacing and it is influenced by high frequency component attenuation and by the problem of aliasing in the wavenumber domain. For this reason, usually, a couple of testing setup, with different spacing, is used to get the necessary information.

The maxima can be easily detected in the frequency-wavenumber domain and hence the derivation of the experimental dispersion curve is rapid and straightforward and it can be easily automated. Moreover this method is less influenced by ambient noise and by body wave effects, hence also a single shot signal can be used to infer the dispersion curve, without any particular need for stacking processes.

Theoretically the information related to the peaks in the frequency-wavenumber domain are modal values. Hence, considering the absolute and local maxima, it should be possible to get information related to the different modes of propagation that compose the wave-packet. In practice it has been shown, with numerical simulations, that, using the typical receiver spacing suitable for usual geotechnical surveys (few meters), a single dispersion curve can be extracted by the field data [Foti, 2000]. Indeed the short distances do not allow modal separation in the wave packet generated by an impulsive source. The obtained phase velocity has to be considered an effective or apparent value [Lai, 1998] arising from mode superposition. The consequences on the inversion process are very important, indeed, as for the SASW test [Gukunski and Woods 1992], it must be conducted considering the mode superposition effects.

EXPERIMENTAL RESULTS

To evaluate the possibilities given by the *fk* analysis of surface waves and to compare its performances with the classical approach of the SASW test, a series of experimental measurements were carried out using a testing configuration designed for multistation methods. It must be considered that signals from a multistation session can be analysed with the classical SASW two-station procedure, just considering pairs of geophone responses.

The testing site is located in Saluggia (VC) in the northern part of Italy, close to the Dora Baltea River and it is part of a large flat area, that is composed essentially of fluvial sediments. The soil is composed basically of gravels and gravelly sands, with the presence of fine sand and clayey silt, in the form of lenses. The water table fluctuates seasonally between 2 and 3 meters below the ground surface.

The data acquisition was performed using a traditional seismic equipment: 24 vertical geophones and a 24 channel seismograph Mark6 (by ABEM). The seismic sources used were a 6 kg hammer (light source) and a 130 kg weight drop (heavy source). Two different test arrays were used (see Table 1, which symbols are relative to Fig. 3). To investigate a coherent portion of soil deposit, the two arrays have been located along the same straight line and with a common midpoint. The natural frequency of the geophones is 4.5 Hz.

Table 1 Test setup

Source	D [m]	X [m]
6kg hammer	1	1
130kg weight-drop (h=3m)	3	3

To evaluate the dispersion curve following the usual two-station procedure of the SASW test, 5 couples of receivers were selected among the data collected. The choice was performed in order to have receiver pairs with equal inter-receiver and source-receiver spacing, as commonly used for the SASW test. Couples corresponding to 3m and 6m spacing were chosen from the first testing array, while the couples corresponding to 12m, 18m and 30m were selected from the second testing arrangement.

As an example, Fig. 4 shows the spectral quantities relative to the couple with 18m spacing, selected from the test performed using the weight-drop source. Together with the Cross-Power Spectrum phase, the Coherence function and the Auto-Power spectra at the two receivers are reported. These other quantities give a clear picture of the frequency range in which the most of energy is concentrated and hence there is a high signal-to-noise ratio.

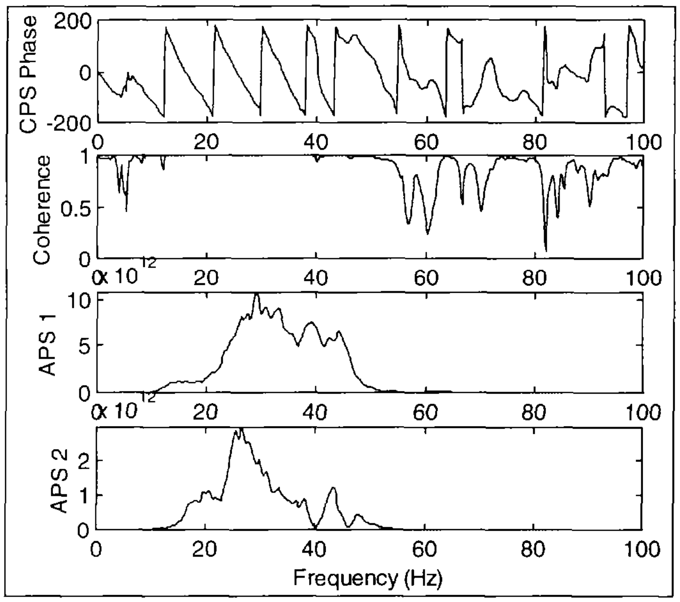

Fig. 4 Example of two-receiver data elaboration (source: 130kg weight-drop, inter-receiver distance 18m)

Assembling the information obtained from the selected receiver pairs the dispersion curve is estimated over a broad frequency range (Fig. 5). Note that the pieces of information related to each receiver couple do not overlap perfectly, this is due both to experimental uncertainties and to the spatial variability of surface wave effective phase velocity [Lai, 1998].
The usual practice is then to reduce the number of points in the dispersion curve considering a given averaging process.

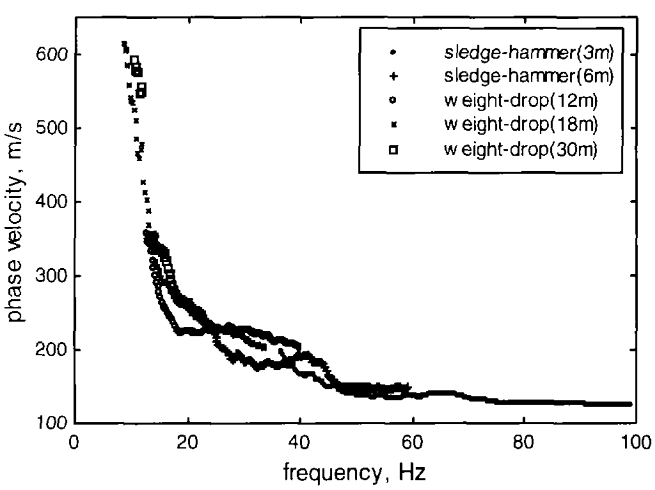

Fig. 5 Experimental dispersion curve from the two-receiver procedure (SASW test)

Concerning the multistation elaboration of the field data, the frequency-wavenumber spectrum has been evaluated applying a 2D-FFT algorithm to the whole ensemble of 24 traces, for each one of the two testing arrays. Each trace has been previously multiplied by the square root of the relative source-receiver spacing, to correct for the geometrical attenuation of surface waves. The obtained *fk* spectra are reported respectively in Fig. 6 and Fig. 7, while Fig. 8 represents the dispersion curve estimated from the spectral maxima. As expected the longer array gives information related to the low frequency range, while the shorter one yields data related to the high frequencies. The range for which there is an overlap of information between the two shows a very good consistency of the phase velocity values.

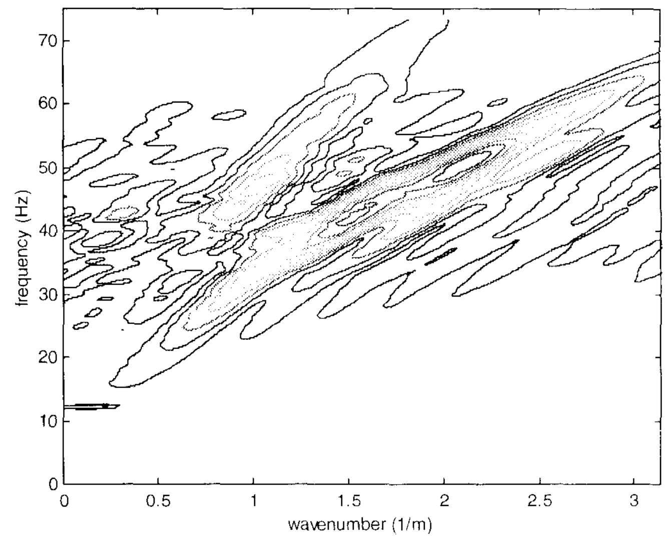

Fig. 6 fk spectrum (source: hammer, inter-receiver distance: 1m)

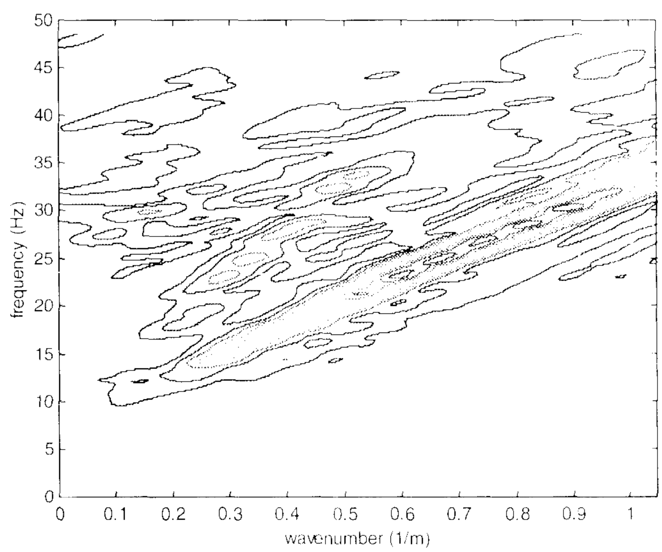

Fig. 7 fk spectrum (source: weight-drop, inter-receiver distance: 3m)

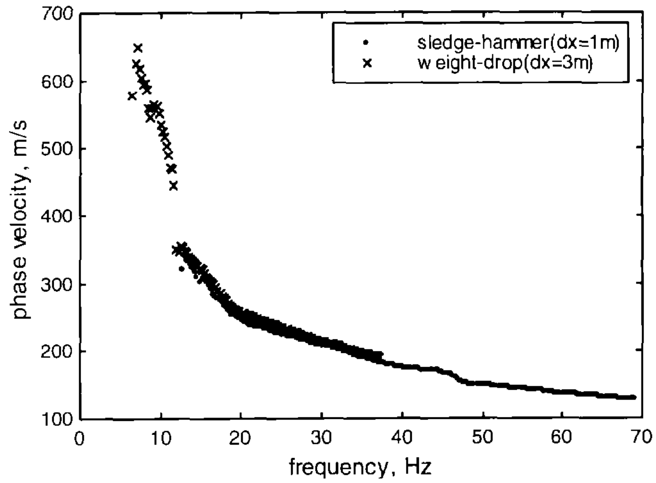

Fig. 8 Dispersion curve from fk analysis

Fig. 9 reports a comparison between the estimate of the experimental dispersion curve obtained with the two procedures: the two-station and the multistation. It is important to remark that both have been obtained from the same set of data and are hence congruent each other. The dispersion curve obtained with the *fk* analysis represents a more stable estimate and it is a sort of average value in between the dispersed values obtained with the two-station procedure. So the use of the *fk* procedure can avoid the need for averaging processes of the data, hence reducing one step of the data processing. Moreover it is important to remember that this method strongly reduces field acquisition time and processing time, also avoiding some crucial steps, such as the unwrapping of the Cross-Power phase.

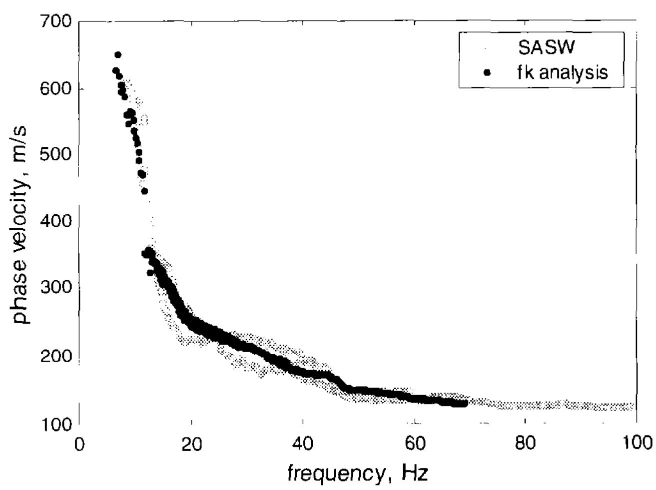

Fig. 9 Comparison between dispersion curves obtained using the two-station procedure (SASW) and the fk analysis

Some remarks must be made also about the outer frequency ranges. Information at low frequency are very important in the view of the subsequent inversion process, indeed they are related to the possibility of characterizing deep layers and usually this is a crucial aspect for surface wave methods. The example reported clearly shows that the estimate obtained by the multistation method is more stable in this range, and this can be a great advantage.

On the other hand, the *fk* analysis does not supply any information for frequency above 70 Hz, this is essentially due to spatial aliasing problems. High frequency components are important for the level of details at very shallow depth. In most cases it is not necessary to have such information, but, if it is the case, they can be obtained using another testing setup with closer geophones.

Finally, the results of the inversion process, conducted using the estimate of the dispersion curve obtained with the *fk* method, are presented in Fig. 10 and Fig. 11. The inversion process has been performed using the program SURF, developed and distributed by R.B. Herrmann of Saint Louis University and his co-workers. The program only accounts for modal phase velocity and it does not account for modal superposition. It has been possible to use this approach because, in this case, the stiffness profile of the site is normally dispersive and hence the fundamental mode is dominant all over the frequency range of interest.

Fig. 10 shows the good fitting existing between the experimental curve and the numerical one, evaluated with the estimated shear wave profile, reported in Fig. 11. The comparison with the results of a Cross-hole test shows a good agreement of the results, especially for shallower layers.

According to the Cross-Hole results, the deeper layers show great oscillations of the shear wave velocity with depth. It is important to remark that the inability to detect such oscillations is implicit for the surface wave method, which looses resolution with depth. Indeed because of its basic principles, surface waves methods can supply a good resolution at shallow depth, but, at great depth, the values of stiffness obtained should be assumed as average values. In the case presented probably the average stiffness is somewhat overestimated at depth higher than 15m.

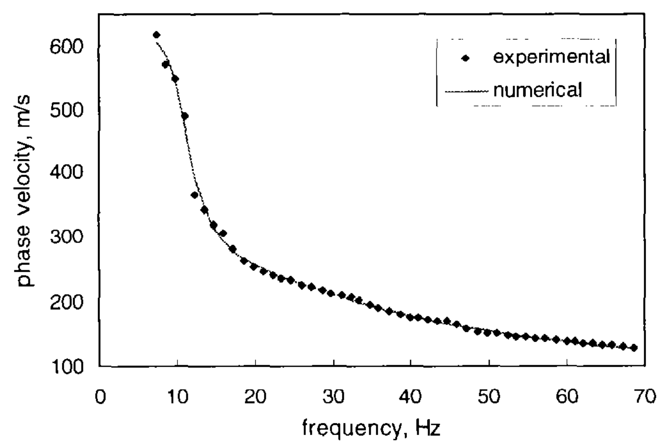

Fig. 10 Experimental and numerical dispersion curve from the inversion process

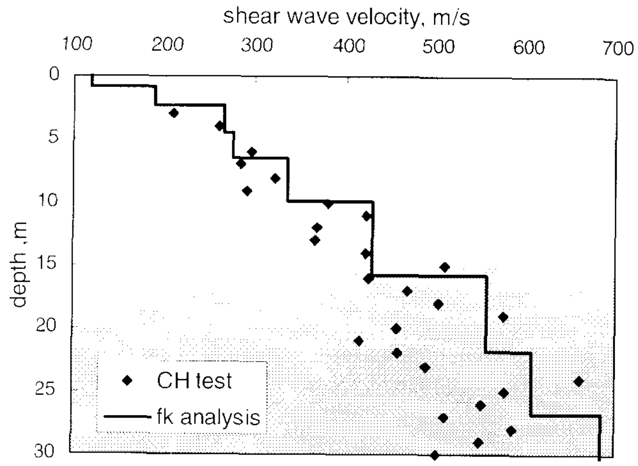

Fig. 11 Shear wave velocity profile from Cross-Hole test and fk analysis of surface waves

CONCLUSIONS

Surface waves based methods for soil characterization are currently used in many engineering projects because of their versatility and the possibility of testing hard-to-sample soils without the need for bore-holes. The two-receiver procedure based on the cross-power spectrum phase is widely adopted to get the estimate of the experimental dispersion curve by which the stiffness profile of the site can be inferred.

In this paper the advantages of using a multistation procedure have been shown with a comparison based on experimental data. A multistation procedure is inherently more robust and stable because is based on simultaneous detection and elaboration of the signals associated to surface waves propagation at a great number of receivers. The dispersion curve obtained with a multistation procedure is an average value over the spatial range interested by the testing setup and it is possible to get the whole frequency range of interest with a reduced number of testing configurations and repetitions of the test.

Probably the most important aspect is the possibility of automating the process of estimation of the experimental dispersion curve. Indeed troublesome processing such as the unwrapping of the cross-power spectrum phase are avoided and the level of judgment is restricted only to the selection of the frequency range over which information from a given experimental configuration can be considered reliable.

The adoption of a multistation procedure can therefore strongly reduce the testing time in the field and part of the data elaboration in the office. Since the evaluation of the dispersion curve is very fast with such procedure it is also possible to have a good estimate directly in the field, and leave to the office only the inversion process. This can be a great advantage in planning and performing the tests.

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