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Local-plate and Distortional Post-buckling Behavior of Cold-formed Steel Lipped Channel Columns with Intermediate Stiffeners

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Abstract

This paper reports the results of an investigation concerning the local-plate and distortional post-buckling behavior of cold-formed steel lipped channel columns with web and flange intermediate stiffeners. They have all been obtained through geometrically non-linear analyses based on a recently developed and implemented Generalized Beam Theory (GBT) formulation that incorporates (i) conventional (shear undeformable), (ii) shear (non-linear warping) and (iii) transverse extension deformation modes. These results, some of which are compared with values yielded by shell finite element analyses performed in the code ABAQUS (mostly for validation purposes), provide the evolution, along a given local-plate or distortional post-buckling equilibrium path, of the column deformed configuration and relevant displacement profiles and/or stress diagrams. In order to assess the influence of the member end support conditions, one also compares the distortional post-buckling behaviors of columns having pinned/free-to-warp and fixed/warping-prevented end sections. Taking full advantage of the GBT unique modal features, all the above results are discussed in great detail and it becomes possible to unveil, explain and/or shed some new light on several interesting and scarcely known behavioral aspects. In particular, one is able to provide very clear and structurally meaningful explanations for the qualitative differences existing between the local-plate and distortional post-

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buckling behavior of lipped channel columns with and without intermediate stiffeners.

Introduction

The structural efficiency of a given cold-formed steel member can only be adequately assessed after obtaining in-depth information about its local-plate and distortional buckling and post-buckling behaviors, a task which involves (i) the identification of the relevant buckling modes, (ii) the evaluation of the corresponding critical bifurcation stresses and (iii) the determination the associated post-buckling equilibrium paths and/or strength values (taking into account the presence of unavoidable initial imperfections and, possibly, also of mode interaction phenomena). This information also plays a crucial role in the elaboration, validation and calibration of design formulae and/or procedures for cold-formed steel members, since only reliable and physically based models are capable of generating rational and efficient (*i.e.*, economic and safe) design tools. In particular, a vast amount of research work has been recently devoted to the development of methods specifically aimed at the design of thin-walled members made of high-strength steels, which exhibit a material behavior characterized by large yield stresses and practically no strain-hardening (*e.g.*, Yang & Hancock 2003, Yang 2004).

On the other hand, subdividing the walls (mostly flanges and/or webs) of a given cold-formed steel member into smaller plate elements, through the incorporation of intermediate V-shaped stiffeners, has been shown to lead to a considerable load-carrying capacity increase. Issues related to the evaluation of the minimum intermediate stiffener rigidity required to preclude the local-plate buckling of compressed members were first studied by Desmond (1977) and Höglund (1978). A few years later, Hoon *et al.* (1993) and Bernard *et al.* (1993) reported the results of experimental investigations, carried out with the aim of developing design rules for stiffened plates or thin-walled members. Concerning specifically the post-buckling behavior and load carrying capacity of cold-formed steel lipped channel columns with flange and web intermediate stiffeners, it is worth noting the recent numerical and experimental work due to Lepistö *et al.* (1996), Teter & Kolakowski (1996, 2004), Yang & Hancock (2003), Yang (2004) and Narayanan & Mahendran (2004).

Up until very recently, the "exact" geometrically non-linear behavior of a thin-walled member could only be accurately determined by resorting to computer-intensive numerical analyses employing either (i) the finite element method, adopting quite fine shell element meshes (*e.g.*, Ville de Goyet 1995, Young &

Yan 2002), or (ii) the spline finite strip method (*e.g.*, Kwon 1992, Prola 2001). This situation has now changed, after the formulation of a genuinely geometrically non-linear Generalized Beam Theory (GBT), which is based on equilibrium equations valid in the large deformation range (Silvestre & Camotim 2003) – the previously available GBT equations, developed by Schardt (1994), only apply to problems involving small deformations and moderate rotations, which precludes their use to perform post-buckling analyses (they are ideally suited for bifurcation analyses, though – *e.g.*, Kesti & Davies 1999).

The objective of this paper is to present and discuss the results of an investigation concerning the local-plate and distortional post-buckling behaviors of cold-formed steel lipped channel columns exhibiting web and flange V-shaped intermediate stiffeners and having pinned/free-to-warp or fixed/warping-prevented end sections. All these results were obtained by means of geometrically non-linear analyses based on a very recently derived and numerically implemented GBT formulation, which includes (i) *conventional* (no shear deformation), (ii) *shear* (non-linear warping) and (iii) *transverse extension* deformation modes (Silvestre & Camotim 2003). For validation purposes, some of them are compared with values yielded by finite element analyses (i) performed in the code ABAQUS (HKS 2002) and (ii) adopting fine 4-node shell element meshes to discretize the columns. The above GBT formulation/implementation has been shown to be computationally very efficient and, as a result of its unique modal nature, it also provides the ideal instrument for an in-depth interpretation of several scarcely known behavioral aspects. The results included in the paper make it possible (i) to assess the effect of the intermediate stiffeners on the member local-plate and distortional post-buckling behaviors and, moreover, (ii) to reconcile the apparently contradictory results recently reported by (ii_1) Prola (2001) and Prola & Camotim (2002) and by (ii_2) Yang & Hancock (2003) and Yang (2004), which concern the distortional post-buckling asymmetry of cold-formed steel lipped channel columns without and with intermediate stiffeners.

GBT Cross-Section (Modal) Analysis

By expressing the cross-section displacement field as a combination of *deformation modes*, a GBT formulation leads to member equilibrium equations that (i) are written in a rather convenient and unique form and (ii) make it possible to perform cross-section *modal* analyses providing a much clearer understanding of the member structural behavior. Since the main objective of this work is to assess how the presence of web and flange intermediate stiffeners affects the post-buckling behavior of cold-formed steel lipped channel columns, one must analyze columns (i) *with* and *without* intermediate stiffeners (*stiffened* and *plain* lipped channels) and (ii) otherwise similar. The cross-section dimensions and material constants considered are given in figure 1(a) and ensure a minimum distortional buckling stress much lower than its local-plate counterpart. Moreover, it should be mentioned that they have already been used in a recent paper by the authors (Silvestre & Camotim, 2004), dealing with the distortional post-buckling behavior of plain channel members.

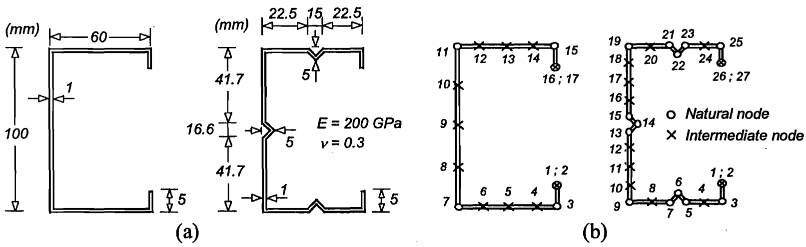

Fig. 1. Lipped channel (a) dimensions and (b) GBT discretizations

Figure 1(b), on the other hand, shows the GBT discretizations adopted, which (i) involve *17* (*6* natural, *11* intermediate) and *27* (*15* natural, *12* intermediate) nodes, respectively for the plain and stiffened channels, and (ii) lead to the identification of the following deformation mode sets (Silvestre & Camotim, 2003): *17 conventional* (*4* global, *2* distortional, *11* local-plate), *14 shear* and *19 transverse extension*, for the plain channel, and *27 conventional* (*4* global, *11* distortional, *12* local-plate), *25 shear* and *32 transverse extension*, for the stiffened channel. The main features of the *31* most relevant stiffened channel modes (*13* conventional, *10* shear, *8* transverse extension) are displayed in figures 2–4. Due to the paper space limitations, similar figures for the plain channel modes are not present here. However, they can be found in two recent papers by the authors – Silvestre & Camotim, 2003, 2004).

The *conventional* modes are based on Vlasov's assumption of null membrane shear strains ($\gamma_{xs}^{M}=0$), constitute the core of GBT and comprise

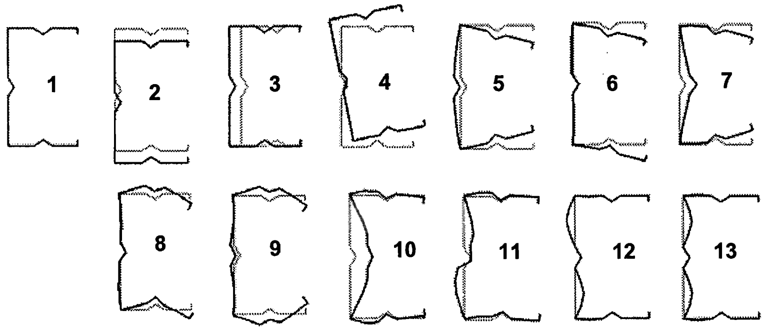

Fig. 2. Stiffened channel 4 global, 7 distortional and 2 local-plate modes

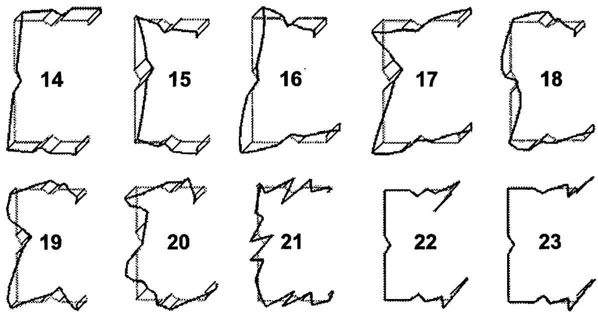

Fig. 3. Stiffened channel 10 most relevant shear modes (warping)

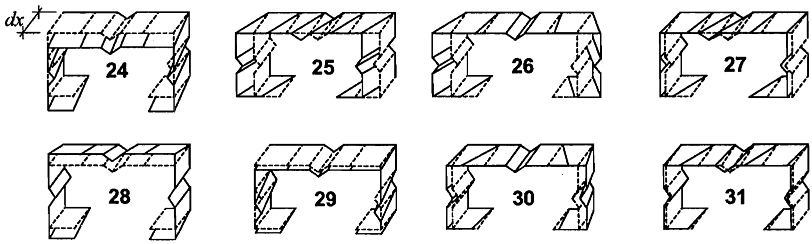

Fig. 4. Stiffened channel 8 most relevant transverse extension modes

(i) extension (**1**), major/minor axis bending (**2**, **3**), torsion (**4**), distortion (**5** to **11**), and local-plate (**12**, **13**) modes. As for the *shear* and *transverse extension* modes, which were very recently introduced by the authors (Silvestre & Camotim, 2003), they deserve a few additional comments:

(i) The *shear* modes deal exclusively with the *non-linear* variation of warping along the cross-section mid-line (no cross-section in-plane deformation takes place). Each cross-section wall (plate) element exhibits *non-null* membrane shear strains ($\gamma_{xs}^{M}=\partial u/\partial s\neq 0$ – u is the warping displacement and s is the cross-section mid-line coordinate), which means that the shear modes violate Vlasov' assumption. Since they are included in the GBT analyses *in addition* to the conventional modes, which are all shear undeformable, their joint participation is able to provide a clear assessment of the effect of shear deformation on the member post-buckling behaviour.

(ii) The *transverse extension* modes satisfy the null warping assumption (*i.e.*, involve only in-plane displacements) and are characterized by non-null transverse extensions ($\varepsilon_{ss}^{M}=\partial v/\partial s\neq 0$ – v are the membrane transverse displacements). These deformation modes are coupled with the conventional ones and their joint participation accounts for the "bowing effect" due to transverse plate bending, a phenomenon associated with the local (cross-section) deformations and mainly occurring in the advanced post-buckling stages.

Finally, it is still worth noting that only a specific *fraction* of the above deformation modes participate in the deformed configuration of a given member. The composition of this deformation mode subset depends on the length and applied loading of that member.

GBT Buckling Analysis

Most of the results presented in this paper concern the local-plate (LP) and distortional (D) post-buckling behavior of plain and stiffened lipped channel columns with pinned/free-to-warp (PFW) end sections and the cross-section dimensions shown in figure 1(a)[1]. The GBT longitudinal discretization involved *8* beam finite elements (Silvestre & Camotim, 2003) and preliminary stability analyses provided the minimum LP and D critical bifurcation stresses and associated member lengths. Figures 5(a) and 5(b) show the variation of the bifurcation stress and buckling mode shape (expressed in terms of its GBT modal decomposition) with the member length L, for the plain and stiffened channel columns. Both curves exhibit two local minima, corresponding to single-wave LP or D buckling and defined by (L and σ_{cr} values in *mm* and *MPa*): (i) $L_{LP}=80$, $\sigma_{cr.LP}=95$ (plain C), (ii) $L_{LP}=30$, $\sigma_{cr.LP}=560$ (stiffened C), (iii) $L_{D}=300$, $\sigma_{cr.D}=76$ (plain C) and (iv) $L_{D}=400$, $\sigma_{cr.D}=100$ (stiffened C). From the

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[1] To avoid LP-D mode interaction phenomena, the columns analysed have very short lips (*5 mm*), which greatly increases the gap between the D and LP critical buckling stresses.

observation of figure 5(a), it becomes clear that the incorporation of the intermediate stiffeners (i) leads to a considerable $\sigma_{cr.LP}$ increase (almost six times) but (ii) does not alter $\sigma_{cr.D}$ too much (only a *33%* increase). In

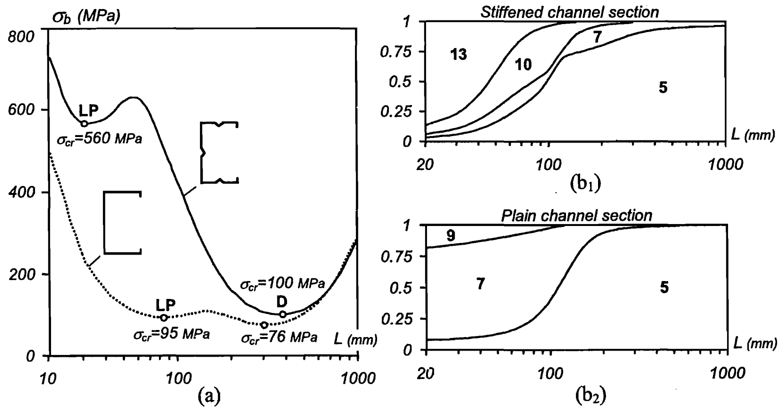


Fig. 5. Plain and stiffened channel PWF columns: (a) single-wave σ_b vs. L curves and (b) GBT modal analysis of the buckling modes

addition, figures 5(b_1) and 5(b_2) show that the stiffened column buckling modes involve more GBT modes than the plain column ones[1]. For this particular stiffened column, one observes that (i) the D buckling mode includes a major contribution of mode **5** and a minor one from mode **7** and (ii) the symmetric LP buckling mode combines a large contribution of mode **13** with small participations of modes **10**, **7**, **5** (all distortional).

Generally speaking, it is fair to say that columns with intermediate stiffeners may buckle in symmetric or anti-symmetric LP buckling modes, which are associated with very close bifurcation stress values. Moreover, in cross-sections with "more efficient" V-shaped stiffeners (larger inclinations and depths), the LP critical buckling mode configuration is *anti-symmetric*, due to the prevalence of modes **12** (local-plate) and **11** (distortional).

Finally, it was decided to perform post-buckling analyses of stiffened and plain stub columns with lengths *L=40 mm* (LP) and *L=300 mm* (D). The critical bifurcation stresses read (i) $\sigma_{cr.LP}$=*570 MPa* and $\sigma_{cr.D}$=*106 MPa* (stiffened C) and (ii) $\sigma_{cr.LP}$=*160 MPa* and $\sigma_{cr.D}$=*76 MPa* (plain C). All the analyses include critical-

[1] In general, one may say that an increase in the number of cross-section walls leads to a similar increase in the number of GBT modes participating in the buckling modes.

mode imperfections with amplitudes $w_0=0.15\cdot t$ (LP) or $v_0=\pm 0.15\cdot t$ (D), where w_0 and v_0 are the (i) maximum initial web flexural displacement and (ii) initial lip transverse membrane displacement.

Local-Plate Post-Buckling Behavior (PFW Columns)

Figure 6(a) displays the LP post-buckling equilibrium paths of the plain and stiffened PFW channel columns, which were obtained by means of (i) FEM-based (ABAQUS + S4 shell element discretization) and (ii) GBT-based (all deformation modes included) geometrically non-linear analyses. Then, while figure 6(b_1) depicts FEM-based deformed configurations of both columns, figure 6(b_2) concerns only the stiffened column and shows how the GBT modal decomposition of its deformed configuration evolves as post-buckling progresses (*i.e.*, the relevant mode "*participation factor*" evolution). The analysis of these results prompts the following remarks:

(i) The GBT and FEM-based post-buckling results virtually coincide.

(ii) In the later pre-buckling stages, one observes that the "normalized"

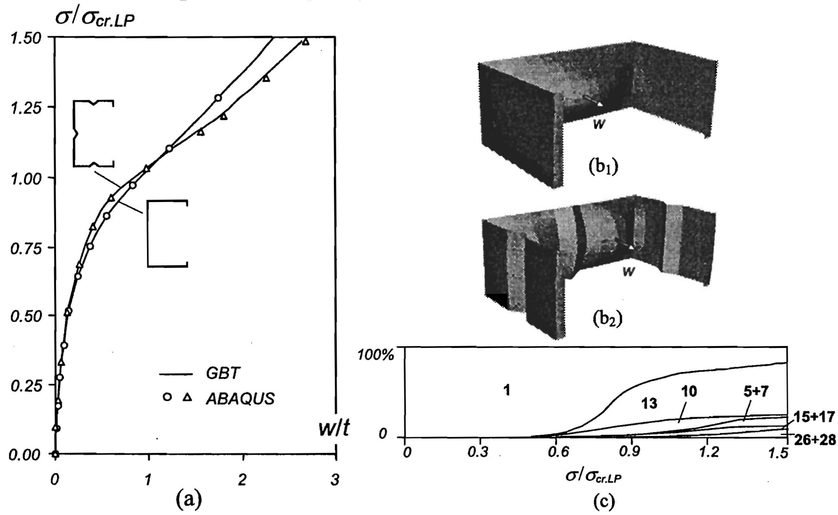


Fig. 6. PFW column local-plate (a) post-buckling paths, (b) deformed configurations and (c) GBT modal decomposition (stiffened column)

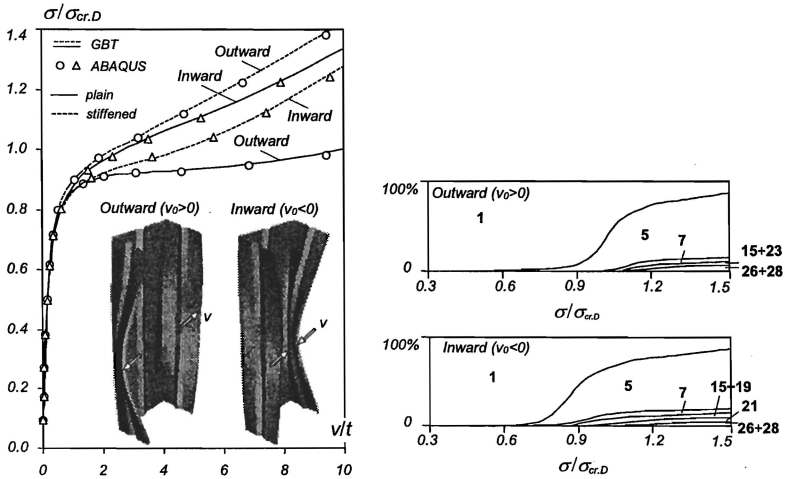
(w.r.t. $\sigma_{cr.LP}$) LP behaviors of the plain and stiffened columns differ slightly, the latter being a little stiffer. As the applied stress increases, the situation changes and the stiffened column progressively becomes the most flexible one. This stems from the joint participation of the distortional modes **5** and **7**, which is much more relevant than in the plain column (Silvestre 2004).

(iii) Concerning the stiffened column, figure 6(c) shows that, as it would be expected, the extension mode **1** governs the early pre-buckling stages, but its participation progressively fades as the applied stress increases. At $\sigma \approx 0.60\ \sigma_{cr.LP}$, modes **10** and **13** come into the picture and quickly gain predominance – their joint participation grows up to about *82%*. On the other hand, the two distortional modes **5** and **7** only become relevant for $\sigma > 0.90\ \sigma_{cr.LP}$, as their joint participation goes over *5%*. Finally, the shear (**15**+**17**) and transverse extension (**26**+**28**) modes only appear at more advanced stages ($\sigma > \sigma_{cr.LP}$ and $\sigma > 1.20\sigma_{cr.LP}$, respectively) and play a minor role in the column LP post-buckling behavior – the participation of modes **15**+**17** always lies below *4%* and the participation of modes **26**+**28** is even smaller.

Distortional Post-Buckling Behavior (PFW Columns)

The four equilibrium paths shown in figure 7(a) describe the *distortional* post-buckling behaviour of two pairs of PWF plain (dashed curves) and stiffened (solid curves) lipped channel columns. The columns in each pair differ only in the "sign" of the initial geometrical imperfection, which may be either (i) *positive* ($v_0 = +0.15t$), if the lips move *outward*, or (ii) *negative* ($v_0 = -0.15t$), when the lips move *inward*. Like in the LP case, these post-buckling equilibrium paths were determined by means of FEM-based (ABAQUS + S4 shell element discretizations) and GBT-based (all deformation modes included) analyses. Figure 7(a) includes also FEM-based deformed configurations of the stiffened columns with outward an inward lip deformations. The diagrams presented in figure 7(b) provide information concerning the evolution of the GBT modal decomposition of the stiffened column deformed configurations, along the outward and inward post-buckling paths. From the observation of these results, it is possible to conclude that:

(i) Once more, there is an almost perfect agreement between the GBT and FEM results. For $v/t < 10$, all the differences are below *2.6%*.



(a) (b)

Fig. 7. PFW column distortional (a) post-buckling paths and (b) GBT modal decomposition (stiffened columns)

(ii) Both the plain and stiffened columns exhibit a clear post-buckling *asymmetry* with respect to the v_0 sign. However, this asymmetry is *opposite* for the two cross-section configurations.

(iii) The *plain* column post-buckling behavior is stiffer (*i.e.*, exhibits a higher post-critical strength) for $v_0<0$, associated with an *inward* lip motion. This phenomenon was first detected and reported by Prola (2001) and Prola & Camotim (2002), who attributed it to marked differences in the flange-lip warping behavior. Very recently, this issue was further investigated by Silvestre & Camotim (2004), who found that the *warping* displacement evolutions for out and inward post-buckling are (iii_1) very similar at the flange-lip corner but (iii_2) quite different at the lip free end (much stiffer inward behavior).

(iv) Unlike its plain column counterpart, the post-buckling behavior of the *stiffened* columns is stiffer for $v_0>0$ (*outward* lip motion). This asymmetry was recently unveiled by the experimental studies of Yang & Hancock (2003) and Yang (2004), carried out for columns with fixed and warping-prevented end sections. These results were subsequently confirmed by FEM-based (ABAQUS) analyses and the authors were somewhat puzzled to find qualitatively differences between the plain and stiffened column behaviors.

(v) The diagrams presented in figure 7(b) show that mode **1** governs in the early pre-buckling stages, after which it is gradually "replaced" by mode **5**.

Moreover, the participation of this distortional mode is quite similar for columns with *outward* and *inward* lip motions: in both cases, it appears at $\sigma \approx 0.60\sigma_{cr.D}$ and its participation gradually becomes predominant – about *91%* (outward) and *83%* (inward)[1].

(vi) On the other hand, the shear mode participation differs considerably for columns with inward and outward lip motions. In the first case, the relevant modes are **15, 19, 21**, which emerge at $\sigma \approx 0.90\sigma_{cr.D}$ and reach a joint participation of about *9%*. In the second case, the only relevant modes are **15, 23**, which appear later (for $\sigma > 1.02\sigma_{cr.D}$) and have a contribution that never exceeds *4%*. This difference in the shear mode participation is responsible for the asymmetry exhibited by the curves shown in figure 7(a). In particular, note the relevance of the mode **21** contribution to this difference: the high compressive stress increase occurring at the web and flange stiffeners, when the lips move *inward*, is totally absent for the *outward* lip motion.

Finally, figures 8 (plain C) and 9 (stiffened C) provide the evolution of the mid-span normal stress distribution $\sigma(s)$ for different applied stress levels $\sigma/\sigma_{cr.D}$. After observing these four sets of diagrams, the following conclusions can be drawn (for these particular cross-section geometries):

(i) As expected, the stress distributions are almost uniform in the pre-buckling stages (mode **1** governs in all cases).

(ii) In the plain column undergoing *outward* lip motion (figure 8(a)), the lip free end compressive stresses grow very fast as buckling progresses. This fact has a major detrimental effect on the column strength, which is essentially governed by the *lip* behavior. In the stiffened column undergoing *outward* lip motion (figure 9(a)), the lip free end compressive stress increase is somewhat smaller.

[1] Note that this similarity does not occur for plain columns (Silvestre & Camotim 2004).

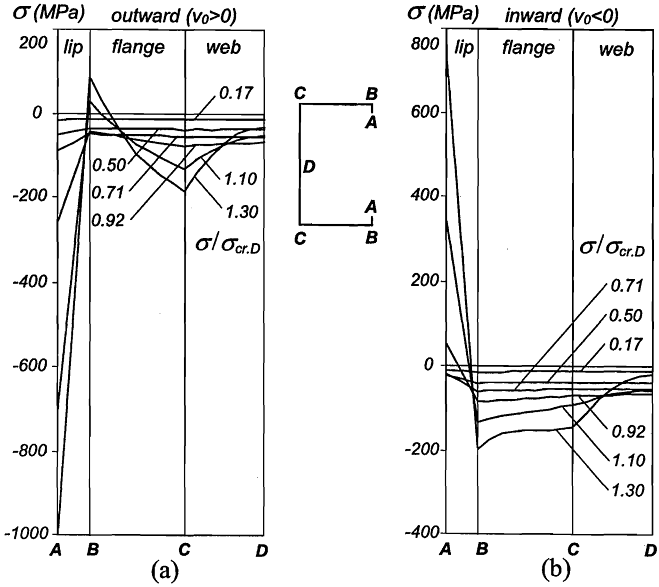


Fig. 8. Plain column mid-span stress distributions: (a) $v_0>0$ and (b) $v_0<0$

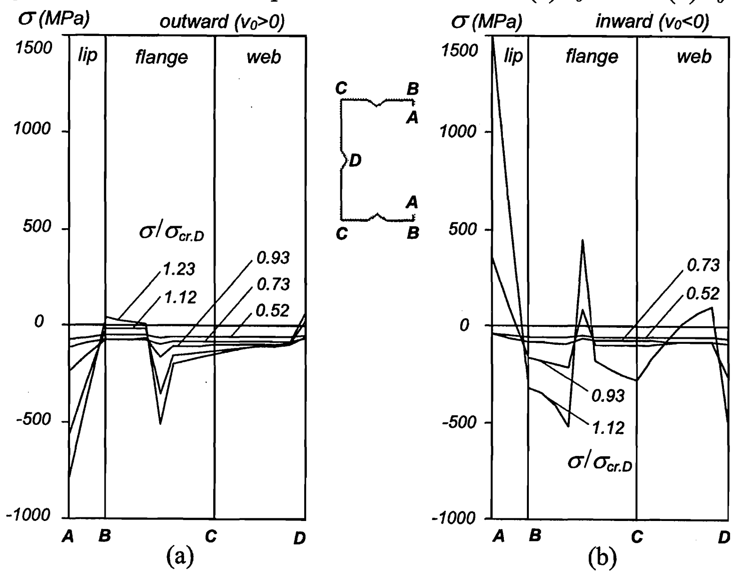


Fig. 9. Stiffened column mid-span stresses: (a) $v_0>0$ and (b) $v_0<0$

(iii) In the plain column undergoing *inward* lip motion (figure 9(a)), tensile stresses develop near the lip free ends and grow very rapidly as buckling progresses. This fact has a major stiffening effect and mostly accounts for the higher post-critical strength clearly shown in figure 7(a) (Silvestre & Camotim 2004). Although even higher and faster-growing tensile stresses develop near the lip free ends of the stiffened column undergoing *inward* lip motion (figure 9(b)), the associated stiffening effect plays a much less relevant role, as far as the post-critical strength is concerned (see the next item).

(iv) In the stiffened column undergoing *inward* lip motion (figure 9(b)), a significant compressive stress increase occurs at the web stiffener, web-flange corner and adjacent "half-flange". This fact has a very pronounced weakening impact, which completely overcomes the lip stiffening effect mentioned in the previous item. Indeed, the column strength is now mostly governed by the *web stiffener* and *stiffened flange* behaviors. Moreover, the high compressive stresses acting on the web stiffener trigger a "lip+flange+half-web" joint rotation (about the web stiffener), which leads to a more flexible behavior. In the plain column undergoing *inward* lip motion (figure 8(b)), on the other hand, the compressive stresses increase much more moderately in the flanges and even decrease at the mid-web area. Thus, there is no significant weakening effect to "oppose" the stiffening due to the lip tensile stresses addressed in item (iii).

Distortional Post-Buckling Behavior (FWP Columns)

In this section, one investigates the post-buckling behavior of stiffened columns with fixed and warping prevented (FWP) end sections and, in order to enable a more fruitful comparison with the experimental and numerical results reported by Yang & Hancock (2003) and Yang (2004), the same cross-section shape and dimensions are considered (identified by these authors as "*LC2000*"– long columns: $L=2000\ mm$). Moreover, these stiffened columns have been fabricated from cold-reduced high-strength steel ($E=220\ GPa$ and $\nu=0.3$) of thickness $0.42\ mm$. Initially, a GBT linear stability analysis was performed, which showed (confirmed) that these columns buckle for $\sigma_{cr.D}=66\ MPa$, in single-wave distortional modes. Concerning the post-buckling behavior, the two FWP columns analyzed contained initial geometrical imperfections defined by $v_0=+t$ and $v_0=-t$ (*outward* and *inward* lip motions). The corresponding post-buckling equilibrium paths, obtained by means of GBT and FEM-based analyses, are depicted in figures 10(a) ($\sigma/\sigma_{cr.D}$ vs. v, where v is the lip transverse membrane displacement at mid-span) and 10(b) ($\sigma/\sigma_{cr.D}$ vs. u, where u is the column axial shortening). Regarding these post-buckling equilibrium paths, the following remarks are appropriate:

(i) The GBT and FEM (ABAQUS) results virtually coincide again.

(ii) Like their PFW counterparts, the FWP columns also exhibit an *asymmetric* distortional post-buckling behavior. However, if one compares the curves shown in figures 7(a) and 10(a), it becomes clear that this asymmetry is now much less pronounced. This fact stems, to a large extent, from the very marked difference between the post-buckling slopes of the columns undergoing an *inward* lip motion: while the PFW curve is practically horizontal, the FWP one exhibits non-negligible slope values.

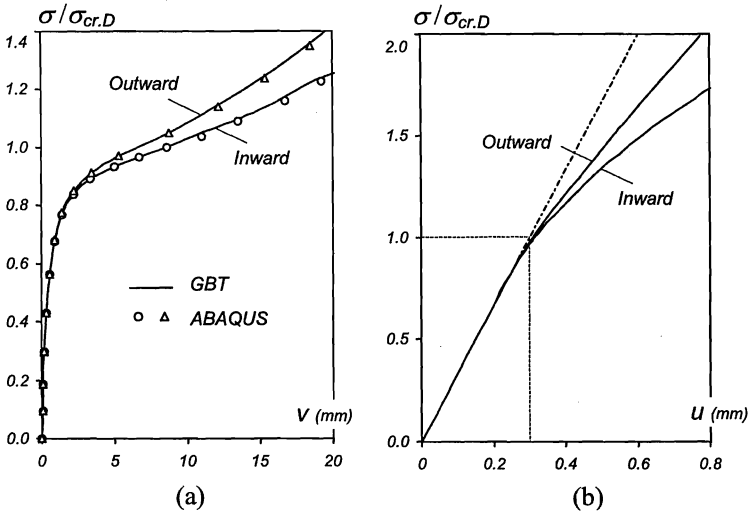


Fig. 10. FWP column distortional post-buckling paths for (a) v and (b) u motion: while the PFW curve is practically horizontal, the FWP one exhibits non-negligible slope values.

(iii) The deformed configuration GBT modal decompositions and mid-span normal stress distributions of the FWP columns follow the same general trend of the PFW ones (they are not presented here).

(iv) The $\sigma/\sigma_{cr.D}$ vs. u equilibrium paths are also asymmetric and it is interesting to note that, unlike its *outward* counterpart, the curve associated with *inward* lip motion is clearly non-linear in the post-buckling range. These equilibrium paths are qualitatively similar to the ones obtained by Yang (2004), through FEM-based elastic-plastic analyses.

(v) The tests carried out and reported by Yang & Hancock (2003) and Yang (2004) provided clear experimental evidence concerning the distortional post-buckling asymmetry addressed here. Moreover, these authors attributed this asymmetric behavior to the presence of interaction effects between local-plate and distortional buckling modes. However, since the local-plate bifurcation stress of this column is $\sigma_{b.LP}=80\ MPa$ (*i.e.*, *23%* above $\sigma_{cr.D}$) and corresponds to a 48-wave buckling mode, this explanation does not seem to be plausible. Indeed, the GBT results presented here show that the aforementioned asymmetry is exclusively due to the distortional post-buckling mechanics, namely the different role played by the shear modes, mostly associated with non-linear warping.

(vi) Yang & Hancock (2003) and Yang (2004) also reported that the specimens containing *inward* and *outward* initial imperfections exhibited distinct elastic-plastic failure modes: while the collapse of the former is triggered by the *web*, the failure of the latter stems from the *lips*. The (elastic) GBT results presented here provide a very logical explanation for these behaviors. Indeed, they showed that the column distortional post-critical strength is controlled (v_1) by the *web stiffener*, for *inward* imperfections, or (v_2) by the *lip* stiffness reduction, for *outward* imperfections.

Conclusion

The results of a GBT-based investigation concerning the local-plate and distortional post-buckling behaviors of cold-formed steel lipped channel columns with web and flange V-shaped intermediate stiffeners were presented and discussed. Both columns with pinned/free-to-warp and fixed/warping-prevented end sections were analyzed and, mostly for validation purposes, several equilibrium paths were also obtained by means of FEM analyses performed in the code ABAQUS and adopting S4 shell elements to model the columns– a virtual coincidence was observed in all cases. From this rather extensive and in-depth study, the following conclusions deserve to be specially mentioned:

(i) In order to obtain accurate GBT-based distortional post-buckling results, it is indispensable to incorporate shear and transverse extension modes into the analysis.

(ii) The distortional post-buckling behavior of lipped channel columns with or without intermediate stiffeners is clearly *asymmetric* with respect to the initial imperfection sign (*inward* or *outward* lip motion). This asymmetry stems mostly from shear deformation effects related to non-linear warping, which completely alter some displacement and/or normal stress post-buckling patterns.

(iii) The distortional post-buckling *asymmetry* of plain and stiffened lipped channel columns is *qualitatively different*: while the former display a stiffer behavior when the initial imperfections involve *inward* lip motion, the latter exhibit a larger post-critical strength if the initial imperfections are related to *outward* lip motion.

(iv) In qualitative terms, the GBT-based results presented in this paper were found to be in excellent agreement with the distortional post-buckling investigations previously reported by (iv_1) Prola (2001) and Prola & Camotim (2002), who employed the spline finite strip method to analyze plain columns, and by (iv_2) Yang & Hancock (2003) and Yang (2004), who carried out experimental and FEM-based analyses of stiffened columns.

(v) By taking advantage of the GBT unique modal features, it was possible to provide an in-depth understanding of the mechanics involved in the distortional post-buckling behavior of plain and stiffened lipped channel columns. In particular, it was possible to shed some new light on the interpretation of the experimental results reported by Yang & Hancock (2003) and Yang (2004).

References

Bernard E.S., Bridge R.Q. and Hancock G.J. (1993). "Tests of profiled steel decks with V-stiffeners", *Journal of Structural Engineering* (ASCE), **119**(8), 2277-2293.

Desmond T.P. (1977). *The Behaviour and Strength of Thin-Walled Compression Members with Longitudinal Stiffeners*, Report No. 369, Department of Structural Engineering, Cornell University.

Hibbit, Karlsson and Sorensen Inc. (2002). *ABAQUS Standard* (Version 6.3-1).

Höglund T. (1978). *Design of Trapezoidal Sheeting Provided with Stiffeners in the Flanges and Webs*, Manus Stockholm, Sweden.

Hoon K.H., Rhodes J. and Seah L.K. (1993). "Tests on intermediately stiffened plate elements and beam compression elements", *Thin-Walled Structures*, **16**(1-4), 111-143.

Kesti J. and Davies J.M. (1999). "Local and distortional buckling of thin-walled short columns", *Thin-Walled Structures*, **34**(2), 115-134.

Kwon Y.B. (1992). *Post-Buckling Behaviour of Thin-Walled Channel Sections*, Ph.D. Thesis, School of Civil and Mining Engineering, University of Sydney.

Lepistö J., Nikula S. and Niemi E. (1996). "Optimum design of cold-formed sections using generalized beam theory", *Coupled Instabilities in Metal Structures* (CIMS'96 – Liège, 5-7/9), J. Rondal, D. Dubina, V. Gioncu (eds.), Imperial College Press, 101-108.

Narayanan S. and Mahendran M. (2003). "Ultimate capacity of innovative cold-formed steel columns", *Journal of Constructional Steel Research*, **59**(4), 489-508.

Prola L.C. (2001). *Local and Global Stability of Cold-Formed Steel Members*, Ph.D. Thesis, Civil Engineering Department, TU Lisbon. (in Portuguese)

Prola L.C. and Camotim D. (2002). "On the distortional post-buckling behavior of cold-formed lipped channel steel columns". *Proceedings of SSRC Annual Stability Conference*, (Seattle, 24-27/4), 571-590.

Schardt R. (1994a). "Generalized beam theory – an adequate method for coupled stability problems", *Thin-Walled Structures*, **19**(2-4), 161-180.

Silvestre N. (2004). *Thin-Walled Member Analysis Using Generalized Beam Theory*, Ph.D. Thesis, Civil Engineering Department, TU Lisbon. (in Portuguese)

Silvestre N. and Camotim D. (2003). "Non linear generalised beam theory for cold-formed steel members". *International Journal of Structural Stability and Dynamics*, **3**(4), 461-490.

Silvestre N. and Camotim D. (2004). "GBT distortional post-buckling analysis of cold-formed steel lipped channel columns and beams", *Book of Abstracts of 17th ASCE Engineering Mechanics Conference* (EM2004 – Newark, 14-16/7) and full CD-ROM paper. (in press)

Teter A. and Kolakowsky Z. (1996). "Interactive buckling of thin-walled open elastic beam-columns with intermediate stiffeners", *International Journal of Solids and Structures*, **33**(3), 315-330.

Teter A. and Kolakowsky Z. (2004). "Interactive buckling and load carrying capacity of thin-walled beam-columns with intermediate stiffeners", *Thin-Walled Structures*, **42**(2), 211-254.

Ville de Goyet V. (1995). "An advanced nonlinear spatial finite element", *Structural Stability and Design*, (Sydney, 30/10-01/11), S. Kitipornchai, G. Hancock, M. Bradford (eds.), Balkema, 249-254.

Yang D. (2004). *Compression Stability of High Strength Steel Sections with Low Strain-Hardening*, Ph.D. Thesis, School of Civil and Mining Engineering, University of Sidney.

Yang D. and Hancock G. (2003). "Compression tests of cold-reduced high

strength steel channel columns failing in the distortional mode", *Advances in Structures* (ASSCCA'03 – Sydney, 23-25/6), G. Hancock, M. Bradford, T. Wilkinson, B. Uy, K. Rasmussen (eds.), Balkema, 303-308.
Young B. and Yan J. (2002). "Finite element analysis and design of fixed-ended plain channel columns", *Finite Elements in Analysis and Design*, **38**(6), 549-566.