1898

Hydraulic air compressors

Euart Carl Torrence

THESIS

for the degree of

B. S.

in Civil Engineering.

1898.

E. C. Torrence.

HYDRAULIC AIR COMPRESSORS.

The rapid increase in the application of compressed air is manifested by the many contrivances for converting various forms of power into that represented by compressed air. The chief method of utilizing the power of a water-fall to compress air has been by an air compressor driven by a water-wheel. This is an exceedingly wasteful method as the quantity of air compressed depends on the effeciency of the compressor and the effeciency of the water-wheel, after deductung the loss due to the friction in the connections. This loss will be considerable; as the water-wheels usually run at a high speed and the compressor at a low speed there will be a great loss of energy in transforming from the high speed of the water-wheel to the low speed of the compressor. Suppose the effeciency of the water-wheel is 75%, and neglect the loss in connections-these being indeterminate - then 75% of the total energy of the water-fall is delivered to the compressor. It requires about one-half the work to compress air isothermally that it does to compress it to an equal volume adiabatically; while the water jacket removes about one-half the heat due to compression. This means that, neglecting all losses in the compressor due to friction, motion of the reciprocating parts, &. the efficiency of the compressor will be considerably below unity. Suppose the efficiency of the compressor is 70% (See Frank Richard's "Compressed Air", page 97). Neglecting the loss occuring in the connections the water-wheel delivers 75% of the total energy of the water-fall to the compressor, and the compressor delivers 70% of this to the compressed air chamber. This means that only 52.5% of the energy of the water-fall is utilized as an air compressing plant. As it requires about twice as much work to compress adiabatically ~~that~~ as it does isothermally the efficiency of the plant as a power transformer cannot exceed 26.25%, but we have neglected one great source of loss, the loss of friction of the connections necessary to transform the high speed of the water-wheel to the low speed of the compressor. We would be safe in saying that the actual efficiency of such a plant as a power transformer could not possibly exceed 20% and would ususally be much less.

If we could compress the air as small bubbles in contact with the water, and without the use of a water-wheel, we would avoid the three great sources of loss- the loss in the water-wheel, the loss in the compressor, and the loss in the connections. The chief method devised to accomplish this is shown in figure (1), known as the "Frizzell System."

In this system the water passes down the pipe A.B; A plate FG prevents direct contact of the water at the top of the pipe with the air; through FG a number of small air pipes pass. The water passing down the pipe causes a relief of pressure at the top and thus acting as an asperator, draws the air through the small pipes and carries it down as bubbles. These bubbles are carried down the pipe by the water with a continual increase of pressure, to the air chamber C. The pressure here is represented by the ~~hydraulic~~ hydrostatic head of the tail water ED. This system avoids all the sources of loss occuring in the water-wheel and compressor system, but introduces some new sources of loss which it is now proposed to investigate..

There are three great sources of loss: (1) the slip of the bubbles;(2) friction of water in pipes;

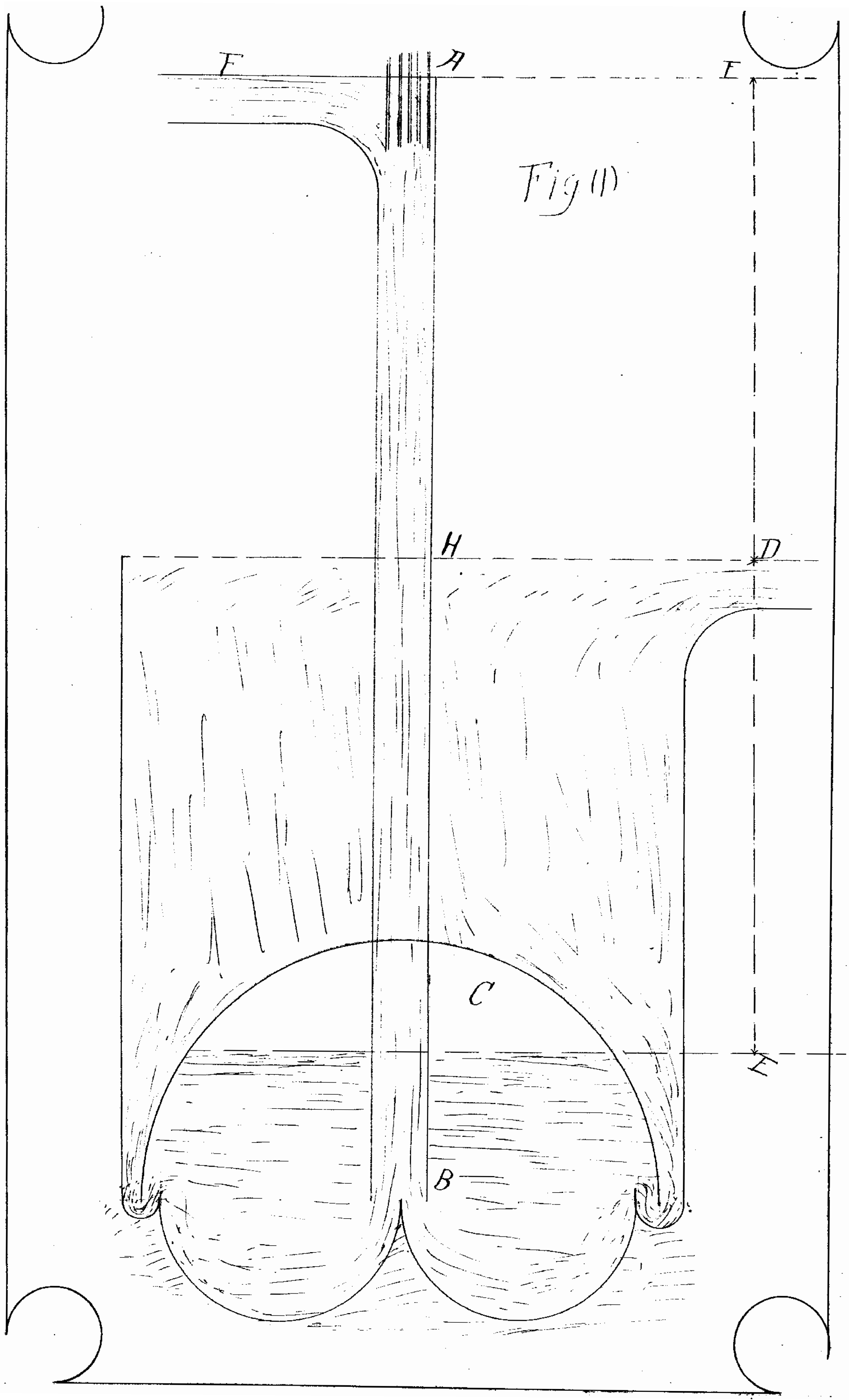


Fig (1)

(3)velocity of discharge at B. All the bubbles entering will not be of the same size; and the larger the bubbles the greater will be its upward velocity relative to the water. As the large bubbles move upward, relative to the water, faster than the small ones they will soon begin catching their little brothers and swallowing them. This cannibalistic tendency of the big brothers causes them to increase in size with a corresponding increase of strength and appetite; and upward they bound in their watery element in quest of prey-_ swallowing their kindred or by their kindred swallowed. Thus the largest bubbles will evidently occur at the point of discharge B, notwithstanding the compression that will occur. This means that the water at B must have a greater velocity than at any other point if the bubbles are to descend with a uniform velocity. The pipe AB will usually be of uniform crossection, so the above is a condition that need not be considered. The efficiency of the plant will vary directly as the quantity of air compressed. The quantity of air compressed will therefore be a function of the velocity with which the water passes down the pipe. The problem is now to find the velocity of the water that will give the maximum efficiency.

Let U= mean velocity of the bubbles relative to the water (to be determined by experiement).

Let L= length of pipe AB.

" Pn=Pressure per sq.ft. at the top of the pipe.

" Pm= " per sq.ft. " " foot " " "

" Vn= Volume of air at pressure Pm taken in per sec., measured in Cu.ft. Pn ?

" V =mean velocity of the water. As the volume of air carried down per sec. will be very small compared to volume of water, the velocity may be taken as constant.

Let W =weight of water psssing per sec.

" Vs= Mean volume of air.

" Pr= Pressure when volume of air is a mean.

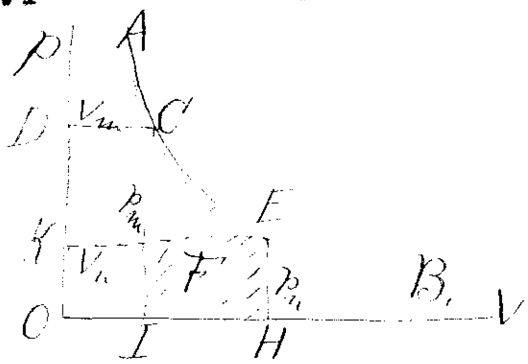
" F= Coefficient of friction.

" H = height of water fall.

$\frac{1}{V-u}$ = Time required for a bubble to descend the pipe

Before beginning the solution it will be necessary to find the value of P_s in terms of Pn and Pm, i.e. to find the hyperbolic mean of two numbers.

Fig.(2).



Let AB be an hyperbola, the axis P and V its asymptotes, fig.(2)

Area F = Pv log. $\frac{Pm}{Pn}$

$$P_s = \frac{PV \log \frac{Pm}{Pn}}{Vn-Vm} \text{------------------------------(1)}$$

The area DCEK = area ICEH.

$$\therefore V_s = \frac{PV \log \frac{Pm}{Pn}}{Pm-Pn} \text{------------------------------(2)}$$

The above equations are true for all values of P and V; in each eqaution let P and V have the values Pm and Vm.

From (1)

$$P_s = \frac{Pm \log \frac{Pm}{Pn}}{1-\frac{Pn}{Pm}} \text{------------------------------(3)}$$

From (2)

$$V_s = \frac{Vm \log \frac{Pm}{Pn}}{1-\frac{Pn}{Pm}} \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(4)}$$

Suppose that in (2) $V=V_s$ and P =Pr, the simultaneous value of P.

$Pr= \frac{Pm-Pn}{\log_e \frac{Pm}{Pn}}$ --(5)

This is the value of P that makes V a mean.

62.5 V_s= average force urging the bubbles up.

$\frac{ul}{V-u}$ =the distance through which the above force acts per sec.

62.5 $V_s \frac{ul}{V-U}$ = work done per sec. by slip of the bubbles.

$=K_1 V_s \frac{1}{V-u}$ ----------------------------(6)

$\frac{wv^2}{2g}$= kinetic energy in the water at discharge.

$= K_2V^2$ ---(7)

$\frac{FlV^2W}{2gd}$ = energy lost in overcoming friction;

d= diameter of the pipe.

? The quantity of water discharged per sec.$=\frac{W}{62.5}$ cu.ft.

$\therefore \frac{W}{62.5} = 1/4\, \pi\, d^2\, V.$

or $d = \sqrt{\frac{4w}{62.5\, \pi V}}$

Substituing this value of d in the above we obtain

$\frac{F\, L\, V^2 \sqrt{W\, \pi 62.5}}{4g} = K_3V^{\frac{5}{2}}$ -------------------------(8)

The actual amount of work done on the air per sec. is equal to the total energy per sec. of the water fall minus these losses.

P V log $\frac{Pm}{Pn}$= work done on the air per sec.

But when V is a mean, V_s,P has a value P_r(See 5).

$\therefore \frac{(Pm-Pn)}{\log \frac{Pm}{Pn}}$ Vs log $\frac{Pm}{Pn}$ =(Pm-Pn)V_s= Work done on the air.

$=(Pm-Pn)V_s=K_4V_s$------------------------(9)

$\therefore K_4V_s=wh-K_1V_s\frac{1}{V-u} -K_2V^2-K_3V^{\frac{5}{2}}$

$V_s = \frac{(wh-K_2V^2-K_3V^{\frac{5}{2}})\,(V-u)}{K_4(V-u)+K_1}$ ---------------(10)

? Since the work done in compressing the air is a maximum when the ~~value~~ volume of the air compressed is a maximum, we may equate the first derivative of (10) to zero and solve for the value of V that will make V_s a maximum (~~See page~~

K_1=62.5 U L.

$K_2=\frac{W}{2g}$

$K_3=\frac{Fl\ 62.5\ \pi W}{4g}$

K_4 =(Pm-Pn)

Perform ing the above operation on (10) we obtain (11) under any given set of conditions all quantities in (11), except v, become known constants. The value of V which will make (11) equal to zero is the velocity in feet per sec. that will give the maximum effeciency under the given conditions. *

$$\frac{dV_s}{dV}=\sqrt{\frac{\left\{K_4(v-u)+K_1\right\}\left\{(v-u)(-2K_2V-\tfrac{5}{2}K_3V^{3/2})+(wh-K_2V^2-K_3V^{5/2})\right\}-(wh-K_2V^2-K_3V^{5/2})(v-u)K_4}{\left[K_4(v-u)+K_1\right]^2}}$$

$$0=\left\{K_4(V-U)+K_1\right\}\left\{(v-u)(-2K_2V-\tfrac{5}{2}K_3V^{3/2})\right\}+K_1(wh-K_2V^2-K_3V^{5/2})\text{---}(11)$$

Suppose we have a fall of 20 ft discharging 20 Cu.ft. of water per sec.,and wish to compress air to 60lbs.per sq.in,gauge pressure-74.7 lbs absolute. Suppose the air is taken in at 14 lbs. pressure, absolute. To give a pressure of 60 lbs. gauge will require the air chamber to be 138.24' ft. below tail water,i.e.

DE= 138.24 ft.(See fig.(1)).

L= 158:24= total lenght of pipe AB

Pn =2016 lbs. per sq.ft.

Pm.= 10756.8 lbs. per sq. Ft.

W= 1250 lbs.

Suppose u=5ft.per sec.

f=.02.

K_1=49450

K_2= 19.53

K_3=12.25

* The radical does not belong to the equation.

K_4=8740.8

Substituting these values in (11) gives (12)

$$0=\left\{8740.8(v-5)+49450\right\}\left\{(v-5)(-39.06V-30.625V^{\frac{3}{2}})\right\}+49450(25000-19.53v^{2}-12.25V^{\frac{5}{2}}) \quad \text{------------------------------}(12)$$

This equation changes sign for some value of V between 10.8 and 10.7; therefore the velocity that will give the maximum efficiency lies between 10.8 and 10.7ft. per sec. Let us take V=10.7 ft. per sec. substituting this value of V in(10) we obtain the volume of air compressed when pressure gives the mean volume.

V_s=1.0434 Cu.ft. per sec.

Substituting these values of V and V_s in (6),(7), and (8) we obtain the energy lost per sec.

Work by slip of the bubbles per sec.=9051.95 ft. lbs.

Work done against friction----------=4586.18 " "

Kinetic energy in water at discharge=2235.99 " "

Total work lost per sec.------------=15874.12

The total energy of the water fall is wh ft.lbs.per sec. =25,000 ft. lbs. per sec.

25000-15874.12= 9120.15

The plant thus only utilizes 9120.15 ft. lbs.per sec. as an air compressing plant. To check this result we will take the volume of air compressed and let it expand isothermally.

$$PV \log \frac{Pm}{Pn}=9120.15 \text{ ft. lbs per sec.}$$

The two results,obtained from independent sources, check within5.73 ft.lbs. . In the solution very large numbers have been used and we have not carried the decimalspast the second and third place,and this has probably caused the small difference. This gives the actual work done on the air in compressing it from 14 lbs. per sq.in.to 74.7 lbs. per sq.in..The efficient work as an air compressing plant will be that which we can obtain by expanding the air from 74.7lbs per sq.in to 14.7 lbs. per sq.in.

$PV \log_e \frac{74.7}{14.7} = 8852.6$ ft. lbs. The efficient work as an air compressing plant.

$\frac{25000}{8852.6} = 28.3\%$ efficiency.

Unless energy is supplied to this air from some external source, all the work we can actually get from it will be by adaibatic expansion from 74.7lbs. to 14.7lbs. As the work of adiabatic expansion is about one half that of isothermal expansion, the efficiency of the plant as a power transformer will be about 14.15% In this we have only taken U as 5ft. per sec., being unable to find any data on the velocity of bubbles in water, it will probably be greater; while we have taken F equal to .02, and it will be greater owing to the breaking of continous flow of the water and the formation of eddies.

In the Frizzell System we have found three great sources of loss: (1) that due to the slip of the bubbles (2) kinetic energy in the water due to the velocity of discharge; (3) work necessary to overcome friction. If we could design a direct hydraulic air compressor with the air intake at the foot of the pipe, and were able to reduce the velocity to zero during compression we would avoid the losses due to the slip of the bubbles and to the velocity of discharge; while we could reduce the work lost in overcoming friction to a mimimum by making the pipe as large as economy would permit. We will now investigate this problem.

Fig.(3) shows the general arrangement of apparatus A plate A prevents contact of the air and the water at the top of the pipe. At the point D the pipe is contracted until it gives the absolute pressure at

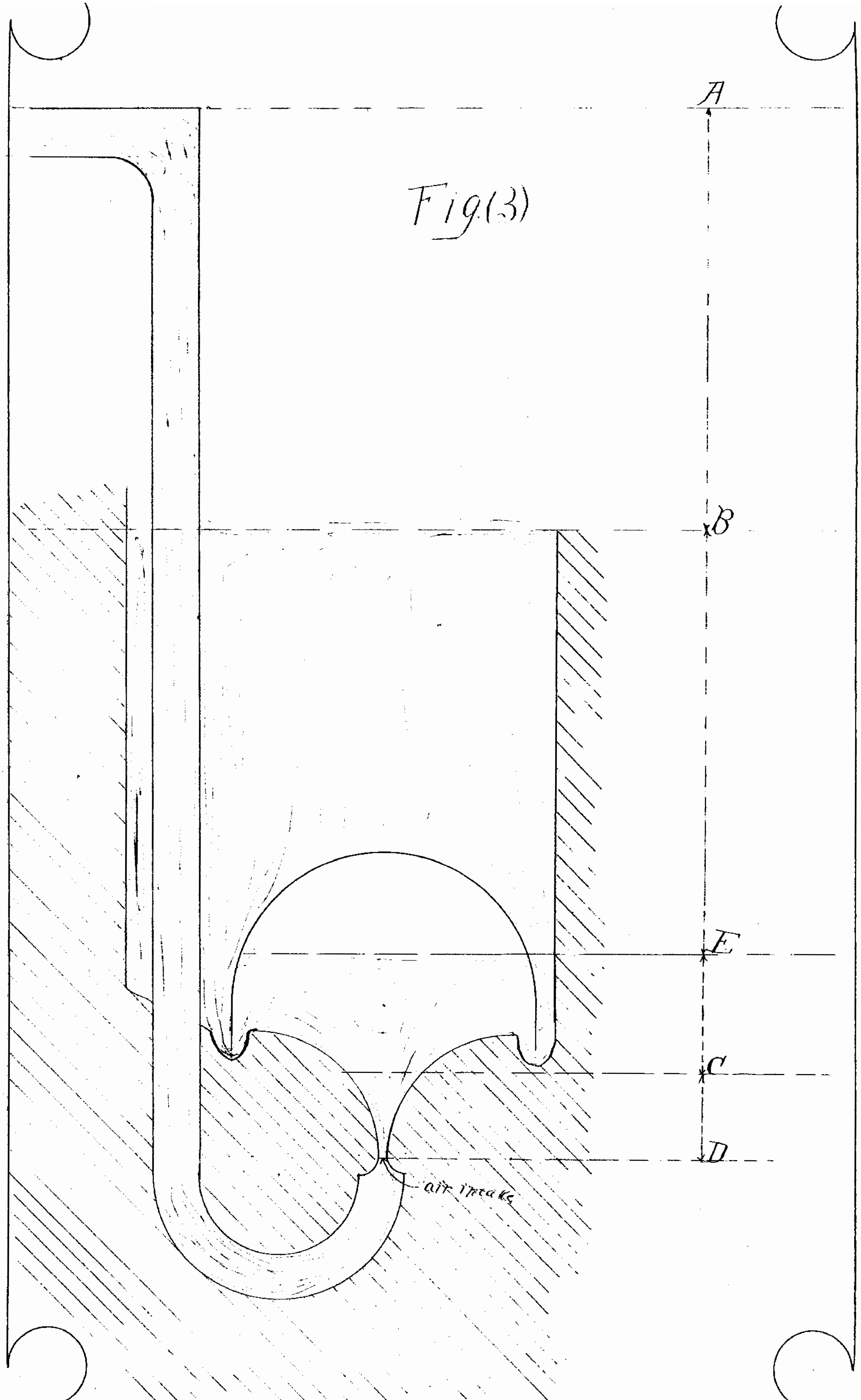


Fig.(3)

which we wish to take in the air in-less than atmospheric
From the point of D the pipe becomes a flaring nozzle whose diameter increases according to some law such that in some leogth DF the diameter becomes equal to infinity At this point the velocity becomes zero and the pressure equal to that due to the hydraulic head of the tail water =FB. Suppose this compression occurs in a nozzle of length DF. It is required to investigate the relation between the velocity and pressure in the nozzle. The only source of loss is the friction in the pipe, and as the pipe may be of any desired diameter this becomes known.

Let. K_1=work done against friction per sec.

" W= weight of water discharged per sec.

" H= height of water fall.

" P_0=absolute pressure at the intake B.

" P_N= " " " any point C.

" P_m= " at F(a known Quantity).

" H_n=DC

" DF= S

" V_0=velocity at D.

" V_n= " " any point C.

As the length of the nozzle, DF, will always be very short we may neglect the loss due to friction in it. Take the datum plane through D The total energy passing D per sec.-due to pressure head, velocity head, and potential head,-after deducting the loss due to friction-is equal to $W(1+34)-K_1=K_2$.

The total amount of work done on the air per sec. in compressing it is equal wh-K_1.

The total amount of energy passing all sections of the nozzle per sec. must be the same; i.e. the power represented by the pressure head, velocity head, potential head, and the isothermal compression of the air must be the same for all sections.

$W\frac{V_n^2}{2g}$ =Kinetic energy of the water at any point C.

$W\frac{P_n}{62.5}$ =The power represented by the pressure head at any point C.

$w\,h_n$= potential energy represented by the potential head h_n.

$PV\log_e\frac{P_n}{P_o}$ =the work required to compress the air from a pressure P_o to a pressure P_n of any point C.

The sum of these must be equal to the total power at the point D.

$$K_2 = w\frac{V_n^2}{2g} + w\frac{P_n}{62.5} + wh_n + PV\log\frac{P_n}{P_o} \text{ ------------------ (13)}$$

This equation shows the relation that must exist between the pressure and velocity in the nozzle at any point C. We may assume either of these to vary according to any law and solve for the corresponding values of the other.

Constant? $\Big\{$ $PV\log\frac{P_m}{P_o} = wh_n K_1$ = Constant ----------------------- (14)

From the relation shown in equation (14) we can calculate the volume of air, V_n, for any point C. Knowing the volume of air and the volume of water passing any point per sec., and the velocity at that point, we can calculate the diameter of the nozzle at that point. From a sufficient number of such points we can construct the nozzle in which the pressure, or velocity, will vary according to the assumed law. Suppose the pressure is to increase uniformly from P_o to P_m in the nozzle of length S (See fig. 3).

$\frac{P_m - P_o}{S}$ =increase in pressure per units length.

$$P_n = P_o + \frac{P_m - P_o}{S}H_n = \frac{H_n P_m + (S - H_n)P_o}{S}$$

Substituting this value of P_n in (13) and solving for V_n we get

$$V_n = \sqrt{K_3 - K_4\left\{\frac{H_n P_m + (S - H_n)P_o}{S}\right\} - 2gH_n - K_5\log\left\{\frac{P_o}{\frac{H_n P_m + (S - H_n)P_o}{S}}\right\}} \text{ (15)}$$

where $K_3 = \frac{2g\,k_2}{w}$

where $K_4 = \frac{2g}{62.5}$

" $K_5 = \frac{2g\ PV}{W}$

Substituting in this equation any number of values of h_n we obtain the corresponding values of V_n. From the total volume of air and water passing the point we can calculate the diameter of the nozzle at that point; and with a sufficient number of such points plot the crossection of the nozzle in which the pressure will increase directly as the distance from D.

Suppose we are to have uniform retardation. We must solve for the law of pressure that will give uniform retardation.

Let a = the retardation per sec.

Take the sum of the heads at any point C; the datum plane being taken through D.

$V_n = \sqrt{2a(S-H_n)}$.

$$\frac{V_0^2}{2g} + \frac{P_0}{62.5} = \frac{2a(s-h_n)}{2g} + \frac{P_n}{62.5} + H_n$$

$$P_n = \frac{62.5}{2g} V_0 + P_0 - 62.5h_n - 62.5\frac{a}{g}(S-H_n) \text{------------------} (16)$$

This equation gives the pressure at any point in the nozzle if there is uniform retardation. Substituting this value of P_n in equation (13) and solving for V_n we get equation (17). in which K3, K_4, and K_5 have the same values they have in equation (15).

$$V_n = \sqrt{K_3\ K_4\left\{\frac{62.5}{2g}V_0^2 + P_0\ 62.5\ h_n - \frac{62.5}{g}(s-h_n)\right\} - 2gh_n - K_5 \log \frac{\frac{62.5}{2g}V_0 + P_0 - 62.5\ h_n - \frac{62.5a}{g}(s-h_n)}{P_0}} \text{----------------} (17)$$

Treating equation (17) similarly to (15) we obtain the crossection of the nozzle in which there is uniform retardation.

Let us illustrate the advantage of this over the "Frizzell System." Let us take the same case we took before, and let the pipe be three feet in diameter.

$f = .02$

$w \frac{f\ lV}{2gd}$ = work lost in obercoming friction.

= 998.75 ft.lbs per sec.

This is the chief source of loss, and the only one that can be calculated. If we take the last case considered uniform retardation, we would surely be safe in saying that not over 5% could possibly be lost in the nozzle. The total power of the water fall is 25000 ft.lbs. per sec.. This allows 1250 ft.lbs per sec. to be lost in the nozzle; a total loss of 2248.75 ft.lbs. per sec. This leaves 22751.25 ft. lbs. per sec. as the work done on the air. The air is taken in at 14 lbs. per sq.in. pressure and compress it to 74.7 lbs. per sq.in.

$\therefore PV \log \frac{P_m}{P_o} = 22751.25$

or $144 \times 74.7\ V_m \log \frac{P_m}{P_o} = 22751.25$

$V_m = 1.261$ cu.ft. of air per sec.

The efficient work as in air compressor will be represented by expanding this isothermally from 74.7 to 14.7 lbs. per sq.in.

$144 \times 74.7 \times 1.261 \times \log \frac{74.7}{14.7} = 22046.09$

This gives the efficient work of the plant as an air compressor.

$\frac{22046.09}{25000} = 88.2\%$ efficiency.

This on the assumption that 5% is lost in the nozzle, which is surely in excess of what would actually occur. ? Substituting $H_n = 0$ in (15) or (17) we obtain the velocity when $P_n = P_0$, and knowing the volume

of water passing the section, we can calculate the diameter of the nozzle at the intake that will give a pressure P_0.