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# Derivation of some empirical formulae, showing relation between breaking strength and various functions which enter into cement testing

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DERIVATION OF SOME EMPIRICAL FORMULAE, SHOWING RELATION BETWEEN BREAKING STRENGTH AND VARIOUS FUNCTIONS WHICH ENTER INTO CEMENT TESTING.

Although we find a great many results of experimental work in cement testing, and some curves plotted, there seems to have been very little attempt made toward fitting equations to these curves. This then leaves an opening for original investigation.

During my experiments quite a variety of interesting points have been made manifest. Most of them are of no practical benefit.but merely of interest in themselves.

My work has been on a few of the most important ones, which will be given subsequently. On these and a few others lies the foundation of cement testing, as given by all writers on the subject.

The tensile strength seems to be of primary importance rather than crushing, so my work has been to derive with experimental data

the relation existing between breaking strength in pounds per square inch, and other functions which vary according to conditions given in each experiment.

#### $-APPARATUS -$

Following is a list of apparatus used, and a brieff description of the more important ones:

> 1 Moulds 2 Testing Machine 3 Sieves 4 Others of minor importance, such as:

thermometer, scales, etc.

Moulds used were Tinius Olsen & Co's "Standard Type". The testing machine was of Rheile's make.

 $(1)$ 

Sieves. They are of "standard mesh". Used to get various sizes· of sand and also in testing fineness of the cement.

#### $-MATERIAT-$

The materials for my work were sand and cement.

The cement was a good portland sement. It was taken from the barrel in sufficient quantity to make a series of one hundred to one hundred and fifty briquettes. This was placed in a box and thoroughly mixed. The object of mixing being to have all the material in each briquette as nearly an average as possible. For instance, if the cement was used directly out of the barrel, we might get cement of quite varying quantities for different briquettes, due to conditions of packing the cement and also of the differenee between center of barrel and outer surface.

#### $-SA$ HD $-$

The sand was not as good a quality as was desired, but best at hand. It was all well washed before using.

The sand was not analyzed, but apparently had some material in it, such as churt, which injures its quality as a material for making cement mortars.

#### $-$ PRELIMINARY TESTS-

Fineness: Cements depend greatly upon fineness of grinding for some of their good qualities, therefore, the test for fineness is generally made.

The steves used for the test of fineness were: viz. No. 50 (2500 meshes to the square inch): No. 80 (6400 meshes to the square inch): No.100 (10000 meshes to the square inch).

In the particular cement used in the following experiments, the average of several results were:-

Percentage rejected by a No. 50 sieve,  $.34\%$ ; percent that passed through a No. 50 and was rejected by No.80,4.40%; percentage that

(2)

passed through a No. 80 and was rejected by No.lOO, 7.5%.

#### $-$ SOUNDNESS $-$

This test was made by usual method. The pat made from the cement used gave no signs of gracking or bulging.

#### - MORTARS-

 $\gamma_1$ ,  $\gamma_2$  and and cement  $\frac{1}{2}$  weighed, instead of taking parts by volume.

After weighing out amount desired for condition of each mortar, the sand and cement were thoroughly mixed before being made into briquettes. The proportion a sed will be given in the details of experiments.

#### -EXPERIMENTS-

The experiments carried out will first be taken up in a general way.

The object of each experiment was to find, if possible, a relation between breaking strength and some other one of the varying functions, keeping all other conditions constant.

The temperature of the water and room was recorded before mixing a batch of briquettes.

#### EXPERIMENT NO 1.

This experiment was to find a relation between tensile strength per square inch and time after mixing.

The method followed in this case was to mix the cument, with amount of water. Quantity of cement and temperature of room and water constant.

The amount of water used was 375 grs to 1500 grs of cement, or  $25%$ 

In order to get a good average, the cenent was made up in batches of ten briquettes each,each batch requiring about 1500 grs. Ten of these batches were made. After being made,they were allowed to set in open air for 24 hours, then they were placed under water

to remain until time of breaking.

In breaking I took one from each batch at the end of a glven time after mixing. To explain more fully, say, I made up batches A. B.C etc. at different times. At the end of ten days after mixing each batch respectively,suppose a briquette from each batch is broken, then this would give the data from the point at end of ten days,etc.

These average results will be recorded in the discussions given on the curve.

#### EXPERIMENT NO 2.

The object of this experiment was the determination of relation between breaking strength per square inch and time after mixing of a 1 to 1 mortar, all other conditions being constant as in experiment No.1.

Sand used for the above was all that passed through a  $No. 30$ sieve and remained on No.40.

These briquettes were mixed as in previous experiment. The sand and cement were weighed separately and then thoroughly mixed before being moulded.

This time I had an average of seven instead of ten, as there was a sudden change in temperature after last batches were made up and before they set,so I discarded them,as they were not ready to be put in water at the end of 24 hours. Even with the average of seven briquettes the results plotted nicely.

A 1 to 1 mortar was used but it seems rational to suppose that the conditions for any other mortar ought to follow the same law, the difference being the value of the 80nstants involved in the equation.

#### EXPERIMENT NO 3.

Here the ratio of constituents of the mortar varied,all other

(4)

things being constant.. The mortar varied from a neat cement to a

4 to 1.

There were thirteen points in all for this curve. The method of mixing and breaking was as follows: In order to get best average, the lots were made up one briquette at a time,making conditions for the average the same as in  $No.1$  and  $No.2.$ 

The temperature of room, water, the quantity of water etc. were noted. The quantity of water was about 25% for neat, decreasing as more sand was used.

#### EXPERIMENT NO 4.

Size of sand and breaking strength were the varying functions in this experiment. A 1 to 1 mortar was used throughout.

After sand reached a fairly large size, it run into something like churt and gravel, hence a neat curve was not expected of the result of this experiment.

The mixing etc. was similar to that of experiment No.3. The number of grades of sand used was seven.

#### EXPERIMENT NO 5.

Among other functions ,which' enter breaking strength is amount of water used in mixing.

This experiment necessitates the batches being made up without any loss of water. The moulds rested on Glass as in other experiments and were sealed around the edge on lower side with white lead.

Two different relations may be determined by this experiment. First, relation between breaking strength and amount of water used. Second, relation between area of cross section of briquettes and water used.

In the first case the briquettes that were made up with a very large amount of water were short in crossection, but in plotting they were all reduced to the basis of one square inch.

DISCUSS OF CURVE NO 1.

The curve on Plate 1 was plotted from data on (page 11) Strength per square inch being plotted as ordinates and time as abscissae. A curve drawn through these points resembles that of an equlateral hyperbala, referred to  $\underline{x} \underline{y}$  as  $ax \underline{y}$ s.

The curve is shown in figure 1 and all reference will be made  $\frac{1}{2}$ to it during derivation of the equation.



Let us assume that the equation referred to  $\underline{X} \underline{Y}$  axis is:-

c being a constant.

Let  $s = 0.5$  = the average strength in pounds per square inch after mixing.

 $t = 0$  k = time in days after mixing, before the briquettes were broken.

 $a = D$  H = distance of x' axis from X axis.

 $\mathbf{a} = \mathbf{F} \mathbf{B} = \mathbf{m} \mathbf{m} \mathbf{y} \mathbf{m} \mathbf{m} \mathbf{y} \mathbf{m}$ 

s' average strength of ten briquettes at one day after mixing.  $t^*$ =time of breaking of first batch after mixing.

As the curve does not pass through the origin 0, we can not deal with it readily unless we take some known point,such as <sup>B</sup> for reference. The coordinates of this point being  $s^*$  and  $t^*$ .

Let us pass the  $x'$  and  $y'$  axis through B and use it as all origin, transferring the equation to these new axes.

The values of x and y for any variable point on the curve such as G are:

> $x = (b + t - t')$  $y = (a-s + s')$

substituting these in equation (1) we have

 $(bt-t')$   $(a-s-s') = c$  ------------------------------- (2).

The value of  $xy = c$  is (ab) for the point B. Therefore equation  $(2)$  becomes:-

$$
\begin{bmatrix} b + (t - t^{\dagger}) \end{bmatrix} \begin{bmatrix} a - (s - s^{\dagger}) \end{bmatrix} = ab \begin{bmatrix} a - (s - s^{\dagger}) \end{bmatrix}
$$
  
\nMultiplying out,  
\n $ab - b (s - s^{\dagger}) + a (t - t^{\dagger}) - [t - t^{\dagger}) (s - s^{\dagger}) = ab$ .  
\n0ancelling out the ab's and collecting,  
\n $(t - t^{\dagger}) (a - (s - s^{\dagger}) = b (s - s^{\dagger})$   
\nDividing through by  $(t - t^{\dagger})$   
\n $a - (s - s^{\dagger}) = \frac{b (s - s^{\dagger})}{a (t - t^{\dagger})}$ 

Dividing next by (s-s')

 $\frac{a}{\sqrt{2}(s-s')}$   $-\frac{s-s'}{D(s-s')}$   $=\frac{1}{t-t'}$ 

 $or$ 

 $\frac{a}{b}(\frac{1}{5-5})-\frac{1}{b}=\frac{1}{5-t}$ Dividing (4) by  $a/b$  we have  $\left(\frac{1}{s-s'}\right) = \frac{b}{a} \left(\frac{1}{t-t'}\right) + \frac{1}{a}$ 

 $(9)$ 

Equation (5) is in the general form for a straight line,  $\frac{h}{a}$ being the slope and  $1/a$  the intercept on  $1/s-s$ ' axis.

If we then plot  $1/s-s$ ' as ordinates and  $1/t-t$ ' as abscinsae we will get a strathat line if our assumption is true.

The points plotted from values of  $\frac{1}{5-5}$ ,  $\pm \frac{1}{5-t}$  (given in Table of Data for this curve) are shown on Plate 1.

We can see from this plot that a line will apparently satisfy the points.

Let us next investigate the meaning of the constants involved in this equation.

From the intercept on the  $\frac{l}{\delta-\delta}$  axis the value of  $1/a$  taken from the plot is.0016 from which  $a = 625$ .

Likewise b/a  $\equiv \tan \alpha$  taken from the plot is =  $\frac{\partial \partial \phi}{\partial \partial \theta} = \frac{\partial \partial I}{\partial \alpha} = \rho \partial \theta$ From the value of  $\underline{a} = 625$  and  $b/a$  ton  $\alpha = \frac{0.0}{2}$  we find  $b = a$  ton  $\alpha$ 3.125.

s' and t' are the other constants that enter but whose value are given in the Tables.

Having these constants determined we may simplify equation (5) by putting in thier values.

S' = 73.4  
\nt' = 1  
\na = 625.  
\nb = 3.125  
\n(5) then becomes 
$$
\frac{1}{5-73.4} = \frac{3.25}{625} \left(\frac{1}{t-1}\right) + \frac{1}{625}
$$
  
\nor  $S = \frac{698.4t-469}{t+3.120}$ 

*<sup>l</sup> <sup>i</sup>* <sup>I</sup> tI By making s=0 we get  $t=.67$  dys = 14 hours 7.4' which shows from calculation and also from the plot of the curve that there is <sup>a</sup> certain length of time after mixing before cement acquires any strength.

This being the case it seems rational to call this  $(t)/$  at which the cement begins to show strength, the time of setting. Calling this the point at which the cement sets, we then have a new means by which time of setting may be determined.

The equation (5) is a general one for the cement used in experiment No.1 and probably can be applied to any neat cement. The constants, however, varying for each case in hand.

<sup>I</sup> should like to have had more data in order that the results of my experiment might be fully verified. It is to be hoped and seems most likely that equation (6) is a law rather than just a relation existing between breaking strength and time.

One of the most favoring features of equation  $(6)$  is that the strength approaches a finite limit at an infinite time.

In the present discussion this finite value is =  $(a+s') = 625 +$  $73.4-698.4$  lbs. per square inch.

This result is a very rational one for the curve approaches very nearly a horizontal line after,comparitively short time.





#### Discussion of (Curve No. 2.

This curve is shown on Plate 11 table-page14, and when plotted it had the appearance of being of the same type as curve No. 1.

This ju stified the assumption of a similar equation to that of last case, the derivation of which need not be repeated.

If we then use equation(5) of experiment No.1. we will get another straight line on plotting  $\frac{1}{\delta-5}$ / +  $\frac{1}{\epsilon-1}$ 'as before if our second assumption is correct. The values of these quantities being given in Table 11.

The result of this plot is whown on Plate 11. A straight line apparently satisfies all the points. This being the case our assumption in experiment No.1 holds good for experiment No.2.

The symbols s, t, &c, involved in the following discussion, are strength, time etc. as before. The equation then is  $\left\langle \right\rangle$ 

$$
\frac{1}{s-s'} = \frac{t}{s} \left( \frac{1}{t-t'} \right) + \frac{1}{a} \cdots - \cdots - \cdots - \cdots
$$

The constants of this equation are determined by same method as used in previous discussion.

 $1/a$  the intercept on  $1/s-s'$  axis: *oulf* from which a =540.5.

 $b/a = tcm$  B = to the slope of the line 100.5

Combining the values of (a &  $b/a$ ) we findb=5.43.

 $A + S'$  gives the strength that the briquettes would reach at an infinite time.  $a + s'$  being the distance from X axis to X axis. X axis from our assumption being one asymptote of the curve.

This is also shown directly from the equation by making  $t = \infty$ thus' $-$ 

 $\frac{1}{1-\overline{s}}$ , =  $\frac{b}{a}$  $\left(\frac{1}{t-t'}\right)$  +  $\frac{1}{a}$  $b\hat{c}$  omes from which  $\frac{1}{s-s'} = \sigma + \frac{1}{\alpha}$  $\sim$  or s-a  $+$ s' = 17.7  $+$  540.5 = 558.2 To determine the other asymptote which is axis Y may be done in

either of two ways. First, by knowing the value of b & t' taken from

#### $(12)$

the plot? Second by making  $s = \infty$  in the equation of the curve.

The equation then becomes

$$
\frac{1}{\infty - 5'} = \frac{b}{\alpha} \left( \frac{1}{t - t'} \right) + \frac{1}{\alpha}
$$

 $rr 0 = [f(\frac{1}{1-t})r]$ Dividing through by  $1/\mathbf{a}$  and multiplying out gives:-

#### $t-t' - b = 1 - 5.43 - 4.43$

The curve fully drawn shows that for any time after the curve crosses the axis X that the strength is negative but has apparently no physical meaning.

The equation in its reduced form obtained by the substitution of the known values  $\bullet$  the constants is :-

 $S = \frac{1032.7t - 854.7}{1.85t + 8.1}$  --------------(2).

t becomes equal to 19 hours-55 min. which By making  $s = 0$ shows that the same thing for this experiment, that was shown in experiment No.  $(1)$ . viz:- that there is some certain time elapses

after mixing before the cement acquires any tensile strength.

This gives a means of determining the time of setting of cement mortars (allowing this time to be what we call time of setting) which cannot be satisfactorily done by means of the arbitrary methods used for neat cements, the most common being the wire needles.

It will be noticed that the value of the time, when  $s=0$  is greater in this experiment than in expermment No.1. This seems rational from the experimental fact which is shown throughout, viz:that it takes longer for a briquette of mortar to reach a given strength than one of neat cement.

As the proportion of the mortar increases, the time of setting apparently increases. This then gives us another interesting point of investigation, viz; to determine the relation that exists be tween amount of sand used and time of setting.



 $T$ able  $T$ <br>For Curve No. 2.



#### (15) DISCUSSION OF CURVE NO (3a).

This curve gives a result which seems to justify the assumption  $\overline{\mathscr{C}}_n^{\neq}$  the equation to an equilateral hyperbola referred to X Y as axes.

The strength was plotted as ordinates and proportions of sand as abscissae.

The derivation of the curve was as follows:-



k being a constant.

let sz average strength of ten briquettes in pounds per square inch.

 $p =$  proportion of sand.

a &  $b = distance$  between  $y Y$  & x X axes respectively.  $S' = 0$  F (fig.2) = strength of neat cement.

Let us now take point F for reference.

Let B be any point on the curven whose coordinates are  $x$  &  $y$ referred to  $X \, Y$  axes.

Then  $A B < x \ge p + a$ 

$$
B \otimes = y = s + b
$$

Substituting in equation (1) we have

 $(p + a)$   $(s + b)$   $\mathbb{F}$ K----------------(2)

To find value of K in terms of the known quan: s' an/unknown constants a & b, take the point F on thecurve. The x & y of this point are a &  $b \downarrow s'$  respectively.

Then K for this point  $F = x y_a$  (s' + b). Putting this vaue in  $(2)$  we have.

 $(p+a)$   $(s+b)$  = a  $(s'+b)$ 

Multiplying out and reducing we get 

we ought to get a straight line.

(a) is the slope and-b the interfept on  $s$  axis.

The points were so scattered in part of this line that the method of least square was used in locating the true position of the  $(1)$ 

This

a = 
$$
\frac{\frac{2}{p} \left( \frac{s'-s}{p} \right) \frac{2}{2} s - \frac{12 \sum \left( \frac{s}{p} \right) \frac{s-3}{p}}{2}}{\frac{\sum (s'-s)}{p} \frac{2}{2} \frac{s'-s}{p} \frac{2}{2} \frac{s'-s}{p} - \frac{2s \sum (s'-s)}{2}}{2}} = -\frac{\frac{1}{\sqrt{6}}}{\frac{\sum (s'-s)}{2} \frac{s'-s}{2}}}{\frac{\sum (s'-s)}{2} \frac{s'-s}{2}} = -\frac{\frac{1}{\sqrt{6}}}{\frac{1}{\sqrt{6}} \frac{s'-s}{2}}}{\frac{1}{\sqrt{
$$

The value of a & b having been determined equation (3) may  $S = \frac{837.7 - 46.66}{6 + 1.48}$ be written as follows:

By making  $s =$  zero we get  $p = 17.97$  proportion. of sand that would give a zero breaking strength.

By making  $s = \sqrt{w}$  get the value of p which located the x  $axis = -a$ :

**(17)**

Thus  $S = \mathcal{A} \left( \frac{S - S}{P} \right) - b$  $ps = as' - as - ps$  $5 (p+a) = a s' - b b$ 

From which

$$
p + a = \frac{a^{5} - b^{5}}{5} = \frac{a^{5} - b^{5}}{5} = 0
$$
  
 
$$
\therefore p = -a =
$$

By making  $p_z \infty$  we locate the X axis;-



CURVE NO (Ib)

In this curve the percentage of sand was plotted instead of proportion of sand. Plot shown on Plate (111h)

The following conclusions drawn from the above, without a y attempt to derive an equation were:-

As the perfentage of sand increases the strength drops off'. The curve evidently has two finite intercepts, one being the strength with zero percentage of sand and one being percent age of sand at a zero strength.

It is very evident that the curve intercepts the horizont al axis before the percen tage of sand reached 100.

The intercept ought to give a percentage that would give the. same proportion of sand as the intercept on the p axis in Curve (3a).







#### DISCUSSION OF CURVE NO 4.

The plot of the curve will be found on Plate 1V Table on page21 On account of such a change in condition of sand, due to impurities, the curve does not show as much infornation as was hoped.

**From** the location of the points by plotting the strength as ordinates and reciprocals of meshes as abscissae, we get apparently a curve which has a maximum for breaking strength and a change of

mexh producing a change in breaking strength either way from this maximum point. The dropping off in case of smaller number of mesh being used may be due to the change of the sand into chart or some other material.

As the sand gets smaller and smaller the amount of surface  $ex$ pop8d becomes greater and there may be a place where this surface exposed would become so great that no setting would take place.

#### DISCUSSIOH OF eURVE NO. %.

This curve was plotted by using breaking strength as ordinates and amount of water as abscissae.

It will be found on Plate <sup>V</sup> Data on Page 21.

I plotted amount of water per batch instead of one briquette, but the curve will be of the same character as if the amount per briquett was used because they are directly prpportional.

The conclusions to be drawn from this curve are: that there is a certain amount of water which would give a maximum strength.

I did not have enough data to show this, however, it is reasonable to suppose that the strength becomes equal zero on the left hand side of the curve when there is not enough water added to cause any setting. From the right hand portion we can see that with a large amount of water the strength drops off rapidly.

#### (19)

From the experiments carried out many other relations might be obtained. As the above experiments were being performed new points of interest arose, some of which have been mentioned.

If the new points are devaloped there will in all probabilities be some remarkable results accomplished.

VJ.







## 1911-111.

# Neat Cement.







 $\mathcal{L}_{\text{max}}$ 



 $[III, II].$ Cenient Mortar-170/by Wt.<br>
Sempt of Room = 17.9 | Batch Tempt of Room = 18.5 Batch *Tempt.of Room* =  $17\%$ <br>  $\propto$   $32\%$   $\frac{1}{22}$   $\frac{1}{40}$  =  $20\%$ Time in Strength in<br>Days. Ibs. perseu-in Time in Strengthin  $\mathcal{N}$ o, 'Nо. Iws.per/squ-in Ibs.persgu-in. Days.  $30$  $3\lambda$  $\sqrt{2}$  $128$  $118$  $\lambda$  $\boldsymbol{\lambda}$  $\mathcal{Z}$  $\boldsymbol{\lambda}$  $184$  $\boldsymbol{\beta}$  $\lambda$  $196$  $\overline{\mathbf{3}}$  $\overline{\mathbf{3}}$  $\boldsymbol{\acute{\delta}}$  $\mathcal{L}$  $282$  $259$  $\overline{\mathbf{r}}$ 4  $\mathcal{S}^{\prime}$  $\overline{7}$  $312$  $\boldsymbol{7}$  $254$  $\overline{b}$  $\boldsymbol{\ell}$  $\boldsymbol{q}$  $379$  $\overline{\mathbf{r}}$  $276$  $\boldsymbol{q}$  $\overline{7}$  $14$  $421$  $\mathcal{E}$  $380$  $21$  $\mathcal{L}$  $32$  $414$ 







Batch Tempt. of Room =  $143$ <br> $\theta$ ,  $\theta$ ,  $\theta$ ,  $\theta$ ,  $\theta$ , =  $153$  $H_2D = 75.3$ <br>Strength  $I_{II}$ <br>Ibs.persqu-in. Time in<br>Pays.  $Na$ 10  $\hat{\mathcal{E}}$  $\AA$  $\hat{\mathcal{X}}$  $128$  $\boldsymbol{\hat{\theta}}$  $\bar{3}$  $282$  $\overline{\mathcal{V}}$  $\acute{\bm{b}}^{\vee}$ ゲ  $\tilde{a}$  $321$ q  $\pmb{6}$  $351$  $\overline{7}$  $423$  $14$  $\mathbf{r}$ 444  $\lambda$  $8.3$  $\boldsymbol{q}$  $3x$  $564$  $\sqrt{0}$  $63$ 

WATA

Cement Mortar With Varying Quantities of Sand.

Tempt. of Room Lot  $\pi$ , 0 Timein Strength III  $\delta e \tau$ bs.pr squ.in.  $\boldsymbol{a}$ lo 9 d  $631$  $\boldsymbol{b}$  $2<sub>2</sub>$  $\mathbf c$  $644$  $\lambda$  $\boldsymbol{d}$  $620$  $2<sub>2</sub>$  $531$  $\boldsymbol{\ell}$  $2<sub>2</sub>$  $533$  $\boldsymbol{f}$  $2<sup>2</sup>$  $\boldsymbol{\mathcal{J}}$ 492  $2<sup>2</sup>$ h  $310$  $2<sup>2</sup>$  $3/2$ 27  $236$  $2<sub>2</sub>$  $\boldsymbol{J}$  $\overline{\mathbf{k}}$  $200$  $22$  $\boldsymbol{\ell}$  $152$  $\mathfrak{p}$  $130$ m ??





















