Oct 14th, 12:00 AM

GBTUL- a Code for the Buckling Analysis of Cold-formed Steel Members

Rui Bebiano

Nuno Silvestre

Dinar Camotim

Recommended Citation

GBTUL – A Code for the Buckling Analysis
of Cold-Formed Steel Members

Rui Bebiano[1], Nuno Silvestre[2] and Dinar Camotim[3]

Abstract

This paper presents GBTUL 1.0β, a code to perform buckling and vibration analyses of open-section cold-formed members that is now available online as freeware. This code, developed at the Department of Civil Engineering and Architecture of the Technical University of Lisbon (ICIST/IST – UTL), constitutes the numerical implementation of a recent Generalized Beam Theory (GBT) formulation – GBT is a thin-walled beam theory that incorporates local deformation and discretizes a member deformed configuration (*e.g.*, a buckling or vibration mode shape) into a linear combination of *cross-section deformation modes* with longitudinally varying amplitudes. After presenting a very brief overview of the GBT formulation, one addresses the GBTUL 1.0β graphic user interface and describes its main commands. Finally, the paper closes with an illustrative example: the application of the code to analyze the buckling behavior of a lipped channel cantilever beam – particular attention is paid to the quality of the code graphic outputs (2D and 3D mode shape representations).

Introduction

Most thin-walled members exhibit high global and local slenderness values, a feature responsible for a rather complex structural behaviour, strongly affected by various instability phenomena, such as local-plate, distortional and global (Euler – flexural, torsional or flexural-torsional) buckling. In the particular case of cold-formed steel members, a considerable amount of research work has been recently devoted to the development of safe and economic design rules, notably in Australia and the USA (*e.g.*, Hancock *et al.* 2001). One of the main fruits of this intense research activity was the

---

[1] Ph.D. Student, Assistant Professor[2] and Associate Professor[3], Department of Civil Engineering and Architecture, IST/ICIST, Technical University of Lisbon, Portugal.

Direct Strength Method (DSM − *e.g.* Schafer 2005, 2008), which can handle the combined effect of local-plate, distortional and global buckling, and has already been included in the current North American (AISI 2004) and Australian/New Zealander (SA-SNZ 2005) specifications for cold-formed steel member design. Since the application of the DSM provisions, as well as virtually all other existing design rules concerning the buckling ultimate limit states of cold-formed steel members (*e.g.*, Eurocode 3 − CEN 2005), requires knowing the member buckling stresses and mode nature, it is indispensable for practitioners to have wide access to accurate and easy-to-use tools to obtain this information. Bearing this in mind, researchers from the University of Sydney (under the leadership of Greg Hancock) and Ben Schafer (first at the University of Cornell and later at Johns Hopkins University) developed the codes THIN-WALL (Papangelis & Hancock 1998) and CUFSM (Schafer 2007), both based on the semi-analytical finite strip method (SAFSM − *e.g.*, Cheung & Tham 1998) − note that the latest CUFSM version already includes the so-called "constrained finite strip method" (*e.g.*, Ádány & Schafer 2006). However, the SAFSM codes currently available can only be applied to simply supported members (end sections locally/globally pinned and free to warp) acted by uniform applied internal force and/or moment diagrams − for instance, non-uniform bending problems are excluded. Then, the assessment of the local and global buckling behavior of thin-walled members with other loading and/or end support conditions is only possible through shell finite element analyses, usually performed in powerful commercial codes (*e.g.*, ABAQUS, ANSYS or ADINA).

Generalised Beam Theory (GBT) may be viewed as either (i) a *bar theory* that incorporates cross-section in-plane and out-of-plane deformations or (ii) a *folded-plate theory* that includes plate rigid-body motions (*e.g.*, Schardt 1989 or Camotim *et al.* 2004, 2007). By expressing the member buckling/vibration modes or deformed configurations as linear combinations of longitudinally varying cross-section *deformation modes*, which account for cross-section rigid-body motions and deformations, GBT provides a general and elegant approach to obtain accurate solutions for several structural problems involving prismatic thin-walled members − moreover, one also obtains the contributions of each deformation mode, a feature enabling a much clearer interpretation of the structural response under consideration. In recent years, Camotim and his co-workers at the Technical University of Lisbon developed and implemented GBT formulations to perform (i) first-order, buckling and post-buckling analyses, and (ii) vibration and dynamic analyses of *isotropic* and *orthotropic* thin-walled members (*e.g.*, Camotim *et al.* 2004, 2006a,b, 2007). However, an important limitation to a more widespread use of GBT analyses has been the lack of an accessible and easy-to-use computer code to perform them.

The aim of this paper is to report on the development and capabilities of the code GBTUL 1.0β (acronym for "GBT at the TU Lisbon" − Bebiano *et al.* 2008), which is intended to fill the aforementioned gap and implements a recent GBT formulation

developed by Bebiano *et al.* (2007). It is available online at *http://www.civil.ist.utl.pt/gbt* (as *freeware*) and performs elastic buckling and vibration analyses of thin-walled members (i) with open cross-sections, (ii) with several end support and/or loading conditions, and (iii) made of isotropic or special orthotropic materials – since the member walls may exhibit different properties, hybrid steel profiles and composite steel-concrete members can also be handled. The code features include (i) the modal decomposition and identification (local-plate, distortional, global) of the member buckling or vibration mode, (ii) the possibility of performing analyses involving any number of selected deformation modes, (iii) 3D high-quality visualisation effects and (iv) user-friendly data input and output.

Generalized Beam Theory: A Brief Overview

The main distinctive feature of GBT is the approximation of the cross-section displacement field by a linear combination of deformation modes with a clear structural meaning. Any GBT-based (buckling or vibration) analysis involves the sequential procedure depicted in fig. 1.

For given material properties and cross-section geometry, one begins by performing the *cross-section analysis*, which leads to the identification of its deformation modes and the evaluation of the corresponding modal mechanical properties. In order to provide a brief illustration of this step, consider the lipped channel beam (longitudinal axis X) shown in fig. 2(a) – in each wall, one defines local axes x-s-z, associated with the displacement components u, v and w. The beam is made of steel ($E=210\,GPa$, $v=0.3$, $\rho=7.800\,kg/m^3$), and its cross-section geometry and discretization are depicted in figs. 2(b)-(c) – the number and nature of the deformation modes obtained depends on the cross-section discretization into
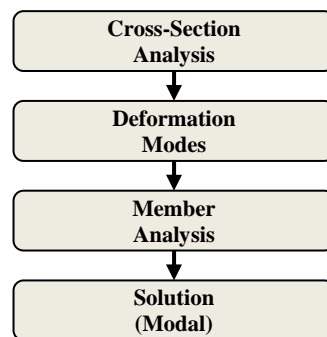

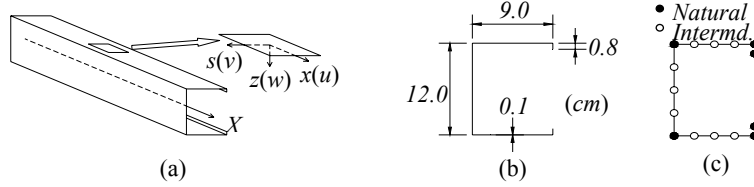
Fig. 1: GBT-based analysis procedure

Fig. 2: Lipped channel member: (a) overall view an local axes, and cross-section (b) geometry and (c) GBT discretization

natural nodes (all wall ends) and *intermediate* nodes (within the walls – selected by the user to obtain the desired accuracy).

The modal representation of the member displacement field can be expressed as

$$u(x,s) = u_k(s)\phi_{k,x}(x) \quad v(x,s) = v_k(s)\phi_k(x) \quad w(x,s) = w_k(s)\phi_k(x) \quad , \quad (1)$$

where (i) $u_k(s)$, $v_k(s)$ and $w_k(s)$ are the modal displacement profiles (defined along the cross-section mid-line) and (ii) $\phi_k(x)$ are their longitudinal amplitude functions (the summation convention applies to subscript k).

In this case, the cross-section discretization adopted (see fig. 2(c)) leads to a set of $N_d=17$ deformation modes – fig. 3 depicts the in-plane deformed shapes of the *12* most relevant ones (mode **1** involves no in-plane motions – axial extension). In order to solve a given problem, one may choose to include any subset of n_d ($1 \leq n_d \leq N_d$) deformation modes in the buckling/vibration analysis.
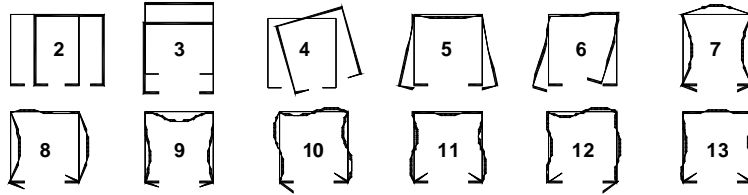

Fig. 3: In-plane configurations of the cross-section deformation modes **2-13**

Then, the *member analysis* yields the solution of the buckling or vibration problem, namely the corresponding eigenvalues (buckling loads or natural frequencies and eigenvectors (buckling or vibration mode shapes) – the latter provide the coefficients of the modal amplitude functions $\phi_k(x)$. In buckling analyses, the system to be solved reads where (i) C_{ik}, D_{ik}, B_{ik} are the cross-section linear stiffness matrices,

$$C_{ik}\phi_{k,xxxx} - D_{ik}\phi_{k,xx} + B_{ik}\phi_k - \lambda\left[X_{jik}\left(W_j^0\phi_{k,x}\right)_x - X_{jki}^\tau\left(W_{j,x}^0\phi_k\right)_x + W_{j,x}^0 X_{jik}^\tau\phi_{k,x}\right] = 0 \quad , \quad (2)$$

(ii) X_{jik}, X^{τ}_{jik} are geometrical stiffness matrices (concerning normal and shear stresses), and (iii) $\phi_k(x)$ are the longitudinal amplitude functions of the deformation modes. Vector W^0_j ($W^0_j(x)$, with $j=1...4$) contains the resultants of the pre-buckling applied stresses, namely (i) axial force ($W^0_1(x) \equiv N(x)$), (ii) major and minor bending moments ($W^0_2(x) \equiv M_Y(x)$, $W^0_3(x) \equiv M_Z(x)$), and (iii) bimoment ($W^0_4 \equiv B$).

The solution of (2), which yields the buckling load parameters λ and mode shapes $\phi(x)$, may be obtained either (i) *analytically* (simply supported members acted by longitudinally uniform stress resultants – sinusoidal $\phi_k(x)$ functions) or (ii) *numerically* (any members, by means of a longitudinal discretization into GBT-based beam finite elements). These procedures are addressed in the next sub-sections.

Analytical Solution. For members acted by loads that cause longitudinally uniform stress resultants (*i.e.*, $W^0_j(x) \equiv W^0_j$), system (2) becomes

$$C_{ik}\phi_{k,xxxx} - D_{ik}\phi_{k,xx} + B_{ik}\phi_k - \lambda X_{jik}W^0_j\phi_{k,xx} = 0 \qquad . \quad (3)$$

For simply supported members, this system has exacts solutions ($k=1...n_d$)

$$\phi_k(x) = A_k \sin\left(\frac{n_h \pi x}{L}\right) \qquad . \quad (4)$$

Introducing (4) into (3), one defines a system whose solution provides the sought eigenvalues λ and eigenvectors A_k. Since the dimension of this system ($n_d \times n_d$) is fairly low, only a small computational effort is required to obtain the solution – this is why an analytical solution is always preferable to a numerical one (computationally much more involved).

Numerical Solution – GBT-Based Finite Element. The GBT-based beam finite element is derived on the basis of the variational (or weak) form of the equilibrium equation system. Within the finite element length L_e, the amplitude functions $\phi_k(x)$ are approximated by means of linear combinations of Hermite cubic polynomials, *i.e.*,

$$\phi_k(x) = d^e_{k.1}\Psi_1(\widetilde{x}) + d^e_{k.2}\Psi_2(\widetilde{x}) + d^e_{k.3}\Psi_3(\widetilde{x}) + d^e_{k.4}\Psi_4(\widetilde{x}) \qquad , \quad (5)$$

where $d^e_{k.1} = \phi_{k,x}(0)$, $d^e_{k.2} = \phi_k(0)$, $d^e_{k.3} = \phi_{k,x}(1)$, $d^e_{k.4} = \phi_k(1)$, $\widetilde{x} = x/L_e$ and

$$\Psi_1 = L_e(\widetilde{x}^3 - 2\widetilde{x}^2 + \widetilde{x}) \qquad\qquad \Psi_2 = 2\widetilde{x}^3 - 3\widetilde{x}^2 + 1$$

$$\Psi_3 = L_e(\widetilde{x}^3 - \widetilde{x}^2) \qquad\qquad \Psi_4 = -2\widetilde{x}^3 + 3\widetilde{x}^2 \qquad , \quad (6)$$

which means that each finite element has 4 degrees of freedom per mode, hence a total of $4 \times n_d$. If the member is discretized into n_e such finite elements, the total number of degrees of freedom is approximately equal to $2 \times n_d \times (n_e + 1)$.

When a numerical solution is carried out, the GBT modal nature makes it possible to consider distinct boundary conditions for different deformation modes. Standard

boundary conditions, *i.e.* those involving the full restraint of displacements (ϕ_k=0) and/or its derivatives ($\phi_{k,x}$=0), are easily taken into account when assembling the member equilibrium eigensystem. GBTUL 1.0β offers the possibility of assigning 4 different support conditions, namely (i) simply supported ("S-S"), (ii) clamped-free (or cantilever, "C-F"), (iii) clamped-clamped ("C-C") and (iv) clamped-pinned ("C-S") – see table 1.

Table 1: The 4 member end support conditions available in GBTUL1.0β

End Support Conditions	
S-S	$\phi_k(0)=\phi_k(L)=0$
C-F	$\phi_k(L)=\phi_{k,x}(L)=0$
C-C	$\phi_k(0)=\phi_{k,x}(0)=\phi_k(L)=\phi_{k,x}(L)=0$
C-S	$\phi_k(0)=\phi_{k,x}(0)=\phi_k(L)=0$

These four types of support conditions can be applied independently to the following deformation modes are: (i) mode **2** (major axis bending), (ii) mode **3** (minor axis bending), (iii) modes **4**+**D** (torsion and distortional modes), and (iv) modes **LP** (local-plate modes).

Modal Participation Factors. In order to assess the contribution of a given deformation mode to a member buckling or vibration mode, one provides the value of its *participation factor P_i,* given by

$$P_i = \int_L \left|\phi_i(x)\right| dx \Big/ \sum_{k=1}^{n_d} \int_L \left|\phi_k(x)\right| dx \times 100\% \qquad , \quad (7)$$

where L is the member length. It corresponds to the ratio between (i) the total area limited by the mode amplitude function $\phi_i(x)$ and (ii) the sum of the areas limited by all the modal amplitude functions (i=1…n_d) – P_i quantifies the relative importance of deformation mode i to the buckling or vibration solution.

Code GBTUL: Scope and Structure

Scope. The code GBTUL (more specifically, its 1.0β version) performs elastic buckling (bifurcation) or vibration analyses of thin-walled members with arbitrary open cross-sections (*i.e.*, excluding cross-sections with *closed cells*). The member walls can be made by one or several *isotropic* or *specially orthotropic* materials (the latter include, for instance, pultruded FRP profiles) – this means that *heterogeneous* members (*e.g.*, hybrid steel or composite steel-concrete members) can also be handled. In order to enable the user to benefit from the GBT modal features, a tool making it possible to visualize and select the deformation modes is included in the code.

As mentioned earlier, the code covers four types of member end support conditions: (i) simply supported (pinned-pinned), (ii) fixed-free (cantilever), (iii) fixed-fixed and (iv) fixed-pinned – moreover, recall that it is also possible to specify different support conditions for the various deformation modes (*e.g.*, bending and torsion). On the other hand, the (pre-buckling) applied stresses may stem from arbitrary combinations of (i) end bending moments, (ii) axial or transverse point loads (acting at any cross-section along the span), (iii) axial or transverse uniformly distributed loads and (iv) uniform bimoments – however, the transverse loads must act on a plane containing the shear centre.

The user may indicate an arbitrarily long list of member lengths (*L* values), so that the code produces a curve describing the variation of λ_b (buckling load parameter) or ω (natural frequency) with *L* as well as the corresponding modal participation diagrams P_i *vs L* – in either case, the plot scale and limit values are freely chosen by the user. The buckling or vibration modes are represented by means of either (i) 3D deformed configurations of the entire member, combined with interactive visualization tools, or (ii) 2D deformed configurations of any cross-section – it is always possible (i) to select the deformation modes employed to obtain the representation (out of the n_d included in the analysis, selected after performing the cross-section analysis) and (ii) to specify the displacement scale. Finally, the code output is also saved in formatted text files, thus making the task of processing it by means of spreadsheet applications (*e.g.*, Microsoft Excel) very easy and straight forward.

The GBTUL interface was conceived to minimize the amount of data inputs and also the occurrence of the most common mistakes (some inputs are controlled to detect them). Moreover, "help buttons" associated with most input/output commands are also available.

Structure. The GBTUL code executable program is written in FORTRAN 90 and linked to a graphic user interface developed in VB.NET, and has its 3D representations created in the VRML graphic environment – this graphic user interface involves the sequence of *four screens* shown in fig. 4(b): while the first three deal with data input, the fourth one is related to the result output. This sequence is closely related to the performance of a GBT analysis (see fig. 4(a)): (i) Screens 1, 2 and 3 concern the inputs associated with the cross-section analysis, deformation mode selection and member analysis, and (ii) Screen 4 displays the sought buckling or vibration solution.
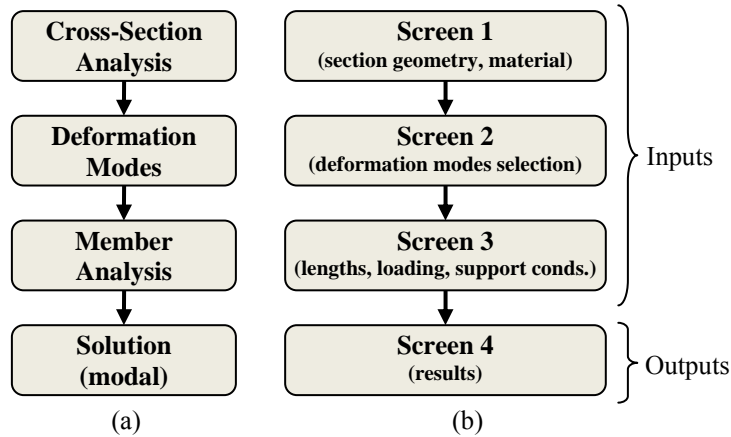
Cross-Section Analysis		Screen 1 (section geometry, material)	
↓		↓	
Deformation Modes		Screen 2 (deformation modes selection)	Inputs
↓		↓	
Member Analysis		Screen 3 (lengths, loading, support conds.)	
↓		↓	
Solution (modal)		Screen 4 (results)	Outputs
(a)		(b)	

Fig. 4: Comparison between (a) the steps involved in a GBT-based analysis and (b) the sequential procedures of the GBTUL graphical user interface.

Input Data

The first three screens ask for the data required to perform the desired analysis – button `Next`, located at the bottom right corner of each of them, directs the user to the subsequent screen. In the next subsections, one provides a brief description of the main commands associated with each screen. Moreover, one illustrates the application of GBTUL to perform the buckling analysis of (i) lipped channel steel cantilevers with the cross-section dimensions and GBT discretization given in figs. 2(b)-(c) and acted by a uniformly distributed transverse load acting along the shear center axis (see fig. 5). One obtains λ_b *vs. L* curves concerning the first three buckling modes and the length range $10 \leq L \leq 1000\,cm$ – all the *17* deformation modes yielded by the GBT cross-section analysis are included in the buckling analyses.
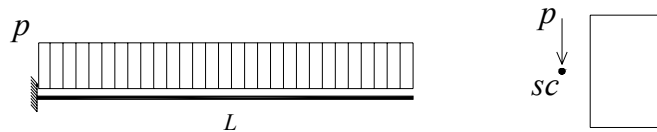

Fig. 5: Illustrative example: lipped channel cantilever beam acted by an uniformly distributed transverse load applied along the shear center axis

Cross-Section Analysis. The first screen, shown in fig. 6, contains (i) several data inputs and (ii) a graphic representation window with a few associated commands. In the `Material Model` field, one introduces the elastic constants of the member

material (or materials) – for isotropic members (*e.g.*, the cantilever of the illustrative example), one uses the template associated with button Isotropic (see fig. 7(a)) and only the values of E, ν and ρ are required (the unit system is arbitrary – in this case, one uses [*kN, cm, s*]). The next field asks for the cross-section wall characterization: the (i) end node coordinates, (ii) material, (iii) thickness and (iii) number of intermediate nodes. For the commonly used cross-section geometries, such as C, U, "Rack", Z, I, T and L (and also a single plate), pre-defined templates
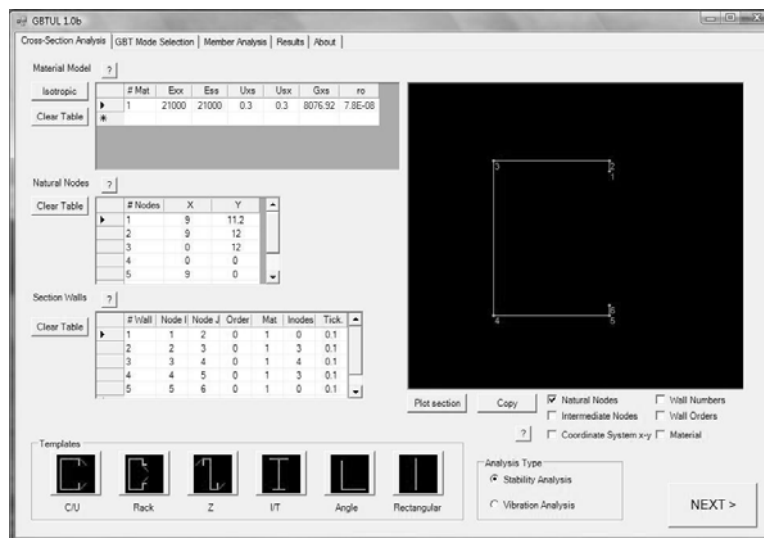

Fig. 6: GBTUL – general view of Screen 1

are available to minimize the amount of input data. In the illustrative example, button C/U activates the window shown in fig. 7(b), which requires the cross-section dimensions – note that, as shown in fig. 2(c), *3* intermediate nodes (Inodes) are considered both in the lipped channel web and flanges.

The cross-section geometry is visualized In the representation window. Using the check boxes shown below, one may choose to represent several additional features, like the intermediate nodes or the wall material references – the illustrative lipped channel section representation is shown in fig. 6. Finally, the type of analysis is selected through button Analysis Mode: either buckling (Stability Analysis) or free vibration (Vibration Analysis).
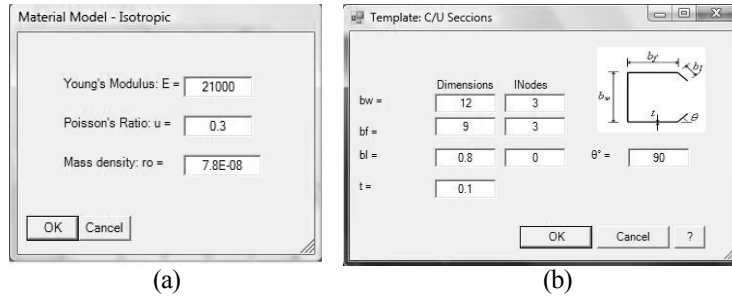
Fig. 7: Screen 1: (a) isotropic material and (b) "C/U" cross-section templates

Screen 2: Deformation Mode Selection. The second screen, depicted in fig. 8, (i) displays the output of the *cross-section analysis* and (ii) asks the user to select the deformation modes to be included in the subsequent *member analysis*. Several cross-section features are presented on the left side: the (i) geometrical properties (*e.g.*, cross-section area, major/minor moments of inertia or warping constant), and (ii) stiffness and mass matrices – they are shown upon a click on the corresponding buttons.

The representation window, located on the right side, makes it possible to visualize each individual cross-section deformation mode – both the in-plane deformed configuration (in-plane displacements – $v_k(s)$ and $w_k(s)$) and the warping profile (warping displacements – $u_k(s)$). The n_d deformation modes to be included in the analysis can be selected in three different ways: (i) button Pick Mode, which adds the mode currently displayed, (ii) button Pick Mode, which adds the mode currently displayed, (ii) button Pick All, which selects all available modes ($n_d=N_d$) or (iii) the text field Mode Selection, where the selected mode numbers can be entered directly.

In the illustrative example, one presents the displacement field associated with deformation mode **4** (torsion): in-plane deformed configuration (fig. 9(a) – v_4 and w_4) and warping profile (fig. 9(b) – u_4). Since all the deformation modes are to be included in the buckling analysis, one clicks Pick All.
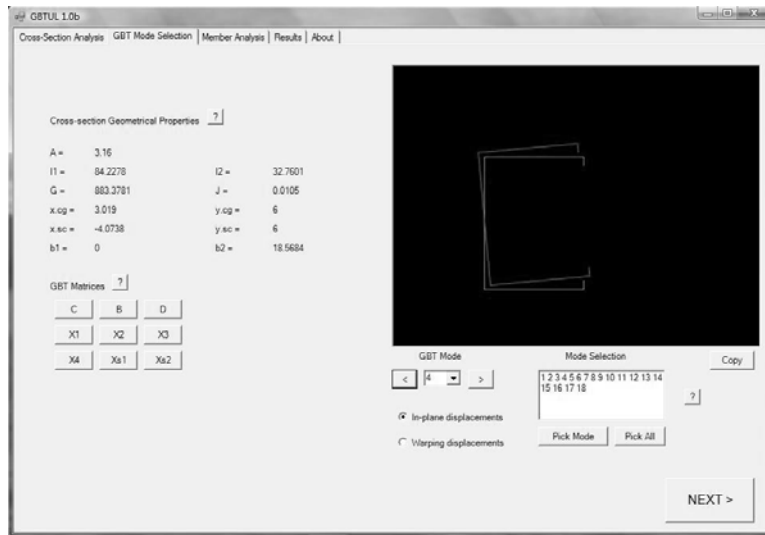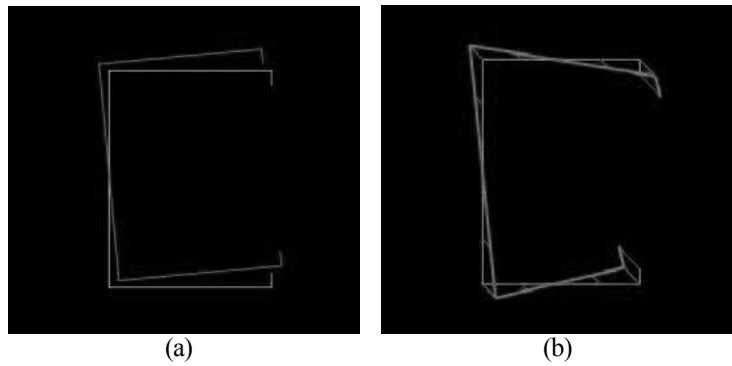
Fig. 8: GBTUL – general view of Screen 2.

(a)   (b)

Fig. 9: Screen 2: (a) in-plane deformed configuration and (b) warping profile of mode **4**

Screen 3: Member Analysis. In the third screen of GBTUL (see fig. 10), the user (i) chooses the type of solution (analytical or numerical) and (ii) specifies the member length, loading and end support conditions. On the left side, there are the tabs `Analytical Solution` (fig. 11(a)) and `Numerical Solution` (fig. 11(b)), which indicate that the equilibrium equations are to be solved analytically (sinusoidal $\phi_k(x)$) or numerically (beam finite elements). Since the illustrative example corresponds to cantilever beams, the numerical solution is the only that can be adopted – moreover, the cantilevers are subjected to non-uniform bending moment diagrams.

When adopting the analytical solution (fig. 11(a)), always computationally more efficient (whenever applicable), the user must provide (i) the parameters defining the acting (uniform) internal force, moments and bimoment (*i.e.*, N, My, Mz and B), (ii) the `Number of half-waves` (*i.e.*, the maximum number of longitudinal half-waves exhibited by the buckling/vibration mode) and (iii) the `Number of intervals` – this number is important for the graphical representations appearing in Screen 4 and addressed further ahead.
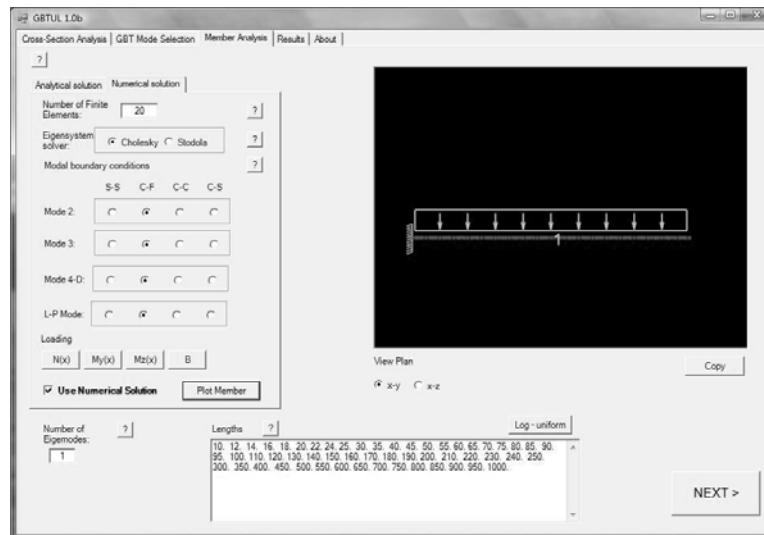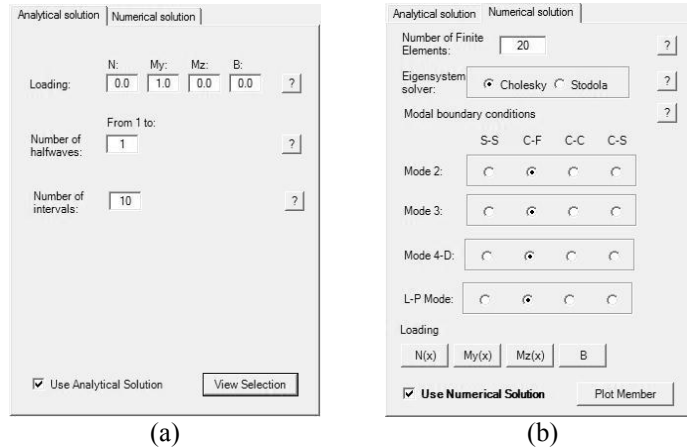

Fig. 10: GBTUL – general view of Screen 3

Fig. 11: Screen 3: (a) analytical solution and (b) numerical solution tabs

On the other hand, if the numerical solution is adopted (`Numerical Solution` tab in fig. 11(b)), the user first indicates the desired `Number of Finite Elements` – *20* in the illustrative example. Then, he must choose the eigenvalue problem solver between (i) Cholesky's factorization method (the one adopted by default) and (ii) Stodola's method. Next, the four rows of buttons `Modal Boundary Conditions` allow for the specification of the member boundary conditions. This can be made independently for (i) mode **2**, (ii) mode **3**, (iii) modes **4**+**D** (modes **4**, **5** and **6** in the illustrative example), and (iv) modes **LP** (modes **7-17** in the illustrative example) – concerning mode **1** (axial extension), it is always fully restrained at the member left end section and completely free at the right one. In the illustrative cantilever beam, it suffices to selects `C-F` in the four button rows. Finally, the buttons `N(x)`, `My(x)`, `Mz(x)` and `B` concern the definition of the loading pattern (buckling analysis only), which may involve stress gradients associated with the axial force and bending moments – fig. 12 shows the illustrative example dialogue box concerning `My(x)` – an unitary value has been assigned to the distributed load parameter `py`. The window on the right makes it possible to visualize and check the correctness of the loading and support conditions specified in the tabs.
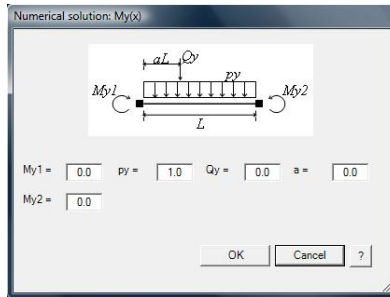
Fig. 12: GBTUL –Screen 3: "My(x) dialogue box"

Lastly, the fields `Number of Eigenmodes` and `Lengths` ask the user to specify (i) the highest order of the buckling or vibration modes sought and (ii) the lengths of the members to be analysed. In the illustrative example, one asks for the *3* first buckling modes of cantilevers with lengths in the interval *10≤L≤1000cm* – a *63*-value length list comprised in that range appears by default in the `Lengths` field (alternatively, the `Log-uniform` button makes it possible to indicate a length list uniformly spaced in a logarithmic scale).

Output Data

The results of the analyses performed are presented graphically in Screen 4 as (i) buckling or vibration curve plots, which provide the variation of the buckling load parameter or natural frequency with the member length *L*, (ii) modal participation diagrams, and (iii) 2D or 3D representations of the member buckling or vibration modes – these data are also recorded in formatted text files, making it very easy to further process them. In the next subsections, one describes these result outputs.

Screen 4: Graphic Outputs. Fig. 13 provides a general overview of Screen 4. While the buckling (λ_b *vs. L*) or vibration (ω *vs. L*) curves are depicted at the upper right side, the modal participation diagrams (P_i *vs. L*) are plotted in the bottom right side. On the left side there are some commands concerning the selection of options associated with the plots presented.

While both plots displayed in fig. 13 correspond to the length range indicated, (i) the results appearing above the upper plot and (ii) the 2D and 3D deformed configurations concern the buckling or vibration mode of a beam with a
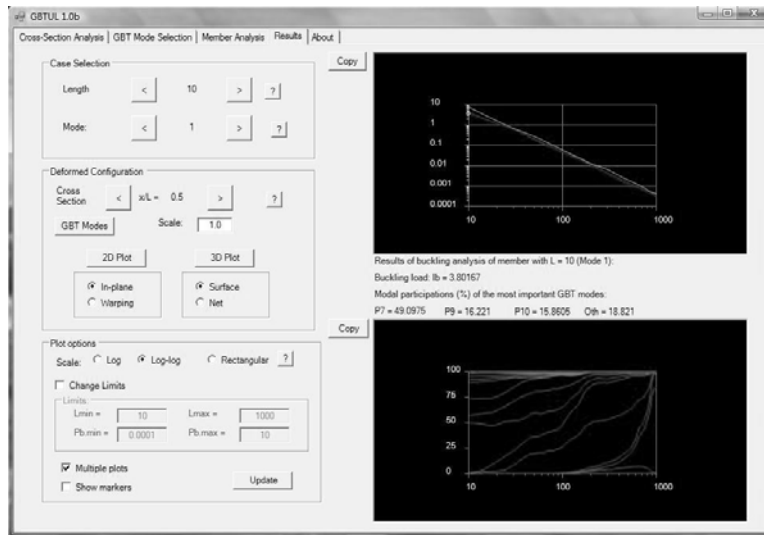
Fig. 13: GBTUL – general view of Screen 4

particular length L – the selection of this beam is made through the commands `Length` and `Mode`, located on the screen top left side. The results displayed are the bifurcation parameter value `lb` (λ_b) and the most important deformation mode contributions to the buckling or vibration mode (P_i). On the other hand, the "location" of that specific member on the two above plots is identified by (i) a small "ball" (lying on top of the λ_b *vs. L* or ω *vs. L* curve under consideration) and (ii) a vertical line (crossing the modal participation diagram at that L value).

By using the `Plot Options`, located at the screen bottom left side, one is able to change some features associated with the visualization of the two plots, namely (i) the scales of the axes, which may be either logarithmic (`Log`), bi-logarithmic (`Log-Log`) or rectangular, (ii) the scale limits (`Limits`), (iii) the number of curves shown (one or more) (`Multiple Plots`), and (iv) the option of showing or hiding the points defining the buckling/vibration curves (`Show Markers`).

Finally, the commands pertaining to the 2D or 3D representations are located at mid-height on the left side – they concern the specific member under consideration and are displayed in separate windows after one clicks on buttons `2D Plot` or `3D Plot`. In the 2D configurations, one (i) uses the command `Cross-Section`, in order to select the sought cross-section (*i.e.*, its x coordinate value), and (ii) selects either `In-plane` (see fig. 14(a)) or `Warping` (see fig. 14(b)) displacements. As for the 3D configurations, the member buckling or vibration mode shape can be plotted either (i) with opaque surfaces (`Surface` – see fig. 15(a)) or (ii) with a line network (`Net` – see fig. 15(b)). In either case, the user may still specify
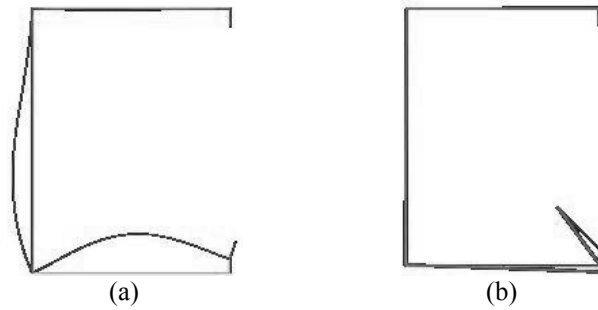
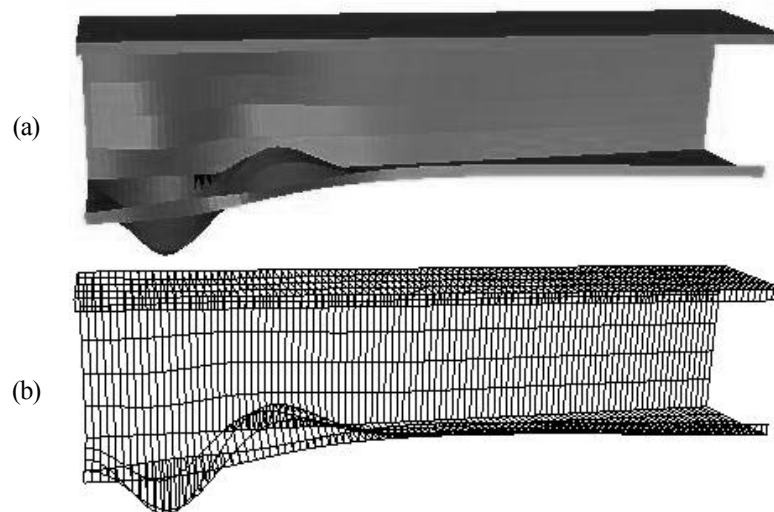Fig. 14: GBTUL –Screen 4: (a) In-plane and (b) Warping buckling mode shape representations ($L=50\,cm$, $x/L=0.25$)

Fig. 15: Screen 4: (a) Surface and (b) Net 3D buckling mode shape representations ($L=50\,cm$)

(i) the subset of the deformation modes included in the analysis (those selected in Screen 2) on which to base the representation, and also (ii) a displacement scale factor.

With respect to the illustrative example, fig. 13 shows (i) the λ_b vs. L curves corresponding to the three first 3 buckling modes (in bi-logarithmic scale) and (ii) the P_i vs. L modal participation diagram associated with the first (critical) buckling mode. As

for figs. 14 and 15, they show 2D and 3D representations of the *L=50 cm* cantilever first buckling mode (for which $\lambda_b \equiv p_b = 19.26\,kN/m$).

Text Files. All the results are saved into three formatted text files, which can be opened and used as input to most data processing applications. These files, created in the folder `GBTUL\calc\Output_Files\`, are the following:

(i)   `Matrices.txt` – contains (i_1) the displacement values (u_i, v_i and w_i) at each cross-section node, for each deformation mode, and (i_2) the components of the GBT matrices.

(ii) `Results.txt` – includes (ii_1) a list of the eigenvalues (buckling load parameters or natural frequencies) associated with every member length and eigenvector (buckling or vibration mode), as well as (ii_2) the corresponding modal participation factors and (ii_3) the number of half-waves they exhibit (only in the case of the analytical solutions).

(iii) `Mafuncs.txt` – contains the longitudinal amplitude functions ($\phi_k(x)$) and their derivatives ($\phi_{k,x}(x)$) associated with every deformation mode included in the analysis, for all member lengths and buckling or vibration modes determined. These functions are defined by their values at a selected (finite) set of cross-sections located along the member length.

Conclusion

This paper presented the code GBTUL 1.0β, which performs buckling or vibration analyses of prismatic thin-walled members with open cross-sections. The code is based on Generalized Beam Theory (GBT) and is available *online* as *freeware*. Initially, a very brief overview of the performance of a GBT structural analysis was provided, focusing on its unique modal features. Then, the sequential procedure involved in using GBTUL 1.0β was addressed – it consisted of going through its four interactive screens, explaining and illustrating, by means of the buckling analysis of a cantilever acted by an uniformly distributed transverse load (non-uniform bending), all the steps and the options associated with the performance of a specific member buckling or vibration analysis.

Finally, one last word to mention that the code and its documentation, including user manuals and tutorials, are available for free download on the website *http://www.civil.ist.utl.pt/gbt*.

Acknowledgements

References

Ádány S. and Schafer B.W. (2006). "Buckling mode decomposition of single-branched open cross-section members via finite strip method: derivation". *Thin-Walled Structures*, 44(5), 563-584.

AISI (American Iron and Steel Institute) (2004). *Appendix I of the North American Specification (NAS) for the Design of Cold-Formed Steel Structural Members: Design of Cold-Formed Steel Structural Members with the Direct Strength Method*, Washington DC.

Bebiano R., Pina P., Silvestre N. and Camotim D. (2008). *GBTUL 1.0β – Buckling and Vibration Analysis of Thin-Walled Members*, DECivil/IST, Technical University of Lisbon (http://www.civil.ist.utl.pt/gbt).

Bebiano R., Silvestre N. and Camotim D. (2007). "GBT formulation to analyze the buckling behavior of thin-walled members under non-uniform bending", *International Journal of Structural Stability and Dynamics*, 7(1), 23-54.

Camotim D., Silvestre N., Gonçalves R. and Dinis P.B. (2004). "GBT analysis of thin-walled members: new formulations and applications", *Thin-Walled Structures: Recent Advances and Future Trends in Thin-Walled Structures Technology* (Loughborough, 25/6) J. Loughlan (ed.), Canopus Publishing, Bath, 137-168.

Camotim D., Silvestre N., Gonçalves R. and Dinis P.B. (2006a). "GBT-based structural analysis of thin-walled members: overview, recent progress and future developments", *Advances in Engineering Structures, Mechanics and Construction* (SMCD 2006 – Waterloo, 14-17/5) M. Pandey, W.-C. Xie, L. Xu (eds.), Springer, Dorderecht, 187-204.

Camotim D., Silvestre N., Dinis P.B., Bebiano R. and Basaglia C. (2006b). "Recent progress in the numerical analysis of thin-walled steel members and Frames", *Proceedings of International Symposium on Innovative Design of Steel Structures* (Hong Kong, 10/11) B. Young (ed.), 63-104.

Camotim D., Silvestre N. and Bebiano R. (2007). "GBT local and global vibration analysis of thin-walled members", *Dynamics of Plated Stuctures: Analysis and Design*, N.E. Shanmugam and C.M. Wang (eds.), Woodhead Publishing Ltd., Cambridge, 36-76.

Cheung Y.K. and Tham L.G. (1998). *The Finite Strip Method*, CRC Press, Boca Raton.

CEN (Comité Européen de Normalisation) (2005). *Eurocode 3: Design of Steel Structures - Part 1.3: General Rules and Supplementary Rules for Cold-formed Members and Sheeting*, Brussels.

Hancock G.J., Murray, T. and Ellifritt D. (2001). *Cold-Formed Steel Structures to the AISI Specification,* Marcel Dekker, New York.

Papangelis J.P. and Hancock G.J. (1998). *THIN-WALL 2.0*, Center for Advanced Structural Engineering, School of Civil and Mining Engineering, University of Sydney.

SA-SNZ (Standards of Australia and Standards of New Zealand) (2005). *Australian/New Zealand Standard on Cold-Formed Steel Structures* − AS/NZS 4600 (2nd edition), Sydney-Wellington.

Schafer B.W. (2005), *Direct Strength Method Design Guide*, American Iron & Steel Institute (AISI), Washington DC.

Schafer B.W. (2008). "Review: the Direct Strength Method of cold-formed steel member design", *Journal of Constructional Steel Research*, **64**(7-8), 776-778.

Schafer B.W. (2007). *CUFSM 3.12 – Finite Strip Buckling Analysis of Thin-Walled Members*, Department of Civil Engineering, Johns Hopkins University (http://www.ce.jhu.edu/bschafer).

Schardt R. (1989). *Verallgemeinerte Technische Biegetheorie*, Springer-Verlag, Berlin, 1989. (German)