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Floor System Design for Distortional Buckling Including Sheathing Restraint

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Floor system design for distortional buckling including sheathing restraint

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ABSTRACT

The objective of this paper is to describe how to include the beneficial rotational restraint, provided by sheathing to the compression flange of a cold-formed steel floor joist, to partially or fully retard the formation of distortional buckling. The design method for checking distortional buckling adopted in the 2007 AISI Specification (AISI-S100-07) provides a means to include a rotational restraint term, k_ϕ, to account for sheathing restraint. A series of cantilever tests were conducted to determine the rotational stiffness, k_ϕ, between a joist and attached sheathing. Tests were conducted for different joist thicknesses, depths, and flange widths, two fastener types, and plywood, oriented strand board, and gypsum board sheathing. The testing lead to (a) the development of a proposed design method, and (b) improvements to the AISI test standard for cantilever tests; both of which are presented herein. The focus of the design method and the improvements to the test standard are the separation of the rotational stiffness, k_ϕ, into contributions from the sheathing and from the local fastener (connector) deformations. It is shown that the sheathing stiffness is well correlated with tabled bending rigidity values, and the connector stiffness is primarily derived from the thickness of the flange. The developed recommendations have been proposed for the next edition of AISI standards and are presented in an Appendix.

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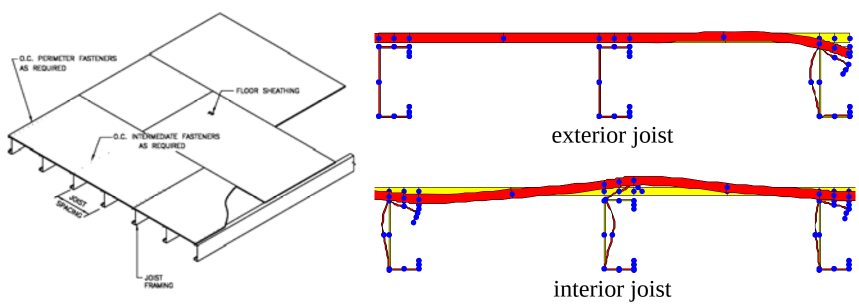
INTRODUCTION

Lateral-torsional buckling, local buckling, and distortional buckling are the three key member instabilities that may limit the ultimate strength of a floor joist. The most common concern is lateral-torsional buckling of the joist; blocking and bridging combined with fastened sheathing is employed to stabilize the joist from the global translation and twist associated with lateral-torsional buckling, as shown in Figure 1a. Local buckling, where the strength and rigidity of portions of the member are partially lost due to plate buckling, must also be accounted for. The strength in local buckling is largely independent of the floor framing details as the instability occurs over a short length of the joist.

The final member instability of concern is distortional buckling (Figure 1b); distortional buckling may be conceptualized as an instability driven by flexural-torsional buckling of the compression flange, involving large rotations of the flange and large plate bending deformations in the web. The floor sheathing provides a beneficial restraint for the joist against distortional buckling, but the magnitude of this restraint is poorly understood. This paper summarizes recent testing which characterizes the rotational restraint from sheathing and a related procedure which allows this restraint to be included in design.



(a) typical floor system (SFA 2000) (b) distortional buckling of a sheathed floor joist

Figure 1 Floor system and distortional buckling

An investigation into the restraint that sheathing provides against distortional buckling is timely as new provisions to account for distortional buckling have recently been adopted in the cold-formed steel specification: AISI-S100-07 (AISI 2007). These provisions, section C3.1.4 of AISI-S100-7, were developed through a series of 4-point bending tests conducted by Yu and Schafer (2003, 2006) which examined distortional and local buckling of bending members. The distortional buckling tests, as shown in Figure 2, did not include any compression flange restraint and resulted in distortional buckling failures (Figure 2b). When the metal panel shown in the shear spans of Figure 2a was extended into the center region and fastened to the compression flange with pairs of fasteners, the failure mode changed to local buckling. In these latter tests the metal panel was engaged and distortional buckling was restricted. The rotational restraint provided by the metal deck was the key to avoiding distortional buckling. The new provisions for distortional buckling in C3.1.4 of AISI-S100-07 include a stiffness term, k_ϕ, which increases the distortional buckling capacity as a function of available rotational restraint (stiffness).

(a) unrestrained distortional buckling test setup of Yu and Schafer (2006)

(b) unrestrained 800S200-054 C exhibiting distortional buckling

Figure 2 Tests on distortional buckling of C-sections

In the early 1980's the Metal Building Manufacturer's Association (MBMA) examined available rotational restraint in their systems: purlins fastened through insulation to metal deck. MBMA developed the "F" test (MRI 1981, Hausler and Pabers 1973) which later was formalized as AISI TS-1-02 (AISI 2002). The test uses a small cantilevered segment of panel with a purlin attached, and pulls on the free flange of the purlin such that a moment and rotation is induced at the panel-purlin connection. This test provides an estimate of the panel-purlin rotational restraint, k_ϕ. The k_ϕ results are critically dependent on purlin thickness (LaBoube 1986). The important role of thickness in the conducted tests (as opposed to purlin depth, deck thickness, insulation, etc.) suggests that the panel-purlin connection flexibility, and local flange deformations at the connection, played a dominant role in the behavior.

The restraint provided by metal deck was further explored in Yu's thesis (Yu 2005) and the existing MBMA tests were found to provide a conservative prediction of developed restraint and suggested for use as k_ϕ in the distortional buckling (Section C3.1.4) commentary of AISI-S100. However, no equivalent data for cold-formed steel framing systems, such as floor joists, is available. The work summarized herein uses an augmented version of the AISI-TS-1-02 tests to examine cold-formed steel framing systems: steel joists sheathed with plywood and OSB, as well as steel joists sheathed with gypsum board as might exist in walls and ceilings.

CHARACTERIZING SHEATHING RESTRAINT

The basic test setup for measuring the sheathing rotational restraint is shown in Figure 2. The setup is similar to that used in AISI TS-1-02 (AISI 2002) but has been modified and expanded to reflect the specific needs of this testing program. Based on the measured load, P, the moment, per unit width is:

$$M = (P/w)h_o \tag{1}$$

This definition for M is exact only for the undeformed state. The total rotation, θ_2, of the sheathing-connector-joist assembly considers only Δ_v and h_o where:

$$\theta_2 = \tan^{-1}(\Delta_v/h_o) \tag{2}$$

Based on these definitions for M and θ the rotational stiffness is defined as

$$k_{\phi 2} = M/\theta_2 \tag{3}$$

where $k_{\phi 2}$ has units of (force·distance/length)/radian or simply force/radian.

(a) line drawing of test setup (b) photo during test of plywood sheathed specimen

Figure 3 Test setup for rotational restraint, k_ϕ, measurement

Component stiffness calculations

AISI-TS-1-02 only considers k_ϕ of Eq. 3, but due to the large variability in the stiffness of typical sheathing, the methodology was expanded to separate the rotation into sheathing and connection components. The rotation due to the sheathing, θ_w, may be removed from the total rotation by assuming a simple beam theory model for the sheathing and measuring the horizontal displacement, Δ_h. The lateral deflection at the point of moment application in the linear elastic range assuming standard beam theory for the sheathing deformation is:

$$\Delta_h = ML^2/(2EI_w) \tag{4}$$

and the rotation at the point of moment application is

$$\theta_w(\text{at } \Delta_h)=ML/(EI_w) \tag{5}$$

Using Eq. 4 and 5 the sheathing rotation is defined as

$$\theta_w=2\Delta_h/L$$

The rotational stiffness of the sheathing (wood) may then be determined via:

$$k_{\phi w}=M/\theta_w=M/(2\Delta_h/L) \tag{6}$$

The simplest definition of the connector rotation, θ_{c2}, assumes that only the sheathing rotation should be removed from the total rotation, i.e.:

$$\theta_{c2} = \theta_2 - \theta_w \tag{7}$$

which results in a connector stiffness of:

$$k_{\phi c2} = M/\theta_{c2}=M/(\theta_2 - \theta_w) \tag{8}$$

Note, this definition of the connector stiffness includes flexibilities from bending of the joist and the loading apparatus. This component model is consistent with a spring in series model, thus:

$$k_{\phi 2}=(1/(1/k_{\phi c2} + 1/k_{\phi w})) \tag{9}$$

EXPERIMENTAL RESULTS

The measured rotational restraint from the tests ($k_{\phi 2}$) is reported in Table 1for the 36 tests conducted (which covered 24 different sets of parameters, due to multiple tests for some parameter sets). To provide an overview of the conducted experiments, results for tests on an 800S200-54 joist with #6 fasteners spaced 12 in. on-center attached to OSB, plywood, and gypsum sheathing (24 in. long, 54 in. wide) are provided in Figure 4. The stiffness results (slope of the M-θ lines) indicates that OSB provides the most robust response, plywood can undergo significant rotation, but is much more flexible than OSB, and gypsum provides a stiff response, but with low rotation capacity.

Table 1 Parameters of conducted rotational restraint tests

$k_{\phi 2}$ (lbf-in./in./rad)											
Sheathing -->	Plywood					OSB		Gypsum			
Joist Spacing (L) -->	12"				24"	24"		12"		24"	
Fastener # -->	6		10		6	6	10	6	10	6	10
Fastener Spacing -->	6"	12"	6"	12"	12"	12"	12"	12"	12"	12"	12"
362S162-33		40						75			
362S162-68		42						94			
800S200-54	41	34		33	18	57	44	76	60	53	58
800S250-54		53		43							
800S200-97			47	44			66		58		
1200S200-54		34				44					
1200S200-97				59			75				

(joist designation, e.g., 362S162-33, in SSMA nomenclature, www.ssma.com, 1 in. = 25.4 mm)
(1) average values reported when multiple tests conducted
(2) re-tests of specimens not included in average value calculations (only original test)

As presented (Table 1, Figure 4), the rotational restraint includes deformations from the sheathing and connector. Figure 5 provides the M-θ relations for the isolated sheathing and connector components for the same three tests as given in Figure 4. Figure 5 shows that the difference between the plywood sheathed specimens and the OSB and gypsum sheathed specimens is due to the plywood, not the connection. In fact, the connection stiffness for all three specimens (slope of the M-θ_{c2}), which have nominally the same joist dimension, joist thickness, fastener size, and fastener spacing are quite similar despite varying attached sheathing types. Complete experimental results for all testing conducted are provided in Schafer et al. (2007).

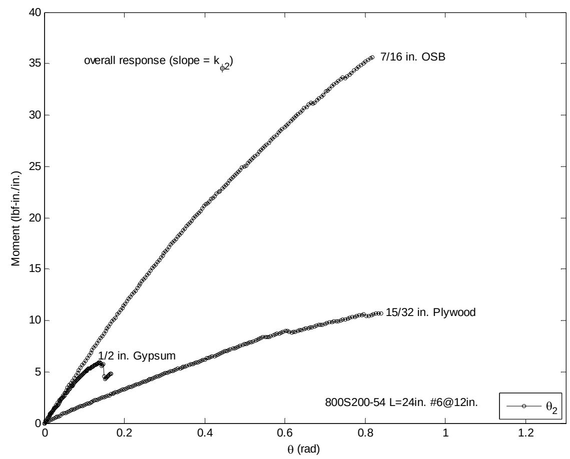


Figure 4 Typical moment-rotation results for overall stiffness (1 lbf = 4.448 N)

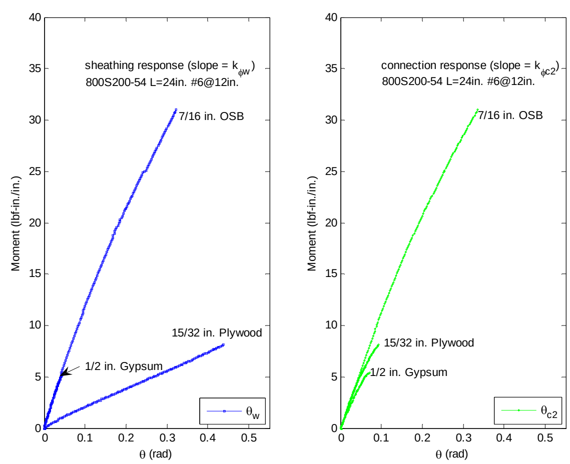
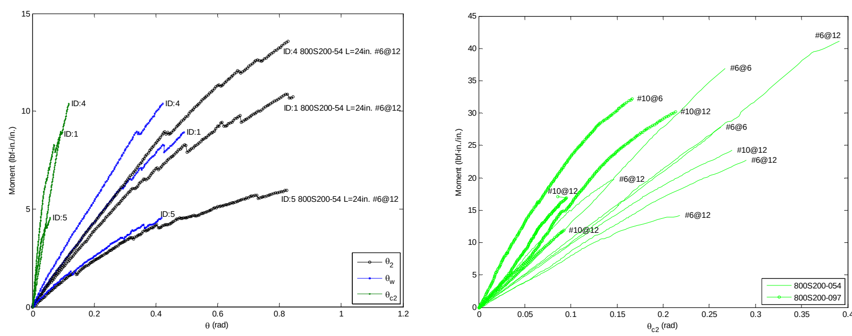


Figure 5 Typical moment-rotation results for sheathing and connection stiffness

Plywood Sheathing

Significant variability was observed in the sheathing stiffness in the plywood sheathed specimens. For example, Figure 6a provides the results for the three plywood sheathed specimens nominally identical to that of Figure 4. Interestingly, the variability derives from variation in the sheathing stiffness, not the connection stiffness (compare M-θ_w with M-θ_{c2} in Figure 6a).

An example of the comparisons provided in Schafer et al. (2007) for the plywood sheathed specimens is provided in Figure 6b, which shows the influence of joist thickness and fastener details on the observed connection response of 800S200 joists (slope of the lines is $k_{\phi c2}$). Careful study shows that joist thickness is a more significant variable than fastener size or spacing. However, close spacing does provide an improved (stiffer) connection response.



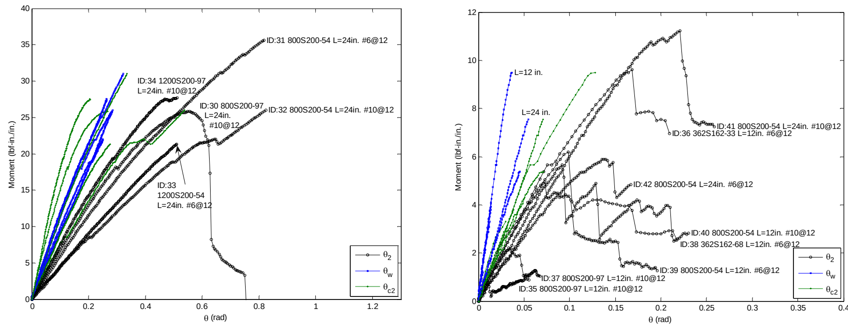
(a) response of 3 nominally identical plywood sheathed joists

(b) Connector M-θ_{c2} for 800S200 joists with varied thickness and fastener details

Figure 6 Moment rotation response of plywood sheathed specimens

OSB Sheathing

Overall moment-rotation response, and hence stiffness (slope of the M-θ_2 curve in Figure 7a), shows significant variation in OSB sheathed joists. However, the observed variability is primarily attributed to connection and joist details, not the OSB – which generally provides a consistent response. In addition, in one of the OSB sheathed specimens a pull-through failure was observed, thus indicating the possibility of this failure mode in OSB. However, the observed pull-through failure did not occur until approximately 0.5 rad (29 deg.), which is well beyond the anticipated rotational demands in distortional buckling up to and including collapse. See Schafer et al. (2007) for further discussion.



(a) OSB (b) gypsum

Figure 7 Moment rotation response of OSB and gypsum sheathed specimens

Gypsum Sheathing

The response of the joists sheathed with gypsum was significantly different than the OSB or plywood sheathed specimens: at low rotations the fasteners pulled-through the gypsum board and failed the specimens (Figure 7b and Figure 8). Figure 7b provides the moment-rotation results for the gypsum sheathed specimens. As the joist thickness increases, the rotation capacity decreases. The observed behavior suggests that while gypsum board may be able to resist distortional buckling of walls and ceilings at service loads, it is unreliable at ultimate strength levels as it has inadequate rotation capacity.

(a) large separation between joist and gypsum board (b) pull-through failure and fracture of gypsum board

Figure 8 Response of 800S200-54 joist sheathed to gypsum board with #10s @ 12 in.

Significantly more detail for all of the testing conducted is provided in Schafer et al. (2007). Utilization of the tested rotational stiffness in design is the focus of the remaining sections of this paper.

DEVELOPMENT OF DESIGN MODEL

It is proposed that the total rotational restraint, k_ϕ, needed for the distortional buckling calculation in AISI-S100 C3.1.4 be found using $k_{\phi 2}$ of Eq. 9. Thus, requirements for design are the sheathing rotational stiffness, $k_{\phi w}$, and the connection rotational stiffness, $k_{\phi c2}$. Based on the experiments reported herein, it is determined (below) that industry provided sheathing stiffness values are conservative for determining $k_{\phi w}$, and that a simplified empirical expression may be used for the connection stiffness, $k_{\phi c2}$.

Sheathing stiffness compared with industry tables values

Employing Eq. 4, the displacement, Δ_h, and the load, P, may be used to back-calculate the experimentally observed sheathing bending rigidity EI_w. The observed EI_w are compared to industry provided values in Table 2. The results indicate that the measured values are generally consistent with industry provided values, but industry provided values are typically more conservative than the average measured response. The relationship between the bending rigidity (EI_w) and the sheathing rotational stiffness ($k_{\phi w}$) is depicted in Figure 9 where it is shown to be a function of joist spacing and location. The expressions for interior and exterior joists given in Figure 9 are recommended for design.

Table 2 Sheathing bending rigidity
(a) sheathing stiffness determined from testing

	EI_w (lbf-in.^{2}/ft of panel width)				
	mean	C.O.V.	n	min	max
Plywood*	9000	0.3	27	4000	14000
OSB*	31000	0.1	5	26000	35000
Gypsum	41000	0.1	7	37000	43000

*stress perpindicular to strength axis

(b) sheathing stiffness available from standards

	EI_w (lbf-in.^{2}/ft of panel width)	
	mean	source
32/16 Plywood*	8100	APA, Panel Design Spec. (2004)
24/16 OSB*	16000	APA, Panel Design Spec. (2004)
32/16 OSB*	25000	APA, Panel Design Spec. (2004)
Gypsum (min)	18000	Gypsum Assoc, GA-235-01, (2001)
Gypsum (max)	48000	Gypsum Assoc, GA-235-01, (2001)

*stress perpindicular to strength axis

(1 lbf-in.^{2}/ft = 9.476 kN-mm^2/m)

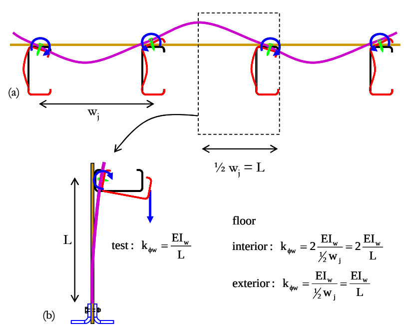


Figure 9 Sheathing stiffness for interior and exterior joists and comparison to conducted tests

Connection stiffness and design simplification

The average connection stiffness using Eq. 8, measured in the testing reported here, is provided in Table 3. The two parameters found to have the most influence on the connection rotational stiffness are joist thickness and fastener spacing (see Schafer et al. 2007 for additional analysis and discussion on this point). From a practical standpoint industry has shown a reluctance to move towards fastener spacing less than 12 in. on center, so the focus of the results are on the 12 in. on-center tests. For those tests, joist thickness is varied from 0.033 in. to 0.097 in. and the resulting measured connection rotational stiffness is reported in Figure 10.

Table 3 Average measured connection rotational stiffness

$k_{\phi c2}$ (lbf-in./in./rad)											
Sheathing -->	Plywood					OSB		Gypsum			
Cantilever (L) -->	12"				24"	24"		12"		24"	
Fastener # -->	6		10		6	6	10	6	10	6	10
Fastener Spacing -->	6"	12"	6"	12"	12"	12"	12"	12"	12"	12"	12"
362S162-33		81						100			
362S162-68		102						137			
800S200-54	116	109		97	137	113	77	103	77	91	99
800S250-54		116		124							
800S200-97			269	167			159		144		
1200S200-54		78				85					
1200S200-97				215			195				

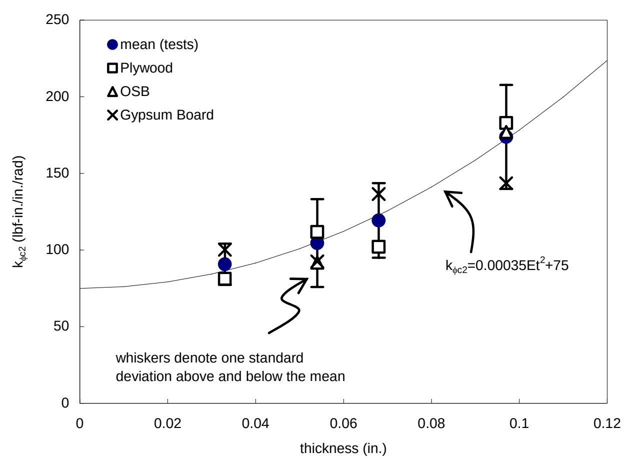


Figure 10 Connection rotational stiffness as a function of joist thickness

Figure 10 shows that an empirical relationship exists between the joist thickness and the connection rotational stiffness, largely independent of sheathing type (sheathing influence is captured through $k_{\phi w}$), in Imperial units:

$$k_{\phi c2} = 0.00035Et^2 + 75 \tag{10}$$

where: $k_{\phi w}$ = sheathing rotational stiffness in units of lbf-in./in. width / radian, E = 29,500,000 psi, and t = nominal joist thickness in inches. Eq. 10 has no mechanical basis, and is merely a mathematical convenience. To date, simple dimensionally consistent mechanical models that have been investigated (see Schafer et al. 2007) have lead to poor correlation with the data.

Comparison of the design method with the measured total rotational stiffness is provided in Table 4. Use of average tested values for the sheathing material leads to relatively high standard deviations for the plywood, but given the variability of plywood this seems acceptable. Simplification of the connection stiffness to values based on the thickness of the joist increases the variability of the predictive method for OSB and gypsum, but leaves the average test-to-predicted values within acceptable ranges. Use of Eq. 10 for $k_{\phi c2}$ is statistically equivalent to using the average tabled values for connection stiffness. Use of design values for the sheathing bending rigidity (i.e., based on APA or GA tables) introduces conservatism and increases variability of the predictive method, but is nonetheless recommended for design practice at this time.

Table 4 Test-to-predicted ratio for total rotational stiffness $k_{\phi 2}$

		plywood		OSB		gypsum board	
$k_{\phi w}$	**$k_{\phi c2}$**	**ave.**	**st. dev.**	**ave.**	**st. dev.**	**ave.**	**st. dev.**
Table 2a	tested values	0.97	0.21	1.00	0.06	1.00	0.02
Table 2a	thickness only*	0.98	0.22	0.97	0.14	0.92	0.16
Table 2a	Eq. 10	0.98	0.22	0.97	0.14	0.92	0.16
Table 2b, min values	Eq. 10	1.03	0.23	1.47	0.26	1.30	0.21

* $k_{\phi c2}$ is determined from the average tested values for a given joist thickness

The developed design model, in Specification language, is provided in the Appendix to this paper.

DISCUSSION AND DESIGN GUIDANCE

From the standpoint of simplifying design, the desired rotational restraint is the k_{ϕ} that will eliminate the distortional buckling limit state. For the sections tested in this experimental program, the k_{ϕ} such that M_n for distortional buckling per C3.1.4(b) of AISI-S100 (2007) is always greater than M_n for a fully laterally braced (L_b=0) section is determined and reported in Table 5. Comparison with Table 1 indicates the provided k_{ϕ} in floor systems is typically not high enough to completely eliminate the distortional buckling limit state from consideration.

At longer unbraced lengths, lateral-torsional buckling will control and distortional buckling will not matter even if k_{ϕ}=0, thus Table 5 also reports the unbraced length L_b at which M_n for distortional buckling per C3.1.4(b) of AISI-

S100 (2007) is greater than M_n (per C3.1.2) for lateral-torsional buckling (LTB). The length at which distortional buckling does not control is relatively short, so if blocking or bracing is spaced at lengths greater than L_b of Table 5 and that length is used for the LTB strength, then distortional buckling can be ignored.

Table 5 Minimum k_ϕ and L_b to avoid distortional buckling for example sections

	avoid distortional bucking via	
	k_ϕ	**L_b**
Section	(lbf-in./in./rad)	(ft)
362S162-33	36	4.4
362S162-33 (50ksi)	76	4.2
362S162-68	*DB never controls*	*DB never controls*
362S162-68 (50ksi)	*DB never controls*	*DB never controls*
800S200-33	31	6.6
800S200-33 (50ksi)	30	5.3
800S162-54	92	4.1
800S162-54 (50ksi)	190	4.1
800S200-54	300	6.1
800S200-54 (50ksi)	326	6.0
800S250-54	190	7.8
800S250-54 (50ksi)	233	7.1
800S200-97	*DB never controls*	*DB never controls*
800S200-97 (50ksi)	400	3.8
1200S200-54	128	5.9
1200S200-54 (50ksi)	123	5.6
1200S200-97	118	4.1
1200S200-97 (50ksi)	770	4.4

Finally, the first author of this paper recently completed a Technical Note for the Cold-Formed Steel Engineers Institute that provides additional tables, design aids, and extensive example calculations for distortional bucking. Designers and interested readers are referred to that document, as of this writing it is currently in press (complete and approved, but not yet printed) but should be available at www.cfsei.org. by the time of the conference.

CONCLUSIONS

Distortional buckling of cold-formed steel members in bending can be significantly retarded, or even altogether precluded, depending on the rotational restraint provided by sheathing or other attachments to the compression flange. A series of cantilever tests on sheathed joists was conducted to assess the rotational stiffness provided by plywood, OSB, and gypsum board sheathing to typical cold-formed steel joists in use in North America. The tests indicate that plywood and OSB can provide beneficial restraint, but gypsum has inadequate rotational capacity due to a pull-through failure which occurs at low strength and rotation. The traditional cantilever testing protocol (AISI TS-1-02) was successfully extended to include additional displacement measurements which were then used to separate the rotational stiffness into a sheathing component and a connection component. Evaluation of the connection stiffness indicated that joist thickness and fastener spacing are the most influential variables for predicting the available stiffness. A simple design method for predicting the component stiffness values was developed and shown to provide reasonable and conservative agreement with the conducted tests. This design method is recommended for use in the design of cold-formed steel framing systems where sheathing partially restraints distortional buckling.

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APPENDIX: PROPOSED DESIGN MODEL

Based on the results presented herein, this Appendix provides a method for calculating the rotational stiffness for use in distortional buckling calculations in "proposed" Specification language:

Calculation of the nominal distortional buckling strength in flexure per C3.1.4 of AISI S100, or per Appendix 1 of AISI S100 may utilize the beneficial system affect of sheathing fastened to the compression flange of *floor joists*, *ceiling joists*, *roof rafters*, or *wall studs* through the calculation of the rotational stiffness provided to the bending member, k_ϕ.

Calculation of the nominal distortional buckling strength in compression per C4.2 of AISI S100, or per Appendix 1 of AISI S100 may utilize the beneficial system affect of sheathing fastened to both flanges of *floor joists*, *ceiling joists*, *roof rafters*, *or wall studs* through the calculation of the rotational stiffness provided to the bending member, k_ϕ.

The rotational stiffness k_ϕ shall be determined via

$$k_\phi = (1/k_{\phi w} + 1/k_{\phi c})^{-1} \quad \text{(A1)}$$

where the sheathing rotational restraint $k_{\phi w}$ is calculated

for interior members (joists or rafters) with sheathing fastened on both sides as

$$k_{\phi w} = EI_w/L_1 + EI_w/L_2 \quad \text{(A2)}$$

for exterior members, or members with sheathing fastened on one side as

$$k_{\phi w} = EI_w/L_1 \quad \text{(A3)}$$

and:

EI_w = sheathing bending rigidity,
for plywood and OSB use APA (2004) as given in Table A1(a),
for gypsum board use min values of GA (2001) as given in Table A1(b);
note, gypsum may be used for serviceability, but not for strength

L_1, L_2 = one half the joist spacing to the first and second sides respectively, as illustrated in Figure A2

where the connection rotational restraint $k_{\phi c}$ is calculated for fasteners spaced 12 in. o.c. or closer in plywood, OSB, or gypsum

$$k_{\phi c} = \text{values per Table 2} \quad \text{(A4)}$$

Table A1 Sheathing Bending Rigidity

(a) Plywood and OSB bending rigidity per APA, Panel Design Spec. (2004)
divide table values by 12 to convert to lbf-in.^{2}/in. of panel width

RATED PANELS DESIGN CAPACITIES

Span Rating	Stress Parallel to Strength Axis: Plywood 3-ply	Plywood 4-ply	Plywood 5-ply	OSB	Stress Perpendicular to Strength Axis: Plywood 3-ply	Plywood 4-ply	Plywood 5-ply	OSB
PANEL BENDING STIFFNESS, EI (lb-in.^{2}/ft of panel width)								
24/0	66,000	66,000	66,000	60,000	3,600	7,900	11,000	11,000
24/16	86,000	86,000	86,000	78,000	5,200	11,500	16,000	16,000
32/16	125,000	125,000	125,000	115,000	8,100	18,000	25,000	25,000
40/20	250,000	250,000	250,000	225,000	18,000	39,500	56,000	56,000
48/24	440,000	440,000	440,000	400,000	29,500	65,000	91,500	91,500
16oc	165,000	165,000	165,000	150,000	11,000	24,000	34,000	34,000
20oc	230,000	230,000	230,000	210,000	13,000	28,500	40,500	40,500
24oc	330,000	330,000	330,000	300,000	26,000	57,000	80,500	80,500
32oc	715,000	715,000	715,000	650,000	75,000	165,000	235,000	235,000
48oc	1,265,000	1,265,000	1,265,000	1,150,000	160,000	350,000	495,000	495,000

(b) Gypsum board bending rigidity (modified to APA units) Gypsum Assoc., GA-235-01 (2001)

Effective Stiffness (EI)* (typical range)		
Board Thickness (in.)	Lb-in^2/in of width	N-mm^2/mm of width
1/2	1500 to 4000	220,000 to 580,000
5/8	3000 to 8000	440,000 to 1,160,000

* EI is dependent on board density, relative humidity, type of board, paper type, direction of board during testing and the amount of handling prior to measurement. In general the value of EI follows the following relationships:
Type X Gypsum Board > Regular Gypsum Board
Denser Gypsum Board > Less Dense Gypsum Board
Machine Direction > Cross Direction
Low Relative Humidity > High Relative Humidity

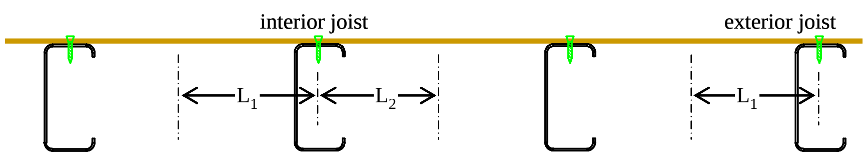

Figure A2 Illustration of L_1, L_2 for sheathing rotational restraint

Table A2 Connection Rotational Restraint

t (mils)	**t (in.)**	**$k_{\phi c}$ (lbf-in./in./rad)**	**$k_{\phi c}$ (N-mm/mm/rad)**
18	0.018	78	348
27	0.027	83	367
30	0.03	84	375
33	0.033	86	384
43	0.043	94	419
54	0.054	105	468
68	0.068	123	546
97	0.097	172	766

(1) fasteners spaced 12 in. o.c. or less

(2) values based on $k_{\phi c} = 0.00035Et^2 + 75$
with E in psi, t in in., $k_{\phi c}$ in lbf-in./in./rad