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Abstract

This paper presents a method for solving for the generation combination that supplies a specific load demand at maximum efficiency in a multigenerator power system. The method accounts for transmission losses using the B constant approach and generating losses using heat rate data. Results are presented for an example test system.

1. INTRODUCTION

Utilities have historically been concerned with operating their systems at minimum cost. Such an operating strategy entails dividing the total system load among generating units in such a manner as to minimize some system cost function. This mode of operation, called economic dispatch, does not necessarily result in maximum operating efficiency. This paper presents a method for determining the generating configuration for overall maximum system efficiency, i.e. minimum total system losses.

2. MATHEMATICAL FORMULATION

System transmission losses can be calculated from an equation of the form

$$P_{TL} = \sum_{i=1}^{n} \sum_{j=1}^{n} P_i B_{ij} P_j + \sum_{i=1}^{n} B_{i0} P_i + B_{00} \quad (1)$$

where P_i = Real generated electrical power for the ith generator

n = no. of generators

This approach is well documented in the literature and methods for computing the B constants are explained by Early [1], Kron [2], Kirchmayer [3], George [4], and others. A generating unit loss may also be approximated with a second order polynomial as:

$$P_{GL_i} = D_{i2} P_i^2 + D_{i1} P_i + D_{i0} \quad (2)$$

See the Appendix for more detail. The total system loss may be written as

$$P_L = P_{TL} + P_{GL}$$

$$= P_{TL} + \sum_{i=1}^{n} (D_{i2} P_i^2 + D_{i1} P_i + D_{i0}) \quad (4)$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} P_i B'_{ij} P_j + \sum_{i=1}^{n} B'_{i0} P_i + B'_{00} \quad (5)$$

where

$$B'_{ii} = B_{ii} + D_{i2} \quad (6)$$

$$B'_{ij} = B_{ij} \qquad (i \neq j) \quad (7)$$

$$B'_{i0} = B_{i0} + D_{i1} \qquad (8)$$

$$B'_{00} = B_{00} + \sum_{i=1}^{n} D_{i0} \qquad (9)$$

As in the economic dispatch problem, the problem may be solved using the method of Lagrangian Multipliers. See reference [5]. Form the Lagrangian $\mathcal{L}$:

$$\mathcal{L} = P_L - \lambda\phi \qquad (10)$$

where the constraint function ϕ is defined as:

$$\phi = \sum_{i=1}^{n} P_i - P_{TL} - P_D = 0 \qquad (11)$$

P_D = Total System Load Demand.

Differentiating $\mathcal{L}$ with respect to each P_i and equating to zero produces the equation set

$$\frac{\partial \mathcal{L}}{\partial P_i} = \frac{\partial P_L}{\partial P_i} - \lambda \left[1 - \frac{\partial P_{TL}}{\partial P_i} - 0\right] = 0;$$

$$i = 1, 2, \ldots, n \qquad (12)$$

where

$$\frac{\partial P_L}{\partial P_i} = 2 \sum_{j=1}^{n} B'_{ij} P_j + B'_{i0} \qquad (13)$$

and

$$\frac{\partial P_{TL}}{\partial P_i} = 2 \sum_{j=1}^{n} B_{ij} P_j + B_{i0} \qquad (14)$$

Define λ_i as:

$$\lambda_i = \frac{\partial P_L / \partial P_i}{1 - \partial P_{TL} / \partial P_i} \qquad (15)$$

Each generating unit is not free to deliver any conceivable power but must operate within limits P_{min_i} and P_{max_i}, imposed by design and operating constraints. The system will operate at its maximum efficiency when the three following conditions are simultaneously met:

1. $\phi = 0$
2. $P_{min_i} \le P_i \le P_{max_i} \quad i = 1, 2, \ldots, n$
3. $\lambda_1 = \lambda_2 = \ldots = \lambda_m = \lambda$ for all eligible units.

A unit is defined as ineligible, if when λ_i is equated to λ, the corresponding P_i is forced to exceed a limit.

3. PROBLEM SOLUTION

The problem can be solved on a digital computer. Assuming some initial generation pattern is established, $\partial P_L/\partial P_i$, $\partial P_{TL}/\partial P_i$, and λ_i can be calculated for each unit from (13), (14), and (15), respectively. An average λ may be calculated, using only eligible units, from

$$\lambda_{av} = \frac{1}{m} \sum_{j=1}^{m} \lambda_j \qquad (16)$$

m = no. of eligible units.

Equations (13), (14), and (15) may be manipulated to solve for $P_i = P'_i$ using λ_{av} for λ_i:

$$P'_i = \frac{\lambda_{av}(1 - B_{i0}) - B'_{i0}}{(\lambda_{av} B_{ii} + B'_{ii})2} - \frac{1}{\lambda_{av} B_{ii} + B'_{ii}} \sum_{\substack{j=1 \\ j \ne i}}^{n} (\lambda_{av} B_{ij} + B'_{ij}) P_j \qquad (17)$$

The equation of constraint (11) must be satisfied; any mismatch in power must be absorbed by the eligible generating units. This adjustment will typically mismatch the λ's, requiring a new λ_{av} and P_i to be calculated. The calculation will iterate until the three conditions for maximum efficiency are simultaneously met. A flow chart summarizing the major steps is shown in Figure 1.

4. AN EXAMPLE APPLICATION

A three generator example was contrived to illustrate the method. Generator data is given in Table I.

Generating Unit	1	2	3
MVA Rating	110	85	140
P_{max} (MW)	110	85	140
P_{min} (MW)	45	40	45
D_2	0.266	0.263	0.309
D_1	1.250	1.350	1.180
D_0	0.180	0.185	0.111

Table I. GENERATING UNIT DATA. D CONSTANTS IN PER UNIT ON GENERATOR RATINGS.

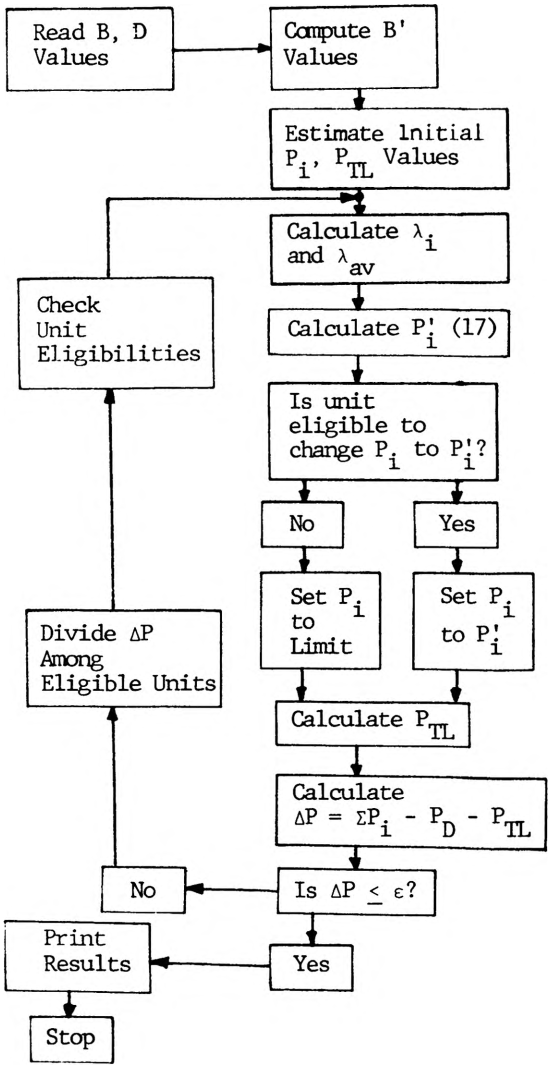

FIGURE 1. FLOW CHART

The system B constants in per unit on a 100 MVA base are:

$$[B_{ij}] = \begin{bmatrix} .0186 & .0016 & .0011 \\ .0016 & .0208 & .0008 \\ .0011 & .0008 & .0244 \end{bmatrix}$$

$$B_{i0} = B_{00} = 0$$

Computer calculated results are presented in Table II.

5. CONCLUSION

The current interest in energy conservation justifies research into operating strategies that minimize power system losses. This operating criteria has the advantage that heat rate versus power output data is readily available, and is fixed, whereas fuel cost data, is required by economic dispatch criteria, is subject to fluctuations. If the cost functions for each unit have the same functional relationship to P_i as does P_{GLi} and all units have the same fuel costs, this method will produce the same optimum generation as does economic dispatch. Transmission losses may also be accounted for by using A constants, as suggested by Hill and Stevenson [7], and others.

6. APPENDIX

It is common practice to supply generating unit efficiency data in terms of Heat Rate (HR) plotted versus

Load	P_1	P_2	P_3	λ_1	λ_2	λ_3	P_L
140	45.0	40.0	56.6	1.516	1.648	1.503	246.7
160	54.1	40.0	68.1	1.571	1.649	1.571	277.5
180	64.4	40.0	78.4	1.632	1.650	1.632	309.6
200	72.6	44.2	86.7	1.683	1.683	1.683	342.7
220	80.0	50.1	94.2	1.728	1.728	1.728	376.8
240	87.4	56.1	101.6	1.774	1.774	1.774	411.9
260	94.8	62.0	109.1	1.820	1.820	1.820	447.8
280	102.2	68.1	116.5	1.867	1.867	1.867	484.7
300	109.7	74.1	124.0	1.914	1.914	1.914	522.5
320	110.0	83.4	135.5	1.918	1.987	1.987	561.5

TABLE II. GENERATION FOR MAXIMUM EFFICIENCY. ALL POWERS IN MW.

generator electrical output power (P_i), where Heat Rate is typically given in BTU/Kw-Hr. Consequently:

$$\text{Efficiency} = \eta = \frac{\text{Output Electrical Power in MW}}{\text{Input Thermal Power in MW}} \quad (A1)$$

$$= \frac{3413}{HR} \quad (A2)$$

Generator Losses (P_{GL_i}) may be written as:

$$P_{GL_i} = \frac{1 - \eta}{\eta} P_i \quad (A3)$$

$$= \frac{HR - 3413}{3413} P_i \quad (A4)$$

Plots of P_{GL_i} versus P_i are typically smooth curves that monotonically increase.

Coefficients for a second order polynomial may be computed by least mean squared error methods [8] to produce a relation of the form

$$P_{GL_i} = D_{i2}P_i^2 + D_{i1}P_i + D_{i0} \quad (A5)$$

Other expressions could be used with only minor modifications to the method; however testing on a variety of generator data showed that the second order expression fit the data quite well. Discontinuities in the data caused by auxilliary equipment can be handled by changing D constants.

7. REFERENCES

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8. BIOGRAPHY

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