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Theoretical and Empirical Approaches To The Analysis of OPEC

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Abstract

Formal theoretical models of world oil price determination have been developed in recent years. In addition, some empirical estimates of expected future prices have been proposed. This paper explains how exhaustible resource and cartel theory are integrated into the opitmal control framework. Some empirical studies in the optimal control tradition are surveyed and evaluated.

1. INTRODUCTION

The important determinants of world oil prices are (1) "market structure" including the cohesiveness of OPEC as a price setting institution, (2) elasticities of substitution, or the reaction to actual prices by non-OPEC producers of oil,natural gas, coal, and other energy resources, and (3) "final product demand elasticities", or the reaction by consumers who make choices based on relative prices. Since the evidence on (1), (2), and (3) is incomplete it is not surprising that opinion differs regarding the magnitude and direction of future energy prices.

The purpose of this paper is related to the determination of world oil prices; it is a condensation of some recent theoretical and empirical analyses which have been used to ascertain likely future trends in world oil prices.

Emphasis is placed upon contributions found in professional journals and related papers. The rationale for choosing this subset of the literature is that most seminal contributions are contained there.

It is important to emphasize the literature not covered very extensively. Studies which assume a specific level of world oil prices and then outline the repercussions on regions, end uses, the use of alternative fuels, and the like, are only briefly considered. The emphasis is upon outlining the variables which determine world oil prices and not upon appraising the effects of specified price levels. In addition, the emphasis is upon economic rather than political explanations of price determination. The reason for this restriction is the belief that prices will not deviate very much or very long from levels which best serve the economic interests of producers, given levels of technology, resource substitutabilities, product demands, and the market structure or state of competition.

The theoretical approaches relevant to the analysis of OPEC include the theory of depletable resources, the theory of cartel behavior, and the theory of individual country behavior, a variant of the theory of the firm. The strategy adopted is to present, in Section 2, a brief account of the exhaustible resources literature and how it relates to OPEC. Section 3 contains a discussion of cartel theory as applied to OPEC. These background discussions are included because they form the basis for the formal theoretical models of individual country or group behavior considered in Section 4 under the heading "Optimal Control Theory." Section 5 contains a summary and critical apprasial of some recent optimal control models of OPEC, and Section 6 is the conclusion.

2. EXHAUSITIBLE RESPURCES LITERATURE

The exhaustible resources literature is based on contributions by H. Hotelling [12], O. Herfindahl [10], A. Scott [18], R. L. Gordon [9], R.G. Cummings [3], J.L. Sweeney [20], and others. It may be viewed as an attempt to formulate a theory to explain the actual course of resource prices or as an effort to assist social planners in allocating an increasingly scarce resource. In the former case one is attempting to explain real world behavior whereas in the latter the analyst attempts to show how some social objective can be attained. The former is designed to determine the level of output a private resource owner will select under various market conditions (e.g., pure competition, duopoly, or monopoly). The latter is designed to ascertain the level of output which a social planner should select for the good of society or a significant subgroup. Most studies of OPEC treat individual countries as "firms" and OPEC as a monopolist, duopolist, oligopolist, or residual supplying firm. The emphasis in this paper is upon the actual course of world oil prices.

The unit of analysis is individualistic and the objective function is generally assumed to be the present value of the decision maker's expected future earnings subject to a constraint upon the total availability of the resource.

The assumptions underlying the model are:

1. there is a limited finite quantity of the resource;

2. technology is given;

3. extraction costs are greater than or equal to zero;

4. costs increase as the reserve is depleted;

5. the demand function is given;

6. the price received for the resource is positive.

Given these conditions the firm seeks to maximize the present value of the stream of profits accruing to it over time.

Hotelling was the first to work out the implications of these assumptions for an individual mine owner under competitive and monopolistic conditions. He simplified the problem somewhat by assuming marginal extraction costs constant and dealt with "net price" or the " ... price received after paying the cost of extraction and placing upon the market ..." [12, 141]. Under free competition, and assuming an initial equilibrium, the net price will rise at the market rate of interest. Under monopoly, and assuming an initial profit maximizing net price, the net price will rise at the market rate of interest. Under monopoly, and assuming an initial profit maximizing net price, the net price will rise less rapidly than the market rate of interest and may eventually be lower than the purely competitive price; in any case production is more protracted under monopoly than under conditions of pure competition [12, 152]. This feature prompted Solow to quip"... that if a conservationist is someone who would like to see resources conserved beyond the pace that competition would adopt, then the monopolist is the conservationists friend." [19, 8].

This relatively simple framework, applicable to an individual firm or mine, is extended to the industry by definition for the monopolist and by a plausible line of reasoning for purely competitive firms. Assuming there are cost differentials among competitive producers the lowest cost producers will operate until their resource deposits are exhausted, then the next higher cost producers will enter the market, and so on [19, 3-5]. This line of reasoning is an extension of that applicable to an owner of a "mine" or "resource deposit" since he will extract his most accessible or least costly reserves first.

If a resource were absolutely indispensible its market price would rise at the market rate of interest to infinity. An ever smaller population would bid for an ever dwindeling quantity available. However, no energy resource is indispensible. Energy resources which are presently being exploited are cost-effectively cheap relative to more or less viable alternatives. Thus, as the "net" and market prices of resources presently exploited rise alternative resources become cost-effectively cheap. The concept of a "backstop technology" is a special case of the general tendency to substitute cost-effectively low-cost resources for high cost ones, regardless of the reasons underlying the cost differentials. Nordhaus defined the backstop technology in a special sense as the " ... ultimate technology - resting on a very abundant resource base - ..." [15, 532].

3. CARTEL THEORY

Cartel theory as applied to OPEC is simply an offshoot of conventional microeconomic theory. The theoretical contributions include those of M. Adelman [1], A. Danielsen [4], and D. Osborne [16]. Cartel theory may be used to explain the course of world oil prices in much the same way that the purely competitive model is used to explain the price of a commodity. It is first necessary to posit the existence of a cartel and that the cartel price is greater than it would be under competitive conditions. Since marginal revenue exceeds marginal costs for each country there are incentives for each country to expand output. But in order to sell more in the short-run a country must have shut-in capacity and be willing to lower or "shave" prices to entice prospective buyers away from other producers. If large volumes of capacity are shut-in and price-shaving becomes wide spread then the collusive arrangement will break down. If price-shaving is largely absent then the collusive or cartel price holds, regardless of how much capacity is shut-in.

Every successful cartel faces the short-run problem of maintaining the extablished cartel price by ensuring that all participants are reasonably content. In the long-run the cartel faces the same set of problems but they are complicated by the fact that additional capacity can be brought on stream. It should be emphasized that each country has a degree of autonomy and that there are inherent conflicts of interest among them regarding the total revenue each should receive vis a'vis the others, their time preferences for income, and their desired growth rates of production and of capacity. In addition, their assessments of world and OPEC demand elasticities may differ, thus complicating an already difficult set of problems.

The success of OPEC in holding the world price of oil above marginal costs is itself a stimulus to the development of excess capacity. This is a continual threat to the maintenance of cartel prices. This problem is composed of two parts, the incentive to substitute non-OPEC for OPEC produced energy, and the incentive for member countries of OPEC to expand capacity beyond the optimum from the standpoint of the cartel. The only thing OPEC can do to influence the former is to establish their price schedules at low levels or promote uncertainty about the maintenance of high prices over time. The latter is a matter of internal cartel discipline and will not be pursued in this paper.

4. OPTIMAL CONTROL THEORY

The basic ideas in the exhaustible resources and cartel literature have been formalized into a theory of individual or group behavior and incorporated into the "optimal control" framework [20], [8], and [5]. The decision-making unit may be an individual country, a sub-group of countries within OPEC, or OPEC considered as a whole (i.e, monopolist or residual supplier). The decision making unit is assumed to exist in a world in which all demand, supply, technological, and market structure relations are given. The task is for him to select a price trajectory through time which will achieve the desired objective, usually maximizing the present value of net revenue. The problem differs only in the constraints relevant for the case of a purely competitive firm, a monopolist, or a cartel.

4.1 PURELY COMPETITIVE MODEL

Under pure competition a resource owner must take into account his (1) total resources (R_0) (2) prices received over time ($p(t)$) (3) extraction costs ($c(t)$) (4) rate of extraction $s(t)$, and (5) discount factor (r). His problem may be posed as maximizing an objective function under a resource constraint:

$$F_C \begin{cases} \text{Max} \int_0^\infty \left\{ [p(t) - c(s(t),t)]s(t) \right\} e^{-rt}dt \\ \text{Subject to:} \\ \quad s(t) \geq 0 \\ \quad \int_0^\infty s(t)\,dt \leq R_0 \end{cases}$$

where price is a function of time, cost is a function of the extraction rate and time, r and R_0 are constants, and $s(t)$ is selected to maximize F_C.

The optimal extraction path may be denoted as $s^*(t)$. The familiar profit maximizing rule of equating marginal revenue to marginal cost is altered by the fact that the resource will ultimately be depleted. Using up the resource means foregoing its use in the future, The profit maximizing condition may be expressed as:

$$(1) \quad p(t) = c' + \lambda e^{rt}$$

where $c' = \partial[c(s(t)]/\partial s(t)$ is marginal extraction cost, and λe^{rt} is interpreted as the opportunity cost of using the resource at t. Since $\lambda > 0$ it follows that price exceeds marginal extraction cost. It is also easy to see that if extraction costs are zero price will rise at the rate r; similarly, if one deals with "net price" as Hotelling did, then net price or $p(t)-c'$ will rise at the rate r.

4.2 MONOPOLISTIC MODEL

A monopolist is faced with a similar problem, the main difference being that the market price is dependent upon the total demand and remaining supply; it is therefore dependent on all previous output as well as the rate at which the resource is currently extracted. Thus, the objective function for the monopolist is:

$$F_M \begin{cases} \text{Max} \int_0^\infty \left\{ [p(S(t), s(t),t) - c(s(t),t)]\, s(t) \right\} e^{-rt}dt \\ \text{Subject to:} \quad s(t) \geq 0 \\ \qquad \int_0^\infty s(t)dt \leq R_0 \\ \qquad S(t) = \int_0^{t-1} s(t) \end{cases}$$

Since the monopolist owns all deposits he maximizes net present value over all resource deposits rather than over an individual field or well.

Associated with the optimal extraction path, $S^*(t)$, is an optimal price trajectory, $p^*(S(t), t)$ or simply s^* and p^*. A profit maximizing condition similar to equation (1) may be expresed as

$$(2) \quad P' = C' + \lambda' e^{rt}$$

where the primes refer to the partial derivatives of the total revenue and cost functions and λ' is now a complex function indicating opportunity costs for the monopolist.

4.3 CARTEL MODEL

Except for the limiting cases where a cartel is so closely knitted that it functions as a monopolist or so disorganized that the market functions as a set of purely competitive firms the problem is altered. In the intermediate cases where a cartel is viable but not perfectly cohesive it is useful to posit two groups, a cartel group or "dominant group" within the cartel and a competitive fringe of firms who stand ready to capture an increasing share of the market. The model is applicable in two special instances. First, a cartel which includes all producers but has internal dissention may be thought of as a dominant group and a competitive fringe within a cartel. Second, a cartel which has a dominant position in an industry but does not include all producers. In this case the competitive fringe functions outside the cartel. Empirical models of OPEC have been constructed for both cases.

The erosion of market share is a problem for any cartel but in the present contex such erosion is complicated by the fact that the resource is depletable for both the cartel and the competitive fringe. Thus, the problem from the standpoint of the cartel is the maximize a rather complex objective function subject to both internal and external constraints. The cartel or oligopolists problem may be expressed as a limit-pricing problem:

$$F_0 \begin{cases} \text{Max} \int_0^\infty \{[p(t)-c(t)]\ s[p(t),x(t),t]\} e^{-rt}\, dt \\ \text{Subject to: } c(t)=c[R_0-\int^t s[p(t),x(t),t]dt] \\ \dot{x}(t)=k[p(t)-\bar{p}\ (x(t),\partial] \\ \int_0^\infty s[p(t),x(t),t]dt \geq R_0 \\ x(0)=x^0 \end{cases}$$

where x(t) is output by the competitive fringe, $\bar{p}$ is the "limit price" or that price which would totally eliminate additional production by the competitive fringe, and ∂ represents a variety of parametric variables which may influence the rate of technological change, substitutability of other energy resources for oil, etc.

The solution to this problem involves the generalized Hamiltonian multiplier and yields both an optimal price trajectory, p*, and an optimal trajectory of output by the competitive fringe, x*. It follows directly that there is also an optimal trajectory for cartel output, s*.

These formal models are useful since they emphasize the important variables the analyst needs to consider as well as their hypothesized interrelations. The variables have both a spatial and temporal dimension. Ignoring the time dimension and designating cartel or "dominant group" variables by the subscript "a", and competitive fringe variables by "b", the variables may be summarized as:

Reserves	R_a R_b
Costs	c_a c_b
Total Demand	D
Production	s_a s_b
Discount rates	r_a r_b
Prices	p_a p_b

The specific assumptions about these variables vary from one study to another and are the subject matter of Section 5.

5. RECENT EMPIRIAL STUDIES OF OPEC

Empirical studies in the optimal control tradition include those of Nordhaus [15], Kalymon [13], Cremer and Weitzman [2], Hnyilicza and Pindyck [11], Gately, Kyle, and Fischer [8], and Marshalla [14].1/ These studies are similar from a broad conceptual point of view in that each seeks to determine either the most probable path of world oil prices or the probable and maximum price trajectories. For example, Pindyck [17] calculates the potential gains from cartelization by substracting the present value of OPEC revenues in a purely competitive environment from those under monopoly. Hnyilicza and Pindyck [11] seek to determine the extent to which the price trajectory for a two-part cartel will differ from that of a monopoly. Cremer and Weitzman [2] are interested in quantifying what long-term oil prices would be if OPEC maximizes the present value of its net income stream. Gately, Kyle, and Fischer [8] are interested in the same problem but the emphasis is upon the optimal price paths for OPEC under a variety of assumptions. They are all aware that "... the implications of any given price path for OPEC's profits and capacity utilization are very sensitive to the choice of parameter values." [6, 17].

Since the basic structure of the various models is identical, it is useful to specify the general procedure necessary to calculate the price trajectories. In each case the following equations for crude oil are specified:

(A) the objective function(s),
(B) the total world demand,
(C) supply by the competitive fringe,
(D) residual OPEC demand,
(E) costs of production,
(F) total reserves

In addition, the following parameters are specified:

(G) discount rates and,
(H) time horizon.

Each of these will be considered in this section as well as the results of several of the more recent models.

5.1 OBJECTIVE FUNCTION(S)

The objective function is a profit maximizing function of the form F_C as specified in Section 4. The more elementary models such as those of Pindyck [17] and Marshalla [14] focus on OPEC as a unified decision-making entity so that only OPEC has an objective function. These are strictly residual supply models. Hnyilicza and Pindyck [11] are mostly concerned with the weights which enter the objective function of a two-part cartel or how the cartel decides upon a weighted average objective function of two participant groups. This is also a residual supply model but with emphasis on internal bargaining. Cremer and Weitzman [2] posit an objective function for both OPEC and the competitive fringe. In order to empirically derive price trajectories it is assumed that OPEC announces a sequence of all future prices; the competitive fringe then sets its production levels to maximize the present value of its discounted profits. OPEC then becomes the residual supplier. The Cremer and Weitzman formulation is a two-group recursive model. This seems to be a step in the right direction. Other possibilities are n-country recrusive models or two-group or n-group simultaneous equations approach. The computational difficulties or the latter are of course formidable.

5.2 WORLD DEMAND

The empirical models assume relatively simple total demand functions. Hnyilicza and Pindyck specify demand as:

$$(5\text{-}2\text{-}1)\ D_t = 1.0 - .13\ P_{at} + .87\ D_{t-1} + 2.3\ (1.015)^t$$

where the .015 growth rate in the last term corresponds to an assumed 3 percent growth rate of real income and a 0.5 long-run income elasticity. Cremer and Weitzman employ:

$$(5\text{-}2\text{-}2)\ D_t = (21-0.6p)(1+g)^t$$

where g is the growth rate of world demand.

Marshalla's demand function is derived from the Federal Energy Administration IEES model and the Stanford Research Institute-Gulf Energy Model (SRI-GEM) and is of the general form:

$$(5\text{-}2\text{-}3)\ D_t = \Sigma d_{it}$$

$$d_{it} = B_{i1} \log p_{it} + B_{i2} \log d_{it-1}$$

where $i = 1, 2, 3, 4$ regions.

He considers the disaggregated demand and the link to established models one of the principal advantages of his model over previous efforts. However, the specification should still be regarded as relatively simple since the larger models contain few independent variables, a distributed lag structure is assumed but is untested, and the functional form is log-linear and thus yields constant price elasticities of demand. Another innovation by Marshalla is the use of a relative demand function where each variable in equation (5-2-3) is divided by its equivalent generated by the IEES or SRI-GEM models. The results are, therefore, relative deviations from the results obtained in the reference models.

5.3 COMPETITIVE FRINGE SUPPLY

Supply functions of the competitive fringe are also relatively simple functions in the Hynilicza and Pindyck and Marshalla models. The former makes competitive supply a linear function of previous supply, a non-linear increasing function of price, and decreasing function of cumulative supply (S_{bt}). The S_{bt} variable is an alternative to "remaining reserves" which are $R_{bt} = R_o - S_{bt}$. The specific equation is written:

$$(5\text{-}3\text{-}1)\ s_{bt} = (1.1 + .10p_t).(1.02)^{-S_{bt}/7} + .75\ s_{bt-1}$$

Marshalla's functional form and treatment of competitive fringe supply is identical to that of his total demand, including the use of "relative" supply functions.

As indicated before Cremer and Weitzman derive the competitive fringe supply schedule using an objective function. The sequence of prices for all future periods is given by the dominant group within the cartel. A "trace" of the output level used in the optimal solution by the competitive fringe is their level of output or supply over time.

5.4 RESIDUAL OPEC DEMAND

All of the models under review are "residual demand" or "residual supplier" models. The residual is simply total demand less competitive fringe supply. The group which constitutes the residual supplier differs in each model. The residual supplier establishes prices by selecting the price trajectory which maximizes the discounted value of income derived from the sale of its petroleum reserves. A price trajectory which is "too high" will dampen world demand and stimulate competitive supply; a trajectory which is "too low" will have the opposite effect and result in OPEC depleting their resources too early. The optimal price trajectory is neither too high nor too low and is obtained by an iterative procedure.

5.5 - 5.6 PRODUCTION COSTS AND TOTAL RESERVES

Production costs are an increasing function of cumulative production or vary inversely with remaining reserves. The precise equation used by Hnyilicza and Pindyck is:

$$5\text{-}5\text{-}1)\ c_{at} = 250/R_{at}$$

where R is in billions of barrels.

Cremer and Weitzman break costs into "current," "capital", and "transportation" costs for both the dominant cartel group and the competitive fringe. Then current costs (ca) are specified as:

$$(5\text{-}5\text{-}2)\quad c_a = k\, R_{ao}/R_{at}$$

where k is a constant.

Capital and transportation costs are assumed to be constants.

Marshalla uses a "Low Cost" case similar to equation (5-5-1) and a "High Cost" case of the form:

$$(5\text{-}5\text{-}3)\quad c_a = \beta_o + \beta_1 \quad (S_{at})^2$$

where the β's are constants and S_{at} is again cumulative supply.

Thus, in each case the trend of projected costs is plausible but not based on firm geological or engineering appraisals. Marshalla performed a sensitivity analysis by varying reserves by 50 percent but direct sensitivity tests on unit costs have not been very extensive.

5.7 DISCOUNT RATES

A relatively high discount rate makes the early conversion of oil to money more desirable. Hnyilicza and Pindyck use rates of both .05 and .10, Marshalla .04, .06, and .08, and Cremer and Weitzman .05 for the dominant group and .08 for the competitive fringe. The higher rates result in high levels of output and low prices in the early years and a lower price trajectory overall. The discount rate or "rate of time preference" for money is used to distinguish the dominant group from the competitive fringe. The competitive fringe is in all cases assumed to have a higher rate of time preference than the dominant group. Hnyilicza and Pindcyk call the dominant group "saver countries" and the remainder "spender countries." The rational for spender countries having the higher discount rate is based on their large population and revenue needs for economic dedevelopment. Thus, the competitive fringe tends to produce a larger proportion of total output in the near term whereas the dominant group's share eventually predominantes. This presumed discount rate differential has a marked effect on the pattern of crude oil output and upon the general conclusion that prices will not rise very greatly during the next twenty years.

5.8 TIME HORIZON

Theoretically, production will take place to infinity or until depletion. As a practical matter the discount rate ensures that net revenues received much beyond 50 years in the future will not weigh heavily on present value calculations. Thus, in almost all cases an approximate 50-100 year time horizon has been selected. The notable exception is the study by Nordhaus who uses 200 years.

5.9 WORLD PRICE TRAJECTORIES BASED ON EMPIRICAL MODELING

One should not be surprised if the conclusions differ markedly among the various models. Focusing on the world oil price trajectory Pindyck concludes that under monopoly (r=.10) the price would be $14.08 in 1975, decline to $10.19 by 1979 and then rise to $20.52 in 2010. The competitive price trajectory would result in $4.62 in 1975 rising to $25.48 in 2010. The Hnyilicza and Pindyck model results in a price of $14.39 in 1975, declining to $10.30 in 1979 and then rising to $20.61 in 2010.

Cremer and Weitzman deal with discrete 10-year time periods but their conclusions are remarkably similar to those of Hnyilicza and Pindyck. In their "preferred" specification prices rise gently from $9.80 to $10.30 over the period 1975-1995, increase to $14.70 during 1995-2005, and to $20.80 during 2005-2015. In all, they report the results of eleven different parameter specifications as alternatives to the preferred model. The highest prices would occur if the annual growth rate of world demand were 6 percent; prices would rise from $12.30 in 1975-1885 to $27.60 in 2005-2015 under this scenario.

Marshalla does not calculate "probable" or "most likely" price trajectories but rather "efficient ones" under the assumed conditions. His base case uses the high extraction cost function, a 6 percent discount rate, 830 billion barrels for OPEC reserves and the coal-based synthetics reference supply path. The most striking result is that price over the next 15 years is very low compared to current prices. The price for the period, 1976 to 1977, is $4.29, rises to $8.45 by 1990 and reaches a high of $31.85 when OPEC runs out of oil in 2025. It may come as no surprise that present prices far exceed "efficient" world prices and that large monopoly rents are currently being received.

6. CONCLUSION

It would be valid to criticize the efforts to model OPEC on grounds that they are relatively naive representations of the real world. With few exceptions, the data requirements are not very demanding, the basic equations include only a few variables, their functional forms have not been subjected to testing, the spacial and temporal interrelations (including lag relations) have been tested extensively, and there is no proof that the world oil market functions as postulated by the "residual supplier" framework.

On the other hand, the modeling efforts are useful for understanding expected future price trends and for policy-making because they focus upon some of the more important variables which determine prices. The market structure, total resource availabilities, unit costs of production and distribution, and the rates of time preference for money are all important determinants of future energy prices. Those who have constructed formal models have made contributions by highlighting probable interrelations. The much more difficult task is to determine the magnitude of the variables and to more accurately specify the interrelations among countries and variables in the real world. Few analysts are content with either the scope of precision of the theoretical and empirical analyses of OPEC but one may anticipate that more detailed specifications will be based on the formal models developed to date.

Footnote

1/ The studies by Nordhaus and Kalymon which are how several years old and have been reviewed by Fischer, Gately, and Kyle [6], will not be considered in detail here.

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