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Charles Geiger

Milton E. Harvey

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Charles Geiger and Milton E. Harvey
Kent State University
Kent, Ohio

Abstract

Systems theory provides an excellent perspective, as well as methodology, for analyzing energy problems of availability and use. The linear systems format is applied here to a system which focuses on household energy, energy independence, and energy conservation. Z-transform analysis can be used to simplify the mathematical operations required in this model.

1. INTRODUCTION

One of the most important political, economic and social issues today is that of energy. The President's recent speech to the nation described the nation's energy situation as a national crisis only equivalent to that of war. Projections of varying accuracy tend to give the prognostication that the consumption of energy in this country will reach astronomical dimensions in a very short time. Another major area of energy study that has been of concern has been the prospects of substitutability between different energy sources. Most of the studies in this area are econometric studies of the electric utility industry and thus have limited scope and purpose.(6.,p. 217) Many others try to deal with determining the proportions of energy sources used to satisfy future energy demand.

Generally, these models cannot accomodate a major disturbance to either the total energy supply or demand. Also, most models attempt to predict only the demand by industrial or commercial consumers, almost completely ignoring the household, which we believe is the most important energy consuming sector.

This total energy picture can be best described in terms of systems theory, which can deal with such intricate and numerous interrelationships. Put simply, the structure that systems theory assumes is that of "sets of elements standing in interrelation."(1.,p. 38) The candidates for these elements in an energy system are many. Systems of source, transportation, or transmission nodes, or of producers or consumers at whatever level of accuracy could variously be considered. It is important to recognize, however, that each of these is not just another perspective of the same thing, but are different components, or subsystems, of the total energy system.

Since it is not possible to predict our energy future, we need another basis for making decisions concerning the availability and use of energy resources. In modelling this we should take advantage of the system structure of the actual energy picture. Even if we don't take every element of the actual system into account, the model must not ignore their influences. We must be able to focus on whatever part of the total picture we like, and yet be able to effect any type of change in the whole system that is conceivable. Above all, the model must be programmable, and the program must be simple enough to run quickly and often.

Linear systems theory does provide such a model. So far it has been almost exclusively used to model systems in physics and electronics. The few social science studies have worked with extremely simplified closed systems which ignore the system inputs. The use of linear systems theory in modelling social systems or resource systems needs to be developed to a much greater degree that it has so far. (3.,4.)

2. LINEAR SYSTEMS MODEL

The linear system model consists of two state equations:

$$x(t+1) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t) \qquad \ldots(1)$$

where u, x, and y are vectors containing the system parameters, and A, B, C, and D are matrices used in deriving these parameters. The controlling parameter is time,t, which for this particular model is discrete.

As in any system formulation, this system has inputs (u) and outputs (y). The third vector (x) gives the state of the system at the given time. The elements of each of these vectors are variables defining the specific input, output, and state factors being considered. The units of x, y, and u must be identical to make them conformable for addition. The dimension of each vector depends on how many factors the user considers important, and is independent of the size of the other two vectors.

The A, B, C, and D matrices describe the relationships between the variables according to their positions in the state equations, (1). Consequently, they take on the dimensions required by that position. If u, y, and x have lengths n, p, and q respectively, then A is $q \times q$, B is $n \times q$, C is $q \times p$, and D is $n \times p$.

Before the model is used, a certain amount of information is necessary. The values in the initial x vector must be known, and the values,for all the time periods, of u must be known. From that information the equations can be used to recursively solve for the values of y and for the rest of the values of x.

Thus we see that the linear system meets our criteria of simplicity and ease of computing. The computing time will depend on the complexity of the problem, which determines the sizes of the vectors and matrices. For the other criteria we must look at how the parameters are defined with respect to the context of the user's problem.

3. THE ENERGY SYSTEM

Selection of the factors which the variables will represent is admittedly a very subjective process. This is not a liability, however; it indicates the model's usefulness as a powerful, yet flexible, exploratory tool.

The variables in the state vector are:

$$x=\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}\text{energy produced in the U.S.}\\\text{energy imported to the U.S.}\\\text{energy consumed by households}\\\text{energy consumed otherwise}\end{bmatrix} \qquad \ldots(2)$$

These were chosen for a number of reasons. "Energy independence" is a phrase that is often mentioned; our first two variables provide (and use) information about it. This paper is about availability and use of <u>household</u> energy, information contained in <u>the third</u> state variable. By including the fourth state variable, we ignore none of the production or consumption information, even at this broad level.

As inputs to the system we want to include factors which might enter during any time interval, but which are not necessarily dependent on the quantities in the state variables. We chose three such factors:

$$u=\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix}=\begin{bmatrix}\text{technological advances}\\\text{governmental policy changes}\\\text{voluntary adaptations}\end{bmatrix} \qquad \ldots(3)$$

as a way of classifying state-independent inputs to the system. Because of the model's format, this information must be in the units of quantity of energy. This quantity is not to be interpreted as the amount of energy required to reach the change; it is the permanent (or long term) change in the state of the system, x(t+1), caused by the change (state equation #1), and it is the amount of energy saved, y(t), in that time period by the change (state equation #2).

The amount of energy available (the state of the system) at each stage, or time period, of the model depends on the amount available previously, and on the state-independent inputs. Outputs from the system, on the other hand, can be thought of as quantities of energy that are not to be returned to the system at a later time. This does not suggest that consumed energy should be treated as output; that is already accounted for in the state variables, x_3 and x_4. Moreover, because of the structure of the model, the output vector will contain information extracted from other factors, but not influenced by them (directly) in past or future time periods. Therefore, we have only one restriction on choosing the factors to be represented as outputs: they must depend on the inputs and on the system's state, at the present time, to fit the model.

The two outputs we chose were:

$$y=\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}=\begin{bmatrix} \text{energy saved within the home} \\ \text{energy saved in transportation} \end{bmatrix} \quad \ldots(4)$$

where the words 'saved' can be interpreted as 'not used.' These reflect our interest in energy used at the household level and in its conservation.

These are very subjective choices for the variables in our system, molded by our research interests. The choices also depend on the availability of data.

4. DERIVATION OF THE INPUT MATRICES

At least as important as the selection of the variables is the definition of the matrices. These are central to the model, because they set down the relationships between the variables. The only way to explicitly define the relationships (that is, with mathematical expressions and quantities) is through extensive research.

Each matrix, A, B, C, or D, expresses the magnitude and relative weight of elements in the vectors on the left hand sides of the state equations, (1), as a function of the right hand side vector it is multiplied by. Thus, matrix B defines the contribution to the value of x(t+1) of u(t). The dimensions of B are defined by the dimensions of the x and u vectors, and the variables affected by a particular element in B can be found according to the row and column of that element.

The form of the A matrix can be partially determined by studying the variables that have been chosen. This is the only matrix for which this holds, because it is the only matrix which defines the relationship between vectors of the same type (i.e., both state vectors, x).

The variables represented in the state vector suggest certain relationships among themselves. Keeping in mind that time is discrete in this model, there are certain assumptions and definitions which must be presented.

The first assumption is that information is "remembered" for only one time period. Also, the relationship from one time period to the next is assumed to be linear. Thus

$$x_i(t+1)=\alpha_i x_i(t),\ i=1,\ldots,4. \quad \ldots(5)$$

The demand in time period t+1 is defined as the total amount of energy consumed:

$$D_t = x_3(t+1) + x_4(t+1) = \alpha_3 x_3(t) + \alpha_4 x_4(t) \quad \ldots(6)$$

The supply on hand at the beginning of time t+1 is defined as the difference between the amount produced and the amount consumed in the previous time period:

$$S_{t+1} = x_1(t) + x_2(t) - x_3(t) - x_4(t). \quad \ldots(7)$$

This does not include the S_t term, which it should, so for the present study we assume that that stock on hand at the beginning of (t) is included in the internal production, x_1 (t).

Since the variables x_1 and x_2, and x_3 and x_4 form two complementary pairs, both equations in each pair will have approximately the same form.

We will define production($(x_1(t+1)+x_2(t+1))$ as the total demand minus the supply on hand. Thus, internal production will be:

$$x_1(t+1) = D_{t+1} - S_{t+1} - x_2(t+1), \quad \ldots(8)$$

while imported production will be:

$$x_2(t+1) = D_{t+1} - S_{t+1} - x_1(t+1) \quad \ldots(9)$$

But, we need x(t+1) as a function of x(t) to fit the form in equation (1). Substituting from equations (5), (6), and (7) into (8) we get:

$$\begin{aligned} x_1(t+1) &= \alpha_3 x_3(t) + \alpha_4 x_4(t) - x_1(t) + x_2(t) - x_3(t) - x_4(t) - \alpha_2 x_2(t) \\ &= -x_1(t) - (1+\alpha_2)x_2(t) + (1+\alpha_3)x_3(t) + (1+\alpha_4)x_4(t) \end{aligned} \quad \ldots(10)$$

In the same steps, equation (9) becomes:

$$x_2(t+1)=-(1+\alpha_2)x_2(t)-x_2(t)+(1+\alpha_3)x_3(t)+(1+\alpha_4)x_4(t) \quad \ldots(11)$$

Total consumption($(x_3(t+1) + x_4(t+1)$)will be defined as the total available for consumption: the supply on hand plus the amount produced in the current time period. As above, we will distinguish household consumption and other consumption as follows:

$$x_3(t+1) = S_{t+1} + x_1(t+1)+x_2(t+1)-x_4(t+1) \quad \ldots(12)$$

$$x_4(t+1) = S_{t+1} + x_1(t+1)+X_2(t+1)-x_3(t+1) \quad \ldots(13)$$

Again we need x(t+1) as a function of x(t), and so will substitute (5), (6), and (7) into (12) to get:

$$x_3(t+1) = x_1(t) + x_2(t) - x_3(t) - x_4(t) + \alpha_1 x_1(t) + \alpha_2 x_2(t) - \alpha_4 x_4(t)$$
$$= (\alpha_1+1)x_1(t) + (\alpha_2+1)x_2(t) - x_3(t) - (\alpha_4+1)x_4(t) \quad \ldots(14)$$

and, similarly, into (13) to get:

$$x_4(t+1) = (\alpha_1+1)x_1(t) + (\alpha_2+1)x_2(t) - (\alpha_3+1)x_3(t) - x_4(t) \quad \ldots(15)$$

The four equations (10), (11), (14), and (15) can be represented in matrix form as:

$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \\ x_4(t+1) \end{bmatrix} = \begin{bmatrix} -1 & -(1+\alpha_2) & (1+\alpha_3) & (1+\alpha 4) \\ -(1+\alpha_1) & -1 & (1+\alpha_3) & (1+\alpha_4) \\ (1+\alpha_1) & (1+\alpha_2) & -1 & -)1+\alpha_4) \\ (1+\alpha_1) & (1+\alpha_2) & -(1+\alpha_3) & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} \quad \ldots(16)$$

The right side of this represents the first part of the first state equation, (1); the matrix is our A matrix. Each element in this 4 x 4 matrix relates the factors on the left side of the equation with those in the vector on the right. More specifically, the row determines the variable from x(t+1), while the column determines which variable from x(t). Each element has two terms essentially; one is a positive or negative one (1), and the other is α_i. Following the derivation of the matrix, the ones arise from the presence of a supply term, and the α_i's are found where a demand term or a term involving another state variable has been included.

5. Z-TRANSFORM ANALYSIS

We now have a model which is flexible and straight-forward to program and use. However, in its present form it would be necessary to perform a matrix multiplication n times in order to arrive at the state of the system at time n. Using z-transform analysis the user changes the form of the state equations to another which is easier to manipulate mathematically, requiring only simple mathematical operations (5., p. 60). Briefly, the z-transform is defined by:

$$Z(x(n)) = \hat{x}(z) = \sum_{n=0}^{\infty} x(n) z^{-n}, \quad \ldots(17)$$

$$Z(x(n+1)) = z(\hat{x}(z) - x(0)). \quad \ldots(18)$$

Therefore, by applying (17) and (18) to the state equations (1), the transformed system of equations is:

$$z(\hat{x}(z) - x(0)) = A\,\hat{x}(z) + B\,\hat{u}(z)$$
$$\hat{y}(z) = C\,\hat{x}(z) + D\,\hat{u}(z) \quad \ldots(19)$$

Now we can solve the first of these for x (z) and then substitute that into the first term on the right side of the second equation, to get the system in its most general transformed form:

$$\hat{x}(z) = \frac{z\,x(0) + B\,\hat{u}(z)}{(I\,z - A)}$$

$$\hat{y}(z) = \frac{c\,(z\,x(0) + B\,\hat{u}(z)) +}{(I\,z - A)} D\,u(z) \quad \ldots(20)$$

Studying these equations (20), we note that everything on the right sides are information that we have before using the model. Since this is simply a theoretical formulation, we will not go to any depth with an example, however, we can describe the anticipated process.

The right sides in equations (20) will be solved as functions of z. It must be realized that z never takes on an actual numerical value; it is simply a variable letting the user know that he is working with a transformed system of equations (5., p. 61). Given the form of the solved function, it is possible to transform it back to a form which is a function of t. With this the user will be able to determine the contents of the x vector given whatever value of t he chooses.

What this all tells us is that the nature of the z-transform depends on the values inserted in the matrices A,B,C,D, in the input vector u, and in the initial state vector x (0). We will not know what these values will be until we begin researching them, but when we do arrive at some to work with we will have the methodology for analyzing them ready.

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BIOGRAPHIES

Charles Geiger is an M.A. candidate in the Department of Geography at Kent State University. His interests include quantitative methods and resource management.

Dr. Milton E. Harvey is Professor of Geography, Department of Geography, Kent State University. His recent research has focused on regional economic development, diffusion theory and energy conservation behavior.