Extending Direct Strength Design to Cold-formed Steel Beams with Holes

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Abstract

The extension of the American Iron and Steel Institute's Direct Strength Method (DSM) to cold-formed steel beams with holes is nearly in place. DSM was first introduced to the AISI specification in 2004 as an alternative to the effective width method, and is widely considered a major advancement in cold-formed steel component design. In DSM, the beam elastic buckling properties for a general cross-section are obtained with a computer analysis utilizing the finite strip method. A disadvantage of the finite strip method and DSM has been that discrete holes along the member length could not be easily accounted for, although the recent development of simplified elastic buckling approximations including holes has now alleviated the inherent shortcoming. This paper provides an introduction to the DSM approach for cold-formed steel beams with holes, where the critical elastic buckling moments for local, distortional, and global buckling are calculated including the presence of holes, and then input into strength prediction expressions modified to capture the strength reduction from yielding at the net section. A DSM design example of a joist with evenly spaced web holes is provided.

Introduction

Cold-formed joists are a popular structural component in the floor systems of low and midrise buildings. These thin-walled structural steel flexural members are manufactured by cold bending steel sheet into an open cross-section, most commonly a C-section. The joists are provided with evenly spaced web holes to accommodate the passage of electrical conduits, plumbing pipes, and HVAC ducts. Hole sizes and shapes vary by manufacturer, and the hole edges can be either unstiffened (Figure 1a) or stiffened (Figure 1b).

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(a)

(b)

Figure 1 (a) Cold-formed steel joist with unstiffened punched web holes (b) C-section joists with stiffened circular holes (photos courtesy of Don Allen)

The broad range of hole shapes, sizes, and spacings in cold-formed steel construction today has exceeded the original scope of the American Iron and Steel Institute (AISI) design equations developed for beams with holes over the last four decades. The current AISI design equations were derived within the context of the effective width method (Yu 2000), and address the influence of unstiffened holes on local buckling dominated failures. The AISI provisions are written specifically for unstiffened holes in C-section webs, and are limited to a somewhat restricted range of hole sizes and spacings. For example, the effective width equations are only applicable when unstiffened web holes are provided with a centerline spacing of 457 mm (18 in.) or greater, and where the hole depth is less than 63 mm (2.5 in.) regardless of the cross-section dimensions (AISI-S100 2007, Section B2.4).

The AISI specification addresses the influence of unstiffened holes on local buckling through the effective width method, however holes are not currently considered for global buckling and distortional buckling limit states. When unstiffened holes are present in a cold-formed steel beam, the critical elastic flexural-torsional buckling load decreases relative to the same beam without holes, which increases the global slenderness and decreases predicted strength (Moen and Schafer 2009a). For distortional buckling, a form of buckling related to intermediate and/or edge stiffeners commonly observed in open cross-sections, the presence of unstiffened web holes decreases the stabilizing influence of the web on the cross-section, reducing the critical elastic distortional buckling moment and increasing the tendency for distortional buckling to initiate at a hole (Kesti 2000; Moen and Schafer 2008; Moen and Schafer 2009a). A more general design method which considers the influence of holes across all cold-formed steel limit states is needed.

An AISI research program was recently completed that capitalizes on advances in cold-formed steel strength prediction, and specifically the AISI Direct Strength Method (DSM) (AISI-S100 2007, Appendix 1), to deliver a more general design approach for cold-formed steel beams with holes. DSM represents an important advancement in cold-formed steel design because it provides engineers and cold-formed steel manufacturers with the tools to predict member strength for a general cross-section. With the design approach summarized herein, DSM can now safely predict the strength of cold-formed steel flexural members with the ever expanding range of cross-section types, hole sizes, shapes and spacings common in industry. Note that this paper focuses on flexural strength prediction for beams with unstiffened holes. However, the DSM approach is also applicable to beams with stiffened holes, and work is underway to formalize the design equations and elastic buckling framework (Moen and Yu 2010).

The AISI Direct Strength Method

DSM for beams without holes

The AISI Direct Strength Method employs the elastic buckling properties of a general cold-formed steel cross-section to predict strength. For members without holes, the elastic buckling properties are obtained from an elastic buckling curve generated with freely available software, for example CUFSM (Schafer and Ádàny 2006) and GBTUL (Bebiano et al. 2008), which perform a series of eigen-buckling analyses over a range of buckled half-wavelengths. An example of an elastic buckling curve is provided in Figure 2 for a cold-formed steel C-section beam, highlighting the three categories of elastic buckling considered in DSM – local, distortional, and global buckling – where $M_{cr\ell}$, M_{crd}, and M_{cre} are the respective elastic buckling moments.

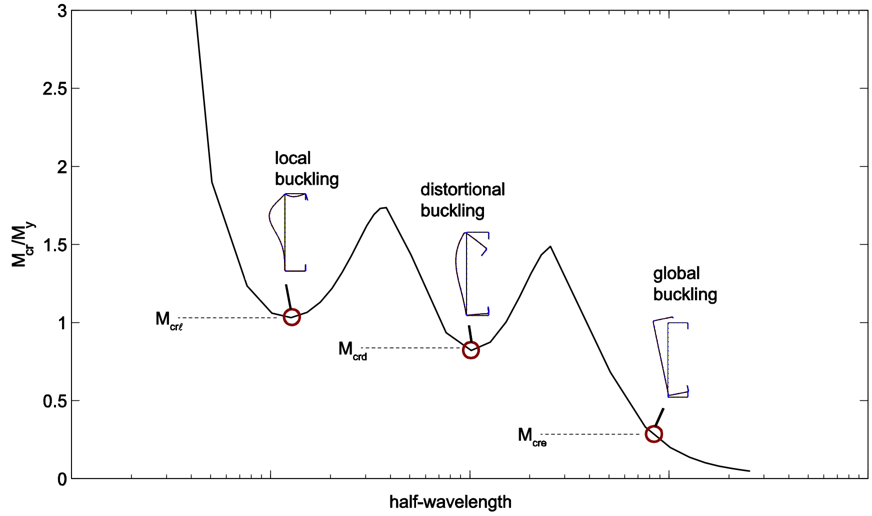

Figure 2 Elastic buckling curve for a cold-formed steel beam without holes

Flexural capacity is calculated with DSM considering three limit states – global buckling, local-global buckling interaction, and distortional buckling (AISI-S100 2007, Appendix 1). The global strength of an unbraced beam span, M_{ne}, is determined with the global slenderness, $\lambda_c=(M_y/M_{cre})^{0.5}$; $M_{n\ell}$ is calculated with the local slenderness, $\lambda_\ell=(M_{ne}/M_{cr\ell})^{0.5}$; and M_{nd} is obtained with the distortional slenderness, $\lambda_d=(M_y/M_{crd})^{0.5}$. When slenderness is high for global or distortional buckling limit states, i.e. when M_{cre} or M_{crd} is small relative to the yield moment of the beam, $M_y=S_fF_y$, flexural strength is limited by elastic buckling. (Note that S_f is the section modulus referenced to the outer fiber that yields first and F_y is the steel yield stress.) When λ_c or λ_d is low, the flexural strength is controlled by inelastic buckling and yielding. Considering the local-global buckling interaction limit state for unbraced beams, the flexural-torsional buckling capacity is reduced from M_{ne} to $M_{n\ell}$ to account for local buckling along the beam span. The minimum strength from the three limit states is taken as the beam's flexural capacity, i.e. $M_n=\min(M_{ne}, M_{n\ell}, M_{nd})$.

Strategy for extending DSM to beams with holes

A logical extension of the Direct Strength approach to cold-formed steel beams with holes is to maintain the assumption that elastic buckling properties can be used to predict strength. For a beam with holes, this means that the elastic buckling moments $M_{cr\ell}$, M_{crd}, and M_{cre}, are calculated including the influence of holes. A suite of simplified methods for obtaining these elastic buckling moments was recently developed as an alternative to cumbersome thin-shell finite element eigen-buckling analysis. The elastic buckling moments, including the influence of holes, can be calculated with finite strip analysis or hand calculations derived from classical buckling solutions (Moen and Schafer 2009a; Moen and Schafer 2009b). The simplified elastic buckling prediction methods are demonstrated in an example at the end of this paper.

It was concluded in the AISI research program that the elastic buckling moments including the influence of unstiffened holes are viable parameters for predicting capacity in a Direct Strength approach (Moen 2008). However, when yielding controls strength, modifications to the existing DSM design expressions for beams without holes were needed to limit flexural capacity to that of the net section, i.e. $M_{ynet}=S_{fnet}F_y$, where S_{fnet} is the section modulus at the net section. Furthermore, the AISI research program concluded that inelastic buckling and collapse at a hole may control flexural strength with intermediate slenderness ranges (Figure 3), requiring a transition from the elastic buckling regime to the net section limit (Moen 2008). DSM distortional buckling design expressions presented in the following section have been modified to provide this transition. For local-global buckling interaction, $M_{n\ell}$ is capped at M_{ynet}, imposing the net section strength limit when flexural capacity is governed by inelastic buckling and yielding, i.e. when λ_ℓ and λ_c are both low.

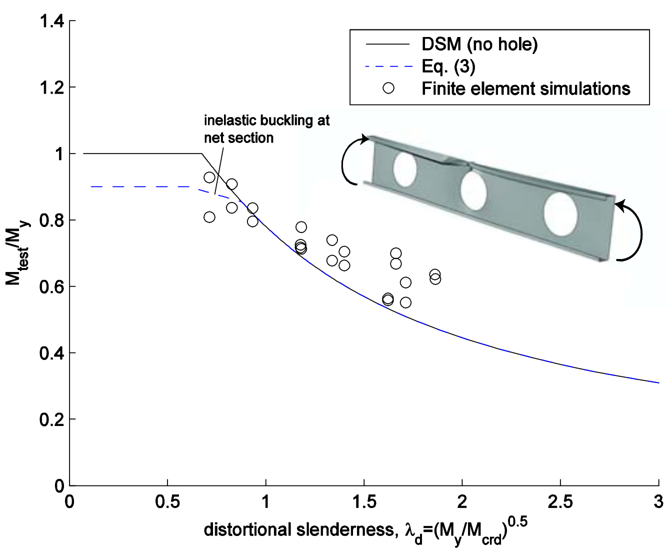

Figure 3 DSM distortional buckling curve for beams with holes

DSM design expressions for beams with holes

The nominal strength of a cold-formed steel beam with holes shall be the minimum of M_{ne}, $M_{n\ell}$, and M_{nd} as given in the following sections.

Global Buckling

The nominal flexural strength, M_{ne}, for lateral-torsional buckling is:

$$
\begin{aligned}
M_{ne} &= M_{cre} \text{ for } M_{cre} < 0.56M_y \\
&= \frac{10}{9}M_y\left(1-\frac{10M_y}{36M_{cre}}\right) \text{ for } 2.78M_y \geq M_{cre} \geq 0.56M_y \\
&= M_y \text{ for } M_{cre} > 2.78M_y
\end{aligned}
\quad (1)
$$

where M_{cre} includes the influence of hole(s).

Local Buckling Interaction

The nominal flexural strength, $M_{n\ell}$, for local-global buckling interaction is:

$$
\begin{aligned}
M_{n\ell} &= M_{ne} \leq M_{ynet} \text{ for } \lambda_\ell \leq 0.776 \\
&= \left[1-0.15\left(\frac{M_{cr\ell}}{M_{ne}}\right)\right]^{0.4}\left(\frac{M_{cr\ell}}{M_{ne}}\right)^{0.4} M_{ne} \text{ for } \lambda_\ell > 0.776\text{ ,}
\end{aligned}
\quad (2)
$$

where $\lambda_\ell=(M_{ne}/M_{cr\ell})^{0.5}$, and $M_{cr\ell}$ includes the influence of hole(s).

Distortional Buckling

The nominal flexural strength, M_{nd}, for distortional buckling is:

$$
\begin{aligned}
M_{nd} &= M_{ynet} \text{ for } \lambda_d \leq \lambda_{d1} \\
&= M_{ynet} - \left(\frac{M_{ynet}-M_{d2}}{\lambda_{d2}-\lambda_{d1}}\right)(\lambda_d - \lambda_{d1}) \text{ for } \lambda_{d1} < \lambda_d \leq \lambda_{d2} \\
&= \left(1-0.22\left(\frac{M_{crd}}{M_y}\right)^{0.6}\right)\left(\frac{M_{crd}}{M_y}\right)^{0.6} M_y \text{ for } \lambda_d > \lambda_{d2}\text{ ,}
\end{aligned}
\quad (3)
$$

where $\lambda_d=(M_y/M_{crd})^{0.5}$, $\lambda_{d1}=0.673(M_{ynet}/M_y)$, $\lambda_{d2}=0.673(1.7(M_y/M_{ynet})^{1.7}-0.7)$, M_{crd} includes the influence of hole(s), and

$$M_{d2} = \left(1-0.22\left(1/\lambda_{d2}\right)^{0.5}\right)\left(1/\lambda_{d2}\right)^{0.5} M_y \,. \qquad (4)$$

Design example

The DSM design approach outlined in Eq. (1) to Eq. (4) is employed to calculate the capacity of a perforated cold-formed steel joist (Figure 4) with an SSMA 550S162-33 lipped C-section (SSMA 2001), where F_y=55 ksi.

Joist geometry, boundary conditions, and loading

The joist carries a uniform vertical load and is assumed to be fully braced against lateral-torsional buckling (Figure 4). Distortional buckling and local buckling are viable strength limit states.

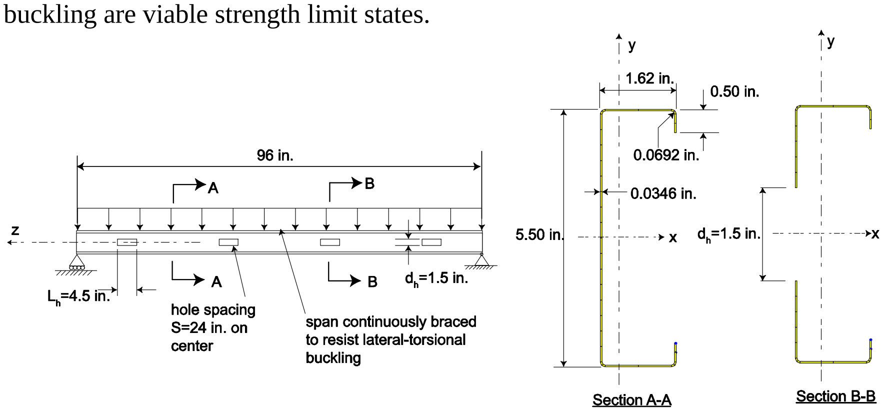

Figure 4 Column dimensions and boundary conditions

Gross and net section properties

The gross section and net section properties (Table 1) are calculated with the section property calculator in CUFSM. To determine the net section properties in CUFSM, assign a thickness of zero to the elements at the location of the perforations, but do not delete them. Assuming 55 ksi steel, M_y=29.15 kip·in. and M_{ynet}=28.95 kip·in.

Table 1 (a) gross section properties, (b) net section properties

(a)

A_g	I_x	I_y	r_x	r_y	J	C_w	x_o	y_o	I_{xo}	I_{yo}	r_{xo}	r_{yo}	r_o
in.^{2}	in.^{4}	in.^{4}	in.	in.	in.^{4}	in.^{6}	in.	in.	in.^{4}	in.^{4}	in.	in.	in.
0.327	1.46	0.11	2.11	0.59	0.000130	0.682	-1.11	0.00	1.86	0.11	2.39	0.59	2.46

(b)

A_{net}	I_{xnet}	I_{ynet}	r_{xnet}	r_{ynet}	J_{net}	C_{wnet}	x_{onet}	y_{onet}	I_{xonet}	I_{yonet}	r_{xonet}	r_{yonet}	r_{onet}
in.^{2}	in.^{4}	in.^{4}	in.	in.	in.^{4}	in.^{6}	in.	in.	in.^{4}	in.^{4}	in.	in.	in.
0.275	1.45	0.10	2.29	0.61	0.000110	0.677	-1.20	0.00	1.84	0.10	2.59	0.61	2.66

Elastic buckling analysis

Local buckling

Local buckling in a cold-formed steel beam with holes is assumed to occur as either buckling in the gross cross-section between holes ($M_{cr\ell nh}$) or buckling of the compressed strip adjacent to a hole ($M_{cr\ell h}$). The buckled mode shape with the lowest critical buckling load defines $M_{cr\ell}$, i.e. $M_{cr\ell}$=min($M_{cr\ell nh}$, $M_{cr\ell h}$). The elastic buckling curve for the gross cross-section (generated with CUFSM, see Figure 5) is used to obtain $M_{cr\ell nh}$. Taking the first minimum on the elastic buckling curve, $M_{cr\ell nh}$= 17.61 kip·in. at a half-wavelength $L_{cr\ell nh}$=3.0 in.

The net-section elastic buckling curve is generated in CUFSM by modifying the gross section node and element geometry such that one finite strip element with t=0 spans across the hole (Figure 6). A reference moment of 1 kip·in. is applied to the cross section and CUFSM (Properties screen) is used to calculate the corresponding stress distribution. The zero thickness element is then deleted, and the two corners of the cross-section in compression are restrained in the CUFSM z-direction. The resulting mode shape and elastic buckling curve is provided in Figure 6. The lowest buckling load of the unstiffened strip occurs at a half-wavelength less than the length of the perforation ($L_{cr\ell h}$ =4.25 in. versus L_{hole}=4.5 in.) meaning that the buckled half-wave can form within the length of the hole, and therefore $M_{cr\ell h}$=10.51 kip·in. (Note that $M_{cr\ell h}$ could be tabulated for standard punchout sizes and shapes as a convenience to the engineer!)

Local buckling is predicted to occur in the net cross section since $M_{cr\ell h}<M_{cr\ell nh}$ and therefore $M_{cr\ell}$= 10.51 kip·in. The local buckling moment is 40% lower at a hole, which means that buckling will tend to occur as unstiffened strip buckling rather than in the web of the gross cross-section between holes.

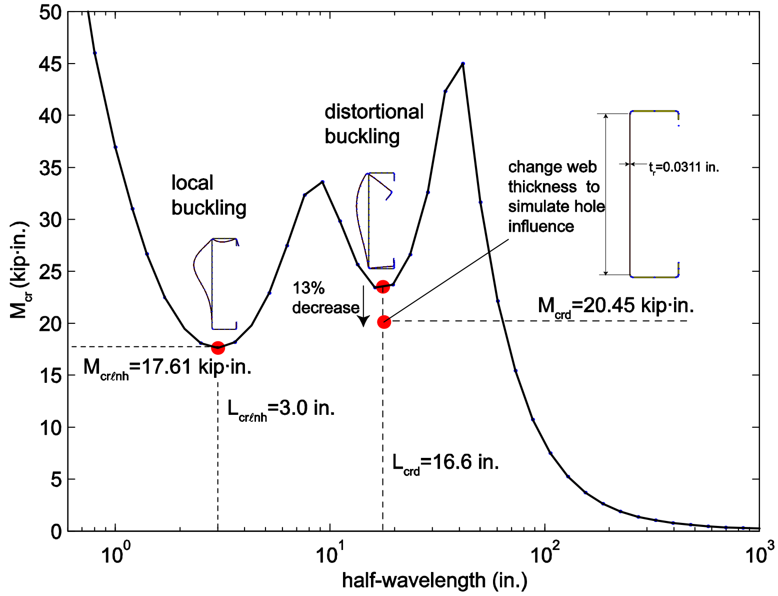

Figure 5 Elastic buckling curve for gross cross-section

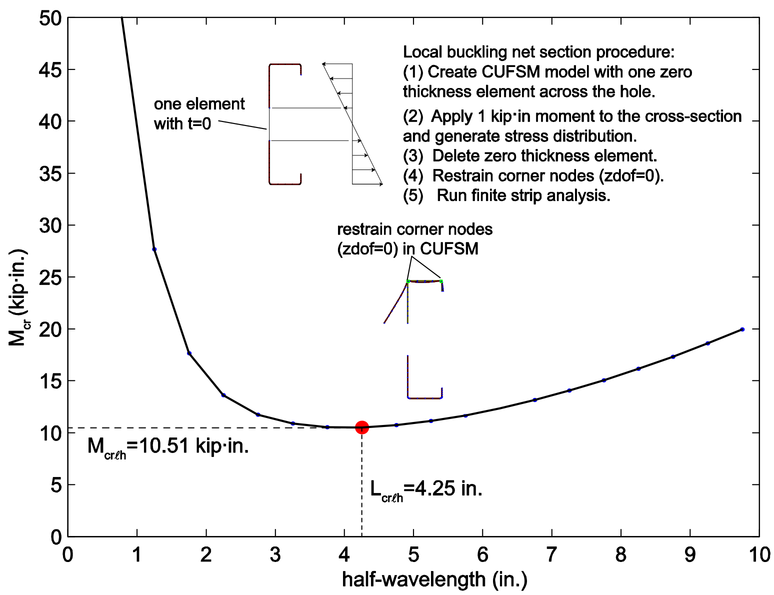

Figure 6 Local buckling curve for net cross-section

Distortional buckling

The critical elastic buckling moment for distortional buckling, including the influence of web holes, is calculated by first obtaining the distortional buckling half-wavelength from a finite strip analysis of the gross cross-section (L_{crd}=16.6 in., see Figure 5). The reduced web bending stiffness caused by a hole over one distortional half-wavelength is simulated by reducing the cross-section thickness of the web (Moen and Schafer 2009a):

$$t_r = \left(1 - \frac{L_h}{L_{crd}}\right)^{1/3} t \quad . \tag{5}$$

For L_h=4.5 in. and t=0.0346 in., t_r=0.0311 in. which is then implemented in a second finite strip analysis (Figure 5) performed just at L_{crd}=16.6 in., resulting in M_{crd}=20.45 kip·in. The presence of perforations reduces M_{crd} by 13% when compared to a distortional buckling moment of 23.43 kip·in for a beam without holes (Figure 5). Note that the beneficial influence of the moment gradient on M_{crd} (Yu 2005) is negligible and not considered because the beam's span length is much longer than L_{crd}.

Ultimate Strength Calculation

Inputs from the elastic buckling analysis include:

$M_y := 29.15 \cdot kip \cdot in$ $\qquad$ $M_{crL} := 10.51 \cdot kip \cdot in$

$M_{ynet} := 28.95 \cdot kip \cdot in$ $\qquad$ $M_{crd} := 20.45 \cdot kip \cdot in$

DSM global buckling strength Eq. (1)

$M_{ne} := M_y$ $\qquad$ beam is fully braced against lateral-torsional buckling

$M_{ne} = 29.15 \cdot kip \cdot in$

DSM local buckling strength, Eq. (2)

$$\lambda_L := \sqrt{\frac{M_{ne}}{M_{crL}}}$$

$\lambda_L = 1.6654$ $\qquad$ local slenderness (including influence of holes)

(subscript "L" = "ℓ")

$$M_{nL} := \begin{cases} \min\left(M_{ne}, M_{ynet}\right) & \text{if } \lambda_L \le 0.776 \\ \left[\left[1 - 0.15\cdot\left(\dfrac{M_{crL}}{M_{ne}}\right)^{0.4}\right]\left(\dfrac{M_{crL}}{M_{ne}}\right)^{0.4}\cdot M_{ne}\right] & \text{if } \lambda_L > 0.776 \end{cases}$$

$$M_{nL} = 17.45\cdot \text{kip}\cdot \text{in}$$

DSM distortional buckling strength, Eq. (3)

$$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}} \qquad \lambda_d = 1.1939$$

distortional slenderness (including influence of holes)

$$\lambda_{d1} := 0.673\cdot\left(\frac{M_{ynet}}{M_y}\right) \qquad \lambda_{d1} = 0.6684$$

$$\lambda_{d2} := 0.673\cdot\left[1.7\left(\frac{M_y}{M_{ynet}}\right)^{1.7} - 0.7\right] \qquad \lambda_{d2} = 0.6865$$

$$M_{d2} := \left[1 - 0.22\cdot\left(\frac{1}{\lambda_{d2}}\right)^{0.5}\right]\cdot\left(\frac{1}{\lambda_{d2}}\right)^{0.5}\cdot M_y \qquad M_{d2} = 25.8\cdot \text{kip}\cdot \text{in}$$

$$M_{nd} := \begin{cases} M_{ynet} & \text{if } \lambda_d \le \lambda_{d1} \\ \left[M_{ynet} - \left(\dfrac{M_{ynet} - M_{d2}}{\lambda_{d2} - \lambda_{d1}}\right)\cdot\left(\lambda_d - \lambda_{d1}\right)\right] & \text{if } \lambda_{d1} < \lambda_d \le \lambda_{d2} \\ \left[\left[1 - 0.22\cdot\left(\dfrac{M_{crd}}{M_y}\right)^{0.6}\right]\left(\dfrac{M_{crd}}{M_y}\right)^{0.6}\cdot M_y\right] & \text{if } \lambda_d > \lambda_{d2} \end{cases}$$

$$M_{nd} = 19.4\cdot \text{kip}\cdot \text{in}$$

Predicted flexural capacity (including holes):

$M_n := \min\left(\left(M_{ne} \;\; M_{nL} \;\; M_{nd}\right)\right)$ LRFD (prequalified section)

$M_n = 17.45 \cdot kip \cdot in$ (M_{nL} controls) $\phi_b := 0.90$

$\phi_b \cdot M_n = 15.7 \cdot kip \cdot in$

Local buckling at a hole is predicted as the governing failure mode, with a decrease in flexural strength of 15% when compared to the same beam without holes. This result is contrary to the AISI Main Specification Section B2.4, which states that when $d_h/h<0.38$, holes do not influence local buckling capacity. It will be difficult to make definitely conclusions on the validity of the Main Specification versus DSM until more experimental data is generated for cold-formed steel joists with unstiffened holes. The elastic buckling prediction of the unstiffened strip employed in DSM is certainly more representative of the actual buckling behavior when compared to the Main Specification as the net section finite strip approach (see Figure 6) considers cross section interaction.

Conclusions

The AISI Direct Strength Method (DSM) for cold-formed steel beams with holes utilizes the critical elastic buckling loads of a beam, including the influence of holes, to predict strength. The elastic buckling predictions are obtained with a suite of recently developed simplified methods that employ finite strip analysis and hand calculations derived from classical buckling solutions. The existing DSM design expressions for beams without holes have been modified to limit flexural capacity to the strength of the net cross section, and in the case of distortional buckling, a transition from the net section capacity to the elastic buckling regime was added to predict flexural strength influenced by inelastic buckling at the net cross section. DSM provides an accessible design approach for cold-formed steel beams that can account for holes across global, local, and distortional buckling limit states with improved accuracy and generality when compared to existing strength prediction methods.

Ongoing research

The DSM approach presented in this paper has been developed and validated primarily with nonlinear finite simulations (Moen 2008) in part because of the lack of experimental data. An experimental program was recently completed by the first author considering cold-formed steel joists with unstiffened holes which will be used to supplement the ongoing validation effort.

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