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Engineering Management and Systems Engineering

01 Jan 2000

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Chung-Li Tseng
Missouri University of Science and Technology

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Chung-Li Tseng, Member, IEEE
Department of Civil Engineering
University of Maryland
College Park, MD 20742

Abstract: In this paper, we use the real options framework to value the operation flexibility of a power plant. The power plant operation is formulated as a multi-stage stochastic problem. We assume that there are hourly spot markets for both electricity and the fuel used by the generator, and that their prices follow some Ito processes. At each hour, the power plant operator must decide whether or not to run the unit so as to maximize expected profit. However, the unit operation is subject to decision lead times and minimum uptime and downtime constraints, so the commitment decision must take into account inter-temporal effects. In this paper, we present power asset valuation using discrete-time price trees for correlated price processes for both electricity and fuel, such as geometric mean reverting processes. With price trees, the valuation problem is solved using stochastic dynamic programming. Numerical results are also presented.

Keywords: Generation asset valuation, unit commitment, real options valuation, financial engineering, deregulated market

I. INTRODUCTION

Recently valuing options embedded in *real* operational processes, activities or investment opportunities that are not financial instruments has become popular. This subject is known as *real options* valuation. Given the popularity of the real options approach, power plants have also been valued using such "financial" methodology in the deregulated environment [8]. The fact is that the core flexibility of a power plant's real options, e.g. committing or decommitting a unit, does not emerge from deregulation. The traditional unit commitment problem (e.g. [7]) is such an example of optimally exercising these operational real options to achieve cost minimization in the regulated environment, though in the "cost-of-service" world savings from efficient dispatch typically accrued to the ratepayers. With deregulation and the introduction of

spot markets for both electricity output and the fuel input, generation asset owners, whether it be a utility or a merchant operator, must reassess the value of their units accounting for the market opportunity costs. Price information must be incorporated to the unit commitment problems in order to capitalize on the profitable opportunities arising in the market. Doing so, utilities and power generators not only optimize their commitment decisions taking into account price stochastics, but also maximize the market value of their power plants over the operating period. In this paper, we discuss using the operational real options to value a power plant.

In [2,3,4], a power plant's valuation is appraised using financial option theory. The idea of these approaches is as follows: A power plant, with its associated *heat rate*, converts a particular fuel to electricity. This conversion involves two commodities with different market prices. When the electricity price is high but the fuel price is low, the power plant should run to capitalize the positive and profitable price spread between the price for power and the unit's cost of generation. If the price spread is negative, then the optimal decision is not to run the unit. Therefore, owning a power plant can be regarded as holding a series of call options of *spark spreads*, defined as the electricity price less the product of the heat rate associated with the generator and the fuel price. Analytical solutions are derived using financial option theory in [2]. Although using option theory to value a power plant is a novel approach, it overlooks the power plant's operational constraints. Without considering the operational constraints, the power plant may be overvalued.

In this paper, we utilize the real options approach to value the operation of a power plant. The power plant operation is formulated as a multi-stage stochastic problem. The uncertainties characterized are the prices for electricity and the fuel consumed by the generator. Operational characteristics of a generating unit such as unit startup/shutdown time and minimum uptime and downtime constraints are taken into account. As opposed to the approaches using financial options, we emphasize the influence due to these *real* operational characteristics. The operator, after observing the market prices of the electricity and fuel, must decide whether to run the generator or not so as to maximize expected profit. Once the commitment or decommitment decision is made, the unit operation is subject to decision lead times, and

minimum uptime and downtime constraints. That is, an on-line (or off-line) unit cannot be turned back down (or on) even if market prices become unfavorable (or favorable) until some minimum uptime (or downtime) requirement is fulfilled.

In [8], the generation asset valuation problem with physical constraints has been tackled using Monte Carlo (MC) simulation. Their approach, similar to backward dynamic programming steps, applies MC simulation to determine optimal decision criteria starting from the last period. The process is then repeated and moved backward: having obtained all optimal decision criteria for the subsequent time periods, MC simulation is applied to the current period. In this paper, a more efficient approach is presented. For given (continuous) price processes for the electricity and fuel, the corresponding discrete-time price trees, much like scenario trees, are obtained. With price trees, the generation asset valuation problem can be solved by backward stochastic dynamic programming.

This paper is organized as follows. In Section 2, we provide an overview of the financial options approach for valuing a power plant. The need of incorporating physical constraints into the valuation is introduced in Section 3. A mathematical model is presented in Section 4. The price model is given in Section 5. We present numerical results in Section 6 and conclude this paper in Section 7.

II. FINANCIAL OPTIONS APPROACH

A power plant consumes a particular fuel and then converts the fuel into electricity. The conversion between electricity and fuel (or heat) is called the (incremental) *heat rate* of a power plant, denoted by H (MMBtu/MWh). To generate 1 MWh electricity, for a generator with a heat rate H, it requires H MMBtu of heat by burning fuel (e.g. gas or fossil fuel). A higher heat rate implies lower unit operating efficiency. This conversion involves two commodities with different market prices.

In [2,3,4], the payoff of a generator is simplified as a linear system. For every 1 WMh electricity generation:

$$\text{Payoff} = P^E - H \cdot P^F, \qquad (1)$$

where P^E ($/MWh) and P^F ($/MMBtu) stand for electricity and fuel prices, respectively. Given spot prices for electricity and fuel, the plant operator decides whether or not to run the generating unit. Obviously, the operator will run the unit only if $P^E \geq H \cdot P^F$. That is, the operator can make a profit by purchasing fuel and using the generator to convert the fuel to electricity, then selling the electricity back to the market. Over a period [0,T], they proposed:

$$\text{Power plant value} = \sum_{t=1}^{T} E_0 [\max(P_t^E - H \cdot P_t^F, 0)]. \qquad (2)$$

That is, owning a power plant can be regarded as holding a series of (European) *spark spread* call options (expiring at t) as in (2). We shall refer to these approaches as the financial options approaches in this paper.

Using (2) to value a power plant implies
1. The unit commitment decisions are made after the prices P_t^E and P_t^F are observed, and a unit can be immediately started up if the market prices are favorable, and vice versa. This implies that there is no decision lead time required.
2. There are no intertemporal constraints for the commitment: a unit can be committed/decommitted at any time.
3. The unit heat rate H is a constant at all levels of power generation.

These assumptions, however, are not true in general. In the following section, we should introduce the operational constraints of a power plant and then present a mathematical model for incorporating the operational constraints to the valuation problem.

III. PHYSICAL CONSTRAINTS FOR A POWER PLANT

The physical constraints of a power plant can be summarized as below.
- Decision lead times
Decision lead times reflect the nonzero response time of the unit. In our case, these are the generator startup time and shutdown time. Namely, the generator takes a nonzero time from startup to full availability, and from normal operation to complete shutdown.
- Intertemporal constraints
A thermal generating unit cannot switch between the on-line mode and the off-line mode at arbitrary frequency, due to both the non-zero response time of the unit and the damaging effects of fatigue. In other words, once a thermal unit is shut down (or started up), it is required to stay off-line (or on-line) for a minimum period, known as the minimum down- (or up-) time, before it can be started up (or shut down) again.
- Variable heat rate
In the real world, the heat rate H of a generator is a function of the generation level. Generally, H increases as the generation level increases. Denote the generation level by q, $H(q)$ is normally modeled as a quadratic function

$$H(q) = a_0 + a_1 q + a_2 q^2, \qquad (3)$$

where a_0, a_1, and a_2 are all positive.

- Additional costs

There are other costs encountered in the unit commitment decision such as startup cost and shutdown cost. Both startup and shutdown costs may account for labor and maintenance costs, which may affect the commitment decision.

All these characteristics complicate the (optimal) commitment decision making, especially under price uncertainties. We shall present a mathematical model of these characteristics in the following section.

IV. MATHEMATICAL MODEL

In the development we first introduce the following standard notation. Additional symbols will be introduced when necessary.

t : index for time ($t = 0, \cdots, T$)

u_t : zero-one decision variable indicating whether the unit is up or down in time period t

x_t : state variable indicating the length of time that the unit has been up or down in time period t

t^{on} : the minimum number of periods the unit must remain on after it has been turned on

t^{off} : the minimum number of periods the unit must remain off after it has been turned off

q_t : decision variable indicating the amount of power the unit is generating in time period t

q^{min} : minimum rated capacity of the unit

q^{max} : maximum rated capacity of the unit

P_t^E : electricity price ($/MWh) in time period t

P_t^F : fuel price ($/MMBtu) in time period t

$C(q_t, P_t^F)$: fuel cost for operating the unit at output level q_t in time period t when the fuel price is P_t^F

S_u : startup cost associated with turning on the unit in time period t

In this paper, the unit of time period is in hours. Let $F(u_t, x_t; P_t^E, P_t^F)$ be the power plant value if the period starts at state x_t in time period t with observed electricity and fuel prices (P_t^E, P_t^F). The recurrence equations can be easily formulated as follows:

$$F_t(u_t, x_t; P_t^E, P_t^F) = (P_t^E q_t - C(q_t, P_t^F) - S_u(1 - u_{t-1}))u_t$$
$$+ \max_{x_{t+1}, u_{t+1}} E_t[F_{t+1}(u_{t+1}, x_{t+1}; P_{t+1}^E, P_{t+1}^F)] \quad (4)$$

subject to the following constraints.

State transition constraints

$$x_t = \begin{cases} \min(t^{on}, \max(x_{t-1}, 0) + 1) & \text{if } u_t = 1 \\ \max(-t^{off}, \min(x_{t-1}, 0) - 1) & \text{if } u_t = 0 \end{cases} \quad (5)$$

Minimum uptime/downtime constraints

$$u_t = \begin{cases} 1 & \text{if } 1 \leq x_{t-1} \leq t^{on}, \\ 0 & \text{if } -1 \geq x_{t-1} \geq -t^{off}, \\ 0 \text{ or } 1 & \text{if } 1 \leq x_{t-1} \leq t^{on}, \end{cases} \quad (6)$$

Initial conditions on u_t and x_t at $t = 0$.

V. PRICE MODEL

In this paper we assume that price processes for both electricity and fuel are given. We focus on the following two processes for electricity and fuel advocated by [1].

$$d \ln(P_t^E) = -\mu^E(\ln(P_t^E) - m_t^E)dt + \sigma^E dB_t^E, \quad (7)$$

and

$$d \ln(P_t^F) = -\mu^F(\ln(P_t^F) - m_t^F)dt + \sigma^F dB_t^F, \quad (8)$$

where B_t^E and B_t^F are two Wiener processes with instantaneous correlation ρ.

The above commodity price models are characterized by mean reversion and lognormally distributed, seasonal prices. Because, to varying degrees, both electricity and fuel have associated storage costs, their prices are determined to a large degree by the forces of producer supply and consumer demand and less so by investor speculation [1].

In this paper, we approximate the two price processes in (7) and (8) by a discrete-time price tree. Assume $y = \ln P$ the logarithm of either the electric price or gas price. The process of y is of the following form:

$$dy = -\eta(y - \bar{y})dt + \sigma dB, \quad (9)$$

called the mean-reverting (MR) processes, e.g. [5], where η is the reversion speed, and $\bar{y}$ is the "mean" level of prices. Representing a mean-reverting process as a price tree has been studied in [6]. Three branching processes are possible at each node in a trinomial model, as depicted in Fig. 1.
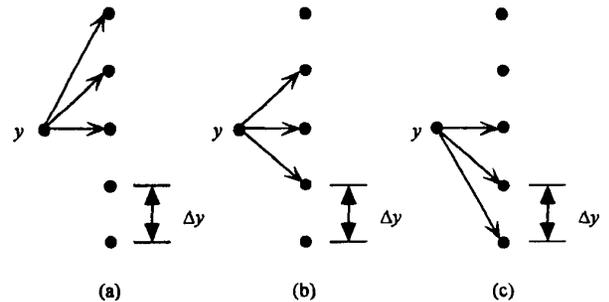

(a)   (b)   (c)

Fig. 1 Alternative branching in a MR trinomial model

We can use an integer variable k to generalize the three cases such that a price y branches into $y+(k+1)\Delta y$, $y+k\Delta y$ and $y+(k-1)\Delta y$, where $k = 1, 0,$ and -1 corresponding to cases (a), (b) and (c) in Fig. 1, respectively. For a small given time step Δt, the price step Δy in the branches was suggested to be [6]

$$\Delta y = \sigma\sqrt{3\Delta t} .\qquad(10)$$

In Fig. 1, cases (a) and (c) represent the reversion case when the prices are far deviated from the mean level $\bar{y}$, while case (b) occurs for normal situations.

We extend the method to a two-dimensional tree to encompass both electricity and gas prices. Let $(y_1, y_2) = (\ln P^E, \ln P^F)$. Each price node in the (y_1, y_2) plane branches into $3 \times 3 = 9$ price nodes in the plane corresponding to the following time period as shown in Fig. 2. The 9 branching probabilities can be obtained by solving a linear program [9].
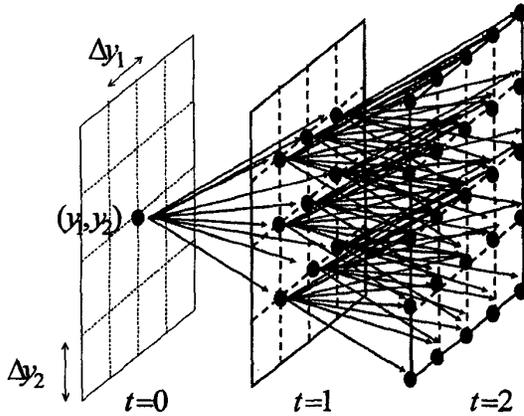

Fig. 2: An illustration of a 2D trinomial price model

With a price tree as shown in Fig. 2, the generation asset evaluation problem in (4) can be solved easily. Assume that (4) is now applied to a node (or a *leaf*) of the price tree, denoted by $(P_t^{E,i}, P_t^{F,i})$, where superscript i denotes some index of nodes at a given time. Let $A_t(i)$ denote the index set of the descendents for node i at time t. Therefore, (i, j) $\forall j \in A_t(i)$ represent all price branches stemmed from node i at time t. Assume that (i, j), $j \in A_t(i)$ is associated with a branching probability $p_t^{i,j}$. The expectation term in (4) can now be rewritten:

$$F_t(u_t, x_t; P_t^{E,i}, P_t^{F,i}) = (P_t^{E,i}q_t^i - C(q_t^i, P_t^{F,i}) - S_u(1-u_{t-1}))u_t$$
$$+ \max_{x_{t+1}, u_{t+1}} \sum_{j \in A_t(i)} p_t^{i,j} F_{t+1}(u_{t+1}, x_{t+1}; P_{t+1}^{E,j}, P_{t+1}^{F,j})]\qquad(11)$$

Equation (11) now reduces to a stochastic dynamic programming recurrence relation, and can be solved efficiently.

VI. NUMERICAL RESULTS AND DISCUSSIONS

We have implemented the proposed method for valuing a power plant in FORTRAN. This section presents numerical test results. The proposed method has been applied to a natural gas-fueled generating unit with the following input-output characteristics:

$$H(q_t) = 820 + 9.023q_t + 0.00113q_t^2\qquad(12)$$

with $q^{\min} = 300$ MW, $q^{\max} = 1000$ MW. We let $t^{on} = t^{off} = 5$. Unless otherwise mentioned, we let $T=168$ hours. Also we assume that the electricity prices and fuel prices both follow the processes described by (7) and (8). The parameters of the price processes are obtained by fitting historical price data series of Nymex natural gas prices and electricity prices from the California Power exchange, taking the logarithm of these prices as our basic data series. For gas we obtain $\mu^F = 6.95 \times 10^{-4}$ and $\sigma^F = 0.019$. For electricity we obtain $\mu^E = 0.072$ and $\sigma^E = 0.27$. As aforementioned, m_t^E captures the cyclical nature of the expected electricity prices. Detailed m_t^E values can be found in [9]. At time 0 suppose prices $P_0^E = 20$ ($/MWh) and $P_0^F = 2.2$ ($MMBtu) are observed. We assume that the instantaneous correlation coefficient between electricity and natural gas is $\rho = 0.5$.

By repeatedly running the program with different parameters, we obtain the following sensitivity analysis results. These are the relations between the power plant value vs. T (in Fig. 3a), power plant value vs. μ^E (in Fig. 3b), and power plant value vs. σ^E (in Fig. 3c).

It can be seen that the power plant value increases approximately linearly as the length of the planning horizon T increases as in Fig. 3a. The power plant value decreases as μ^E increases. This is because with bigger μ^E any price deviation from the mean does not last long, thus there are fewer "lasting" profitable opportunities. Moreover, the physical constraints of the unit place restrictions against the unit to react to these profitable opportunities of short durations. Finally in Fig. 3c, we see that the plant value increases as the price volatility σ^E increases. Moreover, the plant value is extremely sensitive to the price volatility. It can be estimated from Fig. 3c that a 1 % increase in σ^E would result in roughly a 1 % increase of the power plant value. This result

implies that the unit is more profitable in a market place with more volatile prices.

VII. CONCLUSIONS

In this paper we present a method for valuing a power plant using a discrete-time price tree. As opposed to the popular approach using financial options, we utilize the real options approach to value the operation of a power plant. Our presented method can be used to obtain the optimal strategy for exercising the real operational options of a power plant in the deregulated environment.
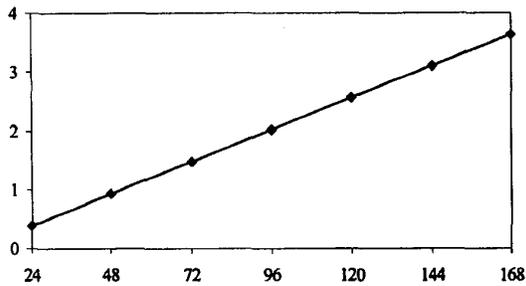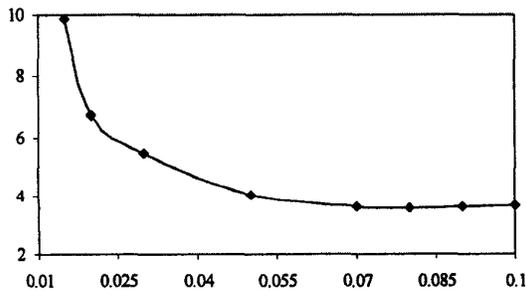

Fig. 3a: Power plant value ($x10^5$) vs. T
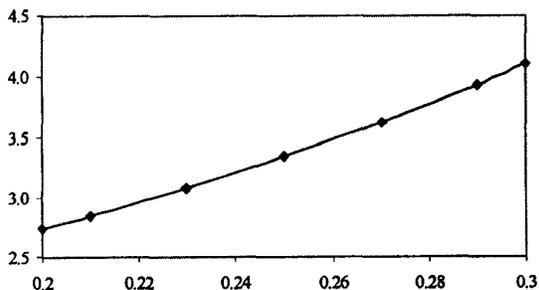

Fig. 3b: Power plant value ($x10^5$) vs. μ^E

Fig. 3c: Power plant value ($x10^5$) vs. σ^E

VIII. ACKNOWLEDGEMENT

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X. BIOGRAPHIES

Chung-Li Tseng (M' 1998) is an Assistant Professor in the Civil Engineering Department at the University of Maryland at College Park. He received a B.S. in Electrical Engineering from the National Taiwan University, a M.S. in Electrical and Computer Engineering from U.C. Davis and a Ph.D. in Industrial Engineering and Operations Research from U.C. Berkeley. Prior to his current position, he worked as an operational research consultant for PG&E, and a risk management analyst for Edison Enterprises. His research interests include financial engineering, project management, and optimization applied to industrial applications.