01 Feb 2008

Application of Design of Experiments to Flight Test: a Case Study

Aaron A. Tucker

Gregory T. Hutto

Cihan H. Dagli
Missouri University of Science and Technology, dagli@mst.edu

Recommended Citation

A. A. Tucker et al., "Application of Design of Experiments to Flight Test: a Case Study," *Journal of Aircraft*, American Institute of Aeronautics and Astronautics (AIAA), Feb 2008.
The definitive version is available at https://doi.org/10.2514/1.40301

U.S. Air Force T&E Days
5 - 7 February 2008, Los Angeles, California

AIAA 2008-1632

Application of Design of Experiments to Flight Test: A Case Study

Major Aaron A. Tucker[1]
United States Air Force, Edwards Air Force Base CA 93524

Gregory T. Hutto[2]
United States Air Force, Eglin Air Force Base FL 32547

and

Cihan H. Dagli[3]
University of Missouri-Rolla, Rolla MO 65409

Modern flight test tends to be a complex, expensive undertaking so any increases in efficiency would result in considerable savings. Design of experiments is a statistical methodology which enables a highly efficient investigation where only the samples needed are collected and analyzed. The application of design of experiments to the design of flight test can result in a significant increase in test efficiency. Increased information is garnered from the data collected while the number of data points required to understand the system is reduced.

In this effort, an actual flight test program serves as a case study to compare and contrast five different designs to explore the flight test envelope: the classic subject-matter-expert (SME) generated survey method, the SME-generated points augmented to a relatively fine mesh orthogonal analysis of variance design, an axial central composite design (CCD), a face-centered CCD plus simplex design, and a Simpson-Landman embedded face-centered CCD. The axial CCD is further expanded by a single point to illustrate the flexibility of the design in response to the interests of the test team. The case study data are analyzed using each designed experiment, and the results are compared and contrasted as a cost-benefit relationship between flight test resources expended (i.e. flight hours) and system understanding gained (i.e. statistical confidence and power).

The design of experiments methodologies, as applied to this case study, generally show a 50 to 80 percent reduction in flight test resources expended to gain similar levels of understanding of the system under test. These savings can be applied to other programs, used to educate design changes before testing an improved system, allow for flexible investigation into areas of interest to the test team, or replicate the test points resulting in a better understanding of systemic error. In an era of restricted budgets and timelines, careful design and thoughtful analysis of flight test experiments can make the difference between a failed or cancelled flight test program and the successful fielding of a needed capability.

Nomenclature

k	=	number of independent variables
M	=	Mach number
α	=	angle of attack
β	=	angle of sideslip

[1] C-5/C-17 Experimental Test Pilot, 418 FLTS/DOO, 300 N Wolfe Ave, Edwards AFB CA 93523, Senior Member.
[2] Wing Operations Analyst, 46 Test Wing, Eglin AFB FL 32542, Member.
[3] Professor of Systems Engineering, Missouri University of Science and Technology, 229 Engineering Management, Rolla MO 65409-0370.

σ = standard deviation
° = degrees

I. Introduction

Modern flight test tends to be complex and expensive so any increases in efficiency would result in considerable savings. Design of experiments is a statistical methodology which enables a highly efficient investigation where only the samples needed are collected and analyzed. The application of design of experiments to the design of flight test can result in a substantial increase in test efficiency. Increased information is garnered from the data collected while the number of data points required to understand the system is reduced.

A typical problem facing the designers of flight test experiment is an investigation of an altitude/airspeed aircraft envelope (e.g. figure 1). Numerous safety and operational limitations (e.g. stall margin, minimum altitude, structural loads, performance) must be enforced throughout a typical test program and don't typically lend themselves to a simple, designed experiment. As a result, most test teams resort to a survey method designed by their subject matter experts. The survey method, however, allows for a simple descriptive study where only limited information can be extracted from a given data set. Generally, the tester only knows how the system responded under the conditions tested with no way to expand those data into a system response model or measure the experimental error. That is, the tester knows only what happened during the test and nothing about how likely those data reflect true system performance.

The generic flight test problem (or any test problem, for that matter) may be viewed as shown in figure 1. The experimenter wishes to vary certain controllable inputs in such a way as to deduce which ones (alone and combined) may affect the output of the process. Furthermore, he would like to vary these factors in such a way as to minimize his chances of drawing an erroneous conclusion such as finding that an inert factor has an effect or failing to detect an active factor. The presence of background variation (fuel state, wind), measurement error, and other noise sources complicate the experimenter's condition. Further complicating the problem is the dauntingly large number of combinations available to be tested – the test envelope. The envelope of the present test, presented in simplified form in figure 1, contains an infinite number of combinations of four continuous variables: altitude, Mach number, angle of attack, angle of sideslip as well as the dichotomous variable representing synthetic jet actuators being turned off and on.

A flight test program planned, executed, and analyzed as a designed experiment would not only provide a model of the system's behavior but also information about how likely those results are representative of the true system behavior (statistical confidence) and how likely the model could detect a response of a given magnitude (statistical power). Deliberate planning decisions can consider the required quality of the system response model resulting in the most efficient use of very expensive, and often limited, flight test resources. For example, a coarse-grained model with a predicted confidence and power would require a given investment of flight test resources (time, money, aircraft, and people). Also, a known increase in the model's resolution could be purchased for a calculable increase in resources according to the specific needs of the customer. Further, the data set gathered under one flight test program using a designed experiment could serve as the baseline for the next system modification. Such savings are particularly attractive in current spiral development model of defense acquisitions. Finally, the system response model with its accompanying statistical confidence and power would be provided to decision-makers in order to educate their technical, safety, and programmatic decisions with rigorous analyses. The result would enable efficient stewardship of the nation's flight test resources.

A. Description of Designed Experiments

> Experiment—a test or series of tests in which purposeful changes are made to the input variables of a process or system so that we may observe and identify the reasons for changes that may be observed in the output response.[1]

Design of experiments (DOE) is the application of statistics to analyze both the main effects of input variables and interactions of those effects on response (output) variables. The experimental design provides a measure of the experiments' ability to determine the probability that the observed system response is representative of the true behavior—statistical confidence. Also, design of experiments gives the probability that a certain sample size was adequate to detect defined changes in the level of the response under test—statistical power.

Design of experiments is the simultaneous solution to the twin challenges of flight test: testing both deeply and broadly while maintaining confidence and power.[2] Designed experiments are used to optimize the functional relationship between an input and an output variable, and their generalized framework has been instrumental in closing the gap between engineering and statistics.[3]

Design of experiments allows for planning and conducting experiments with the goal of analyzing the resultant data so that valid and objective conclusions are obtained. Through each phase of flight test, design of experiments addresses the ultimate objective of achieving a robust flight test—a process affected minimally by external sources of variability.[4] Designed experiments enhance traditional flight test procedures and allow systems engineers and program managers to take advantage of objective, verifiable, and traceable empirical models that reveal both main effects and their interactions, if they exist.[5] Advantages of design of experiments as applied to flight testing:

1) Design of experiments provides a structured planning process that can be used to involve stakeholders and synergize individual subject matter knowledge to generate test and analysis plans that are comprehensive and efficient.
2) Sequential testing and analysis leads to immediate system discovery and understanding (accelerated learning).
3) Empirical statistical models can be used for estimation and prediction.[6]

A short description of the background of DOE and the intent of this paper is in order. DOE had its roots in the hallmark work of Dr. (Sir) Ronald Almyer Fisher in the 1920s and made its formal entry into the world of science with his original work, *The Design of Experiments*, first published in 1935. Fisher pioneered the discipline of how to choose test conditions, how to examine the mathematical properties of the chosen test design, how to order the runs to prevent bias from unknown background conditions, and finally how to analyze the results of the experiment to state what conclusions can logically be drawn from the results.[7] His innovations were revolutionary at the time and are still underutilized. Fisher invented the factorial design, the Analyses of Variance and Co-variance (ANOVA and ANCOVA), regression, and times series methods. Since those days, DOE has proven its worth in many fields of science and engineering and is beginning to make contributions in the field of military test and evaluation in digital simulations, hardware in the loop and integration lab testing, and both developmental and operational flight test.

This paper will address just a small portion of Fisher's work – the experimental design and the properties of such designs, especially their scope, and statistical power and confidence. A few other topics will be covered such as the models that can be fit from such designs, the proper control of test points in execution, and the ability to fractionate such designs to improve the tester's ability to efficiently screen out variables that have little effect in the present investigation. The authors believe that by taking an actual flight test and retrospectively re-designing it in multiple ways, we can illustrate the efficacy and efficiency of Fisher's approach to the challenges of test.

B. Description of the Case Study

The case study was a flight test program charged with demonstrating the effect of active flow control on targeting pod-induced buffet on an F-16B aircraft's ventral fin.[8] A targeting pod shape was equipped with six synthetic jet actuators designed to inject high velocity jets of air into the flow field at a specific frequency. The Air Force Research Laboratory had conducted wind tunnel and preliminary flight research on the effects of the synthetic jet actuators.[9, 10, 11] These jets of air were intended to actively control the buffet levels experienced by the F-16B's ventral fin approximately 13 feet downstream of the targeting pod.

In order to test the effect of the synthetic jet actuators on ventral fin buffet, a modified ventral fin was installed with strain gage and static pressure port instrumentation. The test aircraft was stabilized at a specific test condition with a specified altitude, Mach number (M), angle of attack (α), and angle of sideslip (β). Two, 10-second data points were then collected: 10 seconds with the synthetic jets turned on and 10 seconds with the synthetic jets turned off. The case study experiment did not detect any significant change in ventral fin buffet level across a wide range of flight conditions in 10 flight test sorties and 14 flight hours despite good statistical power.

The baseline case study was executed in traditional fashion with no planned controls to account for day-to-day variability. It lacked the randomization and blocking that Fisher recommended in order to protect against the "unknown unknowns." Further, the baseline case was not designed to produce a data set supporting a nonlinear model of the behavior of the fin vibration across the envelope of flight test explored. All the statistical designed experiments considered in the case study below are specifically designed to fit an empirical model that describes how the vibration varies across the flight envelope investigated. However, we do not consider in detail the ability of a statistically designed experiment to rapidly reach the conclusion that the sought-after goal cannot be achieved in the current test. In the baseline case, a statistically designed experiment would have likely reached the conclusion that the jets were ineffective in only a few sorties, saving the additional test resources to explore other test problems.

II. Application of Designs of Experiment to a Case Study

This paper examines the results of applying various designs of experiment on the case study's flight test envelope investigation. The case study test plan was originally planned and executed as a classic subject matter expert (SME)-designed survey, and the data available for this study were limited to that collected during those test flights. Operational flight envelope limitations (α, M, altitude) were honored during the design of each experiment in order to illustrate the challenge of applying experimental methods to a real flight test envelope.

A. Baseline Experiment.

The case study applied a typical grid-based survey method to a mach-altitude flight envelope.[12] In figure 1, blue dots indicate a typical survey grid placed at the edges of the flight envelope and at regular intervals within the flight envelope. These blue dots represent a test condition at the level-flight angle of attack, α_{level}+{0°}, and zero angle of sideslip, β={0°}. The case study subject matter experts also selected specific areas of the flight envelope in which to further investigate the system response. Therefore, variations in angle of attack and angle of sideslip were applied in these flight regions. Red ✦s indicate that data were collected at α_{level}+{0°, 2°, 4°}, β={0°}, and green triangles indicate that data were collected at α_{level}+{0°, 2°, 4°} × β={−2°, 0°, +2°}. At each test condition, data were recorded with the synthetic jet actuators operating and turned off. The trials consisted of 14 flight hours in an instrumented F-16B flight test aircraft over 16 sorties for a total of 324 test points.

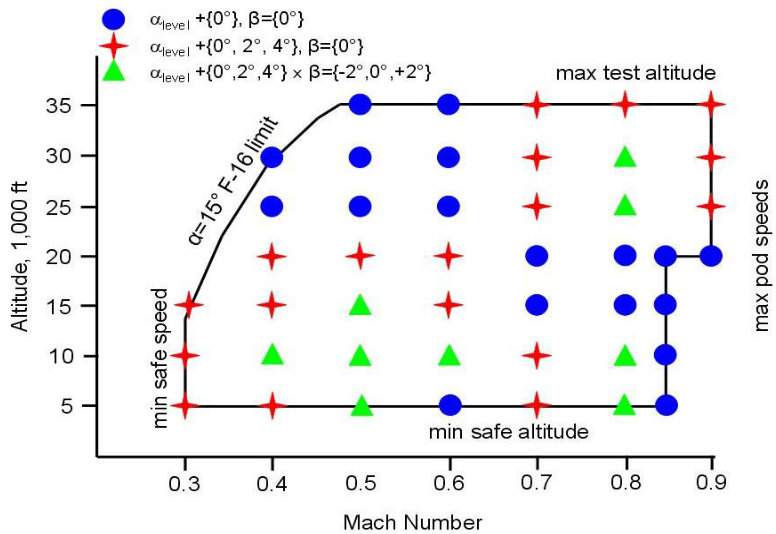

Figure 1. Baseline Experiment

B. Central Composite Design (CCD).

Figure 2 shows the central composite design (CCD) in altitude, Mach number, angle of attack and angle of sideslip. The CCD was invented in the 1950s by Fisher's son-in-law, Dr. George Box,[13] to address the problems of chemical and industrial research and development testing. It efficiently fits a second order polynomial with interactions to describe the effects of the control variables (Mach number, altitude, α, β, synthetic jet operation) on the vibration response. The CCD requires $2^k + 2k + 3$ center points (without fractionating the factorial portion). With four variables (k=4) this is 16 + 8 + 3 = 27 points. Multiplied by two for each on/off pod condition, the total number of test points is 54, 17% of the baseline experiment. A half-fraction Resolution III design requires 2^4 points, (8 + 8 + 3) × 2 = 38 test points. An effective approach to this problem would be to fractionate the CCD resulting in a factorial with center points then add axial points in factors which prove to be active. Not only does the designed experiment required fewer resources but will better characterize the main part of the flight envelope since each altitude/mach number condition is investigated through the full ranges of angle of attack and sideslip. A drawback to this design is that the axial points on the edges of the envelope are remote from the main factorial portion, and the corners of the envelope are not fully explored. Figure 3 illustrates, however, that the center of the envelope receives much more operational utility than the edges of the envelope.[14]

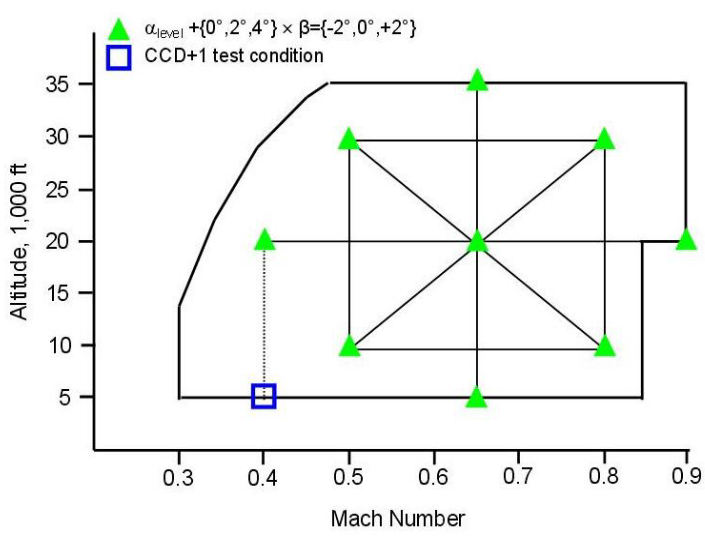

Figure 2. Central Composite Design

Another benefit to the CCD is the flexibility of the design, in particular, its ability to be efficiently augmented in numerous ways to explore interesting or unexpected behavior. Points can be added that enable the analysis of an entirely new orthogonal matrix, a CCD+*n* design. If the test team noted interesting results in the low altitude, low mach number region of figure 2, another two-level factorial could be analyzed by adding a single altitude-mach test condition, the blue square, resulting in a CCD+1 design. The resultant system response model would have higher fidelity in an area of regular operational utility (figure 3). If variations in angle of attack and sideslip warranted investigation, the entire α_{level}+{0°, 2°, 4°} × β={–2°, 0°, +2°} data set could be added to the experimental design with the addition of 9 test conditions, a CCD+9 design. Fractionating techniques could result in a specific number of additional test conditions (between one and nine) with a known loss of fidelity quantified by the entirety of the CCD design.

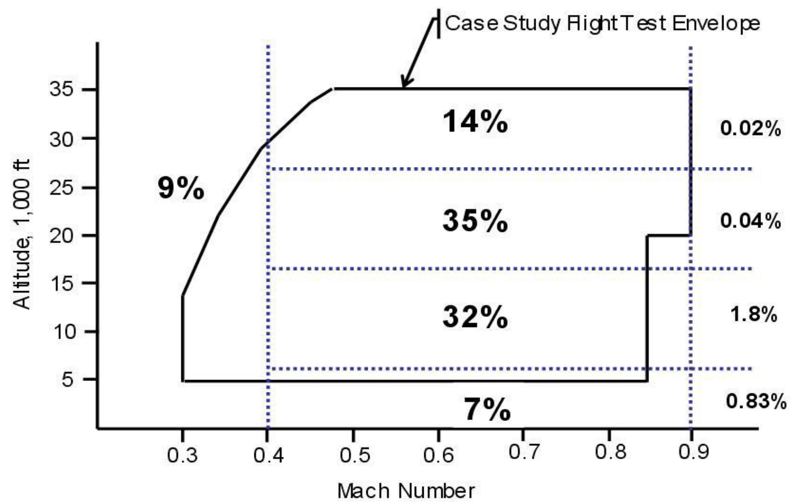

Figure 3. F-16CG operational employment spectra

Such flexibility should be anticipated by the flight test team with test and safety planning that preserves the comprehensive evaluation of risk but allows for flexibility in the placement of specific test points. Consider that similar factorials could be created to further refine the model at the various edges of the envelope, often a necessity to releasing an operational capability to the warfighter.

C. Face-Centered Central Composite Design (FCD).

Another design (same number of runs) choice would be the face-centered CCD (FCD) where the axial points of the CCD are drawn into the face of the factorial (Figure 4). The FCD was developed by Box[15] to address the problem of physical limitations on the test article that did not allow the axial points to be placed remotely from the main portion of the design. It has the added benefit of efficiently exploring a hyper-cubic test region such as the Mach number/altitude envelope commonly encountered in loads, vibrations, limit cycle oscillation, and environmental characterization testing. While this approach degrades a mathematical property of the design known as rotatability, rotatability is not particularly important in this experiment. As with the CCD, the design allows an explicit model of quadratic responses, characterization and quantification of experimental noise and repeatability, and experimentation with confidence and power. If desired, smaller adjacent matrices can be constructed as shown in the low Mach number/altitude corner of the envelope (figure 4 dashed lines). Each FCD again consists of 54 test conditions, 17% of the case study.

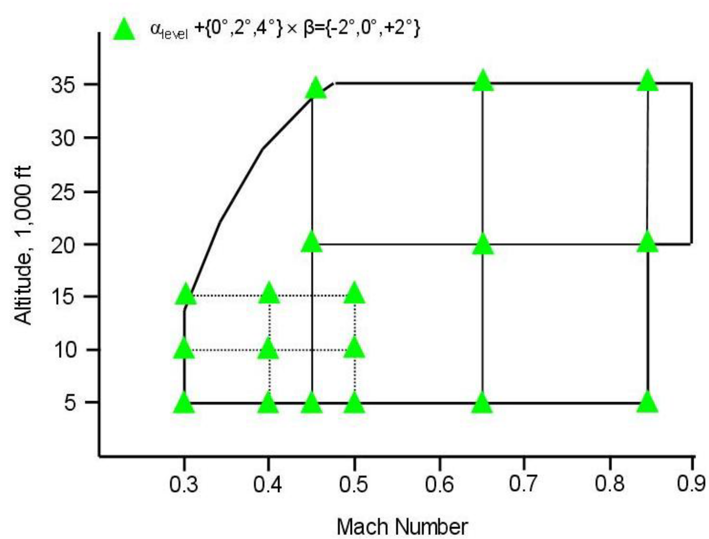

Figure 4. Face-Centered Central Composite Design

Consider that both matrices could be planned in a flight test program and undergo the required test and safety planning processes with flexibility in the placement of the points. The major FCD would be executed, and the data reduced and analyzed before placing the points for the minor FCD. This allows the test team to learn from the fully-modeled system response, better understand the system, and apply that knowledge to the minor FCD and realize the benefits in test efficiency, known risks, and confidence in system response.

D. Embedded FCD.

In the past two years, Dr Drew Landman of Old Dominion and NASA Langley, and Dr Jim Simpson from Florida State[6] have been considering a new class of designs for modeling highly nonlinear processes. As shown in figure 5, one can consider embedding two FCDs – one within the other (blue and green triangles in Fig. 5). The resulting composite design allows up to third-order polynomials to be fit through the experimental region at modestly increased cost. The two 54-test-point FCDs can be run together for a total of 108 test points (minus 3 repeated common center points.)

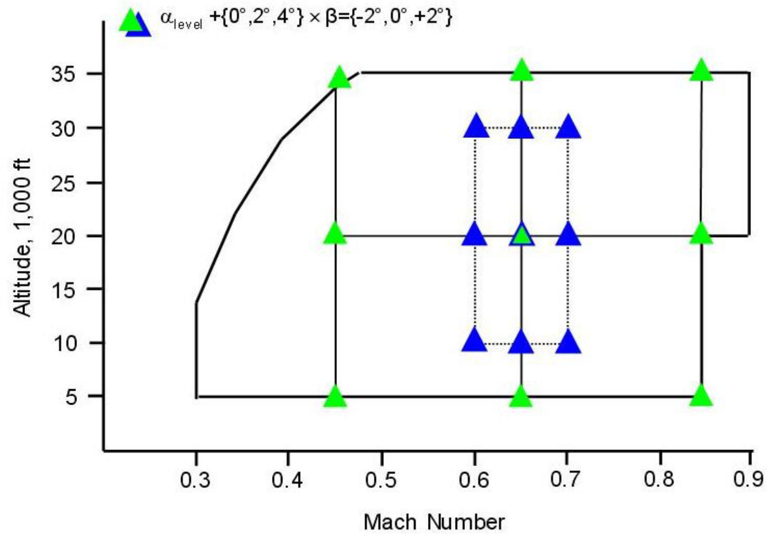

Figure 5. Embedded FCD

E. Fractional FCD.

For efficiency, a further refinement of the CCD and FCD designs is to fractionate the center or factorial portions of the design. Fractionating the factorial portion of the experiment increases efficiency in execution at the cost of slightly increased complexity in the analysis, and the possible need to run several additional test points to resolve uncertainties in the analysis. In practice, fractional factorials are commonly employed with five or more test variables. In this case study, we might consider running a Resolution IV half fraction of the 4 continuous variables yielding a design with 24 – 1 + 2 × 4 + 3 center points for a total of 19 test conditions, yielding 38 data points for the entire experiment. As before, we can fit a quadratic model through each of the variables and explicitly model uncertainty and reproducibility. With these fractional FCDs, we expend about 12% of the flight test resources expended in the baseline case.

F. 3-Level Design.

If the experimenter believes the variables might work together nonlinearly (quadratic interactions) a suitable choice might be a 3-level design referred to as the *3k*-class of designs. For these problems, one often decides to fractionate the resulting design because the powers of 3 grow so rapidly with the number of test conditions. For the four variables under consideration, we are contemplating a 3^4 full factorial or 81 test conditions, repeated for both on and off jet conditions yielding a total of 162 data points – half the size as the baseline but with vastly improved modeling possibilities. If this portion is fractionated we are considering a one-third fraction for a total of 27 × 2 = 54 test conditions. Generally, 3-level designs are not the best choice for nonlinear modeling but are certainly among those that may be considered.

G. Box-Behken Designs.

Invented by George Box in the 1950s, another response surface choice is a class of designs located on a sphere in the experimental region. Though not particularly appropriate for this largely rectangular test region, the Box-Behken designs add to the toolbox of the experimenter seeking to characterize nonlinear responses efficiently. As with other designed experiments, the Box-Behken designs can be fractionated to deal with four or more dimensions.

III. Results

In order to compare the efficiency and efficacy of five experimental methods, the data from the case study were formed into data sets as prescribed by each designed experiment. However, the original case study did not include some test conditions that a designed experiment would have prescribed, particularly in the β={–2°, +2°} data set. Therefore, the β={0°} data sets for the various designed experiments were completed using interpolation, extrapolation, and hand-fairing of system response curves. The case study's β={–2°, +2°} data set, however, did not have enough original data to be completed and could not be completed. Analyses of Variance (ANOVA) were used to analyze only the β={0°} data and statistical power was calculated for each design. The data sets used are included in the Appendix.

A. Statistical Confidence.

As discussed in the introduction, a tester can make one of two errors in a test program – determine that a factor makes a difference in the response when it does not (measured by statistical confidence) or to fail to detect a difference when such a difference really exists (measured by statistical power.) Unfortunately for aerospace testers, the probability of making the first type of error is referred to by statisticians as “alpha,” normally reserved for angle of attack. The protection from making the alpha error is referred to as (1 – alpha) or confidence. The second type of error is again, unfortunately, referred to as “beta,” usually reserved for angle of sideslip, and (1 – beta) is referred to as power. So, the tester would like to have high confidence in his conclusions, those conditions determined to make a difference really do make a difference, while preserving high power, the ability to detect real differences when they exist.

Table 1 presents the experimental designs and the ranges of the five independent variables in the data set: altitude, Mach number, angle of attack (α), angle of sideslip (β), and synthetic jet operation. Note that there are two data points (synthetic jet on and off) for every test condition in a designed experiment.

Table 1 includes a p value for the effect of the system under test on each system response: S1, P1 158 Hz, and P1 225 Hz. Note that none of the synthetic jet p values reach a level of significant effect ($p < 0.05$). This is important to the tester because it shows the synthetic jets were unable to affect the system response in the flight envelope of interest. The conclusion is that the synthetic jets did not significantly affect the vibration of the F-16 ventral fin. An analysis of the 372 data points gathered in the baseline experiment reached the same conclusion, with the same confidence as the designed experiments. Importantly, the designed experiments provided the same conclusion with 16% to 30% of the test resources expended.

Table 1. Results of ANOVA for Designed Experiments

Experimental Design	Altitude (1,000 ft)	Mach number	α	β	Synthetic jet operation	Number of points in data set	Response Variable	synthetic jet p value
Baseline β=0°	5 - 35	0.3 - 0.9	α_{level} α_{level} +2° α_{level} +4°	0°	off, on	324	S1 80 Hz P1 158 Hz P1 225 Hz	0.77 0.45 0.95
CCD	5 - 15	0.4 - 0.9	α_{level} α_{level} +2° α_{level} +4°	0°	off, on	54	S1 80 Hz P1 158 Hz P1 225 Hz	0.65 0.98 0.77
CCD+3	5 - 15	0.4 - 0.9	α_{level} α_{level} +2° α_{level} +4°	0°	off, on	60	S1 80 Hz P1 158 Hz P1 225 Hz	0.65 0.94 0.82
Face-Centered CCD	5 - 35	0.3 - 0.9	α_{level} α_{level} +2° α_{level} +4°	0°	off, on	54	S1 80 Hz P1 158 Hz P1 225 Hz	0.66 0.89 0.82
Simpson-Landman CCD	5 - 35	0.5 - 0.9	α_{level} α_{level} +2° α_{level} +4°	0°	off, on	108	S1 80 Hz P1 158 Hz P1 225 Hz	0.62 0.88 0.79
½ Fractional S-L FCD	5 - 35	0.5 - 0.9	α_{level} α_{level} +2° α_{level} +4°	0°	off, on	53	S1 80 Hz P1 158 Hz P1 225 Hz	0.65 0.88 0.92

B. Statistical Power

Figure 6 compares the statistical power achieved by the several types of experiments. The SME-designed baseline experiment has the statistical power to detect very small changes in ventral fin vibration. For example, a synthetic jet actuator effect of 0.50 standard deviation, 0.50σ, has a 90% probability of being detected by the baseline experiment. While a CCD only have a 40% probability of detecting a 0.50σ effect or would take a 1.0σ effect to have a 90% probability of detection. It should be noted that such small changes in the response (half the noise level or lower) are typically not of much practical interest in a flight test program. One might take the position that the SME-generated baseline was *too* powerful for the intended purposes (finding a solution to ventral fin buffet) and therefore wasteful of test resources.

Statistical power is a crucial commodity in flight test. One can make the case that power is the "gold standard" by which any test program should be judged. A test that lacks power will fail to detect the very behavior it was resourced to explore, while an over-powered test simply wastes resources. The baseline experiment can detect very small effects with a high probability of detection but comes at a cost of six times the invested resources of the CCD experiment. The test team can select a required statistical power for a given effect size and design and resource a test program with the statistical rigor to successfully investigate the system response. The alternative to a designed experiment is to blindly collect and inspect data with no insight into whether or not that test could have detected the effect of interest.

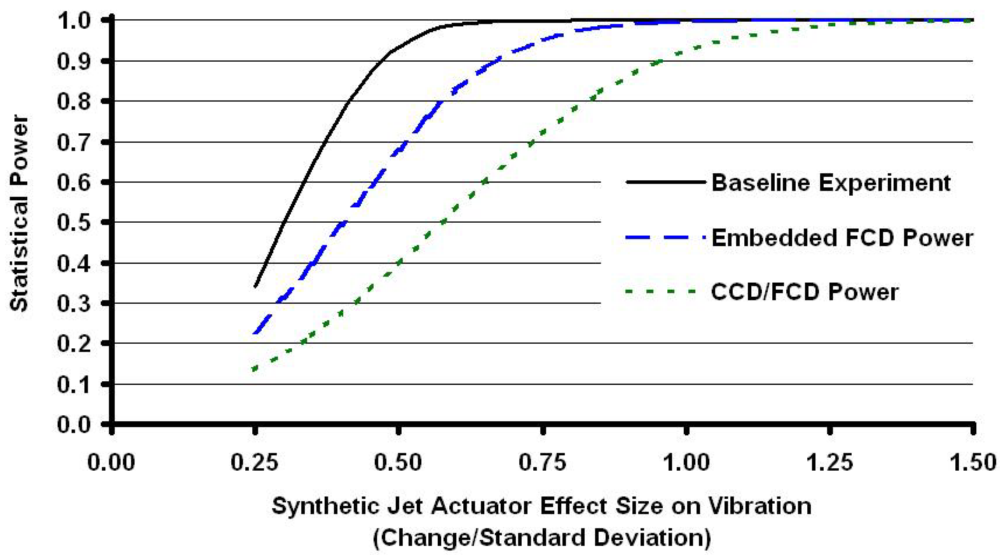

Figure 6. Experimental Statistical Power

IV. Conclusion

The application of the design of experiments to a flight test investigation is valuable by both increasing the information gained from the data collected and helping to ensure efficiency of test. The information gained from the data set is fully exploited to generate a system response model with a known statistical confidence and power. The test is efficient because no more data are collected than required to produce that system response model.

The flexibility in designed experiments allows for test teams to apply safe test practices, investigate interesting results, and continually build on the data set in a structured manner. Most safety planning addresses a build-up approach. Designed experiments can assist that effort by producing insight into the system response in the initial data collection which will educate the judgment of the test team and mitigate risk through increased confidence in the predicted system response as the data collection progresses closer to the predicted edge of the operating envelope. Also, the flexibility of the experimental designs allow the test team to analyze a complete data set, note areas of interest, and completely investigating those regions. Often, the test team can use part of the existing data set instead of launching an entirely new test program. In a designed experiment, data is collected in such a manner that the effect of an independent variable can be isolated from the system response. Therefore, future test efforts can use previous data collected in a designed experiment as part of the structure of another designed experiment.

In times of enormously expensive flight test programs, the efficiencies realized through the application of designed experiments to flight test could mean the difference between the timely delivery of a needed capability to the warfighter; an over cost, late, under-performing system; or outright cancellation of the system. Design of experiments has the capability to make flight test safer and more efficient.

Appendix

The data used for the statistical confidence and power calculations are included in Tables 1-3.

Table 1. Ventral Fin Strain (S1) Data

Ventral Fin Strain S1, $\mu in^2 \cdot in^{-2} \cdot Hz^{-1}$

α_{level}+0°, β=0°, pod=on

Altitude, ft	Mach 0.30	0.40	0.45	0.50	0.60	0.65	0.70	0.80	0.85	0.90
35000			102.0	60.751	8.251	10.294	12.337	25.229	50.192	75.155
30000		65.739	42.369	18.999	15.924	20.462	25.000	42.971	79.383	115.795
25000		82.562	48.387	14.211	25.620	38.936	52.252	63.533	107.082	150.630
20000	66.196	43.651	32.379	21.106	31.913	47.384	62.854	110.797	167.993	338.878
15000	42.065	22.762	24.550	26.337	54.008	77.688	101.368	172.225	195.874	
10000	31.885	22.082	29.270	36.458	72.763	110.938	149.112	218.741	320.662	
5000	6.213	31.481	44.115	56.749	105.599	137.331	169.062	264.847	405.271	
0										

α_{level}+0°, β=0°, pod=off

Altitude, ft	0.30	0.40	0.45	0.50	0.60	0.65	0.70	0.80	0.85	0.90
35000			123.0	73.053	7.655	11.732	15.809	22.282	43.515	64.747
30000		63.805	41.015	18.225	13.697	16.452	19.207	41.916	68.979	96.042
25000		83.885	49.174	14.462	27.405	32.548	37.690	63.613	117.252	170.890
20000	40.514	31.613	27.163	22.712	27.427	43.469	59.510	125.656	178.993	299.669
15000	34.812	18.011	23.419	28.826	57.371	72.911	88.451	187.146	242.799	
10000	26.094	25.636	33.840	42.044	78.642	109.040	139.437	194.140	272.077	
5000	26.38	33.912	37.678	41.444	136.250	148.253	160.255	249.179	356.727	
0										

α_{level}+2°, β=0°, pod=on

Altitude, ft	Mach 0.30	0.40	0.45	0.50	0.60	0.65	0.70	0.80	0.85	0.90
35000			141.0	90.0	33.0	22.6	27.725	37.890	58.160	78.430
30000		168.0	108.0	70.0	30.0	27.0	29.798	78.241	121.0	174.322
25000		125.0	80.0	55.0	30.0	35.0	44.107	89.319	149.067	208.815
20000	130.493	80.614	55.675	30.735	29.296	41.7	65.1	125.0	176.0	250.0
15000	36.860	31.024	32.496	33.968	49.173	66.0	85.0	152.0	200.0	
10000	44.729	30.908	35.135	39.361	68.370	89.229	110.088	181.685	225.0	
5000	29.0	31.592	40.796	50.0	88.035	108.250	128.465	211.369	273.0	
0										

α_{level}+2°, β=0°, pod=off

Altitude, ft	0.30	0.40	0.45	0.50	0.60	0.65	0.70	0.80	0.85	0.90
35000			143.0	90.0	33.0	22.6	29.603	41.286	57.041	72.795
30000		168.0	108.0	70.0	30.0	27.0	24.350	72.079	103.391	134.702
25000		125.0	84.0	63.0	55.0	61.0	79.089	92.602	149.061	205.520
20000	113.130	68.072	45.543	23.014	36.464	43.189	65.1	125.0	176.0	250.0
15000	35.473	29.800	30.537	31.274	60.001	73.0	92.0	152.0	200.0	
10000	50.205	28.418	33.742	39.065	66.231	88.831	111.430	182.187	225.0	
5000	26.0	31.357	40.679	50.0	86.059	122.018	157.976	203.773	273.0	
0										

α_{level}+4°, β=0°, pod=on

Altitude, ft	Mach 0.30	0.40	0.45	0.50	0.60	0.65	0.70	0.80	0.85	0.90
35000			170.0	128.0	97.0	90.013	87.043	81.103	111.228	141.352
30000		200.0	122.0	80.0	60.0	58.0	56.917	130.324	159.296	188.268
25000		185.0	113.0	69.0	53.0	69.0	106.325	149.052	194.571	240.089
20000	292.330	163.725	99.423	35.120	42.716	65.0	109.0	178.0	222.0	277.0
15000	128.0	67.173	54.154	41.135	51.398	66.0	117.0	207.0	251.0	
10000	60.033	41.863	38.0	35.0	61.540	125.052	188.563	240.0	279.0	
5000	91.0	73.047	50.0	49.0	83.897	135.303	186.708	250.693	308.0	
0										

α_{level}+4°, β=0°, pod=off

Altitude, ft	0.30	0.40	0.45	0.50	0.60	0.65	0.70	0.80	0.85	0.90
35000			177.0	128.0	97.0	88.920	83.395	72.346	106.818	141.290
30000		223.0	143.0	96.0	60.0	58.0	64.073	140.297	141.999	143.700
25000		201.0	126.0	84.0	64.0	69.0	80.056	135.641	167.696	199.751
20000	310.230	175.113	107.555	39.996	40.696	57.0	96.0	170.0	210.0	259.0
15000	93.796	73.436	63.256	53.076	51.817	66.0	107.0	195.0	239.0	
10000	53.018	40.071	38.0	35.0	60.267	96.021	131.774	222.0	268.0	
5000	128.866	88.933	50.0	49.0	101.416	138.648	175.880	233.382	285.0	
0										

points outside flight envelope | 0.000 inter/extrapolated points | 0.0 hand-faired points

Table 2. Ventral Fin Static Pressure Port (P1) 158 Hz Data

Ventral Fin Static Pressure Port P1 158 Hz, $psi^2 \cdot Hz^{-1}$

α_{level}+0°, β=0°, pod=on — Mach Number

Altitude, ft	0.30	0.40	0.45	0.50	0.60	0.65	0.70	0.80	0.85	0.90
35000			0.6	1.005	1.312	1.885	2.457	2.289	3.634	4.979
30000		0.469	1.065	1.660	2.408	3.050	3.692	3.301	6.035	8.768
25000		1.019	1.687	2.354	3.779	3.686	3.592	4.793	8.055	11.317
20000	0.839	1.879	2.399	2.919	6.339	5.767	5.195	9.098	13.170	22.347
15000	0.689	2.086	3.649	5.211	5.634	7.204	8.773	15.719	21.992	
10000	1.372	2.728	4.721	6.714	8.080	10.533	12.985	20.322	33.775	
5000	2.0	3.242	5.685	8.128	15.171	16.438	17.705	31.507	46.652	
0										

α_{level}+0°, β=0°, pod=off

Altitude, ft	0.30	0.40	0.45	0.50	0.60	0.65	0.70	0.80	0.85	0.90
35000			0.6	0.939	1.184	1.700	2.215	2.029	3.421	4.813
30000		0.461	1.160	1.858	2.592	2.841	3.090	3.574	5.849	8.123
25000		0.918	1.788	2.658	4.149	3.656	3.162	5.433	9.236	13.038
20000	0.855	1.831	2.319	2.807	5.395	5.264	5.132	9.700	13.961	19.654
15000	0.686	2.250	3.960	5.670	6.340	7.328	8.315	15.054	22.184	
10000	1.373	2.918	5.401	7.883	8.880	10.225	11.570	19.896	35.446	
5000	2.0	3.431	6.155	8.878	14.995	16.092	17.188	31.189	48.967	
0										

α_{level}+2°, β=0°, pod=on — Mach Number

Altitude, ft	0.30	0.40	0.45	0.50	0.60	0.65	0.70	0.80	0.85	0.90
35000			1.1	1.5	2.0	2.1	2.342	3.229	3.806	4.382
30000		1.1	1.3	2.0	2.9	3.3	3.524	5.330	8.059	10.787
25000		1.6	2.0	3.0	5.0	5.3	5.676	6.102	8.881	11.660
20000	0.091	1.713	2.524	3.335	6.523	7.7	8.8	10.6	13.0	16.0
15000	0.529	2.530	5.069	7.608	9.970	10.5	11.0	13.0	16.0	
10000	0.979	3.392	6.550	9.708	8.832	10.377	11.922	14.885	21.0	
5000	2.0	3.599	7.5	9.5	11.220	12.033	12.846	23.465	33.6	
0										

α_{level}+2°, β=0°, pod=off

Altitude, ft	0.30	0.40	0.45	0.50	0.60	0.65	0.70	0.80	0.85	0.90
35000			1.2	1.3	1.6	1.7	1.993	3.347	3.806	4.265
30000		1.0	1.5	2.1	2.8	3.3	3.442	4.626	6.946	9.266
25000		1.4	2.1	2.9	4.0	4.9	6.259	6.094	8.947	11.800
20000	0.1	1.708	2.722	3.736	5.075	5.6	6.3	8.6	11.7	17.1
15000	0.464	2.468	4.613	6.757	7.756	8.5	9.7	13.0	16.3	
10000	0.834	3.708	6.219	8.730	11.240	11.815	12.390	15.354	22.0	
5000	1.0	4.239	6.2	8.0	10.485	11.4	12.511	26.718	35.8	
0										

α_{level}+4°, β=0°, pod=on — Mach Number

Altitude, ft	0.30	0.40	0.45	0.50	0.60	0.65	0.70	0.80	0.85	0.90
35000			1.0	1.3	1.8	2.2	2.855	4.009	5.202	6.394
30000		0.6	1.1	1.8	2.6	3.3	4.215	5.995	7.662	9.328
25000		1.0	2.0	3.2	4.6	6.8	8.196	8.416	10.212	12.007
20000	0.5	1.332	2.736	4.139	5.275	7.9	10.6	14.0	17.1	20.1
15000	0.5	2.690	4.170	5.650	9.090	12.1	15.4	20.6	24.6	
10000	0.525	4.293	6.3	8.2	12.233	16.014	19.795	25.4	30.4	
5000	1.0	5.102	7.5	10.3	16.757	20.191	23.624	29.263	36.7	
0										

α_{level}+4°, β=0°, pod=off

Altitude, ft	0.30	0.40	0.45	0.50	0.60	0.65	0.70	0.80	0.85	0.90
35000			0.9	1.0	1.7	2.2	2.938	3.544	4.995	6.445
30000		1.0	1.2	1.4	2.6	3.5	4.368	6.352	6.905	7.457
25000		1.2	2.0	2.6	4.0	5.3	6.564	8.028	9.972	11.916
20000	0.4	1.447	3.049	4.651	5.639	7.8	9.6	13.4	16.3	20.0
15000	0.8	6.760	6.471	6.181	8.312	10.7	13.4	19.1	23.3	
10000	0.577	4.822	6.0	7.5	11.717	15.351	18.984	26.0	31.7	
5000	0.8	5.109	6.5	9.0	14.139	18.241	22.343	30.677	38.0	
0										

points outside flight envelope — 0.000 inter/extrapolated points — 0.0 hand-faired points

Table 3. Ventral Fin Static Pressure Port (P1) 225 Hz Data

Ventral Fin Static Pressure Port P1 225 Hz, $psi^2 \cdot Hz^{-1}$

α_{level}+0°, β=0°, pod=on	Mach Number									
Altitude, ft	0.30	0.40	0.45	0.50	0.60	0.65	0.70	0.80	0.85	0.90
35000			0.8	0.987	1.262	2.187	3.112	4.201	7.926	11.650
30000		0.426	1.041	1.655	2.360	3.712	5.064	6.980	13.945	20.910
25000		0.915	1.614	2.313	3.359	5.131	6.903	11.270	18.350	25.430
20000	0.183	1.680	2.429	3.177	4.694	6.532	8.369	19.740	30.230	42.750
15000	0.590	2.120	3.355	4.590	6.904	9.852	12.800	29.350	45.670	
10000	1.190	2.225	4.681	7.137	10.910	14.940	18.970	43.040	63.920	
5000	2.0	3.446	6.546	9.646	17.840	23.305	28.770	74.630	92.870	
0										

α_{level}+0°, β=0°, pod=off										
Altitude, ft	0.30	0.40	0.45	0.50	0.60	0.65	0.70	0.80	0.85	0.90
35000			0.7	0.849	1.377	2.228	3.078	4.308	8.619	12.930
30000		0.416	0.919	1.421	2.188	3.727	5.266	7.785	13.443	19.100
25000		0.853	1.621	2.388	3.316	5.329	7.341	11.800	19.260	26.720
20000	0.2	1.696	2.622	3.547	5.106	6.281	7.456	18.540	29.150	45.720
15000	0.583	1.889	3.578	5.267	6.950	9.185	11.420	27.970	42.990	
10000	1.232	2.254	5.025	7.795	10.940	14.190	17.440	44.080	62.570	
5000	2.0	3.556	7.103	10.650	18.950	22.685	26.420	66.070	91.050	
0										

α_{level}+2°, β=0°, pod=on	Mach Number									
Altitude, ft	0.30	0.40	0.45	0.50	0.60	0.65	0.70	0.80	0.85	0.90
35000			1.2	1.5	2.0	2.1	2.748	4.425	5.787	7.148
30000		1.1	1.3	2.0	2.9	3.3	4.296	7.459	13.135	18.810
25000		1.5	2.0	3.0	5.0	6.7	8.481	9.871	15.741	21.610
20000	0.2	1.595	2.783	3.970	5.777	7.3	8.8	11.4	16.0	23.9
15000	0.441	1.962	4.092	6.221	9.448	10.5	11.0	15.8	22.1	
10000	0.877	2.833	5.512	8.191	11.860	13.405	14.950	26.060	33.2	
5000	1.0	2.765	6.7	10.5	15.660	16.495	17.330	38.390	51.5	
0										

α_{level}+2°, β=0°, pod=off										
Altitude, ft	0.30	0.40	0.45	0.50	0.60	0.65	0.70	0.80	0.85	0.90
35000			1.2	1.3	1.6	1.7	2.602	4.719	6.142	7.565
30000		1.0	1.5	2.1	3.1	4.0	5.196	7.051	12.436	17.820
25000		1.8	2.4	3.3	5.3	6.3	8.994	10.040	16.250	22.460
20000	0.1	1.696	3.008	4.319	5.357	6.9	9.0	12.6	19.5	28.3
15000	0.411	1.953	4.247	6.541	9.467	11.0	13.2	18.7	24.0	
10000	0.882	3.128	5.894	8.659	16.360	16.99	17.620	26.200	32.7	
5000	2.0	3.607	6.2	9.0	15.780	16.3	17.530	39.310	55.0	
0										

α_{level}+4°, β=0°, pod=on	Mach Number									
Altitude, ft	0.30	0.40	0.45	0.50	0.60	0.65	0.70	0.80	0.85	0.90
35000			1.0	1.3	1.8	2.2	3.208	4.934	6.385	7.836
30000		0.6	1.1	2.0	3.0	3.7	5.008	7.199	10.295	13.390
25000		1.0	2.0	3.5	5.5	7.4	10.320	10.460	13.265	16.070
20000	0.5	1.122	2.580	4.037	5.750	8.5	11.3	16.1	19.6	24.0
15000	0.5	2.281	3.909	5.537	9.550	13.1	16.3	22.8	26.5	
10000	0.404	3.414	5.8	8.0	16.040	20.485	24.930	32.2	38.7	
5000	1.0	4.586	7.5	10.3	23.950	28.565	33.180	47.040	57.6	
0										

α_{level}+4°, β=0°, pod=off										
Altitude, ft	0.30	0.40	0.45	0.50	0.60	0.65	0.70	0.80	0.85	0.90
35000			0.9	1.0	1.7	2.2	3.148	4.508	6.318	8.128
30000		1.0	1.2	1.6	2.8	4.0	4.589	7.418	8.210	9.001
25000		1.2	2.0	2.8	4.7	6.7	8.370	10.500	14.325	18.150
20000	0.4	1.236	2.700	4.164	6.256	9.2	11.4	15.1	17.9	21.6
15000	0.8	4.242	4.841	5.440	10.530	13.6	16.3	21.0	24.4	
10000	0.484	3.408	6.0	7.5	16.850	22.070	27.29	34.4	37.4	
5000	0.8	4.937	6.5	9.0	22.890	30.015	37.140	44.600	49.8	
0										

points outside flight envelope   0.000 inter/extrapolated points   0.0 hand-faired points

Acknowledgments

Major Tucker would like to thank Dr. George Ka'iliwai for his generous support and direction in this line of inquiry and Capt Elwood T. Waddell for his statistical expertise and guidance.

References

[1] Montgomery, Douglas C. *Design and Analysis of Experiments, 5th edition*. New York: John Wiley & Sons, Inc., 2001.

[2] Hutto, Gregory. "Design of Experiments: Meeting the Central Challenge of Flight Test." *American Institute of Aeronautics and Astronautics* 2005-7607, presented at US Air Force T&E Days, 6-8 December 2005, Nashville Tennessee.

[3] Nair, Vijayan N., ed. "Taguchi's Parameter Design: A Panel Discussion." *Technometrics* 34, no. 2 (May 1992): 127-161.

[4] Montgomery.

[5] Simpson, James R. and James W. Wisnowski. "Streamlining Flight Test with the Design and Analysis of Experiments." *Journal of Aircraft* 38 no. 6, American Institute of Aeronautics and Astronautics 0021-8669 (November-December 2001): 1110-1116.

[6] Landman, D., Simpson, J., Mariani, R., Ortiz, F., and C. Britcher. "A High Performance Aircraft Wind Tunnel Test using Response Surface Methodologies." *American Institute of Aeronautics and Astronautics* 2005-7602, presented at the US Air Force T&E Days 2005, Nashville TN.

[7] Fisher, R.A., *The Design of Experiments*, Oxford University Press, (re-issue), New York, 1990. Originally published by Hafner in 1935.

[8] Tucker, Captain Aaron A., Captain Shawn D. Morgenstern, Captain Heather C. Giebner, Captain Dick Wong, Captain Aniello Violetti, and Lieutenant Commander Gregory Burgess. "Limited Characterization of Dual Bimorph Synthetic Jet Actuators: Aeroelastic Load Control—Phase I." *American Institute of Aeronautics and Astronautics* 2007-475, presented at the 45th Aerospace Sciences Meeting and Exhibit, 4-8 January 2007, Reno Nevada.

[9] Shaw, Leonard L., Smith, Brian R. and Seyed Saddoughi. "Active Control of Pod Wake-Mid-Scale Application." *American Institute of Aeronautics and Astronautics* 2005-799, presented at the 43d AIAA Aerospace Sciences Meeting and Exhibit, 10-13 January 2005, Reno Nevada.

[10] Hamburger, Charles L., Smith, Brian R. and Leonard L. Shaw. "Wake Large Eddy Simulation over a Blunt-Base Pod with and without Base Pulsating Blowing." *American Institute of Aeronautics and Astronautics* 2005-5014, presented at the 35th AIAA Fluid Dynamics Conference and Exhibit, 6-9 June 2005, Ontario Canada.

[11] Shaw, Leonard L., Smith, Brian R. and Seyed Saddoughi. "Full-Scale Flight Demonstration of Active Control of a Pod Wake." *American Institute of Aeronautics and Astronautics* 2006-3185, presented at the 3d AIAA Flow Control Conference, 5-8 June 2006, San Francisco California.

[12] Tucker.

[13] Montgomery.

[14] Wickert, Maj Douglas P. "Mitigating Risk at the Edge of the Envelope: Revising Basic Aircraft Limit Test Procedures." *Proceedings of the 50th Annual Society of Experimental Test Pilots Symposium*, September 2006, Anaheim California.

[15] Montgomery.