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New Graph Model for Channel Assignment in Ad Hoc Wireless Networks

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Abstract: The channel assignment problem in *ad hoc* wireless networks is investigated. The problem is to assign channels to hosts in such a way that interference among hosts is eliminated and the total number of channels is minimised. Interference is caused by direct collisions from hosts that can hear each other or indirect collisions from hosts that cannot hear each other, but simultaneously transmit to the same destination. A new class of disk graphs (FDD: interFerence Double Disk graphs) is proposed that include both kinds of interference edges. Channel assignment in wireless networks is a vertex colouring problem in FDD graphs. It is shown that vertex colouring in FDD graphs is NP-complete and the chromatic number of an FDD graph is bounded by its clique number times a constant. A polynomial time approximation algorithm is presented for channel assignment and an upper bound 14 on its performance ratio is obtained. Results from a simulation study reveal that the new graph model can provide a more accurate estimation of the number of channels required for collision avoidance than previous models.

1 Introduction

In wireless networks, radio signals propagate in all directions if omnidirectional antennae are used. Many applications benefit from this characteristic, for example, broadcast in multihop *ad hoc* networks. If all hosts use a single shared channel, transmission from one host will interfere with other hosts within its propagation range. Since the radio hardware does not have the ability to detect collisions, collision avoidance is very important.

Collisions can be avoided by partitioning the given radio spectrum into a set of disjointed channels and assigning channels to transmitters appropriately. This is called the channel assignment problem or the frequency assignment problem. In this paper, we assume interchannel interference is small, so only cochannel interference is considered. Since radio transmission has a limited propagation range, two hosts can use the same channel provided that the two hosts are spaced sufficiently apart. This property has been used to design efficient channel allocation algorithm in cellular networks.

There is a one-to-one correspondence between the channel assignment problem and the vertex colouring problem in graph theory. Formally, the channel assignment problem can be modelled as an appropriate colouring problem on an undirected graph representing the network topology, where vertices correspond to hosts and edges correspond to pairs of hosts that cannot use the same channel. The purpose of channel assignment algorithms is to assign channels to transmitting hosts such that cochannel interference is avoided and the total number of channels used is minimised. There are some other versions of channel assignment problems, for instance, to minimise the total interference for a given set of channels. In this paper, we consider the zero-interference-minimum-span version.

We consider two types of interferences: primary interference and secondary interference. The primary interference is caused by direct collision, due to simultaneous transmissions from hosts that can hear each other. The secondary interference is also called hidden terminal interference, which is caused by hosts outside the hearing range of each other transmitting to the same receiver. In this paper, we present a channel assignment algorithm to eliminate both the primary and secondary interference.

A variety of disk graphs have been used to model the interference of wireless transmissions. In unit disk (UD) graphs, intersection disk (ID) graphs and containment disk (CD) graphs, a host is represented as a single disk. In UD graphs, transmitters have the same transmission range, and an edge exists between two transmitters if and only if they can reach each other; in ID graphs, two hosts are considered interfering if their disks intersect; in CD graphs, two hosts are considered interfering if at least one disk contains the centre of another disk. CD graphs accurately model the direct collisions, but not the hidden terminal collisions. ID graphs include both, but they introduce extra edges when there is no other host in the overlapped area.

Double disk (DD) graphs are more realistic than the single disk graphs, in which each host is represented as two concentric disks, with the inner disk representing the range of the transmitter (or supply area as it is called in cellular networks) and the outer disk representing the interference area. The region between the outer circle and the inner circle represents the area where the signal is not strong enough to be received successfully, but strong enough to interfere with others. Two hosts are interfering if one host's interference area intersects with another host's supply area. DD graphs more accurately model the real networks than ID and CD

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graphs. However, DD graphs have the same problem as ID graphs, i.e., they do not distinguish if there exist other hosts in the overlapped area.

In this paper, we propose a new graph model, i.e., two cochannel hosts are considered to be interfering with each other if and only if the receiver of one transmitter is in the interference area of another transmitter. We call it the interFerence Double Disk graph model. To avoid confusion with the intersect disk (ID) graph and the double disk (DD) graph, we use FDD to denote it. Similar to a DD graph model, this model considers two concentric disks each representing the transmission range and interference range separately, but it more accurately models wireless networks. In an FDD graph, an edge exists between two vertices if the inner disk of one transmitter overlaps with the outer disk of the other transmitter and there exists another node within the overlapped area.

The purpose of this study is to provide an upper bound on the number of channels needed to support a collision-free wireless *ad hoc* network. Without the knowledge of the carrying traffic, we use the worst case estimation in theoretical analysis, i.e., all hosts are active transmitters, and the communication request is possible between an arbitrary pair (unicast) or among an arbitrary group (multicast). It is assumed that once a transmitter is granted a channel, it will use this channel for all transmissions, and receivers will adjust their receiving frequencies for different transmitters. The number of channels required to support certain traffic pattern is largely dependent on the traffic model, which very often cannot realistically model the traffic. To answer how many channels are enough to support a specific traffic pattern, simulation studies may be used to obtain a tight bound.

A traditional way to represent the performance ratio of colouring algorithms is to compare the chromatic number $\chi(G)$ with the clique number $\omega(G)$, because $\omega(G)$ is the lower bound for $\chi(G)$. It has been shown that for any UD graph, the chromatic number is bounded by a clique number times a constant. In this paper, we try to find out if there is an upper bound for $\chi(G)/\omega(G)$ in FDD graphs. We prove that this upper bound exists and $\chi(G) \leq 14(\omega(G)-1)$ for FDD graphs.

2 Related works

The channel assignment problem has been extensively studied and many models and solutions have been proposed since the problem arose in wireless communication. In general, most of the channel assignment schemes fall into three categories: mutual exclusion approach, linear programming approach and graph colouring approach.

Based on the principle of mutual exclusion, Prakash *et al.* [1] proposed a channel allocation strategy for traditional cellular systems. The main idea is to model communication sessions as multi processes requesting a shared resource concurrently. Specifically, a communication request is assigned a channel by the base station if an idle channel is available in its cell; otherwise, it acquires a new channel if it has no interference with the neighbouring cells. A channel is released once the communication session is terminated; the newly acquired channel remains allocated to a cell until it is required to transfer to a neighbouring cell. Nesargi and Prakash [2] extended this strategy to virtual cellular networks, in which the static base stations are replaced with mobile base stations, and the wired links between base stations are replaced with wireless links. In [2], the set of wireless channels is partitioned into two disjointed subsets, one subset is used exclusively for links between mobile base stations and the other subset is used exclusively for links between base stations and mobile hosts. Cochannel interference is considered within each set separately. This separation simplifies the task of channel allocation at the cost of channel utilisation. In [3], the channels are divided into groups, and two levels of blocking granularity are used, group blocking and channel blocking. Since each base station maintains the channel usage table and makes its decision based on messages received from all neighbours, there is no channel transferring at any time, thus it reduces the message overhead and channel acquisition time.

The frequency assignment problem (FAP) can also be formulated as an integer programming (IP) problem (see e.g. [4–7]). IP reveals the structure of the FAP and allows lower bounds on the optimal solution obtained via relaxation technique (see [8]). A major drawback of the IP approach is that the total number of frequencies must be estimated precisely to make the solution feasible. Another drawback is that this approach is not practical for large-scale networks and is not applicable for dynamic distributed environment.

A commonly used approach is to formulate the frequency assignment problem in graph-theoretic terms (e.g. [9–12]. In a graph formulation, the vertex set V of graph G defines the set of wireless nodes, and the edge set E defines the set of interference edges. An edge (v_i, v_j) exists if node v_i and v_j cannot use the same channel. This is cochannel constraint. Metzger [13] first pointed out that the cochannel frequency assignment problem is equivalent to a graph colouring problem, and formulated the minimum-order problem. Hale [14] first recognised the importance of the span of the spectrum, i.e., the difference between the largest and the smallest frequency, and formulated the minimum-span problem based on the results of [13]. Cozzens and Roberts [10] first generalised the channel-constrained frequency assignment problem as a T-colouring of a multigraph and obtained bounds on the minimum-order and minimum-span. The T-colouring is used to consider the adjacent-channel constraints and more general channel constraints, where neighbouring nodes cannot be assigned channels that differ by less than a positive integer k.

In this paper, we consider a cochannel problem and use the graph colouring approach to compute an approximate solution. Next, we will provide some of the preliminary graph-theoretic results.

3 Graph colouring on disk graphs: preliminaries

The colouring of a graph $G=(V, E)$ is an assignment of colours to the vertex set V such that no two adjacent vertices have the same colour [8]. A colouring that assigns k colours to G is termed a k-colouring. The chromatic number of a graph G is the minimum number $\chi(G)$ for which a $\chi(G)$-colouring exists for G. A graph is *k-colourable* if $\chi(G) \leq k$. A graph is k-chromatic if $\chi(G) = k$. For any graph G, the clique number $\omega(G)$ is the lower bound for its chromatic number $\chi(G)$, so the chromatic number is usually represented as a constant times its clique number.

The upper bounds on the chromatic number of graphs have been studied by applying sequential colouring algorithms. A sequential colouring algorithm can colour each graph G with at most $\Delta(G)+1$ colours, where $\Delta(G)$ is the maximum vertex degree of G, regardless of the vertex ordering. Matula and Beck [15] first introduced the smallest-last vertex colouring algorithm and stated that the chromatic number $\chi(G)$ of a graph G is upper bounded by $\hat{\delta}(G)+1$, where $\hat{\delta}(G)$ is the maximum value of all the minimum degrees of the induced subgraphs by smallest-last vertex ordering. Since $\hat{\delta}(G) \leq \Delta(G)$, so smallest-last ordering provides a tighter bound on the chromatic number.

Unit disk (UD) graphs are first used in [14] to model wireless networks. Clark *et al.* [16] proved that the k-colourability problem for UD graphs is NP-complete for $k = 3$. Based on the proof in [16], Gräf *et al.* [17] proved for any fixed number $k \geq 3$, the k-colourability problem in UD graphs remains NP-complete. A UD graph G can be coloured using at most $6\omega(G) - 6$ colours using sequential colouring algorithms. This result can be improved to $3\omega(G) - 2$ if the vertices in the graph are ordered lexicographically [18].

A double disk (DD) graph is used in [19] to model the cochannel interference in wireless networks. The best possible performance bound for DD graph is obtained by the Stripe algorithm, which again must use the geometry model of the graph as an input. For a DD graph with $D_0/d_0 < \sqrt{3}$, where d_0 and D_0 are the lower and upper bounds of the distance between two adjacent vertices, the Stripe algorithm needs at most $3\omega(G)$ colours to colour it. For sequential colouring algorithms, Malesinska *et al.* [19] proved that the chromatic number of a double disk graph is at most $33(\omega(G) - 1) - 2$. No other linear relation between $\chi(G)$ and $\omega(G)$ has been known so far for DD graphs.

Wan *et al.* [20] proposed a new graph model for a channel assignment in wireless networks, and provided a thorough analysis for approximation algorithms based on this model. This graph model and FDD graph model both consider the existence of other nodes in the overlapped area. The difference between the FDD graph model and the model used in [20] is that in FDD graphs, two concentric disks are used to represent each transmitter, while in [20] only a single disk is used.

Next, we will formulate the channel assignment problem using the FDD graph model and study its computational complexity.

4 Problem formulation and computational complexity

4.1 Channel assignment problem

Given a set of nodes V on the Euclidean plane and each node v is associated with two concentric disks with radii r_v and R_v respectively, where $R_v = c \cdot r_v$ and constant $c \geq 1$, build an FDD graph on V and assign each node a colour such that no node has the same colour as its adjacent nodes in the FDD graph. To construct the FDD graph, V is used as the vertex set, and the edge set is constructed in such a way that there exists an edge between two nodes x and y if and only if

1. $x \neq y$, and
2. there exists a node $w \in V$ that satisfies $|xw| \leq r_x$, $|yw| \leq R_y$ or $|yw| \leq r_y$, $|xw| \leq R_x$.

We use $D(v)$ and $d(v)$ to denote the area covered by the outer disk and inner disk of node v, respectively. Since w could be x or y, the above statement is equivalent to:
x and y are connected by an edge in G if and only if at least one of the following is true:

(i) $D(y)$ covers x
(ii) $D(x)$ covers y
(iii) there exists a node $z \in V\backslash\{x, y\}$ that lies in the overlapped area of $d(x)$ and $D(y)$
(iv) there exists a node $z \in V\backslash\{x, y\}$ that lies in the overlapped area of $D(x)$ and $d(y)$.

This graph model includes both direct interference edges and indirect interference edges, therefore an appropriate vertex colouring of an FDD graph can eliminate both direct collisions and hidden terminal collisions.

For the same set of wireless nodes V, the FDD graph is a subgraph of the ID graph built on outer disks and a super graph of the CD graph built on inner disks, i.e., each FDD graph contains a CD graph, and each ID graph contains an FDD graph as its subgraph. The FDD graph is also a subgraph of the DD graph on V. The ID graph is a graph with the given set of nodes as a vertex set and there is an edge between two nodes x and y if and only if there is overlap between the two disks associated with x and y, respectively. The CD graph is a graph with the given set of nodes as a vertex set and there exists an edge between two nodes x and y if and only if the disk centred at x covers y or the disk centred at y covers x.

Whether FDD graphs form a different class from ID graphs and CD graphs is still an open question. It is known that CD is not isomorphic to any ID. Gräf *et al.* [17] proved that $K_{3,3}$ as an instance of a CD graph, is not isomorphic to any ID, but where the new graph FDD fits in the disk graph paradigm remains unknown.

The FDD graph model is expected to be more suitable in *ad hoc* wireless networks than all previous models. ID, CD and DD models are widely used in cellular networks, where each base station covers a fixed area, and the mobile users move around inside the area. So frequency assignment only needs to consider the disk area covered by each base station, regardless of the existence of a mobile user in the overlapped area. However, in *ad hoc* wireless networks, there is no fixed base station, and every node has the same mobility. To compute a tight bound on the number of channels needed, it is important to consider if there are other nodes in the overlapped area. The graph model proposed in [20] also considers the existence of other nodes in the overlapped area, however a single disk model is used. So it can be considered as a special case of an FDD graph, where all outer disks collapse to inner disks.

We next show the computational complexity and approximatability of vertex-colouring for FDD graphs.

4.2 Computational complexity

The following theorem indicates that the frequency assignment problem is NP-complete for FDD graphs:

Theorem 1: It is NP-complete to determine whether a given FDD graph is three-colourable.

Proof: We reduce three-colourability of planar graphs to three-colourability of FDD graphs. As shown in Fig. 1, each edge (u, v) in the original planar graph is replaced by a sequence of equilateral triangles plus one edge. Clearly, when all triangles for each edge have the same size and the size is chosen properly, the resulting graph is an FDD graph. Moreover, the original planar graph is three-colurable if and only if the resulting graph is three-colourable. ∎

Corollary 1: The vertex-colouring in FDD graphs cannot have a polynomial–time approximation with performance ratio less than 4/3 unless NP = P.

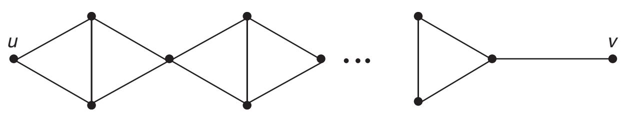


Fig. 1 *Replace each edge with a widget to construct FDD graphs*

Proof: Suppose there exists a polynomial–time approximation $\mathcal{A}$ with performance ratio $\alpha < 4/3$ for vertex-colouring in FDD graphs. For any three-colourable FDD graph, $\mathcal{A}$ will produce a solution using at most $3\alpha < 4$ colours. For any non-three-colourable FDD graph, $\mathcal{A}$ will use at least 4 colours. Therefore, from a solution produced by $\mathcal{A}$, it can be known in polynomial–time whether a given FDD graph is three-colourable. This results in $NP = P$. ■

5 Polynomial time colouring algorithm for FDD graphs

Theorem 2: There exists a polynomial–time 14-approximation for vertex-colouring in FDD graphs.

To show this theorem, we employ a greedy approximation as follows:

Heuristic 1

Step 1. Put all vertices in a list as follows: starting from an empty list, choose a vertex with the lowest degree and put it at the head of the list, then delete it from the graph, repeat until all vertices are included in the list.

Step 2. Colour all vertices as follows: At each iteration, colour the head in the list with the smallest colour not appearing in its neighbours, then delete it from the list; repeat until the list is empty.

This greedy algorithm has performance ratio 14. The detailed proof is in the Appendix (Section 10).

6 Distributed channel assignment algorithm

The distributed implementation of the channel assignment algorithm would require that each node has knowledge of its two-hop neighbourhood, which is obtained within the first three rounds described below. Each node has three states: initial, colouring and coloured.

Heuristic 2

Round 1: A node in 'initial' state would start by broadcasting its own ID, and learn its one-hop neighbours from the information it has received.

Round 2: Once a node receives the IDs from all its neighbours, it broadcasts its one-hop neighbours. Based on the information it has received from all its neighbours, each node learns its two-hop neighbours, and then computes a local FDD graph that spans over its two-hop neighbours. It then enters the 'colouring' state.

Round 3: A node with a stable FDD graph would broadcast its degree (i.e., the number of neighbours in its FDD graph), and relay this information for its one-hop neighbours.

Round 4: To decide a channel number, each node would first build a list from its local FDD graph using the smallest-last order. To get a list of smallest-last order, start with an empty list, pick a node with the smallest node degree, put it at the head of the list, and remove it from the local FDD graph; repeat until all nodes are in the list. A tie is broken in favour of a smaller node ID. The relative order of two nodes that appear at the list is consistent between each other. The node that finds itself at the head of the list would pick the smallest channel number not used by its FDD neighbours (i.e. nodes that share an edge with it on the FDD graph) and announce its channel immediately, and then go to the 'coloured' state. Other nodes once they hear this announcement will remove it from the list, update the FDD graph, and relay it for one hop. Round 4 is repeated until every node is assigned a channel number.

A node in the 'coloured' state would periodically announce its channel number and ID, and relay this information for one hop.

6.1 Mobile node channel assignment

In a mobile environment, a node movement can be pictured as disappearing from one position and reappearing at another. When a node moves to a new position, it announces its current channel number and, from the heart-beating messages it has received from other nodes, it can determine if it has conflict with other nodes; if it does, it will go to the 'initial' state, and the Heuristic described above will kick in again; if not, the node keeps its original channel and stays in the 'coloured' state.

After a node leaves a spot, no additional work needs to be done, since other nodes will realise that it is gone from the absence of its heart-beating message.

6.2 Message overhead analysis

Each node would generate four types of messages to get its channel number. The content and the lengths of the messages are as follows:

in round 1: node ID of itself, message length $O(\log n)$

in round 2: list of one-hop neighbours, message length $O(\Delta \log n)$, where Δ is the maximum node degree

in round 3: degree of itself, message length $O(\log \Delta)$; relay for one-hop neighbours, total message length $O(\Delta \log \Delta)$

in round 4: channel number of itself, message length $O(\log n)$; relay for one-hop neighbours, total message length $O(\Delta \log n)$.

Therefore for all the messages, the message length is at most $O(\Delta \log n)$.

7 Simulation results

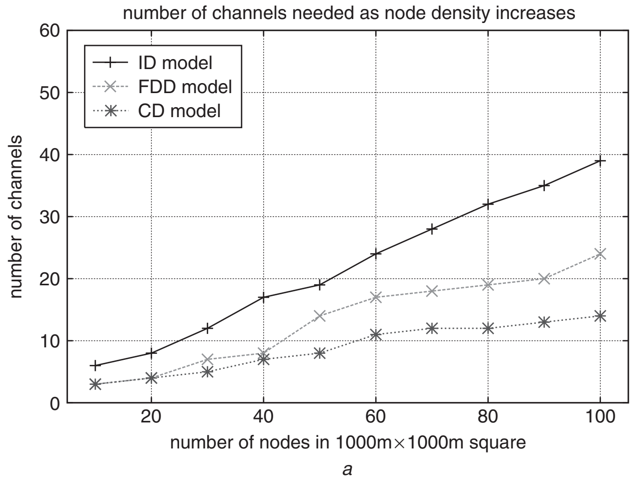
In this Section, we evaluate the performance of the approximation algorithm in terms of the total channels needed for zero-interference communication. Without the knowledge of the carrying traffic, we assume a worst-case traffic pattern, i.e., any node is a potential transmitter, and its intended receiver can be any node; and all nodes are active. We assume the channels are orthogonal to each other, so only the cochannel interference is considered. The mobility of nodes is low and therefore ignored in this simulation.

In this simulation study, all networks are deployed in a $1000m \times 1000m$ square, and node positions are randomly generated. Each simulation is repeated 100 times and the average results are plotted.

7.1 Channel assignment as the network density increases

To study the influence of network densities, we assume the transmission range is $300m$, and the number of nodes varies from 10 to 100. Only connected networks are evaluated.

Figure 2*a* shows the results from different graph models as the number of nodes increase from 10 to 100. For a fixed transmission range $R = 300m$, the ID model computed 64% more in the total number of channels than the FDD model, and the CD model computed 34% less than the FDD model on average.



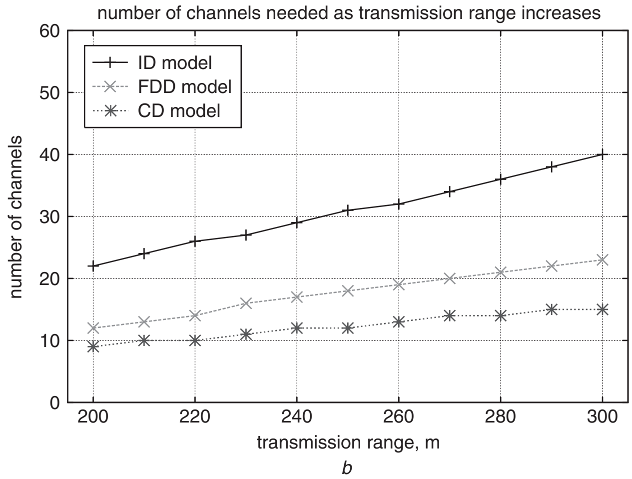


Fig. 2
a Number of channels needed by CD, FDD and ID models with a fixed transmission range 300*m*
b Number of channels needed by CD, FDD and ID models for 100-node networks

7.2 Channel assignment as the transmission range increases

We now study the influence of transmission ranges on the channel assignment. Figure 2*b* shows the results from different graph models as the node transmission range increases from 200*m* to 300*m*. For each transmission range, 100 network instances are used and the results are averaged. Each network instance has 100 nodes randomly deployed in the $1000m \times 1000m$ square. The results show that the total number of channels needed in ID graphs increases much faster than in FDD and CD graphs as the transmission range increases. On average, ID graphs use 73% more channels and CD graphs use 30% less channels than the FDD graphs do.

This simulation study verified the prediction from the theoretical analysis that ID graphs require more channels because unnecessary edges are introdued when there is no other node in the overlapped area, and CD graphs under-estimate the number of channels needed because they only consider direct collisions. The simulation results provided a quantitative comparison of ID, CD and FDD models.

8 Conclusions and future work

In this paper, we considered the collision-free channel assignment problem in *ad hoc* wireless networks. We modelled the wireless networks by a new class of graphs (interFerence Double Disk Graphs (FDD)). The problem of minimising the number of channels needed to eliminate interference is a graph colouring problem in FDD graphs. We proved its NP-completeness and provided an upper bound for its chromatic number. We designed a centralised channel assignment approximation algorithm and its distributed implementation that can eliminate both direct collisions and hidden terminal collisions. The FDD graph model requires more channels than the containment disk (CD) graph model, and less channels than the intersection disk (ID) and double disk (DD) graph models. FDD graphs model the wireless networks more accurately than CD, ID and DD graphs.

The performance ratio of this algorithm on FDD graphs is 14 when the radii of outer disks and inner disks have a constant ratio. For a more general case where $R_i = c_i r_i$ and $c_i \neq$ constant $\forall v_i \in V$, the performance ratio of the sequential colouring algorithm is still unknown. It is our future interest to find out the performance ratio of the approximation algorithms when c_i is not a constant but bounded, i.e. $C_1 \leq c_i \leq C_2$.

The theoretical bound provided in this paper can be used as a worst-case estimation on the total number of channels needed in wireless *ad hoc* networks. Our future work on channel assignment will consider more efficient channel assignment algorithms to reduce the number of channels needed. Especially when the traffic pattern is given, or the activity factor of nodes is given, channels can be reused between neighbouring nodes when their activity periods have no overlap, or their intended receivers are not interfered by the others.

Channel assignment is especially useful in collision-free broadcast, since other collisions avoidance techniques are not applicable in broadcast. When multichannels are available in broadcast, we can consider the channel allocation along with broadcast tree construction. Vertex degrees can be controlled by adjusting transmission power of each node to meet the channel constraints.

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10 Appendix

To prove theorem 2, we first prove a lemma.

Lemma 1: *For any FDD graph G, there exists a vertex with degree* $\leq 14(\omega(G)-1)-1$, *where* $\omega(G)$ *is the size of the maximum clique in G.*

Proof: Choose a vertex v with the smallest transmission range. We divide the neighbourhood of vertex v into 14 regions such that in each region, all the neighbours of v form a clique.

First, we divide the plane into two parts; the area $N_1(v)$ covered by the inner disk of v (i.e. $N_1(v) = d(v)$), and the remaining area $N_2(v)$. It is obvious that all neighbours of v contained in $N_1(v)$ are adjacent to each other therefore form a clique with v.

Next, we divide the remaining area $N_2(v)$ into 13 regions (see Fig. 3) such that the sectors centred at v have degree $\alpha < 2\arcsin\frac{1}{4}$. Each of these 13 regions has the property that if a vertex u is adjacent to v and lies in one of the 13 regions, then u would be adjacent to any node w in the same region if $|wv| \leq |uv|$. This property is proven in the following.

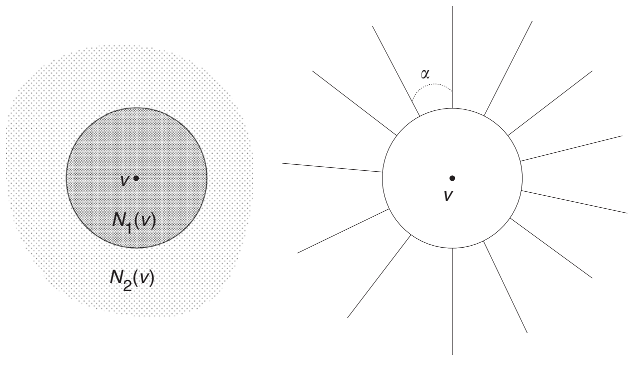

Fig. 3 *Partition the plane into 14 regions*

The fact that u and v are adjacent implies at least one of the following is true:

Case 1: u and v are adjacent because $D(u)$ covers v.

Case 2: u and v are adjacent because $D(v)$ covers u.

Case 3: u and v are adjacent because $D(u)$ overlaps with $d(v)$ and there exists another node x in the overlapped area.

Case 4: u and v are adjacent because $d(u)$ overlaps with $D(v)$ and there exists another node x in the overlapped area.

In case 1 and case 2, any neighbour w within the same region satisfying $|wv| \leq |uv|$ will be adjacent to u.

In case 3 and case 4, if w is located inside of the disk $D(u)$ then w is adjacent to u. We only need to study the case where w is located outside of $D(u)$.

We denote the area that is outside $D(u)$ and $d(v)$ but within the 30° sector as A_w, and we claim that any node $w \in A_w$ must be adjacent to u if $|wv| \leq |uv|$. If this claim holds for $\alpha = 30°$, then in the partition in Fig. 3, when the angles of the sectors are bounded by $\alpha < 2\arcsin\frac{1}{4}$, the same claim still holds.

In case 3, we denote the overlapped area of $D(u)$ and $d(v)$ as A_{uv}, and node x is at the intersection of the two boundary circles. Similarly, in case 4, we denote the overlapped area of $d(u)$ and $D(v)$ as A_{uv}, and node x is at the intersection of the two boundary circles. We claim that in case 3, any node in the area A_w covers A_{uv} with its inner disk; in case 4, any node in the area A_w covers A_{uv} with its outer disk. So w will be adjacent to u in both cases.

Let L denote the line that passes v and forms a 30° angle with the line uv. In the following analysis, we assume w is at the intersection of the line L and the boundary circle of $D(u)$. The distance between any node in A_w and any node in A_{uv} is then bounded by $\max\{|wx|, |vx|\}$. If $|wx| \leq |vx|$, then in both cases the claim that any node in A_w must be adjacent to u holds.

Next we prove that $|wx|$ is indeed $\leq |vx|$. For convenience, we normalise $|vx|$ to 1, and use r and R to denote the radii of $d(u)$ and $D(u)$ respectively.

Case 3: Case 3 is shown in Fig. 4*a*. The radius of the inner disk $d(v)$ is normalised to 1.

$$|wx|^2 = R^2 + R^2 - 2R \cdot \cos\beta, \quad \text{where } \beta = \beta 1 + \beta 2$$

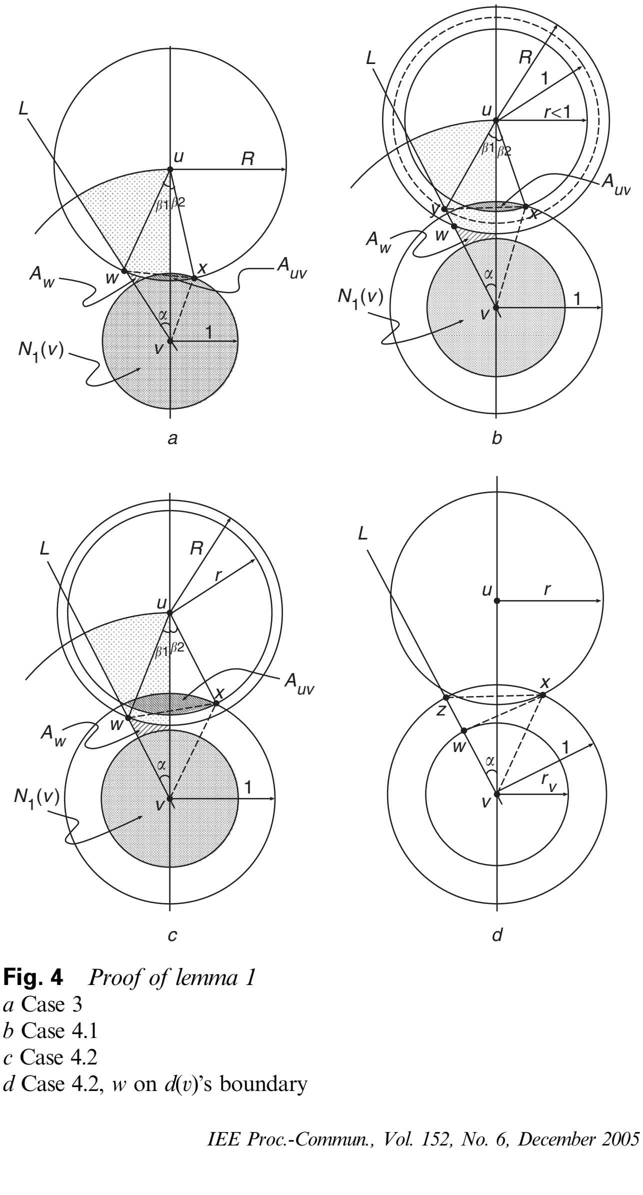

Fig. 4 *Proof of lemma 1*
a Case 3
b Case 4.1
c Case 4.2
d Case 4.2, w on $d(v)$'s boundary

so when R is fixed, $|wx|$ monotonically increases with β. Let $|uv| = d$. Since $\beta 1 + \alpha < 90°$, thus

$$\beta 1 = \arcsin\frac{d \sin\alpha}{R} - \alpha, \quad \beta 2 = \arccos\frac{R^2 + d^2 - 1}{2Rd}$$

$$\frac{d\beta}{dd} = \frac{\sin\alpha}{\sqrt{R^2 - d^2\sin^2\alpha}} - \frac{1 - \dfrac{R^2-1}{d^2}}{\sqrt{4R^2 - \left(\dfrac{R^2-1}{d} + d\right)^2}}$$

Let $A = \sin^2\alpha \cdot (4R^2 - (\frac{R^2-1}{d} + d)^2) - (R^2 - d^2\sin^2\alpha) \cdot (1 - \frac{R^2-1}{d^2})^2$. Since $\sin\alpha = 1/2$,

$$A = 1 - R^2\left(1 - \frac{R^2-1}{d^2}\right)^2 = \left(1 + R\left(1 - \frac{R^2-1}{d^2}\right)\right) \cdot \left(1 - R\left(1 - \frac{R^2-1}{d^2}\right)\right)$$

It is sufficient to show that $\frac{d\beta}{dd} < 0$. Since $R \geq 1$, so $A < 0$ if $(1 - R(1 - \frac{R^2-1}{d^2})) < 0$.

The restriction that w falls out of $d(v)$ implies $|wv| \geq 1$ and $d \geq \frac{\sqrt{3}}{2} + \sqrt{R^2 - \frac{1}{4}}$, as shown in Fig. 5*a*. Therefore, $d^2 > R(R+1)$, which leads to $(1 - R(1 - \frac{R^2-1}{d^2})) < 0$, $\frac{d\beta}{dd} < 0$. Thus $|wx|$ monotonically increases as d decreases. The maximum value of $|wx| = 1$ is achieved when $|wv| = 1$. So $|wx| \leq |vx|$ holds in case 3.

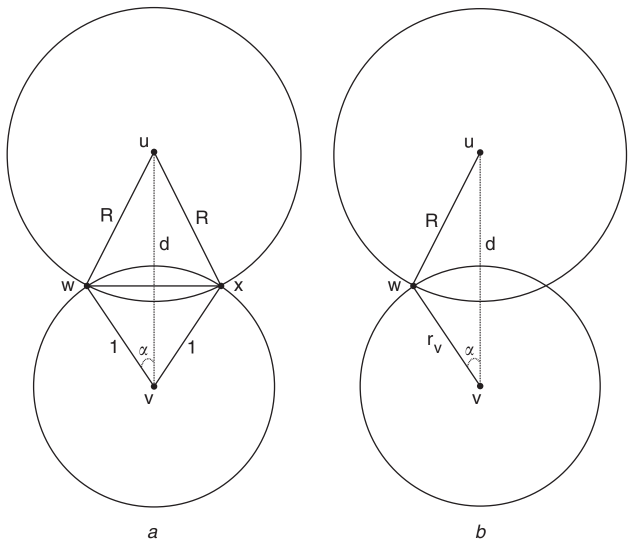

Fig. 5 *Minimum value of d*
a Case 3
b Case 4.2

Case 4: The radius of the outer disk $D(v)$ is normalised to 1. There are two possibilities, $r < 1$ or $r \geq 1$.

Case 4.1: as shown in Fig. 4*b*, $r < 1$, $R \geq 1$.

Consider the disk centred at u with radius 1 (shown as a dashed line); we call it $D_1(u)$. Assume the line L and the boundary circle of the disk $D_1(u)$ intersect at y. Node w is between y and v, therefore $|wx| \leq \max(|vx|, |yx|)$. If $|yx| \leq |vx|$, then $|wx| \leq |vx|$. Now we prove that $|yx|$ is indeed $\leq |vx|$.

$$|yx|^2 = r^2 + 1 - 2r \cdot \cos(\beta 1 + \beta 2)$$

So when r is fixed, $|yx|$ increases monotonically as $(\beta 1 + \beta 2)$ increases. Let $|uv| = d$. Since $\beta 1 + \alpha < 90°$, $|uy| = 1$, thus

$$\beta 1 = \arcsin(d \sin\alpha) - \alpha, \quad \beta 2 = \arccos\frac{r^2 + d^2 - 1}{2rd}$$

Let $\beta = \beta 1 + \beta 2$, so the derivative of β with respect to d is

$$\frac{d\beta}{dd} = \frac{\sin\alpha}{\sqrt{1 - d^2\sin^2\alpha}} - \frac{1 - \dfrac{r^2-1}{d^2}}{\sqrt{4r^2 - \left(\dfrac{r^2-1}{d} + d\right)^2}}$$

Let $A = (\sin^2\alpha) \cdot (4r^2 - (\frac{r^2-1}{d} + d)^2) - (1 - d^2\sin^2\alpha) \cdot (1 - \frac{r^2-1}{d^2})^2 = 1 - (1 - \frac{r^2-1}{d^2})^2$. Since $r < 1$, so $A < 0$. Thus $\frac{d\beta}{dd} < 0$. β is a monotonically decreasing function of d, so is $|yx|$. When $d \geq 1$, $|yx| \leq |vx|$. Therefore, $|wx| \leq |yx| \leq |vx|$.

A necessary condition for the above proof to hold is that the line L must intersect with $D_1(u)$. Since $d(u)$ overlaps with $D(v)$, $D_1(u)$ will always overlap with $D(v)$, so the 30° degree line L will always intersect with $D_1(u)$ at y.

Case 4.2: as shown in Fig. 4*c*, $R \geq r \geq 1$.

$$|wx|^2 = R^2 + r^2 - 2Rr \cdot \cos(\beta 1 + \beta 2)$$

$$\beta 1 = \arcsin\frac{d \sin\alpha}{R} - \alpha, \quad \beta 2 = \arccos\frac{r^2 + d^2 - 1}{2rd},$$

$$\beta = \beta 1 + \beta 2$$

$$\frac{d\beta}{dd} = \frac{\sin\alpha}{\sqrt{R^2 - d^2\sin^2\alpha}} - \frac{1 - \dfrac{r^2-1}{d^2}}{\sqrt{4r^2 - \left(\dfrac{r^2-1}{d} + d\right)^2}}$$

Let $A = \sin^2\alpha \cdot (4r^2 - (\frac{r^2-1}{d} + d)^2) - (R^2 - d^2\sin^2\alpha) \cdot (1 - \frac{r^2-1}{d^2})^2$. Since $1 \leq r \leq R \leq d$, so $A < 0$ if $1 - R(1 - \frac{r^2-1}{d^2}) < 0$.

Let r_v be the radius of $d(v)$. With the restriction that w falls outside of $d(v)$, we get $|wv| \geq r_v$, thus $d \geq \frac{\sqrt{3}r_v + \sqrt{4R^2 - r_v^2}}{2}$, as shown in Fig. 5*b*. Since $R = cr$, $r_v = \frac{1}{c}$, so $d \geq \frac{1}{2c}(\sqrt{3} + \sqrt{4c^4r^2 - 1})$.

$$d^2 \geq \frac{1}{2c^2} + c^2r^2 + \sqrt{3}\sqrt{r^2 - \frac{1}{4c^4}} > r^2 + \sqrt{3}\sqrt{r^2 - \frac{1}{4}} > r^2 + r$$

$$1 - R\left(1 - \frac{r^2-1}{d^2}\right) \leq 1 - r\left(1 - \frac{r^2-1}{d^2}\right) = (r-1)\left(\frac{r(r+1)}{d^2} - 1\right) < 0$$

Therefore $A < 0$, β and $|wx|$ monotonically decreases as d increases. $|wx|$ achieves its maximum value $|wx|_{\max}$ when $d = \frac{\sqrt{3}r_v + \sqrt{4R^2 - r_v^2}}{2}$ (i.e. w is on the boundary circle of $d(v)$). Let z be the intersection of the line L and the boundary circle of $D(v)$, so $|zx| = |vx|$. When d and r are fixed, w approaches v when c increases, and approaches z as c decreases (see Fig. 4*d*), therefore $|wx|_{\max} \leq |vx|$ always holds.

Hence, all neighbours of v in the same region form a clique with v. Therefore, v has degree of at most $(13+1) \cdot (\omega(G) - 1) = 14 \cdot (\omega(G) - 1)$.

Next, we show that we can get a tighter bound by the following improvement: we divide $N_2(v)$ in such a way that some vertex appears on the boundary of two sectors. This implies that the total number of vertices in v's neighbourhood is $14(\omega(G) - 1) - 1$. This completes the proof of lemma 1. ■

Note that the subgraph of an FDD graph is still an FDD graph. Therefore, during the implementation of heuristic 1, at each iteration of step 1, every chosen vertex has degree at most $14(\omega(G') - 1) - 1$ in the remaining graph G''. Since G'' is a subgraph of G, $\omega(G') \leq \omega(G)$. It follows that $14(\omega(G) - 1)$ colours are enough to use in step 2. Since $\omega(G)$ is a lower bound for optimal solution of vertex colouring, the greedy algorithm has performance ratio 14. This completes the proof of theorem 2.