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LOCAL BUCKLING IN CURVED BOX MEMBERS

by

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ABSTRACT

In curved structural members, the flanges of all horizontally curved members and the webs of all vertically curved members are composed of flat plate elements which, for the purpose of analysis and design, can be treated as annular sector plates.

In this study, the classical linear elastic theory of bukcling of sector plates is employed to determine the buckling behavior of the flat plate elements in horizontally and vertically curved box members. The critical buckling stress is computed from theoretical formulas by a semi-analytical semi-numerical method of solution. The procedure employs the classical method of separation of variables with Fourier series and finite difference techniques. Graphical and tabular results are presented for the buckling coefficients of a number of annular sectors under the action of normal stresses.

INTRODUCTION

The problem of local buckling in curved structural members is of both theoretical interest and practical importance. In engineering structures, such as: buildings, curved bridges, hydraulic structures, containers, airplanes, ships, steam turbine diaphragms, etc., the use of curved structural

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members is common due to architectural and structural limitations or due to economic and aesthetic advantages. Because of their widespread usage, it is essential to understand the local buckling problem of such members. However, the area of local buckling in curved structural members has received very little attention.

Local buckling of straight structural members is generally well understood. Extensive investigations have been carried out in both the elastic and inelastic ranges (Refs. 1, 10, 13, 20). Based on this work, design specifications (Refs. 17, 18, 19) limit the width to thickness ratios of the plate elements of structural members. These limitations, which are derived from the buckling analysis of rectangular plate elements, were established in order to insure that local buckling will not occur prior to overall failure of the member or prior to yield of the plate elements.

In curved structural members, the flanges of all horizontally curved members and the webs of all vertically curved members are composed of flat plate elements which, for the purpose of analysis and design, can be treated as annular sector plates (Fig. 1).

When a horizontally curved member is loaded normal to the plane of curvature it bends and twists. In addition to the bending moment, cross bending of the flanges resulting from torsion also occurs. The equal and opposite flange moments, due to this cross bending, also produce normal stresses in the girder flanges. The combined stress state produced by these two effects is shown in Fig. 2. Due to the curvature of the flanges, radial stresses (σ_r) are also developed in the flanges (Fig. 3). Shear stresses ($\tau_{r\theta}$) are also developed at the web-flange juncture.

In vertically curved girders loaded in the plane of curvature and restrained from movement in the direction normal to the plane of curvature, internal forces due only to bending are produced. In addition to the normal stresses (σ_θ) developed in the girder web, radial stresses are also present due to the curvature of the web. Shear stresses are also developed at the web-flange juncture.

The existing studies of the stability problems associated with curved members were primarily concerned with buckling of the member as a whole (Refs. 2, 14, 20). The local buckling problem was addressed by Mikami et al. (Refs. 14, 15) for vertically curved I-girders, and by Culver et al. (Refs. 4, 5) in the case of horizontally curved girders. The results of Culver's studies indicated that in horizontally curved bridges (where curvatures are small), curvature has little effect on local buckling. Based on Culver's findings, the "Task Committee on Curved Girders of the ASCE-AASHTO Committee on Flexural Members" (Ref. 6) recommended to adopt for curved girders the flange width to thickness requirements presently used for straight girders (Refs. 17, 18, 19). This recommendation was made for designs based on the Working Stress Method.

Apart from different types of loads and load related phenomena (e.g. fatigue), the basic stability criteria does not vary from bridges to buildings or other engineering structures. A review of the current literature and the current design codes indicates that, except for the work by Culver and Mikami, no information exists on the local buckling behavior of the flat plate elements of curved structural members.

The objective of this study is to investigate the local buckling of the compression flanges of horizontally curved box members and the webs of vertically curved box members. A simple method, which is semi-analytical in nature, is presented for the solution of the elastic stability problem. The method has been successfully applied to the static and dynamic analysis of sector plates (Refs. 9, 16), and to the buckling problem of rectangular and simply supported sector plates (Refs. 11, 12).

ANALYSIS

Consider an annular sector plate subjected to the action of in-plane forces (Fig. 4). Within the classical small deflection theory of thin plates, the governing differential equation in polar coordinates of a sector element is expressed as (Refs. 1, 4, 20)

$$\frac{\partial^4 W(r,\theta)}{\partial r^4} + \frac{2}{r}\frac{\partial^3 W(r,\theta)}{\partial r^3} - \frac{1}{r^2}\frac{\partial^2 W(r,\theta)}{\partial r^2} + \frac{2}{r^2}\frac{\partial^4 W(r,\theta)}{\partial r^2 \partial\theta^2}$$

$$+ \frac{1}{r^3}\frac{\partial W(r,\theta)}{\partial r} - \frac{2}{r^3}\frac{\partial^3 W(r,\theta)}{\partial r\partial\theta^2} + \frac{4}{r^4}\frac{\partial^2 W(r,\theta)}{\partial\theta^2} + \frac{1}{r^4}\frac{\partial^4 W(r,\theta)}{\partial\theta^4}$$

$$= \frac{t}{D}\ \{\ \sigma_r \frac{\partial^2 W(r,\theta)}{\partial r^2} + \sigma_\theta [\ \frac{1}{r}\frac{\partial W(r,\theta)}{\partial r} + \frac{1}{r^2}\frac{\partial^2 W(r,\theta)}{\partial\theta^2}\ ]$$

$$+ 2\tau_{r\theta} [\ \frac{1}{r}\frac{\partial^2 W(r,\theta)}{\partial r\partial\theta} - \frac{1}{r^2}\frac{\partial W(r,\theta)}{\partial\theta}\ ] \qquad (1)$$

where W(r, θ) is the normal deflection of the plate, σ_r, σ_θ and $\tau_{r\theta}$ are the radial, normal and shear stresses respectively, t is the thickness and D is the elastic rigidity of the plate. Eq. (1) is a partial differential equation with variable coefficients and, in general, the stress components are functions of r and θ. A closed form solution would be difficult if not impossible to obtain. The use of approximate methods is necessary.

The deflection of the sector is assumed in the following form

$$W(r,\ ) = \sum_{m=1}^{\infty} F_m(r) \sin\frac{m\pi\theta}{\gamma} \qquad (2)$$

where $F_m(r)$ is the function in the radial direction and sin $(m\pi\theta/\gamma)$ is the basic function in the angular direction that satisfies the boundary conditions at $\theta = 0$ and $\theta = \gamma$. Substituting Eq. (2) into Eq. (1) and disregarding the radial and shear stresses, then for the m^{th} buckling mode Eq. (1) reduces to

$$\frac{d^4F_m(r)}{dr^4} + \frac{2}{r}\frac{d^3F_m(r)}{dr^3} - (1 + 2\eta_m^2)\frac{1}{r^2}\frac{d^2F_m(r)}{dr^2}$$

$$+ (1 + 2\eta_m^2)\frac{1}{r^3}\frac{dF_m(r)}{dr} + (\eta_m^4 - 4\eta_m^2)\frac{1}{r^4}F_m(r)$$

$$= \frac{\sigma_\theta t}{D}\left[\frac{1}{r}\frac{dF_m(r)}{dr} - \frac{\eta_m^2}{r}F_m(r)\right] \tag{3}$$

in which, $$\eta_m = \frac{m\pi}{\gamma} \tag{4}$$

The finite difference form of Eq. (3) applied at i^{th} point in the deck is given by (Fig. 5)

$$\frac{D}{h}\,(C_{m,i+2}F_{m,i+2} + C_{m,i+1}F_{m,i+1} + C_{m,i}F_{m,i}$$

$$+ C_{m,i-1}F_{m,i-1} + C_{m,i-2}F_{m,i-2})$$

$$= -N_\theta\left(\frac{\phi_i}{2}F_{m,i+1} - \eta_m^2\phi_iF_{m,i} - \frac{\phi_i}{2}F_{m,i-1}\right) \tag{5}$$

in which, $$N_\theta = \frac{k\pi^2D}{(R_o-R_i)^2} \tag{6}$$

$$\phi_i = \frac{h}{r_i} \tag{7}$$

and, $$C_{m,i+2} = 1 + \phi_i$$

$$C_{m,i+1} = -(4+2\phi_i) - (1+2\eta_m^2)(\phi_i^2-\phi_i^3/2) \tag{9}$$

$$C_{m,i} = 6 + 2\phi_i^2(1+2\eta_m^2) + \phi_i^4(\eta_m^4-4\eta_m^2) \tag{10}$$

$$C_{m,i-1} = -(4+2\phi_i) - (1+2\eta_m^2)(\phi_i^2+\phi_i^3/2) \tag{11}$$

$$C_{m,i-2} = 1 - \phi_i \tag{12}$$

BOUNDARY CONDITIONS

In horizontally curved box sections, the boundary conditions for the inner and outer curved edges of the sector depend upon the rotational stiffness of the web. For the condition of zero rotational restraint at the flange-web juncture, the flange can be treated as simply supported, while for complete rotational restraint, the flange is treated as fixed. In vertically cruved girders, the boundary conditions will depend on the rotational stiffness of the flanges.

For simply supported circular edges, the boundary conditons are

$$W(r,\theta) = 0 \qquad \text{and} \qquad M_r(r,\theta) = -D\left[\frac{\partial^2 W}{\partial r^2} + \left(\frac{1}{r}\frac{\partial W}{\partial r} + \frac{1}{r^2}\frac{\partial^2 W}{\partial \theta^2}\right)\right] \tag{13}$$

For fixed edges, the boundary conditions are

$$W(r,\theta) = 0 \qquad \text{and} \qquad \frac{\partial W}{\partial r} = 0 \tag{14}$$

APPLICATION

A FORTRAN computer program was written to solve for the buckling coefficient of annular sector plates with simply supported straight edges and any combination of clamped or simply supported circular edges.

The following non-dimensional parameters were varied to determine their influence on the buckling coefficient.

1. the plate curvature ratio: b/R_a
2. the plate aspect ratio: $\gamma R_a/b$
3. the stress variation coefficient: $= (\sigma_\theta \text{ at } R_i)/(\sigma_\theta \text{ at } R_o)$

The accuracy of the method was established by considering the limiting case of a rectangular plate. The plate curvature ratio was decreased and a value of $b/R_a = 0.001$ was assumed to represent the case of a rectangular plate. The results for this case were identical to those found in the literature (Refs. 1, 10, 20).

The rate of convergence of the present method depends on the plate curvature ratio, the grid spacing and the buckling mode. In general, it was found that seventeen grid points were required for accurate results.

Figures 6-9 show the variation of the buckling coefficient with the plate aspect ratio for $b/R_a = 0.1$ and for different buckling modes and loading conditions. Each figure represents a plate with a particular combination of boundary conditions on the inner and outer circular edges.

Table 1 presents the minimum buckling coefficient of the first three modes for different loading and edge conditions.

The influence of the plate curvature ratio on the buckling coefficient was investigated by varying b/R_a from 0.001 to 0.5. It was found that the maximum difference in the magnitude of the minimum buckling coefficient was in the order of 0.3%.

SUMMARY AND CONCLUSIONS

In this paper, a semi-analytical method was employed to determine the buckling coefficient of the flat plate elements in horizontally and vertically curved box members. The plate elements were treated as annular sector plates with simply supported straight edges and clamped and/or simply supported circular edges. The radial and shear stresses were assumed to be negligible and the angular (or normal) stresses were assumed to vary linearly along the straight edges.

The results indicated that the curvature of the plate has very little effect on the buckling of the flat plate elements in curved box members. In fact, the difference in the magnitude of the buckling coefficient of a rectangular plate and that of curved plate was less than one percent for any curvature ratio (b/R_a) between and 0.001 and 0.5.

NOTATION

D	Flexural rigidity of the plate
$F_m(r)$	Radial function for mode m
R_a	Average radius of sector plate
R_i	Inner radius of sector plate
R_o	Outer radius of sector plate
$W(r,\theta)$	Plate deflection
h	Spacing between nodal points
k	Buckling Coefficient
m	Buckling mode
r	Radial position
t	Plate thickness

γ Opening angle
θ Angular position
ξ Stress variation ratio
σ_r Radial stresses
σ_θ Angular or normal stresses
$\tau_{r\theta}$ Shear stresses

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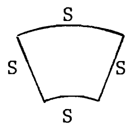
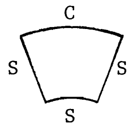
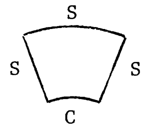
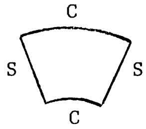
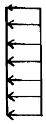
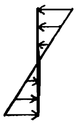
Table 1. Minimum Buckling Coefficient K_{min} ($b/R_a = 0.1$)

Stress Distribution	Buckling mode	 S S S S		 C S S S		 S S S C		 C S S C	
		$\frac{\gamma R_a}{b}$	K_{min}	$\frac{\gamma R_a}{b}$	K_{min}	$\frac{\gamma R_a}{b}$	K_{min}	$\frac{\gamma R_a}{b}$	K_{min}
	1	1.00	3.99	0.80	5.35	0.80	5.24	0.67	6.84
	2	2.00	3.99	1.60	5.35	1.60	5.25	1.33	6.85
	3	3.00	3.99	2.40	5.36	2.40	5.25	2.00	6.85
	1	1.00	7.75	0.80	11.59	0.80	9.27	0.65	13.32
	2	2.00	7.75	1.60	11.58	1.70	9.30	1.30	13.31
	3	3.00	7.75	2.40	11.63	2.40	9.26	2.00	13.32
	1	0.67	23.49	0.50	38.72	0.60	23.75	0.47	38.56
	2	1.35	23.49	0.90	38.60	1.20	23.75	1.00	38.71
	3	2.00	23.49	1.40	38.56	1.80	23.75	1.40	38.57

S = Simply supported
C = Clamped

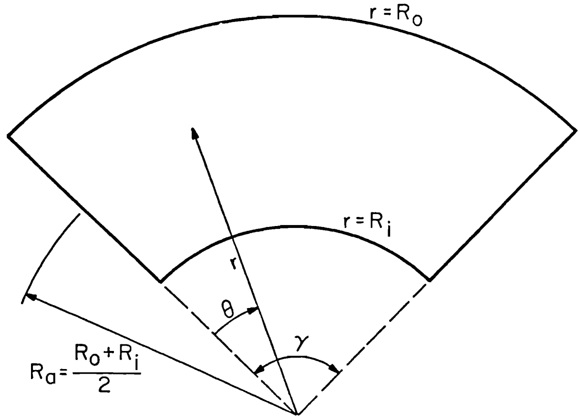


Fig. 1. Annular Sector Subjected to In-plane Loads

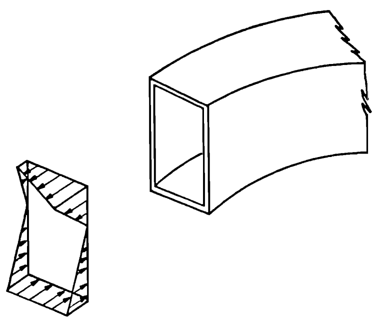


Fig. 2. Elastic Stresses

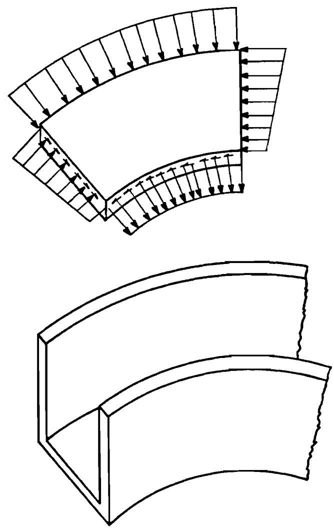


Fig. 3. Free Body of Top Flange

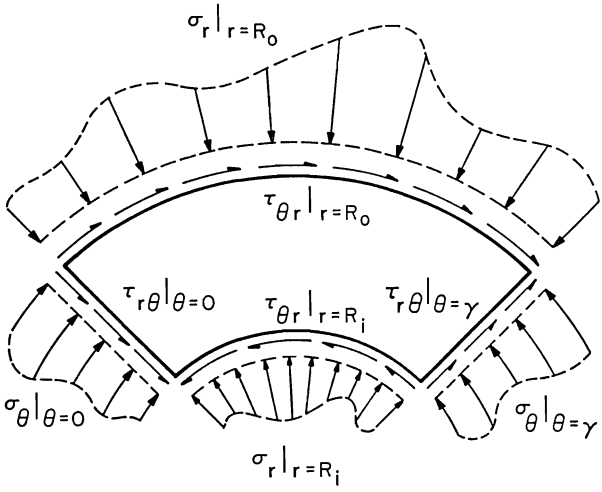


Fig. 4. Annular Sector Plate Subjected to Edge Forces

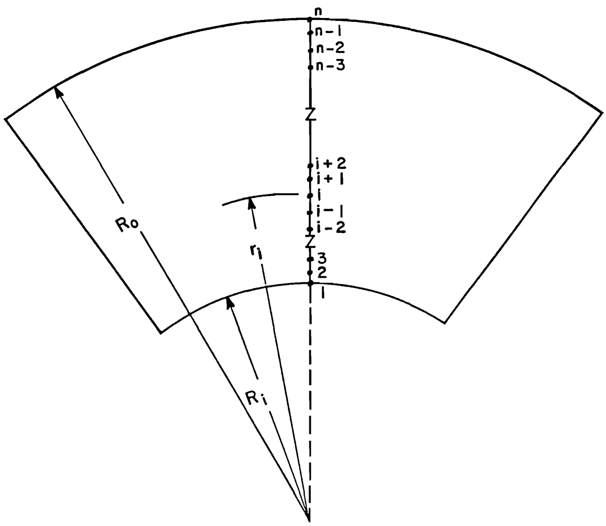


Fig. 5. Nodal Points in the Radial Direction

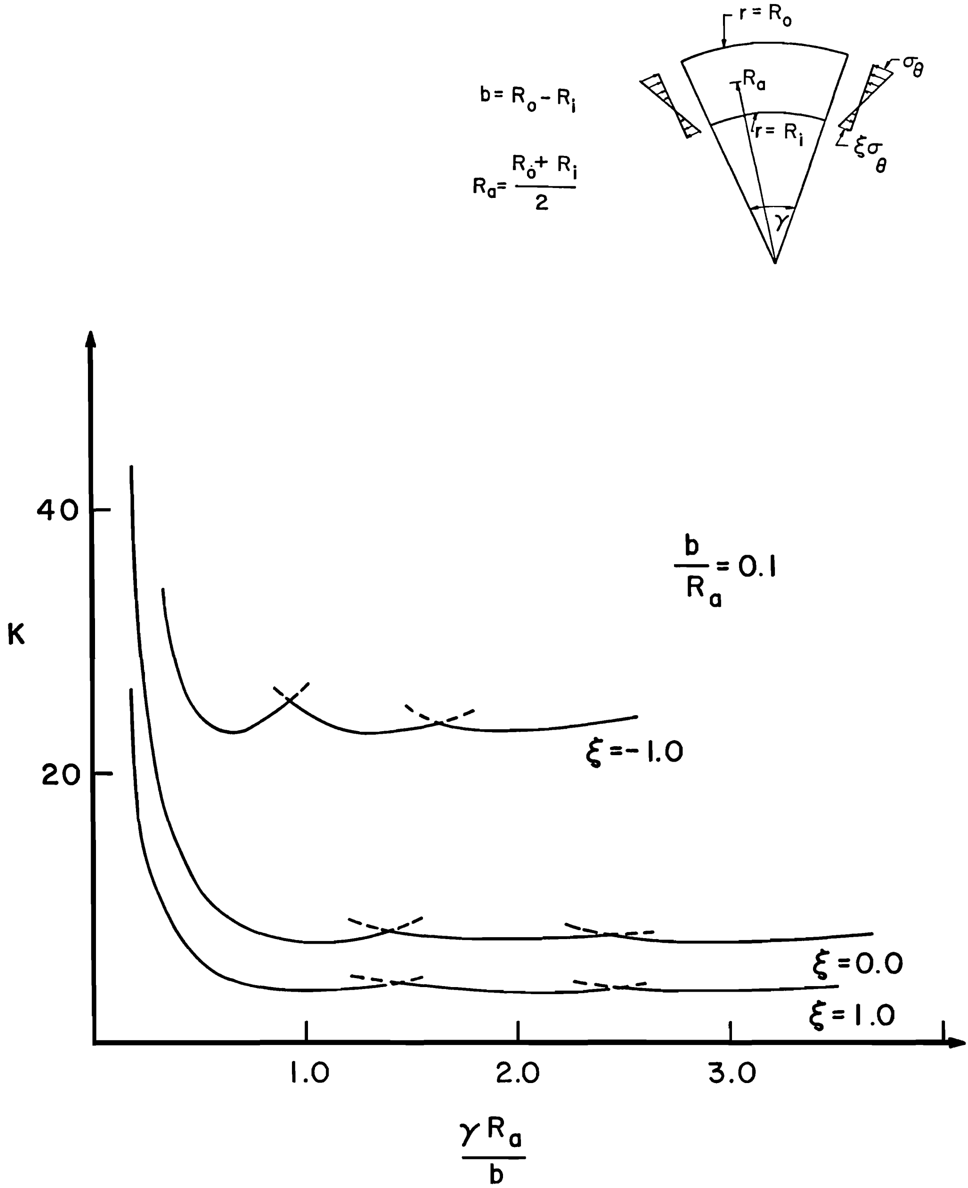


Fig. 6. Buckling Coefficient of a Simply Supported Sector

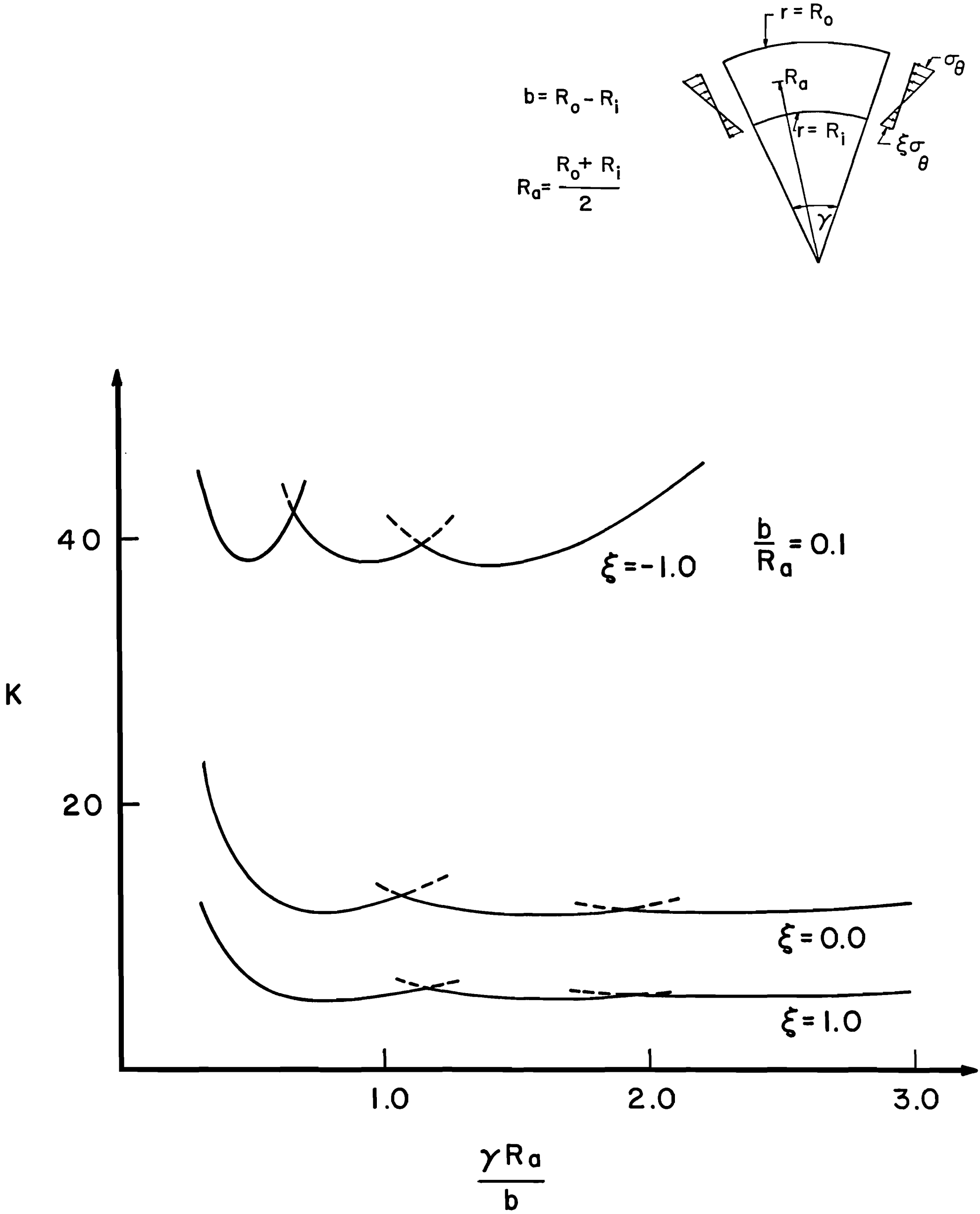
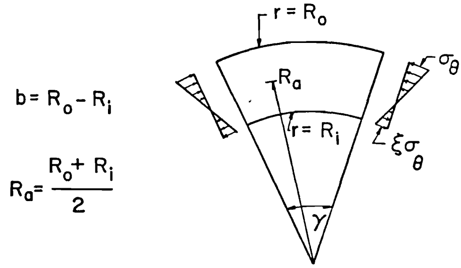


Fig. 7. Buckling Coefficient of a Sector With Outer Edge Clamped and all Other Edges Simply Supported



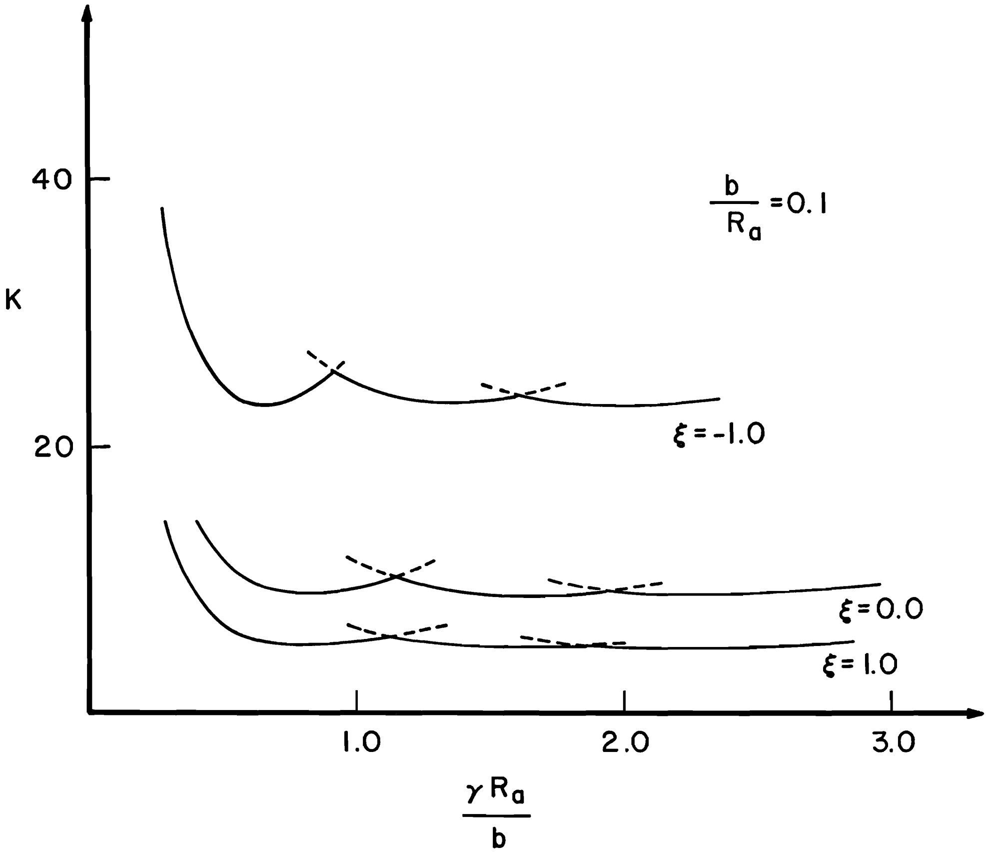


Fig. 8. Buckling Coefficient of a Sector With Inner Edge Clamped and all Other Edges Simply Supported

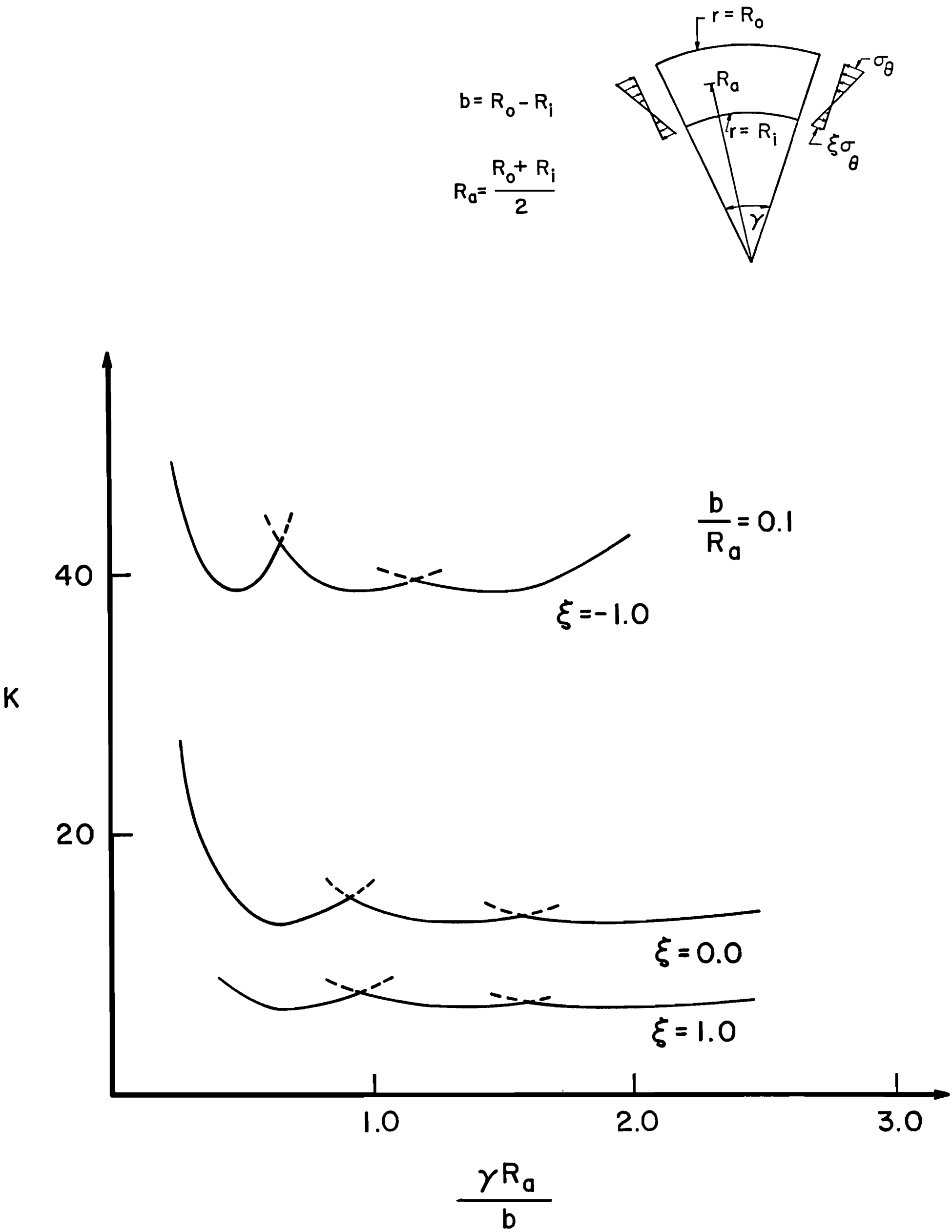


Fig. 9. Buckling Coefficient of a Sector With Clamped Circular Edges and Simply Supported Straight Edges