1922

Applications of method of least squares to the adjustment of level circuits: Adjustment of elevations in Dawson and St. James Townships, Phelps County, Missouri

Clarence Edward Bardsley

Department:

APPLICATIONS OF THE METHOD OF LEAST SQUARES TO THE ADJUSTMENT OF LEVEL CIRCUITS.

ADJUSTMENT OF ELEVATIONS IN DAWSON
AND ST. JAMES TOWNSHIPS,
PHELPS COUNTY,
MISSOURI.

BY

CLARENCE EDWARD SOLOMAN BARDSLEY

A

T H E S I S

submitted to the faculty of the
SCHOOL OF MINES AND METALLURGY OF THE UNIVERSITY OF MISSOURI
in partial fulfillment of the work required for the
D E G R E E O F
CIVIL ENGINEER
Rolla, Mo.
1922.

ASSISTED BY

CHARLES EDWARD COOKE

Approved by _____
Professor of Civil Engineering.

APPLICATIONS OF THE METHOD OF LEAST SQUARES TO THE ADJUSTMENT OF LEVEL CIRCUITS.

ADJUSTMENT OF ELEVATIONS IN DAWSON AND ST. JAMES TOWNSHIPS, PHELPS COUNTY, MISSOURI.

C O N T E N T S .

INTRODUCTION.

P A R T I .

THE INDEPENDENT ADJUSTMENT OF A MULTIPLE LINE.

METHOD OF LEAST SQUARES APPLIED TO THE ADJUSTMENT OF THE LINE A VIA B, C, AND D TO E.

BASED ON THE LINE OF LEVELS RUN BY THE U. S. GEOLOGICAL SURVEY IN 1908 ALONG THE FRISCO R. R. WITH PERMANENT BENCHMARKS, "A" AT ST. JAMES, AND "E" AT KNOBVIEW.

C O N T E N T S .

P A R T I I .

THE LEAST SQUARE ADJUSTMENT OF A LEVEL NET BY METHOD OF OBSERVATION EQUARIONS.

APPLIED TO THE ADJUSTMENT
OF THE
DAWSON-ST. JAMES TOWNSHIPS LEVEL NET,
PHELPS COUNTY,
MISSOURI.
BASED ON THE A VIA B, C, AND D TO E BASE.

C O N T E N T S .

P A R T I I I .

THE LEAST SQUARE ADJUSTMENT OF A LEVEL NET
BY METHOD OF OBSERVATION EQUATIONS.
APPLIED TO THE ADJUSTMENT
OF THE
DAWSON-ST. JAMES TOWNSHIPS LEVEL NET,
PHELPS COUNTY,
MISSOURI.
BASED ON THE LINE OF LEVELS RUN BY THE
U. S. GEOLOGICAL SURVEY IN 1908 ALONG THE FRISCO R. R.
WITH PERMANENT BENCHMARKS, "A" AT ST. JAMES, AND
"E" AT KNOBVIEW.

C O N T E N T S .

P A R T I V .

THE LEAST SQUARE ADJUSTMENT OF A LEVEL NET
BY METHOD OF CONDITION EQUATIONS
AND
CORRELATIVES.

APPLIED TO THE ADJUSTMENT
OF THE
DAWSON-ST. JAMES TOWNSHIPS LEVEL NET,
PHELPS COUNTY,
MISSOURI.

BASED ON THE LINE OF LEVELS RUN BY THE
U. S. GEOLOGICAL SURVEY IN 1908 ALONG THE FRISCO R. R.
WITH PERMANENT BENCHMARKS "A" AT ST. JAMES,
AND "E" AT KNOBVIEW.

APPLICATIONS OF THE METHOD OF LEAST SQUARES TO THE ADJUSTMENT OF LEVEL CIRCUITS.

ADJUSTMENT OF ELEVATIONS IN DAWSON AND ST. JAMES TOWNSHIPS, PHELPS COUNTY, MISSOURI.

INTRODUCTION.

An introduction to a thesis on least squares could be drawn out to considerable length in explanation: as the purpose of this thesis has been an investigation in applications of the method of least squares to the adjustment of level circuits,it will be found in the body of the report considerable space has been devoted to the explanation of the different applications of the method,with a view that these investigations may be of use to engineers in the profession; so the introductory remarks will be brief.

It is well known among engineers and others that measurements made with apparatus of most any kind,chains,rods,scales,verniers, micrometers,etc., and no matter how careful one may be in making measurements,there will be discrepancies among the observations. If a series of benchmarks be connected by lines of levels,some of which are check lines forming with the others complete circuits, it is necessary to adjust the difference in elevation so that all the circuits will close exactly,in order that the difference of elevation between any two benchmarks will be constant when computed through two or more series of lines by the several routes. This is illustrated by several applications of the method of least squares to the adjustment of elevations in Dawson and St. James Townships,Phelps County, Missouri.

It is understood that observations are always made as carefully as possible,for the most probable results and minimum errors. Positive and negative errors of the same magnitude occur with equal frequency; they are equally probable.
Errors of increasing magnitude occur with decreasing frequency.
Small errors occur more frequently;are more probable that large ones.
Very large errors seldom occur;they are likely to belong in the class of mistakes rather in that of accidental errors.
Accidental errors are systematically modified by the circumstances of the observations. The chief circumstances affecting a set of observations are atmospheric conditions,the skill of the observer, and the precision of the instruments.

C O N T E N T S .

P A R T V .

PROPORTIONAL ADJUSTMENT OF INTERMEDIATE BENCHMARKS FROM CORRECTED ELEVATIONS OF ADJACENT JUNCTION POINTS.

APPLIED TO THE ADJUSTMENT OF INTERMEDIATE BENCHMARKS IN THE DAWSON-ST. JAMES TOWNSHIPS LEVEL NET, PHELPS COUNTY, MISSOURI.

BASED ON THE LINE OF LEVELS RUN BY THE U. S. GEOLOGICAL SURVEY IN 1908 ALONG THE FRISCO R. R. WITH PERMANENT BENCHMARKS "A" AT ST. JAMES, AND "E" AT KNOBVIEW.

C O N T E N T S .

C O N C L U S I O N .

B I B L I O G R A P̄ H Y .

I N D E X .

APPLICATIONS OF THE METHOD OF LEAST SQUARES
TO THE ADJUSTMENT OF LEVEL CIRCUITS.
ADJUSTMENT OF ELEVATIONS IN DAWSON
AND ST. JAMES TOWNSHIPS,
PHELPS COUNTY,
MISSOURI.

I L L U S T R A T I O N S .

ILLUSTRATIONS.

The principle from which the term "*least squares*" arises is the following: *In measurements of equal precision, the most probable values of observed quantities are those that render the sum of the squares of the residual errors a minimum.* The errors that may be corrected, however, by this method, are only those accidental errors that are governed by no laws other than those of chance.

The method of least squares takes first rank as an arithmetical process and by its use uniformity is secured in the process of adjustment and comparison, and the most probable balancing of errors of observations may be expected.

It is believed that the method of least squares is not in general use among engineers and that approximate methods are more often substituted for the adjustment of quantities, although far superior results might be expected through the employment of the more exact method. There are those, no doubt, who although versed in mathematical science, might find it difficult to utilize this method, for lack of concentration and patience to carry this process through to completion.

It is recommended, however, whenever precision is demanded and subsequent measurements are dependent upon the results.

In the adjustment of level circuits, the several lines are weighted inversely as their lengths, this causes us to state the above principle as follows: *In observations where unequal weights are applied, the most probable values of the observed quantities are those that render the sum of the weighted squares of the residual errors a minimum.*

The levels in Dawson and St. James Townships were run under my supervision by messrs. E. E. Decker, W. R. Denison, and F. B. Hollow.

In the arrangement of this thesis, and in the computations and construction of drawings, I wish to acknowledge the valuable assistance rendered by Major Charles E. Cooke, Professor of Topographic Engineering.

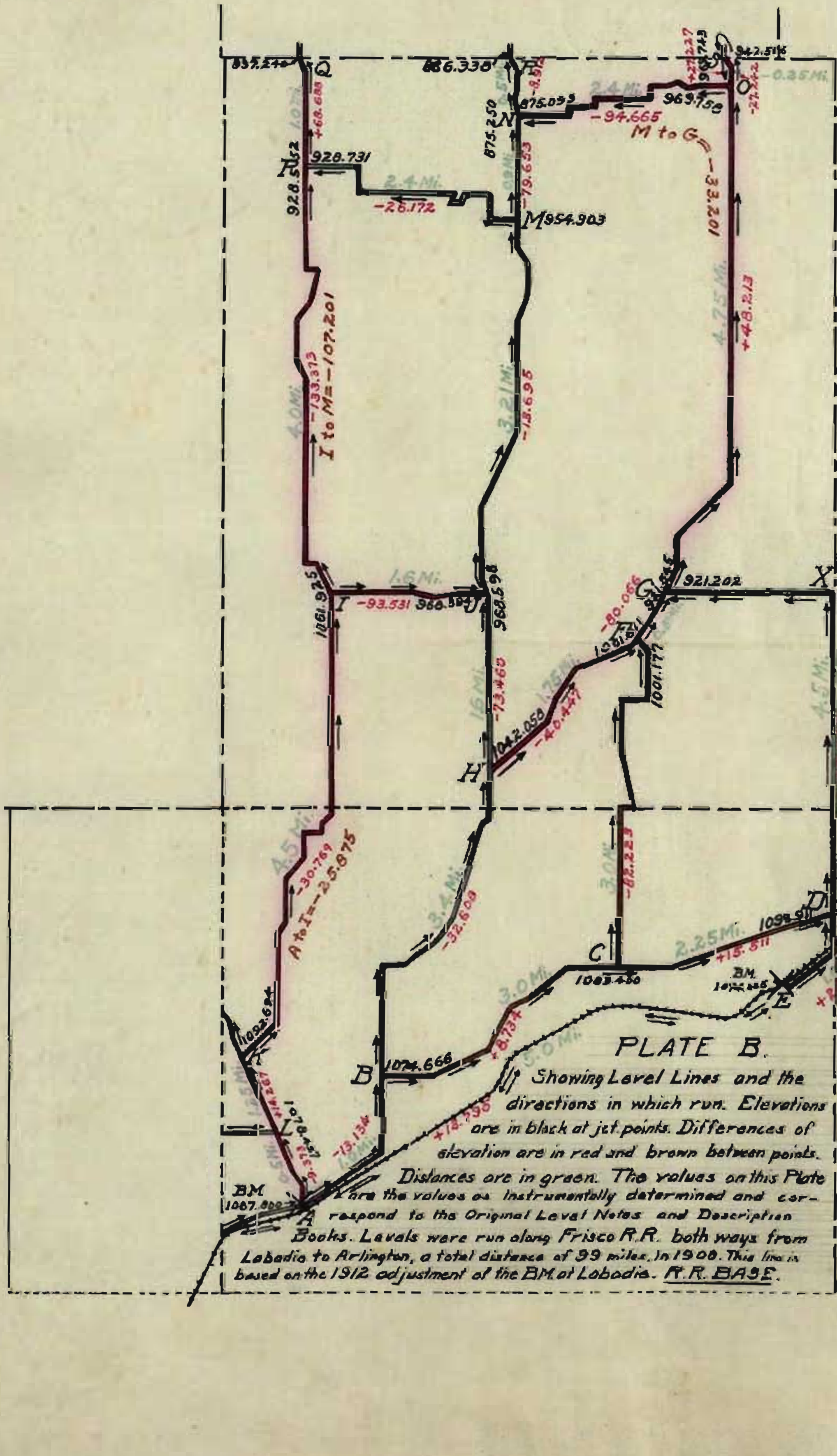
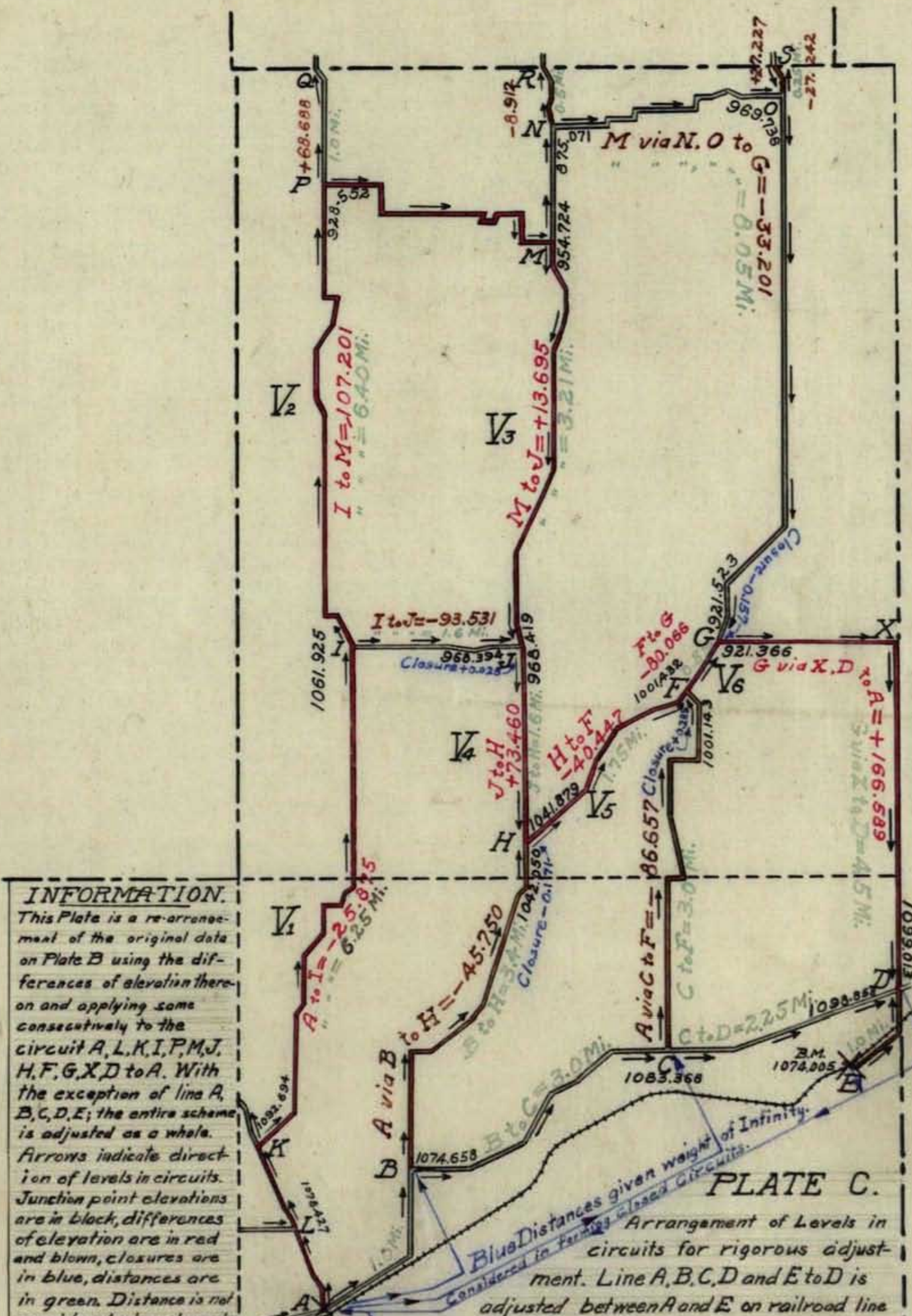


PLATE B.

Showing Level Lines and the directions in which run. Elevations are in black at jct. points. Differences of elevation are in red and brown between points. Distances are in green. The values on this Plate are the values as instrumentally determined and correspond to the Original Level Notes and Description Books. Levels were run along Frisco R.R. both ways from Labadie to Arlington, a total distance of 99 miles. In 1908. This line is based on the 1912 adjustment of the BM at Labadie. R.R. BASE.



INFORMATION.

This Plate is a re-arrangement of the original data on Plate B using the differences of elevation thereon and applying same consecutively to the circuit A, L, K, I, P, M, J, H, F, G, X, D to A. With the exception of line A, B, C, D, E; the entire scheme is adjusted as a whole. Arrows indicate direction of levels in circuits. Junction point elevations are in block, differences of elevation are in red and blown, closures are in blue, distances are in green. Distance is not considered when shown in blue. Lines PQ, NR, and OSO are considered lost in the adjustment scheme.

PLATE C.

Arrangement of Levels in circuits for rigorous adjustment. Line A, B, C, D and E to D is adjusted between A and E on railroad line of levels, first; and the whole scheme to the north is hinged on A, B, C, D as a Base.

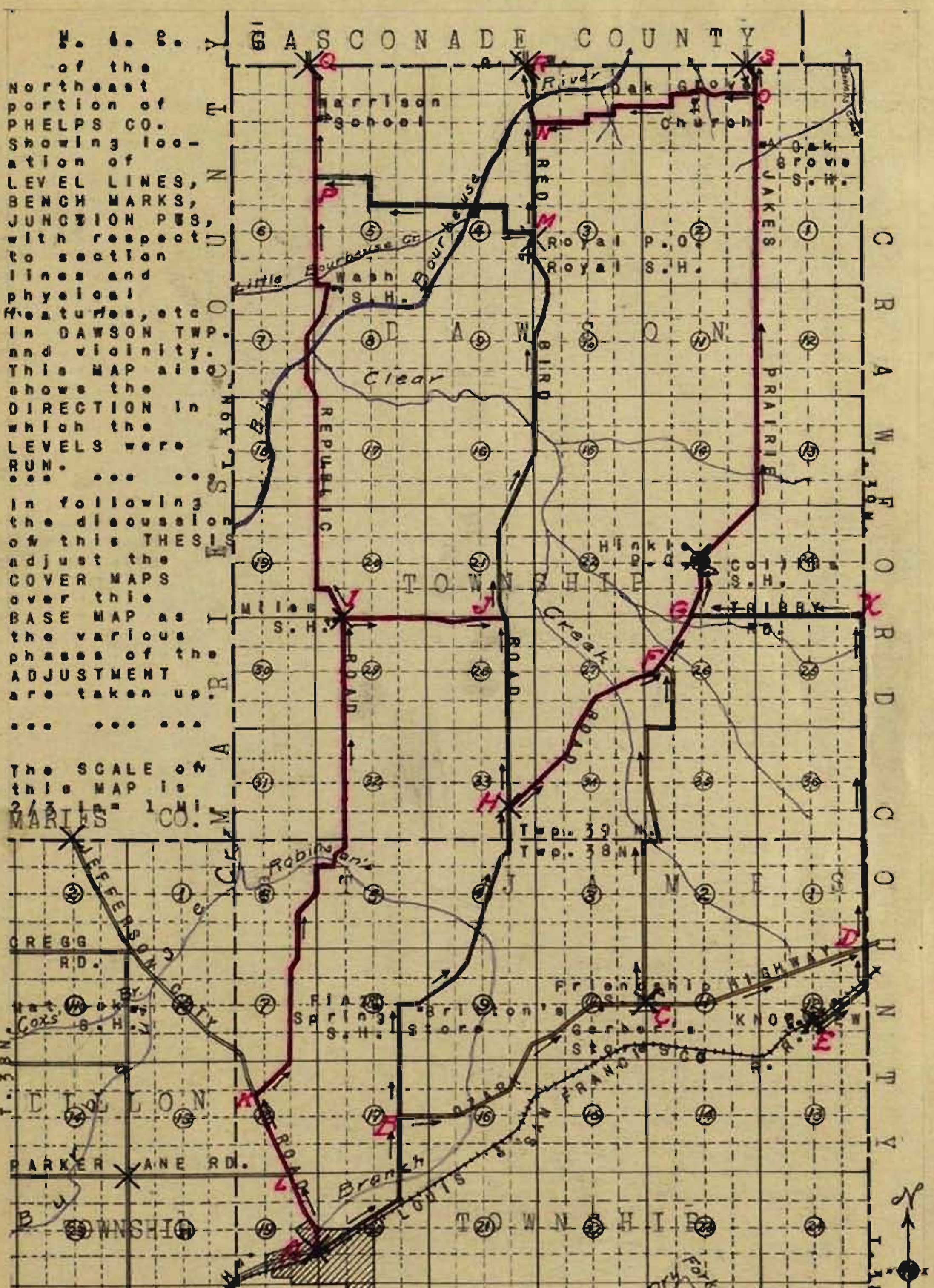

M. A. C.
of the
Northeast
portion of
PHELPS CO.
Showing loc-
ation of
LEVEL LINES,
BENCH MARKS,
JUNCTION PTS,
with respect
to section
lines and
physical
features, etc
in DAWSON TWP.
and vicinity.
This MAP also
shows the
DIRECTION in
which the
LEVELS were
RUN.
In following
the discussion
of this THESIS
adjust the
COVER MAPS
over this
BASE MAP as
the various
phases of the
ADJUSTMENT
are taken up.
The SCALE of
this MAP is
2/3 in = 1 Mi
GASCONADE COUNTY
CRAWFORD COUNTY
MARIES CO.
MARIES COUNTY
DAWSON
TOWNSHIP
JAMES
TOWNSHIP
DILLON
Harrison School
Oak Grove Church
Oak Grove S.H.
Royal P.O.
Royal S.H.
Wash S.H.
Little Bourbeuse Cr.
Bourbeuse
River
RED BIRD
JAKES PRAIRIE
REPUBLIC
Clear
Creek
Hinkle P.O.
Collins S.H.
TRIBBY RD.
Miles S.H.
ROAD
Twp. 39 N.
Twp. 38 N.
Robinson's
GREGG RD.
JEFFERSON CITY RD.
Coxs' S.H.
Friendship S.H.
Brixton's Store
Gerber's Store
HIGHWAY
KNOBVIEW
ST. LOUIS & SAN FRANCISCO R.R.
Branch
PARKER LANE RD.
Forest Grove S.H.
ST. JAMES
Big Dry Fork
Sawyer School
R. 6 W.
T. 38 N.
N

PLATE A.

P A R T I .

THE INDEPENDENT ADJUSTMENT OF A MULTIPLE LINE.

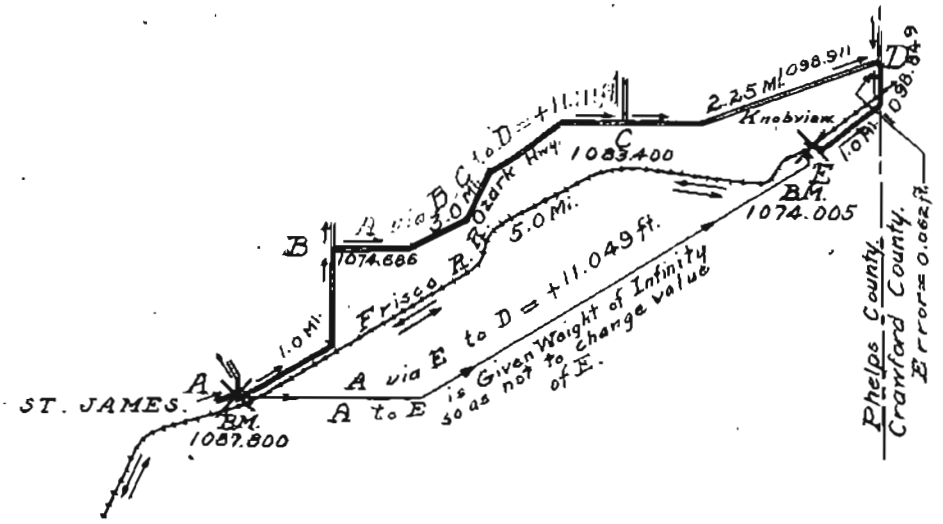
METHOD OF LEAST SQUARES APPLIED TO THE ADJUSTMENT OF THE LINE A VIA B, C, AND D TO E.

(BASED ON THE LINE OF LEVELS RUN BY THE U. S. GEOLOGICAL SURVEY IN 1908 ALONG THE FRISCO R. R. WITH PERMANENT BENCHMARKS, "A" AT ST. JAMES, AND "E" AT KNOBVIEW).

THE DETERMINATION AND ADJUSTMENT OF ELEVATIONS IN PHELPS COUNTY MISSOURI.

(Dawson Township Level Net)

Adjustment of lines A, B, C, D; and E, D as Multiple Lines of Levels. See sketch below and Plate C.



MULTIPLE LINES. By a multiple line of levels is meant a set of two or more lines connecting the same two bench marks by routes of different length. In order to find the most probable value for the difference of elevation between the terminals of a multiple line, it is necessary to weight each constituent line inversely as its length. Having weighted the several lines as thus explained, the case becomes identical with any case of weighted measurements, and hence the probable error of a single measurement of unit weight is given by the formula:

$$r_1 = 0.6745\sqrt{\frac{\sum pv^2}{n-1}}$$

the probable error of any of the lines of the weight p by the formula:

$$r_p = \frac{r_1}{p} = 0.6745\sqrt{\frac{\sum pv^2}{p(n-1)}}$$

and the probable error of the weighted arithmetic mean by the formula:

$$r_{p.a.} = \frac{r_1}{\sqrt{\Sigma p}} = 0.6745 \sqrt{\frac{\Sigma pv^2}{\Sigma p(n-1)}} = \frac{r_1}{\sqrt{\Sigma p}}$$

Our problem is as follows:

Given two lines of levels, as shown in the above figure, which give the following results:

A via B, C, to D, 6.25 mile line, +11.111 ft.

A via E to D, 1.00 mile line, +11.049 ft.
Note: A to E given weight of infinity.

The elevation of B.M. at A = 1087.800 ft., and the elevation of B.M. at E = 1074.005 ft.

Required the most probable elevation of the point D, and the probable error of this result.

S O L U T I O N .

M	p	pM	v	v^2	pv^2
11.111	0.160	1.77776	+0.053	0.002809	0.00044944
11.049	1.000	11.04900	-0.009	0.000081	0.00008100
	$\Sigma p = 1.160$	) 12.82676			$\Sigma pv^2 = 0.00053044$
		11.058		$n = 2.$	

Determination of the above quantities:

A = 1087.800 ft.
D = 1098.911 ft.
+ 11.111 ft.
Diff. Elev A via B, C to D.

A = 1087.800 ft.
E = 1074.005 ft.
- 13.795 ft
Diff. Elev A to E.

D = 1098.849 ft.
E = 1074.005 ft.
+ 24.844
- 13.795
+ 11.049 ft.
Diff. Elev. A via E to D.

D from Ozark Hwy line = 1098.911 ft.
D from Knobview B. M. = 1098.849 ft.
Observed error at D = 0.062 ft.

RESIDUALS: In observations of unequal weight, the difference between the weighted arithmetic mean and the observed value of each observation is a residual. If the observed value is greater than the weighted arithmetic mean, the residual quantity is plus., and if less than the weighted arithmetic mean, the residual quantity is minus.

First observed diff. in Elev M = 11.111 ft.
Weighted arithmetic mean pM = 11.058
First residual error = + 0.053

Second observed diff. in Elev M = 11.049 ft.
Weighted arithmetic mean pM = 11.058

Second residual error = - 0.009 = v.

GENERAL PRINCIPLE: In observations of unequal precision, the most probable values of the observed quantities are those that render the sum of the weighted squares of the residual errors a minimum. It is on account of this principle that the method of least squares has been so named.

THE WEIGHTS P are the reciprocals of the distances for each line. Thus:

A via B, C, to D = 6.25 miles, reciprocal or Wt. = 0.160.

E to D = 1.00 mile , " " " " " = 1.00.

Note: Reciprocals and Squares can be very conveniently looked up in Barlow's Tables of Squares, Roots, Etc.

N = the number of observations.

CHECK: As a check on the solution so far, the summation of weighted residuals must equal zero.

p	v	pv
0.160	+0.053	+0.009
1.000	-0.009	-0.009
	Σpv	= 0.000

Then the probable error of the weighted arithmetic mean is:

$$r_{pa} = 0.6745 \sqrt{\frac{0.00053044}{1.160 \times 1}} = \pm 0.0144 \text{ ft.}$$

1087.800 + 11.058 = 1098.858 ft.,

And the most probable value for the elevation D is:

D = 1098.858 ±0.0144 ft.

INTERMEDIATE POINTS. By an intermediate point is meant one lying only on a single line of levels, and hence having no influence on the general adjustment. These may occur on a line whose ends have been satisfactorily adjusted or on a closed circuit. In either case, the required adjustment is distributed uniformly throughout the line, making the correction between any two points directly proportional to the length between those two points.

ADJUSTMENT OF INTERMEDIATE JUNCTION POINTS B & C.

Observed value of D = 1098.911 ft. (Ozark Hwy. Line)
Adjusted value of D = 1098.858 ft.

Adjustment = + 0.053 ft. lower at D and taper to 0 at A.

As the distance A via B, C to D = 6.25 miles, and the distance A to B = 1.00 mile; then the adjustment to point B is as follows:

$$\text{Adjustment to B} = -\frac{1}{6.25} \times 0.053 = -0.008 \text{ ft.}$$

And the most probable elevation of point B is:

$$= 1074.666 - 0.008 = 1074.658 \text{ ft.}$$

In like manner, as the distance A to C = 4.00 miles,

$$\text{the adjustment to C} = -\frac{4}{6.25} \times 0.053 = -0.034 \text{ ft.}$$

And the most probable elevation of point C is:

$$= 1083.400 - 0.034 = 1083.366 \text{ ft.}$$

* *

CHECK on elevation of D from E.

Elevation of D from E	=	1098.849 ft.
Elevation of E	=	1074.005
Diff. in Elev. E to D	=	24.844 ft.
Correction for 1.00 mile E to D	=	+0.009
		24.853 ft.
Elev. E	=	1074.005
Most probable Elev. of D	=	1098.858 ft
Observed Elev. of D	=	1098.849 ft
Adjusted Elev. of D	=	1098.858
Adjustment	=	0.009 ft. (O.K. for 1 Mile).

* *

See Plate C for the adjusted values for this circuit.

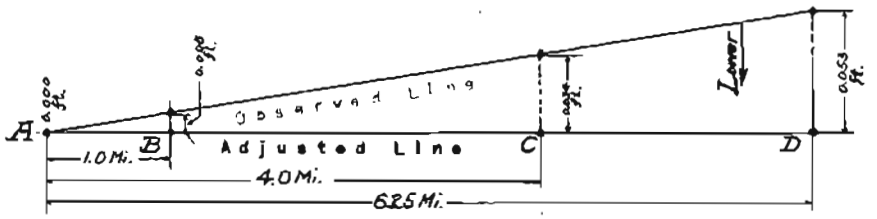


DIAGRAM SHOWING ADJUSTMENT OF POINTS BET. A and D.

From the above diagram it is easily seen how the values along the Observed (red) line are lowered proportionally to the distances from A down to the adjusted (black) line. The diagram also serves as a check on the adjustments of the intermediate points if drawn to scale.

P A R T I I .

THE LEAST SQUARE ADJUSTMENT OF A LEVEL NET BY METHOD OF OBSERVATION EQUATIONS.

APPLIED TO THE ADJUSTMENT
OF THE
DAWSON-ST. JAMES TOWNSHIP LEVEL NET.
PHELPS COUNTY,
MISSOURI.

(BASED ON THE A VIA B, C, AND D TO E BASE).

THE DETERMINATION AND ADJUSTMENT OF ELEVATIONS IN PHELPS COUNTY MISSOURI.

(Dawson Township Level Net)

Referring to the Dawson Township Level Net Diagrams indicated on pages and , the field notes show the following results:

A via L, K to I	= -	25.875 ft.
I via P to M	= -	107.201 ft.
J to H	= +	73.460 ft.
M to J	= +	13.695 ft.
H to F	= -	40.447 ft.
F to G	= -	80.066 ft.
G via X, D to A	= +	166.589 ft.
I to J	= -	93.531 ft.
A via B to H	= -	45.750 ft.
A via C to F	= -	86.657 ft.
M via N, O to G	= -	33.201 ft.

It is desired to find the most probable values for the elevations of the level net junction stations I, M, J, H, F, and G. Upon finding these junction elevations, it is then desired to find all intermediate and remaining elevations in this net. Referring to page 2, it will be noted that the line of levels running from A via B, C, D, to E has previously been adjusted; levels along the Frisco R. R. have also been adjusted: these lines will be considered base lines and their values will remain unchanged. The Dawson Township net will be considered to hinge on the U. S. Geological Survey Stations A (at St. James) and E (at Knobview).

SOLUTION OF PROBLEM BY METHOD OF LEAST SQUARES.

As there are but six unknown Bench Marks (B.Ms.) (I, M, J, H, F, and G), there can be but six independent unknowns in the observation equations. As the lines AI, IM, MJ, JH, HF, and FG, may evidently be selected as the independent unknowns, we may write for the most probable values

of the corresponding differences of elevation:

A via L, K, to I	= −	25.875 +	V_1 ;
I via P to M	= −	107.201 +	V_2 ;
M to J	= +	13.695 +	V_3 ;
J to H	= +	73.460 +	V_4 ;
H to F	= −	40.447 +	V_5 ;
F to G	= −	80.066 +	V_6 .

The conditional equations involved in the several closed circuits may then be avoided by writing all the observation equations in terms of these quantities. Writing the reduced observation equations directly from the diagram on page 1C, we have, by comparison with the observed values,

Route		Residuals	Closures	Weights
A via L, K, to I	=	V_1	= 0.000	(weight 0.160);
I via P to M	=	V_2	= 0.000	(weight 0.156);
M to J	=	V_3	= 0.000	(weight 0.312);
J to H	=	V_4	= 0.000	(weight 0.625);
H to F	=	V_5	= 0.000	(weight 0.571);
F to G	=	V_6	= 0.000	(weight 2.000);
G via X, C, to A	=	$-V_1-V_2-V_3-V_4-V_5-V_6$	=+0.155	(weight 0.222);
A via B to H	=	$V_1+V_2+V_3+V_4$	=-0.171	(weight 0.294);
A via C to F	=	$V_1+V_2+V_3+V_4+V_5$	=+0.289	(weight 0.333);
M via N, O, to G	=	$V_3+V_4+V_5+V_6$	=-0.157	(weight 0.124);
I to J	=	V_2+V_3	=+0.025	(weight 0.625).

To show how the above reduced observation equations were formed, a compilation of the values for elevations of the several level net junction points determined from the successive differences of elevation along the selected circuit, A via L, K, I, P, M, J, H, F, G, X, C, to A is shown below. The closures shown above for the several circuits are extended below, as well as the calculation of the respective weights.

Observed values for the elevations of stations along the Circuit A via L, K, I, P, M, J, H, F, G, X, D, to A, Determined from successive differences of elevation:

A = 1087.800 ft. U.S.G.S. Iron Post at St. James.

- 25.875

I = 1061.925 ft. -------- I = 1061.925 ft.

- 107.201

M = 954.724 ft.

+ 13.695

J = 968.419 ft. -------

+ 73.460

H = 1041.879 ft. ---- [illegible]

- 40.447

F = 1001.432 ft.

- 80.066

G = 921.366 ft.

+ 166.589

A = 1087.955 ft.

= [illegible]

+ 0.155 ft Closure.

93.506 ft.
93.531 ft. I to J.
+ 0.025 ft. Closure.

A = 1087.800 ft.
[illegible]
45.921 ft.
45.750 ft. A via B to H.
- 0.171 Closure.

A= 1087.800 ft.
[illegible]
86.368 ft.
86.657 ft. A via C to F.
+ 0.289 ft. Closure.

M = 954.724 ft.
[illegible]
33.358 ft.
33.201 ft. M via O to G.
- 0.157 ft. Closure.

ROUTE.	DISTANCE.	WEIGHT.
A via L, K, to I	= 6.25 miles	= 0.160
I via P to M	= 6.40 "	= 0.156
M to J	= 3.21 "	= 0.312
J to H	= 1.60 "	= 0.625
H to F	= 1.75 "	= 0.571
F to G	= 0.50 "	= 2.000
G via X, D, to A	= 4.50 "	= 0.222
A via B to H	= 3.40 "	= 0.294
M via N, O, to G	= 8.05 "	= 0.124
I to J	= 1.60 "	= 0.625
A via C to F	= 3.00 "	= 0.333

It will be noted that in each case the respective weights are the reciprocals of the distances.

We next write the normal equations, which are arived at as summarized in the following rule:

RULE: To form the normal equation for each one of the unknown quantities, multiply each observation equation by the product of the weight of that observation and the algebraic coefficient of that unknown quantity in that equation, and add the results.

Having formed the several normal equations, their solution as simultaneous equations gives the most probable values of the unknown quantities.

Then by the above rule; we have:

NORMAL EQUATION IN V_1. (Solution for)

$$0.160V_1 = +0.000000$$
$$0.222V_1+0.222V_2+0.222V_3+0.222V_4+0.222V_5+0.222V_6 = -0.034410$$
$$0.294V_1+0.294V_2+0.294V_3+0.294V_4 = -0.050274$$
$$0.333V_1+0.333V_2+0.333V_3+0.333V_4+0.333V_5 = +0.096333$$

1) [illegible]

NORMAL EQUATION IN V_2.

$$+0.156V_2 = +0.000000$$
$$0.222V_1+0.222V_2+0.222V_3+0.222V_4+0.222V_5+0.222V_6 = -0.034410$$
$$0.294V_1+0.294V_2+0.294V_3+0.294V_4 = -0.050274$$
$$0.333V_1+0.333V_2+0.333V_3+0.333V_4+0.333V_5 = +0.096333$$
$$+0.625V_2+0.625V_3 = +0.015625$$

2) [illegible]

NORMAL EQUATION IN V_3.

$$+0.312V_3 = +0.000000$$
$$0.222V_1+0.222V_2+0.222V_3+0.222V_4+0.222V_5+0.222V_6 = -0.034410$$
$$0.294V_1+0.294V_2+0.294V_3+0.294V_4 = -0.050274$$
$$0.333V_1+0.333V_2+0.333V_3+0.333V_4+0.333V_5 = +0.096333$$
$$+0.124V_3+0.124V_4+0.124V_5+0.124V_6 = -0.019468$$
$$+0.625V_2+0.625V_3 = +0.015625$$

3) [illegible]

NORMAL EQUATION IN V_4.

$$+0.625V_4 = +0.000000$$
$$0.222V_1+0.222V_2+0.222V_3+0.222V_4+0.222V_5+0.222V_6=-0.034410$$
$$0.294V_1+0.294V_2+0.294V_3+0.294V_4 = -0.050274$$
$$0.333V_1+0.333V_2+0.333V_3+0.333V_4+0.333V_5 = +0.096333$$
$$+0.124V_3+0.124V_4+0.124V_5+0.124V_6=-0.019468$$
4) [illegible]

NORMAL EQUATION IN V_5.

$$+0.571V_5 = +0.000000$$
$$0.222V_1+0.222V_2+0.222V_3+0.222V_4+0.222V_5+0.222V_6=-0.034410$$
$$0.333V_1+0.333V_2+0.333V_3+0.333V_4+0.333V_5 = +0.096333$$
$$+0.124V_3+0.124V_4+0.124V_5+0.124V_6=-0.019468$$
5) [illegible]

NORMAL EQUATION IN V_6.

$$+2.000V_6=+0.000000$$
$$0.222V_1+0.222V_2+0.222V_3+0.222V_4+0.222V_5+0.222V_6=-0.034410$$
$$+0.124V_3+0.124V_4+0.124V_5+0.124V_6=-0.019468$$
6) [illegible]

Whence from the solution for the Normal Equations, we write the six following normal equations that are to be solved simultaneously for the values of $V_1, V_2, V_3, V_4, V_5, \& V_6$.

NORMAL EQUATIONS.

1) $+1.009V_1+0.849V_2+0.849V_3+0.849V_4+0.555V_5+0.222V_6=+0.011649$
2) $+0.849V_1+1.630V_2+1.474V_3+0.849V_4+0.555V_5+0.222V_6=+0.027274$
3) $+0.849V_1+1.474V_2+1.910V_3+0.973V_4+0.679V_5+0.346V_6=+0.007806$
4) $+0.849V_1+0.849V_2+0.973V_3+1.598V_4+0.679V_5+0.346V_6=-0.007819$
5) $+0.555V_1+0.555V_2+0.679V_3+0.679V_4+1.250V_5+0.346V_6=+0.042455$
6) $+0.222V_1+0.222V_2+0.346V_3+0.346V_4+0.346V_5+2.346V_6=-0.053878$

Now we are to solve the above equations simultaneously.

SOLUTION OF NORMAL EQUATIONS.

SOLUTION OF NORMAL EQUATIONS.

First Set.

Eliminate V_3.

1): $+1.0090V_1+0.8490V_2+0.8490V_3+0.8490V_4+0.5550V_5+0.2220V_6=+0.011649$

2): $+0.4890V_1+0.9390V_2+0.8490V_3+0.4890V_4+0.3197V_5+0.1279V_6=+0.015713$

7): $+0.5200V_1-0.0900V_2+0.3600V_4+0.2353V_5+0.0941V_6=-0.004064$

2): $+0.8490V_1+1.630V_2+1.4740V_3+0.8490V_4+0.5550V_5+0.2220V_6=+0.027274$

3): $+0.6552V_1+1.1375V_2+1.4740V_3+0.7509V_4+0.5240V_5+0.2670V_6=+0.006024$

8): $+0.1938V_1+0.4925V_2+0.0981V_4+0.0310V_5+0.0450V_6=+0.021250$

3): $+0.8490V_1+1.4740V_2+1.910V_3+0.9730V_4+0.6790V_5+0.3460V_6=+0.007806$

4): $+1.6666V_1+1.6666V_2+1.910V_3+3.1369V_4+1.3329V_5+0.6792V_6=-0.015349$

9): $-0.8176V_1-0.1926V_2-2.1639V_4-0.6439V_5-0.3332V_6=+0.023155$

4): $+0.8490V_1+0.8490V_2+0.9730V_3+1.598V_4+0.6790V_5+0.3460V_6=-0.007819$

5): $+0.7953V_1+0.7953V_2+0.9730V_3+0.9730V_4+1.7912V_5+0.4958V_6=+0.060836$

10): $+0.0537V_1+0.0537V_2+0.6250V_4+1.1122V_5+0.1498V_6=-0.068655$

5): $+0.5550V_1+0.5550V_2+0.6790V_3+0.6790V_4+1.250V_5+0.3460V_6=+0.042455$

6): $+0.4357V_1+0.4357V_2+0.6790V_3+0.6790V_4+0.679V_5+4.6039V_6=-0.105732$

11): $+0.1193V_1+0.1193V_2+0.5710V_5-4.2579V_6=0.148187$

Summary of first set of equations.

7): $+0.5200V_1-0.0900V_2+0.3600V_4+0.2353V_5+0.0941V_6=-0.004064$

8): $+0.1938V_1+0.4925V_2+0.0981V_4+0.0310V_5-0.0450V_6=+0.021250$

9): $-0.8176V_1-0.1926V_2-2.1639V_4-0.6439V_5-0.3332V_6=+0.023155$

10): $+0.0537V_1+0.0537V_2+0.6250V_4-1.1122V_5-0.1498V_6=-0.068655$

11): $+0.1193V_1+0.1193V_2+0.5710V_5-4.2579V_6=+0.148187$

Second Set.

Eliminate V_4.

7): $+0.5200V_1-0.0900V_2+0.3600V_4+0.2353V_5+0.0941V_6=-0.004064$

8): $+0.7112V_1+1.8073V_2+0.3600V_4+0.1138V_5-0.1651V_6=+0.077982$

12): $-0.1912V_1-1.8973V_2+0.1215V_5+0.2592V_6=-0.082046$

8): $+0.1938V_1+0.4925V_2+0.0981V_4+0.0310V_5-0.0450V_6=+0.021250$

9): $-0.0371V_1-0.0087V_2-0.0981V_4-0.0292V_5-0.0151V_6=+0.001050$

13): $+0.1567V_1+0.4838V_2+0.0018V_5-0.0601V_6=+0.022300$

Second set of Equations continued.

9) $-0.8176V_1-0.1976V_2-2.1639V_4-0.6439V_5-0.3332V_6=+0.023155$

10) $\times 3.46224$ / $+.1859V_1+0.1859V_2+2.1639V_4-3.8607V_5-0.5186V_6=-0.237700$

14) $-0.6317V_1-0.0067V_2\qquad\qquad -4.4946V_5-0.8518V_6=-0.214545$

summary of second set of equations:

12) $-0.1912V_1-1.8973V_2+0.1215V_5+0.2592V_6=-0.082046$

13) $+0.1567V_1+0.4838V_2+0.0018V_5-0.0601V_6=+0.022300$

14) $-0.6317V_1-0.0067V_2-4.4946V_5-0.8518V_6=-0.214545$

11) $+0.1193V_1+0.1193V_2+0.5710V_5-4.2579V_6=+0.148187$

....

Third set.

Eliminate V_1.

12) $-0.1912V_1-1.8973V_2+0.1215V_5+0.2592V_6=-0.082946$

13) $\times 1.22017$ / $+.1912V_1+0.5903V_2+0.0022V_5-0.0733V_6=+0.027210$

15) $\qquad -1.3070V_2+0.1237V_5+0.1859V_6=-0.054836$

13) $+0.1567V_1+0.4838V_2+0.0018V_5-0.0601V_6=+0.022300$

14) $\times 0.24806$ / $-.1567V_1-0.0017V_2-1.1149V_5-0.2113V_6=-0.053220$

16) $\qquad +0.4821V_2-1.1131V_5-0.2714V_6=-0.030920$

14) $-0.6317V_1-0.0067V_2-4.4946V_5-0.8518V_6=-0.214545$

11) $\times 5.29505$ / $+.6317V_1+0.6317V_2+3.0235V_5-22.5458V_6=+0.784658$

17) $\qquad +0.6250V_2-1.4711V_5-23.3976V_6=+0.570113$

summary of third set of equations:

15) $-1.3070V_2+0.1237V_5+0.1859V_6=-0.054836$

16) $+0.4821V_2-1.1131V_5-0.2714V_6=-0.030920$

17) $+0.6250V_2-1.4711V_5-23.3976V_6=+0.570113$

....

Fourth set.

Eliminate V_2.

15) $-1.3070V_2+0.1237V_5+0.1859V_6=-0.054836$

16) $\times 1.77108$ / $1.3070V_2-3.0177V_5-0.7358V_6=-0.083826$

18) $\qquad -2.8940V_5-0.5499V_6=-0.138662$

16) $+0.4821V_2-1.1131V_5-0.2714V_6=-0.030920$

17) $\times 0.77136$ / $+.4821V_2-1.1347V_5-18.0480V_6=+0.439762$

19) $\qquad +0.0216V_5+17.7766V_6=-0.470682$

Summary of Fourth set of equations:

18) $-2.8940V_5 - 0.5499V_6 = -0.138662$

19) $+0.0216V_5 + 17.7766V_6 = -0.470682$

....

Fifth set.

Eliminate V_5.

18) $-2.8940V_5 - 0.5499V_6 = -0.138662$

5133.98148

19) $/2.8940V_5 + 2381.7352V_6 = -63.062671$

---

20) $+2381.1853V_6 = -63.201333$

Summary of Fifth set of equations:

20) $+2381.1853V_6 = -63.201333$

Then $V_6 = -0.02654$

....

Substitute this value for V_6 in equation 18) and solve for V_5:

18) $-2.8940V_5 + 0.014594346 = -0.138662$

Then $-2.8940V_5 = -0.153256346$

And $V_5 = +0.052956$

Substitute the values found for V_6 and V_5 in equation 15) and solve for V_2:

15) $-1.3070V_2 + 0.0065506572 - 0.004933786 = -0.054836$

Then $-1.3070V_2 = -0.0564528712$

And $V_2 = +0.0431927$

Substitute the values found for V_6, V_5, and V_2 in equation 11) and solve for V_1:

11) $+0.1193V_1 + 0.00515288911 + 0.030237876 + 0.113004666 = 0.148187$

Then $+0.1193V_1 = -0.00020843111$

And $V_1 = -0.001747$

Substitute the values for V_6, V_5, V_2, and V_1 in equation 7) and solve for V_4:

$+0.3600V_4$

7) $-0.00090844 - 0.003887343 + 0.0124605468 - 0.002497414 =$

$= -0.004064.$

Then $+0.3600V_4 = -0.0092313498$

And $V_4 = -0.025643$

Substitute the values found for V_6, V_5, V_2, V_1, and V_4 in equation 1) and solve for V_3:

1) $-0.001762723 + 0.0366706023 + 0.849V_3 - 0.021770907 - 0.00589188 = +0.011649.$

Then $+0.849V_3 = -0.0249866723$

And $V_3 = -0.02943$

ooooo ooooo ooooo ooooo ooooo

Check these V values thru one of the equations in the first set, and we find:

4) $-0.001483203 + 0.036670857 - 0.02863539 - 0.040977514 + 0.035957124 - 0.00918284 = -0.007819$

And we find that these values will check 0 = 0 thru the fourth decimal place.

Summary of residual values.

$V_1 = -0.001747$ ft;
$V_2 = +0.043193$ ft;
$V_3 = -0.029430$ ft;
$V_4 = -0.025643$ ft;
$V_5 = +0.052956$ ft;
$V_6 = -0.026540$ ft.

Now we find that the most probable values for the differences of elevation are:

A via L, K, to I = -25.875 - 0.0017 = - 25.8767 ft;
I via P to M = -107.201 + 0.0432 = -107.1578 ft;
M to J = + 13.695 - 0.0294 = + 13.6656 ft;
J to H = + 73.460 -0.0256 = + 73.4344 ft;
H to F = - 40.447 + 0.0530 = - 40.3940 ft;
F to G = - 80.066 - 0.0265 = - 80.0925 ft.

Then from A via the route L, K, I, P, M, J, H, F, G, back to A, we have the most probable elevations of the unknown stations.

A = 1087.8000 ft.	M = 954.7655 ft.	H = 1041.8655 ft.
- 25.8767	+ 13.6656	- 40.3940
I = 1061.9233 ft.	J = 968.4311 ft.	F = 1001.4715 ft.
- 107.1578	+ 73.4344	- 80.0925
M = 954.7655 ft.	H = 1041.8655 ft	G = 921.3790 ft.

(O) N O R M A L E Q U A T I O N S .

	V_1	V_2	V_3	V_4	V_5	V_6	Absolute Term	
1)	+1.009	+0.849	+0.849	+0.849	+0.555	+0.222	-0.011649	= 0
2)	+0.849	+1.630	+1.474	+0.849	+0.555	+0.222	-0.027274	= 0
3)	+0.849	+1.474	+1.910	+0.973	+0.679	+0.346	-0.007806	= 0
4)	+0.849	+0.849	+0.973	+1.598	+0.679	+0.346	+0.007819	= 0
5)	+0.555	+0.555	+0.679	+0.679	+1.250	+0.346	-0.042455	= 0
6)	+0.222	+0.222	+0.346	+0.346	+0.346	+2.346	+0.053878	= 0

(Numbers below the red line are redundant numbers).

(P) Solution of Normal Equations.

	1	2	3	4	5	6	Absolute Term
P_1	+1.0090	+0.8490	+0.8490	+0.8490	+0.5550	+0.2220	-0.011649
P_2	(-0.9911)	-0.8414	-0.8414	-0.8414	-0.5501	-0.2200	+0.011545
P_3		+0.9157	+0.7597	+0.1347	+0.0880	+0.0352	-0.017473
P_4		(-1.0921)	-0.8297	-0.1471	-0.0961	-0.0384	+0.019082
P_5			+0.5654	+0.1469	+0.1401	+0.1300	+0.016968
P_6			(-1.7687)	-0.2598	-0.2478	-0.2299	-0.030011
P_7				+0.8257	+0.1627	+0.1202	+0.015782
P_8				(-1.2111)	-0.1970	-0.1456	-0.019114
P_9					+0.8694	+0.1646	-0.041682
P_{10}					(-1.1502)	-0.1893	+0.047943
P_{11}						+2.2172	+0.058803
P_{12}						(-0.4509)	-0.026514

(S)Find most probable values for the differences of elevation.

A	via L,K,to	I	= -25.875	- 0.0016	= - 25.8766 ft;
I	via P to	M	=-107.201	+ 0.0440	= -107.1570 ft;
M	to	J	=+ 13.695	- 0.0304	= + 13.6646 ft;
J	to	H	=+ 73.460	- 0.0257	= + 73.4343 ft;
H	to	F	=- 40.447	+ 0.0530	= - 40.3940 ft;
F	to	G	=- 80.066	- 0.0265	= - 80.0925 ft.

(T) Then from A via the route L,K,I,P,M,J,H,F,G,back to A, we have the most probable elevations of the unknown stations:

A =	1087.8000 ft.,		M =	954.7664 ft.,		H =	1041.8653 ft.,	
	-25.8766			+13.6646			-40.3940	
I =	1061.9234 ft.,		J =	968.4310 ft.,		F =	1001.4713 ft.,	
	-107.1570			+73.4343			-80.0925	
M =	954.7664 ft.,		H =	1041.8653 ft.,		G =	921.3788 ft.	

(q) Solution of Normal Equations.

	2	3	4	5	6	Absolute Term	
q_1	+1.6300	+1.4740	+0.8490	+0.5550	+0.2220	-0.027274	q_1
q_2	-0.7143	-0.7143	-0.7143	-0.4670	-0.1868	+0.009801	q_2
	+0.9157	+0.7597	+0.1347	+0.0880	+0.0352	-0.017473	
q_3	. . . .	+1.9100	+0.9730	+0.6790	+0.3460	-0.007806	q_3
q_4	. . . .	-0.7143	-0.7143	-0.4670	-0.1868	+0.009801	q_4
q_5	. . . .	-0.6303	-0.1118	-0.0719	-0.0292	+0.014973	q_5
		+0.5654	+0.1469	+0.1401	+0.1300	+0.016968	
q_6	. . . . .	. . . .	+1.5980	+0.6790	+0.3460	+0.007819	q_6
q_7	. . . . .	. . . .	-0.7143	-0.4670	-0.1868	+0.009801	q_7
q_8	. . . . .	. . . .	-0.0198	-0.0129	-0.0052	+0.002570	q_8
q_9	. . . . .	. . . .	-0.0382	-0.0364	-0.0338	-0.004408	q_9
			+0.8257	+0.1627	+0.1202	+0.015782	
q_{10}	. . . . .	. . . .	. . . .	+1.2500	+0.3460	-0.042455	q_{10}
q_{11}	. . . . .	. . . .	. . . .	-0.3053	-0.1221	+0.006408	q_{11}
q_{12}	. . . . .	. . . .	. . . .	-0.0085	-0.0034	+0.001679	q_{12}
q_{13}	. . . . .	. . . .	. . . .	-0.0347	-0.0322	-0.004205	q_{13}
q_{14}	. . . . .	. . . .	. . . .	-0.0321	-0.0237	-0.003109	q_{14}
				+0.8694	+0.1646	-0.041682	
q_{15}	. . . . .	. . . .	. . . .	. . . .	+2.3460	+0.053878	q_{15}
q_{16}	. . . . .	. . . .	. . . .	. . . .	-0.0488	+0.002563	q_{16}
q_{17}	. . . . .	. . . .	. . . .	. . . .	-0.0014	+0.000671	q_{17}
q_{18}	. . . . .	. . . .	. . . .	. . . .	-0.0299	-0.003901	q_{18}
q_{19}	. . . . .	. . . .	. . . .	. . . .	-0.0175	-0.002298	q_{19}
q_{20}	. . . . .	. . . .	. . . .	. . . .	-0.0312	+0.007890	q_{20}
					+2.2172	+0.058803	

(r) Solution of Normal Equations.

	V_1	V_2	V_3	V_4	V_5	V_6
r_1	+0.011545	+0.019082	-0.030011	-0.019114	+0.047943	-0.026514
r_2	+0.005833	+0.001018	+0.006096	+0.003860	+0.005019	
r_3	-0.029134	-0.005089	-0.013124	-0.010434	+0.052962	
r_4	+0.021614	+0.003779	+0.006874	-0.025688		
r_5	+0.025549	+0.025194	-0.030365			
r_6	-0.037007	+0.043983				
r_7	-0.001600					

Adjustment Quantities:

$V_1 = -0.0016$ $V_4 = -0.0257$

$V_2 = +0.0440$ $V_5 = +0.0530$

$V_3 = -0.0304$ $V_6 = -0.0265$

Check these V values thru one of the equations in (O), and we find:

1) -0.001614400 +0.037341567 -0.025779885 -0.021809112
+0.029393910 -0.005886108 -0.011649000 = 0

And we find that these values check O=O thru the seventh decimal place.

THE DETERMINATION AND ADJUSTMENT OF ELEVATIONS IN PHELPS COUNTY MISSOURI.

(Dawson Township Level Net)

ADJUSTMENT OF INTERMEDIATE JUNCTION POINTS L, K, P, N, O.

Note: (See discussion on Intermediate Points on page .)

Observed value of I = 1061.925 ft. (Republic Rd. Line)
Adjusted value of I = 1061.923

Adjustment = + 0.002 ft. Lower line at I and taper to 0.00 at A.

dist.

As the A via L, K to I = 6.25 miles, and the distance A to L = 0.5 miles; then the adjustment to point L is as follows:

$$\text{Adjustment to L} = -\frac{0.5}{6.25} \times 0.002 = -0.00016 \text{ ft.}$$

And the most probable elevation of point L is=

= 1078.427 - 0.000 = 1078.427 ft.

In like manner, as the distance A to K = 1.75 miles,

$$\text{the adjustment to K} = -\frac{1.75}{6.25} \times 0.002 = -0.00056 \text{ ft.}$$

And the most probable elevation of point K is=

= 1092.694 - 0.001 = 1092.693 ft.

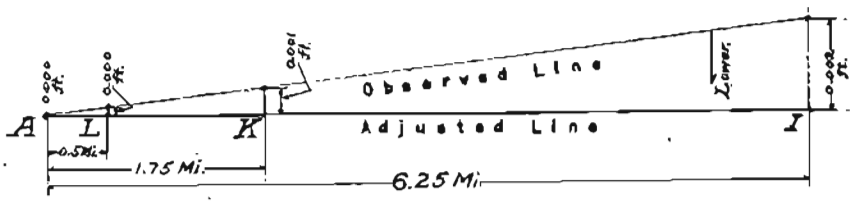


DIAGRAM SHOWING ADJUSTMENT OF POINTS BET. A and I.

From the above diagram it is easily seen how the values along the Observed line are lowered proportional to the distance from A to the Adjusted line. The diagram also serves as a check on the adjustment of the Intermediate Points if drawn to scales horizontally and vertically.

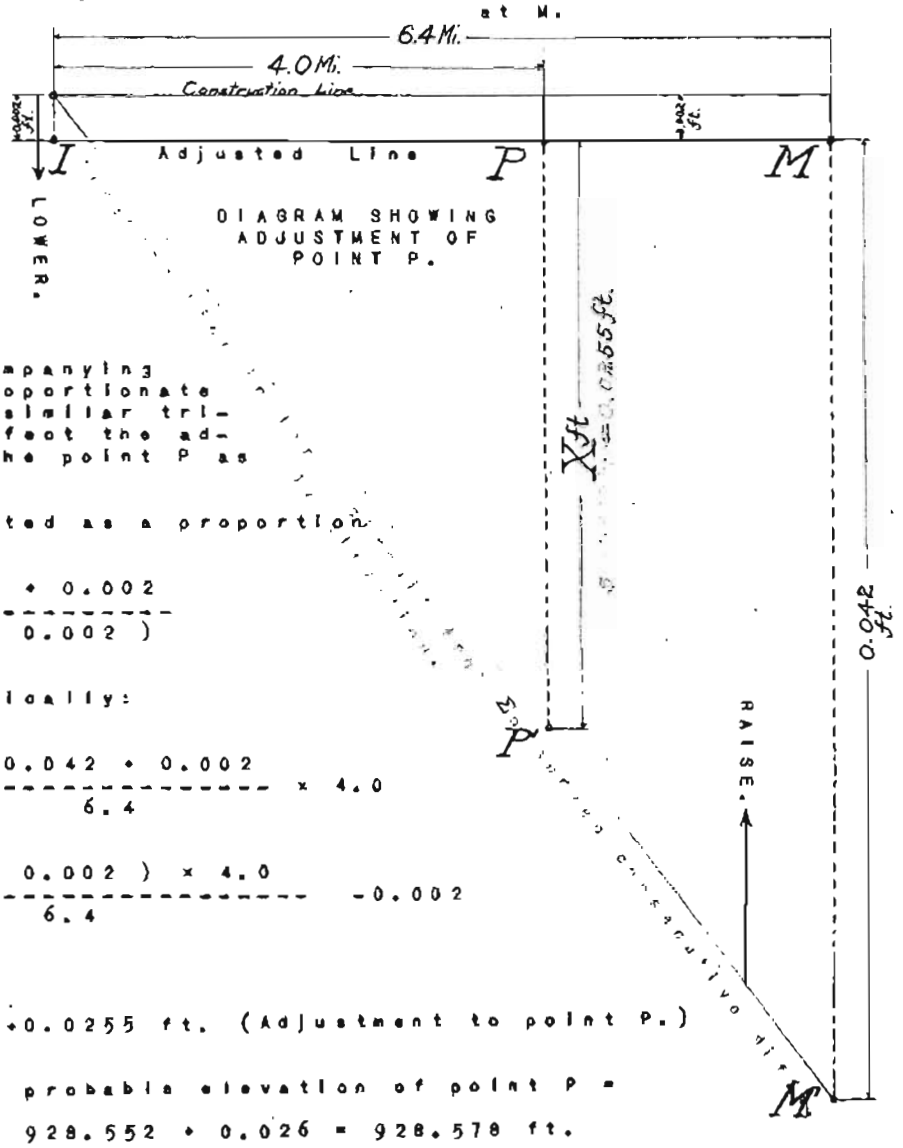
Observed value of point I = 1061.925 ft. (Republic Road)
Adjusted value of point I = 1061.923

Adjustment = + 0.002 ft. Line to be lowered at I.

Arranged value of point M (in large circuit from summation of consecutive diffs. in elevation.) = 954.724 ft. (from the west)

Adjusted value of point M = 954.766

Adjustment = - 0.042 ft. Line to be raised at M.



From the accompanying diagram by proportionate lines in the similar triangles, we affect the adjustment to the point P as follows:

Relations stated as a proportion:

$$\frac{6.4}{4.0} = \frac{0.042 + 0.002}{(X + 0.002)}$$

Solve algebraically:

$$X + 0.002 = \frac{0.042 + 0.002}{6.4} \times 4.0$$

$$X = \frac{(0.042 + 0.002) \times 4.0}{6.4} - 0.002$$

And we find:

X = +0.0255 ft. (Adjustment to point P.)

Then the most probable elevation of point P =

= 928.552 + 0.026 = 928.578 ft.

Arranged value of point M (in large circuit from summation of consecutive diffs. in elevation) = 954.724 ft. (from the west)

Adjusted value of point M = 954.766

Adjustment = - 0.042 ft. Line to be raised at M.

Arranged value of point G (in large circuit from summation of consecutive diffs. in elevation). = 921.523 ft. (from M via N, O to G)

Adjusted value of point G = 921.379

Adjustment = + 0.144 ft. Line to be lowered at G.

From the accompanying diagram by proportionate lines in the similar triangles, we effect the adjustment of points N and O as follows:

ADJUSTMENT OF POINT N.

Relations stated as a proportion:

$$\frac{8.05}{0.9} = \frac{0.144 + 0.042}{(0.042 - X)}$$

By algebra:

$$-X = \left(0.186 \times \frac{0.9}{8.05}\right) - 0.042$$

Solving:

X = 0.0212 ft. Adjustment to point N.

Then the most probable elevation of point

N = 875.071 + 0.021 =

875.092 ft.

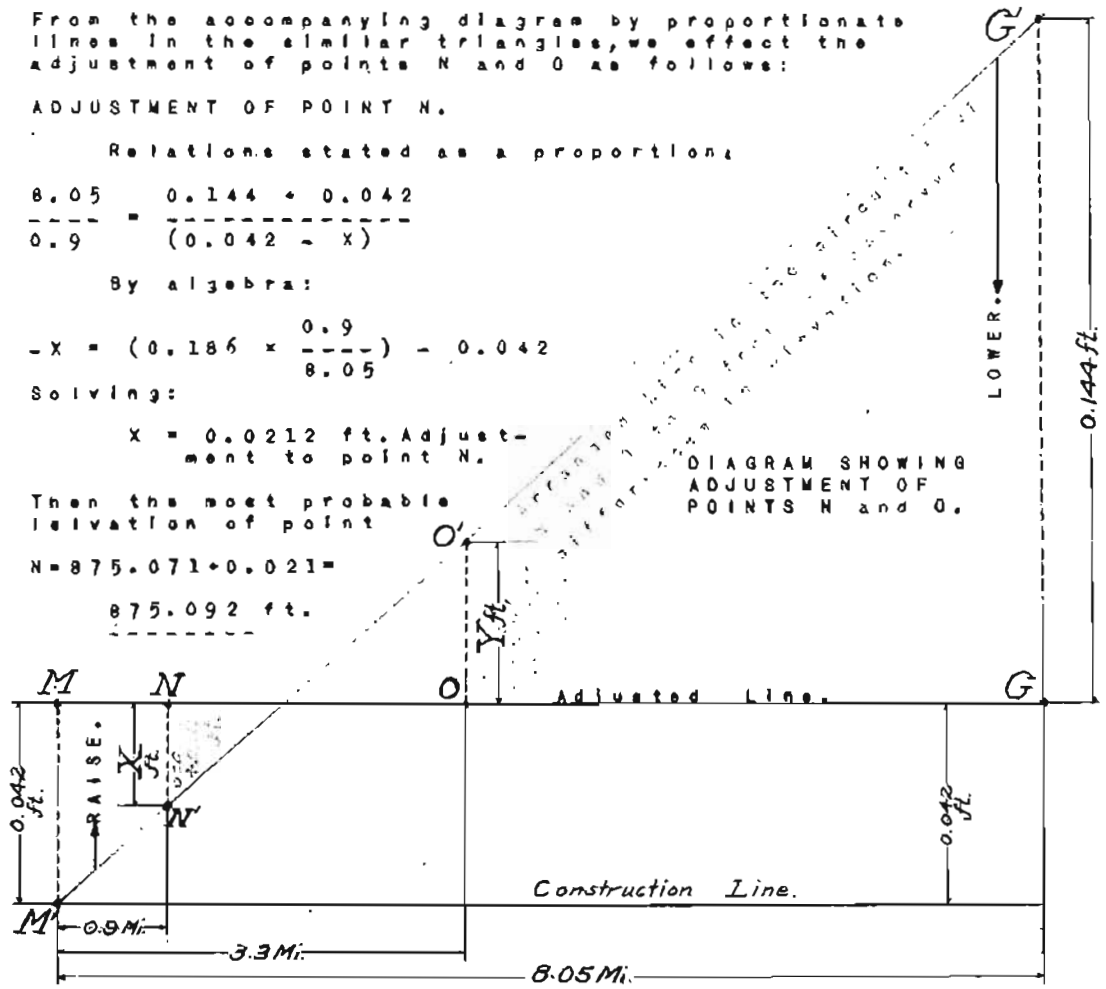


DIAGRAM SHOWING ADJUSTMENT OF POINTS N and O.

ADJUSTMENT OF POINT O. In the same manner as above:

$$\frac{8.05}{3.3} = \frac{0.144 + 0.042}{(Y + 0.042)}$$

Solving: Y = 0.0342 ft. The adjustment to point O.

Then the most probable elevation of point

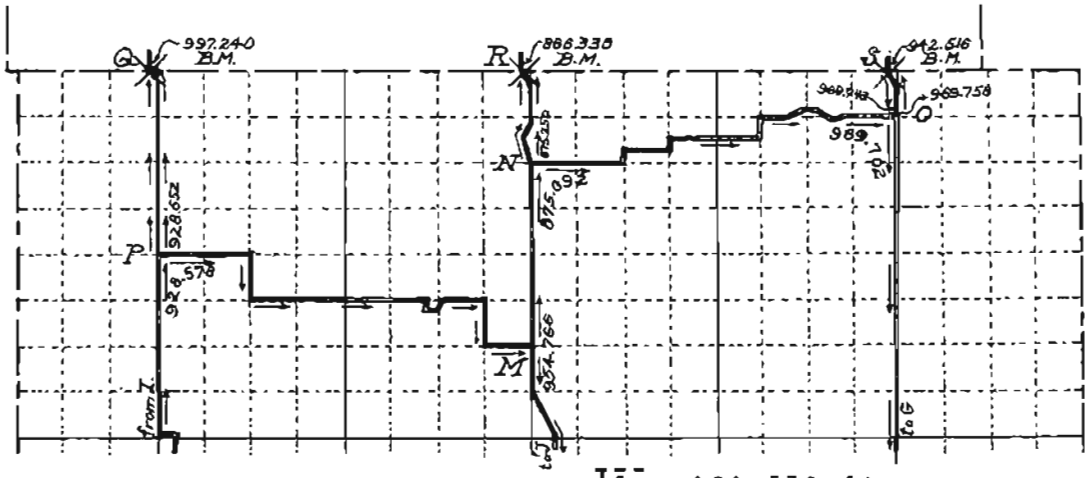
O = 969.736 - 0.034 = 969.702 ft.

THE DETERMINATION AND ADJUSTMENT OF ELEVATIONS IN PHELPS COUNTY MISSOURI.

(Dawson Township Level Net)

Adjustment of lines PG and NR as Parallel Duplicate Lines of Levels, and the adjustment of the line OSO as a Duplicate Line of Levels.

See sketch below and the three Plates of Dawson Township.



Adjusted values are as follows

P	=	928.578 ft.
M	=	954.766 "
N	=	875.092 "
O	=	969.702 "

It is required to find the most probable elevations of the points Q, R, and S.

ADJUSTMENT OF POINT Q.

Parallel Duplicate Lines of Levels: By a parallel duplicate line of levels is meant a line that is run twice over the same route with equal care and in the same direction. Levels are sometimes run in this manner when there are two instruments on the work, as was the case when the line PG was run, also the line NR.

Observed elevation of point	P	= 928.552 ft.	
" " " " " "	Q	= 997.242 "	1st Instrument.
Difference in elevation		= +68.690 " " " "	
Observed elevation of point	P	= 928.552 ft.	2nd Instrument.
" " " " " "	Q	= 997.237 " " " "	
Difference in elevation		= +68.685 " " " "	

UNWEIGHTED MEASUREMENTS. If the difference of elevation of two stations is measured a number of times in the same manner, over the same lenght of line, and under such conditions that the different determinations may be regarded as of equal weight, then the arithmetic mean of the several results is the most probable value of this difference of elevation. The probable error of a single measurement is given by the formula:

$$r_1 = 0.6745 \sqrt{\frac{\Sigma v^2}{n - 1}}$$

The probable error of the arithmetic mean of n measurements by the formula:

$$r_a = \frac{r_1}{\sqrt{n}} = 0.6745 \sqrt{\frac{\Sigma v^2}{n(n - 1)}}$$

ADJUSTMENT OF POINT Q.

Observed Values	v	v^2
68.690 ft.	+0.0025	0.00000625
68.685 ft.	-0.0025	0.00000625
2)137.375	0.0000	0.00001250 $=\Sigma v^2$
z = 68.6875 ft.	Check	n=2. (No.of Obs.)

$$r_a = 0.6745 \sqrt{\frac{0.00001250}{2(2 - 1)}} = \pm 0.0017 \text{ ft.}$$

Then the most probable value for the difference in elevation = 68.6875± 0.0017 ft.

And the most probable elevation of the point Q is as follows:

Adjusted value of point P = 928.578 ft.
Adjusted Diff.in elev.P to Q = 68.688
Most probable elev.point Q= 997.266 ft.

ADJUSTMENT OF POINT R.

Observed elevation of point N = 875.250 ft.
" " " " " " R = 866.383 " 1st Instrument.
Difference in elevation = - 8.867 " " " "

Observed elevation of point N = 875.250 ft.
" " " " " " R = 866.293 " 2nd Instrument.
Difference in elevation = - 8.957 " " " "

ADJUSTMENT OF POINT R.

The line N to R is a parallel duplicate line of levels.

	Observed values	v	v^2
	8.867 ft.	- 0.045	0.002025
	8.957 ft.	+ 0.045	0.002025
2z =	17.824	0.000	0.004050 $=\Sigma v^2$
z =	8.912 ft.	Check	n=2.(No. of Obs.)

$$r_A = 0.6745 \sqrt{\frac{0.004050}{2(2-1)}} = \pm 0.0304 \text{ ft.}$$

Then the most probable value for the difference in elevation = 8.912± 0.0304 ft.

And the most probable elevation of the point R is as follows:

Adjusted value of point N =	875.092 ft.
Adjusted Diff. in elev. N to R =	- 8.912
Most probable elev. point R =	866.180 ft.

ADJUSTMENT OF POINT S.

Duplicate Lines of Levels: By a duplicate line of levels is meant a line that is run twice over the same route with equal care, but in opposite directions.

Letting d represent the discrepancy between the result obtained from the forward line and that obtained from the reverse line, we have:

$$v_1 = + \frac{d}{2} \qquad \text{and} \qquad v_2 = - \frac{d}{2}$$

Substituting these values in the following equation and replacing r_1 with r for the case of duplicate lines, we have the probable error of a single determination (forward or reverse) by a line of the length 1:

$$r = 0.6745 \sqrt{\frac{\Sigma v^2}{n-1}}$$

$$\Sigma v^2 = v_1^2 + v_2^2 = \left(+\frac{d}{2}\right)^2 + \left(-\frac{d}{2}\right)^2 = +\frac{2d^2}{4}$$

$$\Sigma v^2 = + \frac{d^2}{2}$$

Substituting:

$$r_L = 0.6745 \sqrt{\frac{d^2}{2(n-1)}} = \frac{0.6745}{1.4142136} \sqrt{\frac{d^2}{n-1}}$$

In the case of duplicate measurements, $(n - 1) = (2 - 1) = 1$.

Then:

$$r_L = 0.4769 \sqrt{d^2} = \pm 0.4769\ d.$$

And for the error of the arithmetic mean in duplicate leveling:

$$r_A = \frac{r_L}{\sqrt{n}} = \frac{\pm 0.4769\ d}{\sqrt{2}} = \frac{\pm 0.4769\ d}{1.4142136} = \pm 0.3373\ d.$$

$$r_A \text{(approximately)} = \pm 1/3\ d.$$

ADJUSTMENT OF POINT S.

Observed elevation of point O = 969.758 ft (From South)
" " " " " " S = 942.516 "
Difference in elevation = 27.242 (Going North)

Observed elevation of point S = 942.516 ft.
" " " " " " O = 969.743 " (From North)
Difference in elevation = 27.227 (Going South)

Observed Values

27.242 ft. Then: $r_L = \pm 0.4769 \times 0.015 = \pm 0.0071$ ft.
27.227 ft. And: $r_A = \pm 0.3373 \times 0.015 = \pm 0.0051$ ft.
d = 0.015 ft. and 1/2 d = 0.0075 ft.

Then the most probable value for the difference in elevation = 27.242 - 0.0075 = 27.2495 ± 0.0051 ft.

And the most probable elevation of the point S is as follows:

Adjusted value of point O = 969.702 ft. (On Circuit)
Adjusted Diff. in elev. O to S = -27.250 ft.
Most probable elev. point S = 942.452 ft.

SUMMARY OF THE ELEVATIONS OF ALL OF THE JUNCTION POINTS.

A = 1087.8000 ft.	G = 991.4790 ft.	N = 875.0920 ft.
B = 1074.6530 ft.	H = 1041.8655 ft.	O = 969.7020 ft.
C = 1045.3660 ft.	I = 1061.9233 ft.	P = 928.5780 ft.
D = 1038.8580 ft.	J = 968.4311 ft.	Q = 997.2560 ft.
E = 1074.9050 ft.	K = 1092.6040 ft.	R = 865.1800 ft.
F = 1001.4715 ft.	L = 1078.4270 ft.	S = 942.4520 ft.
	M = 954.7656 ft.	

THE DETERMINATION AND ADJUSTMENT OF ELEVATIONS IN PHELPS COUNTY MISSOURI.

(Dawson Township Level Net)

ADJUSTMENT OF INTERMEDIATE TEMPORARY BENCH MARKS BETWEEN THE POINTS A(at St.James) AND I(at Miles School).

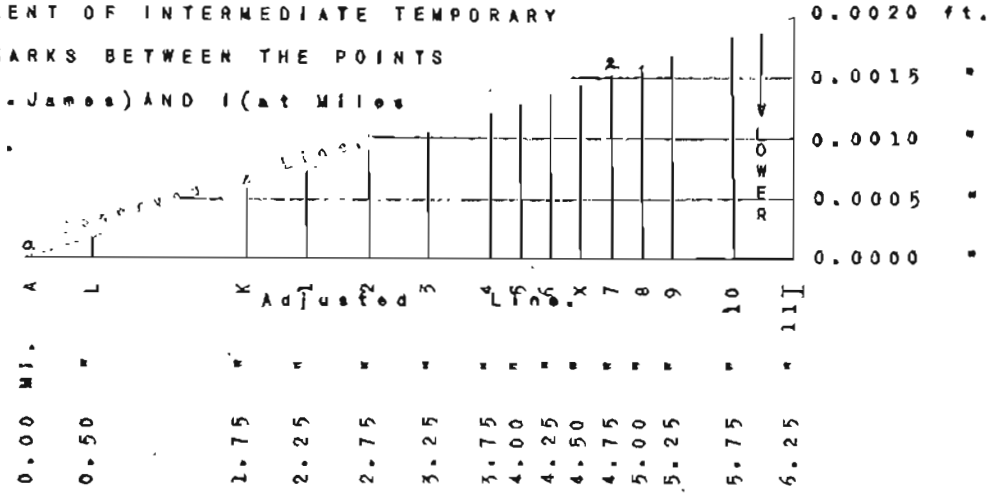


DIAGRAM TO SCALE SHOWING ADJUSTMENT OF ALL POINTS BETWEEN A AND I.

(See diagram on page)

The adjustment of these Intermediate Bench Marks can be effected by scaling the adjustment corrections from the above diagram, and subtracting each correction from the Observed Elevation as it appears in the Level Notes; or, if desired, the adjustment corrections can be computed by the following formula for this section(A to I):

FORMULA:

$$\text{Adjustment to Intermediate Point} = \frac{\text{Dist. A to Inter. Pt.}}{6.25} \times 0.002.$$

In this case, the Adjustment Corrections are to be subtracted from the Observed Elevations, as stated above.

MERAMEC SPRINGS AND RED BIRD QUADRANGLES.

PHELPS COUNTY.

ST. JAMES AND DAWSON TOWNSHIPS.

The elevations in the following list are based on a precise-level line run in 1908 by E.L.McNair from the United States Army Engineers' bench mark at Labadie Mo., the accepted elevation of which is 472.572 feet.

The leveling under adjustment was run in the Fall of 1921 by the Vocational Department of the Missouri School of Mines and Metallurgy from the United States Geological Survey bench marks at St.James and Knobview, the accepted elevations of which are 1087.800 feet and 1074.005 feet respectively.

MERAMEC SPRINGS QUADRANGLE.

(Latitude 37°45'-38°; longitude 91°30'-91°45')

(St James Township)

LINE FROM ST. JAMES NORTHWEST VIA JEFFERSON ROAD TO INTERSECTION WITH REPUBLIC ROAD, THENCE NORTH VIA REPUBLIC ROAD TO PHELPS-GASCONADE COUNTY LINE.

No.	Description	Elevation By Field Book *feet*	Adjustment Correction *feet*	Adjusted Elevation *feet*
A.	St. James, corner of Meramec and Washington Sts., about 70 ft. north of center line of Frisco R.R. track, 25 ft. from corner of Commercial Hotel, in angle between cross walk and side walk; iron post stamped "1098". (See Bulletin #568 page 10.)	1087.800	0.000	1087.800 P.B.M.

RED BIRD QUADRANGLE.

(Latitude 38°-38°15'; longitude 91°30'-91°45')

L.	St. James, 0.5 miles north of, on Jefferson road at intersection of Parker Lane road, concrete culvert, north end wall, top of; chiseled square.	1078.427	0.000	1078.427
K.	St. James, 1.75 miles north of, on Jefferson road, at intersection of Republic road, on south side of road, 10" black jack tree, in base of; copper nail and washer,	1092.694	- 0.001	1092.693

No.	Description	Elevation By Fld. Bk. feet	Cor. feet	Adj. Elev. feet
1.	St. James, 2.25 miles north of, on Republic road, Jefferson road, 0.5 miles north of, culvert, west end, sandstone slab, top of; painted square.	1034.345	-0.001	1034.344
2.	Jefferson road, 1.0 mile north of on Republic road, T road east, at Cox's residence, 100 ft. east of, sandstone boulder in N.E. Cor. of T road; topcorner broken off, painted white.	1024.715	-0.001	1024.714
3.	Jefferson road, 1.5 miles north of, Robinson creek, 0.75 miles south of, Fitzenreider's residence, 100 ft. west of, wagon gate post, in base of; copper nail and washer.	998.469	-0.001	998.468
4.	Robinson Creek, 0.25 miles south of, on Republic road, on east side of road, 10" post oak tree near Kenndy's mail box, in base of; nail.	923.575	-0.001	923.574
5.	Jefferson road, 2.25 miles north of, on Republic road, Robinson Creek, west bank of, 24" white oak tree, in root of; copper nail and washer.	917.604	-0.001	917.603
	(Dawson Township)			
6.	Robinson Creek, 0.25 miles north of, 15 ft. west of road, sandstone boulder on St. James-Dawson Twp. Line, opposite wire fence corner, point is on south hillside, 300 ft. south of Woodchopper's cabin; painted cross on top of boulder.	1022.394	-0.001	1022.393

No.	Description.	Elevation By Fld. Bk. feet	Cor. feet	Adjd. Elev. feet	
7.	Robinson Creek, 0.75 miles north of, on Republic road, Township line, 0.5 miles north of, at north end of ridge, 30 ft. west of road, sandstone boulder, top of; chiseled square. ----------	1039.423	-0.001	1039.422	
8.	Robinson Creek, 1.0 mile north of, on Republic road, at foot of hill, concrete culvert across road, W. end wall, top of; chiseled Sq.---	934.325	-0.002	934.323	
9.	Robinson Creek, 1.25 miles north of, Miles School House, 1.0 mile south of, T road west, center of roads; road elevation.---------	976.005	-0.002	976.003	
10.	Miles School House, 0.5 miles S. of, on Republic road, Marrs Cemetary, 400 ft. north of, east side of road, sandstone boulder at rail fence corner, top of; chiseled square. ----------	1059.077	-0.002	1059.075	
11.	St. James, 6.0 miles north of, on Republic road, Safe, 2.5 miles E. of, Miles School, 25 ft. W. of, S.W. Cor. S.E. Qtr. of the S.W. Qtr. Sec. 20, T. 39N., R. 6W., 60 ft. S.E. of, concrete well curb, top of; bronze tablet stamped" ".		-0.002		P.B.M.
I.	St. James, 6.0 miles north of, on Republic road, Safe, 2.5 miles E. of, cross roads at Miles School, northeast corner of school house, limestone wheel guard, top of; painted square. ----------	1061.925	-0.002	1061.923	

ADJUSTMENT OF INTERMEDIATE TEMPORARY BENCH MARKS

BETWEEN THE POINTS *I* (at Miles School) AND *P* (on Republic road one mile south of County Line.)

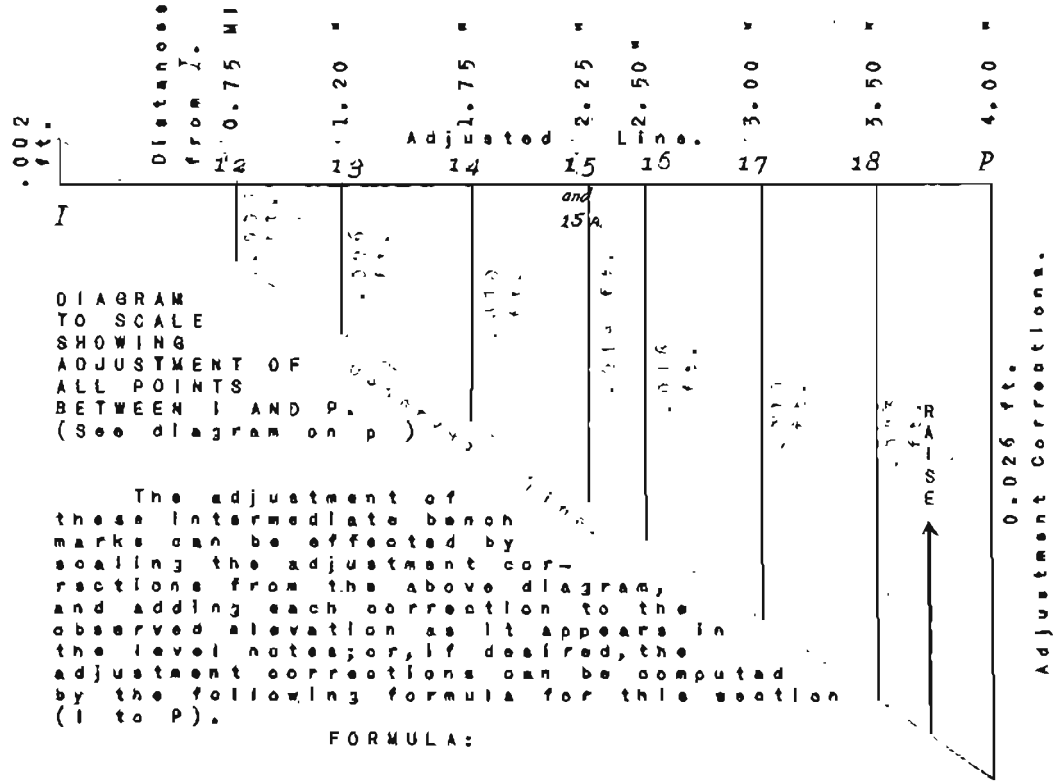


DIAGRAM TO SCALE SHOWING ADJUSTMENT OF ALL POINTS BETWEEN I AND P. (See diagram on p)

The adjustment of these intermediate bench marks can be effected by scaling the adjustment corrections from the above diagram, and adding each correction to the observed elevation as it appears in the level notes; or, if desired, the adjustment corrections can be computed by the following formula for this section (I to P).

FORMULA:

$$\text{Adj. Cor. to Inter. Pt.} = \left(\frac{0.028 \times \text{Dist. I to Inter. Pt.}}{4.0} - 0.002\right)$$

Add these corrections to the observed values.

No.	Description.	Elevation By Fld. Bk. feet	Cor. feet	Adjd. Elev. feet
I.	(See description on page)	1061.925	-0.002	1061.923
12.	Miles School, 0.75 miles north of, on Republic road, W. L. Martin's house, 150 ft. S. E. of, on top of hill, on east side of road, 12" post oak tree, in base of; copper nail and washer. ---	1023.857	+0.003	1023.860
13.	Miles School, 1.2 miles north of, on Republic road, Bourbeuse River, 0.75 miles south of ford, at dim cross road, southeast corner of, sandstone boulder; painted square. ----------	924.251	+0.006	924.257

No.	Description.	Elevation By Fld. Bk. feet	Cor. feet	Adjd. Elev. feet
14.	Miles School, 1.75 miles north of, on Republic road, Bourbeuse River, 0.2 miles south of, farm house, 600 ft. east of, on east side of road, 15" walnut tree, in base of; copper nail and washer. ----------	873.567	+0.010	873.577
15.	Wash School House, 0.75 miles south of, Bourbeuse River, at ford, 100 ft. south of, east bank of river, 30" sycamore tree, in base of; copper nail and washer.	839.998	+0.014	840.012
15a.	Bourbeuse River, at ford, on Republic road, center of river; road elevation. ----------	834.000	+0.014	834.014
16.	Bourbeuse River, 0.25 miles N. of, on Republic road, at T. road east, northeast corner of, in base of fence post; copper nail and washer. ----------	840.106	+0.016	840.122
17.	Bourbeuse River, 0.75 miles N. of, Wash School, 400 ft. S.W. of, J.W. Crossner's house, 300 ft. S.E. of, 8 ft. west of three mail boxes, on west side of road, 14" scaley bark hickory tree, in root of; copper nail and washer. ---	947.596	+0.019	947.615
18.	Wash School, 0.5 miles north of, on Republic road, Little Bourbeuse creek, at ford, on south bank, west side of road, 10" scaley bark hickory tree, in base of; copper nail and washer. -------	835.927	+0.023	835.950

No.	Description.	Elevation By Fld. Bk. feet	Cor. feet	Adjd. Elev. feet
P.	Wash School, 1.0 miles north of, Little Bourbeuse Creek, 0.5 Mi. north of, Harrison School, 0.5 Mi. south of, Phelps-Gasconade County Line, 1.0 mile south of, Austin E. Breuer's house, 200 ft. west of, at T. road east, west side of Republic road, 2 ft. inside of fence, 10" post oak tree, in base of; copper nail and washer. --------	928.552	+0.026	928.578

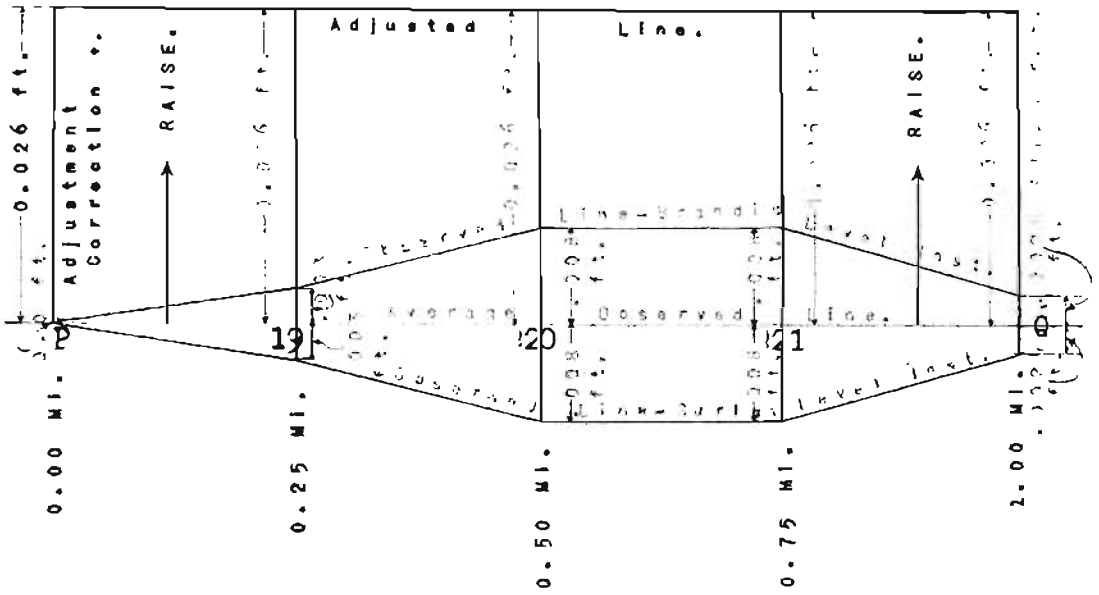


DIAGRAM TO SCALE SHOWING THE ADJUSTMENT OF ALL POINTS BETWEEN P AND Q.

It will be remembered that the line P to Q is a spur line of levels in our scheme of adjustment. The line P to Q was run as a parallel duplicate line of levels, and the most probable values for the elevations of the points on such a line is the average of the several determinations. The above diagram shows a (red) line of levels run by a Brandis Instrument and a (blue) line of levels run by a Gurley Instrument; as these instruments were of equal refinement, equal weight was applied to the work done by each instrument. The (green) line represents the average of the instrumental determinations. In the level net adjustment just preceeding, it was found that the point P was 0.026 ft. low. As the spur line P to Q hinges directly on the point P, then the whole line P to Q will have to be raised 0.026 ft. In the diagram this adjustment is shown by the (black) line.

The adjustment is a constant of 0.026 ft. which is to be added to each of the average instrumental observed values between P and O. This is appearant from the diagram.

FORMULA:

Adj.Cor.to Inter.Pt.=Av.Obs Value + 0.026.

No.	Description.		Elevation By Fld. Bk. feet	Cor. feet	Adjd. Elev. feet
P.	(See description on page 31)		928.552	+0.026	928.578
19.	Nash School, 1.25 miles north of, Little Bourbeuse Creek, 0.75 Mi. north of, Harrison School, 0.25 miles South of, Phelps-Gasconade County Line, 0.75 miles south of, Steve Lorts' house, 70 ft. north of, T. road west (Mail road to High Gate in Maries County), northwest corner of roads, 24" oak stump in fence corner, top of; ten penny nail.	Brandis	962.732		
		Gurley	962.726		
		Average	962.729	+0.026	962.755
20.	Harrison School House, on St. James-Republic road, west side of School Building, north concrete door step, top of; chiseled square.	Brandis	965.674		
		Gurley	965.658		
		Average	965.666	+0.026	965.692
21.	Harrison School House, 0.25 Mi. north of, on Republic road, Phelps-Gasconade County Line, 0.25 miles south of, private T. road west, southwest corner of, 16" blazed black oak tree; bent nail in base of.	Brandis	959.594		
		Gurley	959.578		
		Average	959.586	+0.026	959.612

No.	Description.	Elevation By Fld. Bk. feet	Cor. feet	Adjd. Elev. feet
Q.	Harrison School House, 0.50 miles north of, on Republic road, intersection St. James-Republic road and High Gate-Red Bird road, 0.5 miles south of, Phelps-Gasconade County Line, 28 ft. south of, center of road, 30 ft. west of, 5 ft. inside wire fence, in line with center of road north into Gasconade County, dim T. road west, 50 ft. south of; iron post stamped "997".----- Brandis 997.242			
	Gurley 997.237			
		997.240	+0.026	997.266 P.B.M.

This completes the description and adjustment of the line of levels from St. James northwest via Jefferson Road to intersection with Republic Road, thence north via Republic Road to Phelps-Gasconade County Line.

MERAMEC SPRINGS QUADRANGLE.

(Latitude 37°45'-38°; longitude 91°30'-91°45')

(St. James Township)

LINE FROM ST. JAMES EAST VIA OZARK HIGHWAY TO PHELPS-CRAWFORD COUNTY LINE, SAID POINT ONE MILE NORTHEAST OF KNOB-VIEW RAILROAD DEPOT.

ADJUSTMENT OF INTERMEDIATE BENCH MARKS ALONG THE ROUTE A VIA B AND C TO D. (A is the P.B.M. at St. James and D is the T.B.M. at the intersection of the Ozark Highway and the Phelps-Crawford County Line.)

BELOW IS DIAGRAM TO SCALE SHOWING THE ADJUSTMENT OF ALL INTERMEDIATE POINTS ALONG THE OZARK HIGHWAY BETWEEN "A" AT ST. JAMES AND "D" AT THE COUNTY LINE NEAR KNOBVIEW.

The adjustment of these intermediate bench marks can be effected by scaling the adjustment corrections from the diagram, and subtracting each correction from the observed elevation as it appears in the level notes and description books; or, if desired, the adjustment corrections can be computed by the following formula for the section A to D:

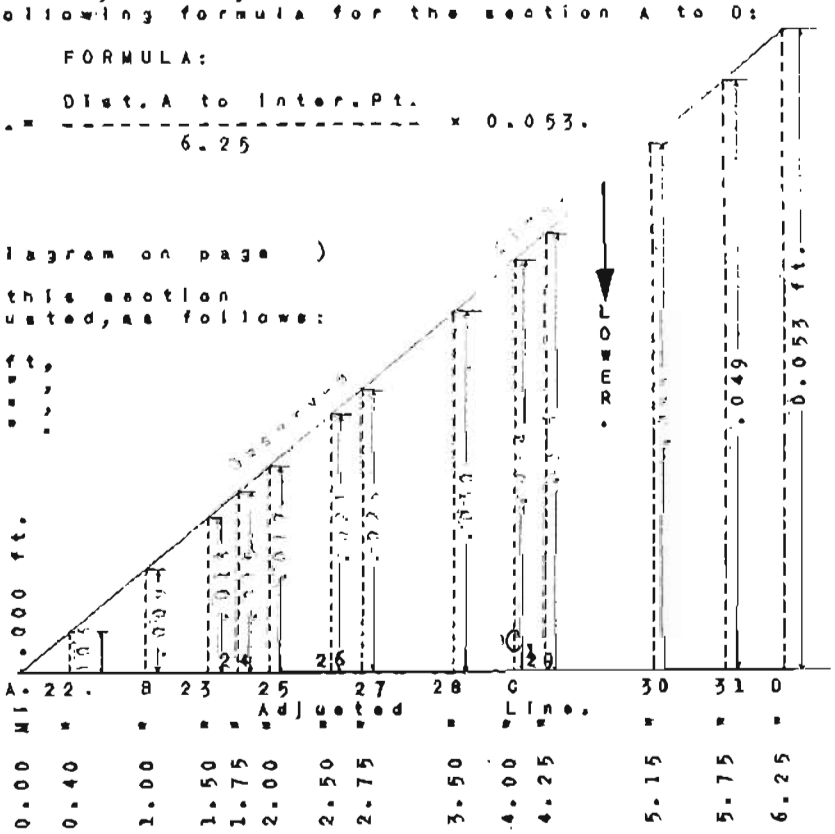
FORMULA:

$$\text{ADJ. OF INTER. PT.} = \frac{\text{Dist. A to Inter. Pt.}}{6.25} \times 0.053.$$

(See Diagram on page)

Elevations on this section previously adjusted, as follows:

A = 1087.8000 ft,
B = 1074.6580 " ,
C = 1083.3660 " ,
D = 1098.8580 " .



No.	Description.	Elevation By Fld. Bk. feet	Cor. feet	Adjd. Elev. feet	
A.	(See description on page 26)	1087.800	0.000	1087.800	P.B.M.
22.	St. James, 0.4 miles east of, on St. James-Cuba road (Ozark Highway), Frisco R.R., 50 ft north of, turn in road to north, 500 ft. W. of, concrete culvert across road, north end wall, top of; chiseled square.	1065.363	-0.003	1065.360	

RED BIRD QUADRANGLE.
(Latitude 38°-38°15'; longitude 91°30'-91°45')
(St. James Township)

B.	St. James, 1.0 mile N.E. of, on Ozark Highway, T. road east (intersection of Ozark Highway and Red Bird road), 400 ft. S. of, Dr. E. A. Scott's house, 300 ft W. of, on W. side of road, 10" elm tree, in root of; copper nail and washer.	1074.666	-0.009	1074.658	
23.	St. James, 1.5 miles N.E. of, on Ozark Highway, 0.5 miles east of Red Bird road intersection, on N. side of road, 12" post oak tree, in base of; copper nail and washer.	1062.669	-0.013	1062.656	
24.	St. James, 1.75 miles northeast of, on Ozark Highway, concrete culvert across road, south end wall, top of; chiseled triangle.	1034.437	-0.015	1034.422	
25.	St. James, 2.0 miles N.E. of, on Ozark Highway, on top of hill, center of road; road elevation.	1054.446	-0.017	1054.429	

No.	Description.	Elevation By Fld. Bk. feet	Cor. feet	Adj'd. Elev. feet	
26.	St. James, 2.5 miles northeast of, on Ozark Highway, Friendship School, 1.5 miles west of, concrete culvert (errected 8-13-20 by T. Hall), north end wall, top of; chiseled square.	1010.057	-0.021	1010.036	
27.	Friendship School, 1.25 miles west of, on Ozark Highway, at T. road N. to Hinkle P.O., Corrugated iron culvert, rock on east end of culvert, top of; chiseled triangle.	1018.926	-0.023	1018.903	
28.	Friendship School, 0.5 miles W. of, on Ozark Highway, Gerber's Store, 600 feet W. of, concrete culvert, north end wall, top of; chiseled triangle.	1062.045	-0.030	1062.015	
C.	Friendship School, on Ozark Highway, southwest corner of concrete porch of school house, top of; chiseled triangle, painted "1083.3"	1083.400	-0.034	1083.366	
C_1.	Friendship School, on Ozark Highway, 50 ft. S.W. of, in School Yard, 10 ft. from fence corner, 30 ft. N. of center of Ozark Highway, 30 ft. E. of center of T. road north; iron post stamped "1083".	1083.458	-0.034	1083.424	P.B.M.
29.	Friendship School, 0.25 miles E. of, private T. road N. to Louis Hall's residence, 75 ft. W. of, concrete culvert, S. end wall, top of; chiseled square.	1037.041	-0.036	1037.005	

No.	Description.	Elevation By Fld. Bk. feet	Cor. feet	Adj'd. Elev. feet
30.	Friendship School, 0.9 miles E. of, on Ozark Highway, Phelps-Crawford County Line, 1.1 miles west of, concrete culvert, south end wall, top of; chiseled square. ----------	1042.297	-0.044	1042.253
31.	Friendship School, 1.5 miles east of, on Ozark Highway, Phelps-Crawford County Line, 0.5 miles west of, Knobview, 0.5 miles north of across field, concrete culvert under Ozark Highway, north end wall, top of; chiseled square.-	1028.412	-0.049	1028.363
D.	Friendship School, 2.0 miles east of, on Ozark Highway, at cross roads on Phelps-Crawford County Line, southwest corner of roads, 14" hickory tree, in base of; copper nail and washer. ----------	1098.911	-0.053	1098.858

This completes the description and adjustment of the line of levels from St. James east via Ozark Highway to the Phelps-Crawford County Line, said point one mile northeast of Knobview Railroad Depot.

P A R T . . I I I .

THE LEAST SQUARE ADJUSTMENT OF A LEVEL NET BY METHOD OF OBSERVATION EQUATIONS.

APPLIED TO THE ADJUSTMENT
OF THE
DAWSON-ST. JAMES TOWNSHIPS LEVEL NET,
PHELPS COUNTY,
MISSOURI.

(BASED ON THE LINE OF LEVELS RUN BY THE U. S. GEOLOGICAL SURVEY IN 1908 ALONG THE FRISCO R. R. WITH PERMANENT BENCHMARKS, "A" AT ST. JAMES, AND "E" AT KNOBVIEW).

Showing elevations and closures when level circuits are arranged as shown in plat on page*40*.

A = 1087.800 (St.James) Adjusted value.
- 25.875

I = 1061.925 (Miles School)
-107.201

M = 954.724 (Royal)
+ 13.695

J = 968.419
+ 73.460

H = 1041.879 (Red Bird and Jakes Prairie Rds.)
+ 32.608

B = 1074.487 (Red Bird Rd.and Ozark Highway.)
+ 8.734

C = 1083.221 (Friendship School)
- 82.223

F = 1000.998
- 80.066

G = 920.932 (Jakes Prairie and Tribby Rds.)
+177.647

D = 1098.579 (Ozark Highway and County Line.)
- 24.844

E = 1073.735 (Knobview Depot.)
E = 1074.005 (" ") Adjusted value.
- 0.270 Closure. At Knobview.

C = 1083.221
+ 15.511
D = 1098.732
D = 1098.579 in arr.Cir.
+ 0.153 Closure at D.

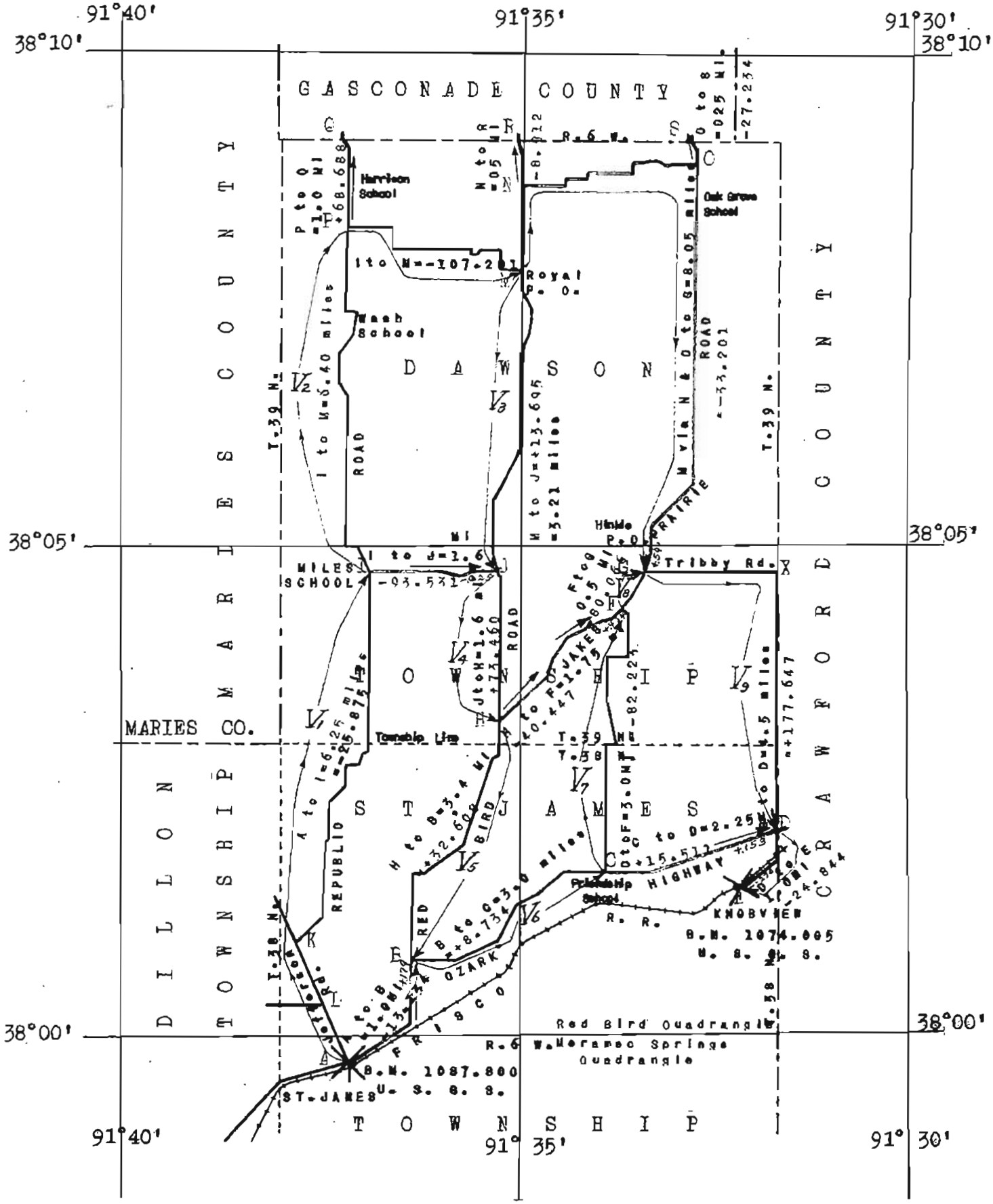
H = 1041.879
- 40.447
F = 1001.432
F = 1000.998 in arr.Cir.
+ 0.434 Closure at F.

I = 1061.925
- 93.531
J = 968.394
J = 968.419 in above arranged circuit.
- 0.025 Closure on circuit above.At J.

M = 954.724
- 33.201
G = 921.523
G = 920.932 in above arranged circuit.
+ 0.591 Closure on circuit above.At G.

A = 1087.800
- 13.134
B = 1074.666
B = 1074.487 in above arranged circuit.
+ 0.179 Closure on circuit above.At B.

Note: The values given on this page are not adjusted values,but only an arrangement to facilitate the adjustment that is to be made on the following pages by the theory of Least Squares.


91°40'
91°35'
91°30'
38°10'
38°05'
38°00'
GASCONADE COUNTY
DILLON TOWNSHIP
MARIES COUNTY
DAWSON TOWNSHIP
CRAWFORD COUNTY
MARIES CO.
ST. JAMES TOWNSHIP
Harrison School
Wash School
Royal P. O.
Oak Grove School
Hinkle P. O.
MILES SCHOOL
Tribby Rd.
Township Line
Friendship School
KNOBVIEW
B.M. 1074.005
U. S. G. S.
B.M. 1087.800
U. S. G. S.
ST. JAMES
Red Bird Quadrangle
Meramec Springs Quadrangle
R. 6 W.
T. 39 N.
T. 38 N.
REPUBLIC ROAD
OZARK TRAIL
PRAIRIE ROAD
BIRD
RED
FRISCO R. R.
HIGHWAY 68

Observation Equations.

(A to I)	$+V_1$								=	+0.000	(weight 0.160)
(I to M)		$+V_2$							=	+0.000	(weight 0.156)
(M to J)			$+V_3$						=	+0.000	(weight 0.312)
(J to H)				$+V_4$					=	+0.000	(weight 0.625)
(H to B)					$+V_5$				=	+0.000	(weight 0.294)
(B to C)						$+V_6$			=	+0.000	(weight 0.333)
(C to F)							$+V_7$		=	+0.000	(weight 0.333)
(F to G)								$+V_8$	=	+0.000	(weight 2.000)
(G to D)								$+V_9$	=	+0.000	(weight 0.222)
(D to E)	$-V_1$	$-V_2$	$-V_3$	$-V_4$	$-V_5$	$-V_6$	$-V_7$	$-V_8$	$-V_9$ =	−0.270	(weight 1.000)
(I to J)		$+V_2$	$+V_3$						=	−0.025	(weight 0.625)
(A to B)	$+V_1$	$+V_2$	$+V_3$	$+V_4$	$+V_5$				=	+0.179	(weight 1.000)
(H to F)					$+V_5$	$+V_6$	$+V_7$		=	+0.434	(weight 0.571)
(M to G)			$+V_3$	$+V_4$	$+V_5$	$+V_6$	$+V_7$	$+V_8$	=	+0.591	(weight 0.124)
(C to D)							$+V_7$	$+V_8$	$+V_9$ =	+0.153	(weight 0.445)

. Formation of the Normal Equations.

Normal Equations are formed in the following manner from the Observation Equations:

RULE: To form the normal equations for each one of the unknown quantities, multiply each observation equation by the product of the weight of that observation and the algebraic coefficient of that unknown quantity in that equation, and add the results.

Having formed the several normal equations, their solution as simple simultaneous equations gives the most probable values of the unknown quantities.

As simple, simultaneous equations of the first degree, the normal equations may be solved by any of the algebraic methods of eliminations; by addition or subtraction, by substitution, or by comparison. In fact, these methods are satisfactory when there are but two equations to be solved. But in larger sets, of three or more, it is possible to shorten the numerical work by taking advantage of the peculiar symmetry which all normal equation possess. It is much easier to solve a set of normal equations than a set of ordinary, simultaneous equations of the same number which do not have this symmetry.

Normal Equation in V_1 .

										Absolute Term
	$+0.160V_1$									$= +0.000000$
	$+1.000V_1$	$+1.000V_2$	$+1.000V_3$	$+1.000V_4$	$+1.000V_5$	$+1.000V_6$	$+1.000V_7$	$+1.000V_8$	$+1.000V_9$	$= +0.270000$
	$+1.000V_1$	$+1.000V_2$	$+1.000V_3$	$+1.000V_4$	$+1.000V_5$					$= +0.179000$
1)	$+2.160V_1$	$+2.000V_2$	$+2.000V_3$	$+2.000V_4$	$+2.000V_5$	$+1.000V_6$	$+1.000V_7$	$+1.000V_8$	$+1.000V_9$	$= +0.449000$

Normal Equation in V_2 .

		$+0.156V_2$								$= +0.000000$
	$+1.000V_1$	$+1.000V_2$	$+1.000V_3$	$+1.000V_4$	$+1.000V_5$	$+1.000V_6$	$+1.000V_7$	$+1.000V_8$	$+1.000V_9$	$= +0.270000$
		$+0.625V_2$	$+0.625V_3$							$= -0.015625$
	$+1.000V_1$	$+1.000V_2$	$+1.000V_3$	$+1.000V_4$	$+1.000V_5$					$= +0.179000$
2)	$+2.000V_1$	$+2.781V_2$	$+2.625V_3$	$+2.000V_4$	$+2.000V_5$	$+1.000V_6$	$+1.000V_7$	$+1.000V_8$	$+1.000V_9$	$= +0.433375$

Normal Equation in V_3 .

			$+0.312V_3$							$=+0.0000000$
	$+1.000V_1$	$+1.000V_2$	$+1.000V_3$	$+1.000V_4$	$+1.000V_5$	$+1.000V_6$	$+1.000V_7$	$+1.000V_8$	$+1.000V_9$	$= +0.270000$
		$+0.625V_2$	$+0.625V_3$							$= -0.015625$
	$+1.000V_1$	$+1.000V_2$	$+1.000V_3$	$+1.000V_4$	$+1.000V_5$					$= +0.179000$
			$+0.124V_3$	$+0.124V_4$	$+0.124V_5$	$+0.124V_6$	$+0.124V_7$	$+0.124V_8$		$=+0.0732840$
3)	$+2.000V_1$	$+2.625V_2$	$+3.061V_3$	$+2.124V_4$	$+2.124V_5$	$+1.124V_6$	$+1.124V_7$	$+1.124V_8$	$+1.000V_9$	$=+0.5066590$

Normal Equation in V_4 .

				$+0.625V_4$						$= +0.000000$
	$+1.000V_1$	$+1.000V_2$	$+1.000V_3$	$+1.000V_4$	$+1.000V_5$	$+1.000V_6$	$+1.000V_7$	$+1.000V_8$	$+1.000V_9$	$= +0.270000$
	$+1.000V_1$	$+1.000V_2$	$+1.000V_3$	$+1.000V_4$	$+1.000V_5$					$= +0.179000$
			$+0.124V_3$	$+0.124V_4$	$+0.124V_5$	$+0.124V_6$	$+0.124V_7$	$+0.124V_8$		$= +0.073284$
4)	$+2.000V_1$	$+2.000V_2$	$+2.124V_3$	$+2.749V_4$	$+2.124V_5$	$+1.124V_6$	$+1.124V_7$	$+1.124V_8$	$+1.000V_9$	$= +0.522284$

Normal Equation in V_5 .

									Absolute Term
				+0.284V_5					= +0.000000
	+1.000V_1	+1.000V_2	+1.000V_3	+1.000V_4	+1.000V_5	+1.000V_6	+1.000V_7	+1.000V_8 +1.000V_9	= +0.270000
	+1.000V_1	+1.000V_2	+1.000V_3	+1.000V_4	+1.000V_5				= +0.179000
					+0.571V_5	+0.571V_6	+0.571V_7		= +0.247814
			+0.124V_3	+0.124V_4	+0.124V_5	+0.124V_6	+0.124V_7	+0.124V_8	= +0.073284
5)	+2.000V_1	+2.000V_2	+2.124V_3	+2.124V_4	+2.989V_5''	+1.695V_6	+1.695V_7	+1.124V_8 +1.000V_9	= +0.770098

Normal Equation in V_6 .

					+0.333V_6				= +0.000000
	+1.000V_1	+1.000V_2	+1.000V_3	+1.000V_4	+1.000V_5	+1.000V_6	+1.000V_7	+1.000V_8 +1.000V_9	= +0.270000
					+0.571V_5	+0.571V_6	+0.571V_7		= +0.247814
			+0.124V_3	+0.124V_4	+0.124V_5	+0.124V_6	+0.124V_7	+0.124V_8	= +0.073284
6)	+1.000V_1	+1.000V_2	+1.124V_3	+1.124V_4	+1.695V_5	+2.028V_6	+1.695V_7	+1.124V_8 +1.000V_9	= +0.591098

Normal Equation in V_7 .

							+0.333V_7		= +0.000000
	+1.000V_1	+1.000V_2	+1.000V_3	+1.000V_4	+1.000V_5	+1.000V_6	+1.000V_7	+1.000V_8 +1.000V_9	= +0.270000
					+0.571V_5	+0.571V_6	+0.571V_7		= +0.247814
			+0.124V_3	+0.124V_4	+0.124V_5	+0.124V_6	+0.124V_7	+0.124V_8	= +0.073284
							+0.445V_7	+0.445V_8 +0.445V_9	= +0.068085
7)	+1.000V_1	+1.000V_2	+1.124V_3	+1.124V_4	+1.695V_5	+1.695V_6	+2.473V_7	+1.569V_8 +1.445V_9	= +0.659183

Normal Equation in V_8 .

								+2.000V_8	= +0.000000
	+1.000V_1	+1.000V_2	+1.000V_3	+1.000V_4	+1.000V_5	+1.000V_6	+1.000V_7	+1.000V_8 +1.000V_9	= +0.270000
			+0.124V_3	+0.124V_4	+0.124V_5	+0.124V_6	+0.124V_7	+0.124V_8	= +0.073284
							+0.445V_7	+0.445V_8 +0.445V_9	= +0.068085
8)	+1.000V_1	+1.000V_2	+1.124V_3	+1.124V_4	+1.124V_5	+1.124V_6	+1.569V_7	+3.569V_8 +1.445V_9	= +0.411369

Normal Equation in V_9 .

$$+0.222V_9 = +0.000000$$
$$+1.000V_1 +1.000V_2 +1.000V_3 +1.000V_4 +1.000V_5 +1.000V_6 +1.000V_7 +1.000V_8 +1.000V_9 = +0.270000$$
$$+0.445V_7 +0.445V_8 +0.445V_9 = +0.068085$$

9) $+1.000V_1 +1.000V_2 +1.000V_3 +1.000V_4 +1.000V_5 +1.000V_6 +1.445V_7 +1.445V_8 +1.667V_9 = +0.338085$

(0)

NORMAL EQUATIONS.

	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	Absolute Term	
1)	+2.160	+2.000	+2.000	+2.000	+2.000	+1.000	+1.000	+1.000	+1.000	−0.449000	= 0
2)	+2.000	+2.781	+2.625	+2.000	+2.000	+1.000	+1.000	+1.000	+1.000	−0.433375	= 0
3)	+2.000	+2.625	+3.061	+2.124	+2.124	+1.124	+1.124	+1.124	+1.000	−0.506659	= 0
4)	+2.000	+2.000	+2.124	+2.749	+2.124	+1.124	+1.124	+1.124	+1.000	−0.522284	= 0
5)	+2.000	+2.000	+2.124	+2.124	+2.989	+1.695	+1.695	+1.124	+1.000	−0.770098	= 0
6)	+1.000	+1.000	+1.124	+1.124	+1.695	+2.028	+1.695	+1.124	+1.000	−0.591098	= 0
7)	+1.000	+1.000	+1.124	+1.124	+1.695	+1.695	+2.473	+1.569	+1.445	−0.659183	= 0
8)	+1.000	+1.000	+1.124	+1.124	+1.124	+1.124	+1.569	+3.569	+1.445	−0.411369	= 0
9)	+1.000	+1.000	+1.000	+1.000	+1.000	+1.000	+1.445	+1.445	+1.667	−0.338085	= 0

(Numbers below the red line are redundant numbers.)

(P) Solution of Normal Equations.

	1	2	3	4	5	6	7	8	9	Absoluto Term	
P_1	+2.1600	+2.00000	+2.00000	+2.00000	+2.00000	+1.00000	+1.00000	+1.00000	+1.00000	−0.449000	P_1
P_2	(−0.4630)	−0.92600	−0.92600	−0.92600	−0.92600	−0.46300	−0.46300	−0.46300	−0.46300	+0.207887	P_2
P_3	. . . .	+0.92900	+0.77300	+0.14800	+0.14800	+0.07400	+0.07400	+0.07400	+0.07400	−0.017601	P_3
P_4	. . . .	(−1.0764)	−0.83206	−0.15931	−0.15931	−0.07965	−0.07965	−0.07965	−0.07965	+0.018946	P_4
P_5	. . . . .	+0.56582	+0.14886	+0.14886	+0.13643	+0.13643	+0.13643	+0.01243	−0.076240	P_5	
P_6	. . . . .	(−1.7673)	−0.26308	−0.26308	−0.24111	−0.24111	−0.24111	−0.02107	+0.134739	P_6	
P_7	. . . . .	+0.83426	+0.20926	+0.15032	+0.15032	+0.15032	+0.05894	−0.083649	P_7		
P_8	. . . . .	(−1.1987)	−0.25084	−0.18019	−0.18019	−0.18019	−0.06065	+0.100270	P_8		
P_9	. . . . .	+1.02177	+0.68361	+0.11261	+0.11261	+0.04416	−0.310480	P_9			
P_{10}	. . . . .	(−0.9787)	−0.66905	−0.66905	−0.11021	−0.04322	+0.303867	P_{10}			
P_{11}	. . . . .	+1.04176	+0.70876	+0.51979	+0.48794	−0.140627	P_{11}				
P_{12}	. . . . .	(−0.9599)	−0.68034	−0.49895	−0.46837	+0.134988	P_{12}				
P_{13}	. . . . .	+1.00456	+0.61116	+0.60097	−0.113038	P_{13}					
P_{14}	. . . . .	(−0.9955)	−0.60841	−0.59827	+0.112529	P_{14}					
P_{15}	. . . . .	+2.39653	+0.34852	+0.004532	P_{15}						
P_{16}	. . . . .	(−0.41727)	−0.14543	−0.001891	P_{16}						
P_{17}	. . . . .	+0.55359	+0.024204	P_{17}							
P_{18}	. . . . .	(−1.8064)	−0.043722	P_{18}							

(q) Solution of Normal Equations.

	2	3	4	5	6	7	8	9	Absolute Term	
a_1	+2.78100	+2.62500	+2.00000	+2.00000	+1.00000	+1.00000	+1.00000	+1.00000	-0.433375	a_1
a_2	-1.85200	-1.85200	-1.85200	-1.85200	-0.92600	-0.92600	-0.92600	-0.92600	+0.415774	a_2
	+0.92900	+0.77300	+0.14800	+0.14800	+0.07400	+0.07400	+0.07400	+0.07400	-0.017601	
a_3	. . . .	+3.06100	+2.12400	+2.12400	+1.12400	+1.12400	+1.12400	+1.00000	-0.506659	a_3
a_4	. . . .	-1.85200	-1.85200	-1.85200	-0.92600	-0.92600	-0.92600	-0.92600	+0.415774	a_4
a_5	. . . .	-0.64318	-0.12314	-0.12314	-0.06157	-0.06157	-0.06157	-0.06157	+0.014645	a_5
		+0.56582	+0.14886	+0.14886	+0.13643	+0.13643	+0.13643	+0.01243	-0.076240	
a_6	. . . . .	. . . .	+2.74900	+2.12400	+1.12400	+1.12400	+1.12400	+1.00000	-0.522284	a_6
a_7	. . . . .	. . . .	-1.85200	-1.85200	-0.92600	-0.92600	-0.92600	-0.92600	+0.415774	a_7
a_8	. . . . .	. . . .	-0.02358	-0.02358	-0.01179	-0.01179	-0.01179	-0.01179	+0.002804	a_8
a_9	. . . . .	. . . .	-0.03916	-0.03916	-0.03589	-0.03589	-0.03589	-0.00327	+0.020057	a_9
			+0.83426	+0.20926	+0.15032	+0.15032	+0.15032	+0.05894	-0.083649	
a_{10}	. . . . .	. . . .	. . . .	+2.98900	+1.69500	+1.69500	+1.12400	+1.00000	-0.770098	a_{10}
a_{11}	. . . . .	. . . .	. . . .	-1.85200	-0.92600	-0.92600	-0.92600	-0.92600	+0.415774	a_{11}
a_{12}	. . . . .	. . . .	. . . .	-0.02358	-0.01179	-0.01179	-0.01179	-0.01179	+0.002804	a_{12}
a_{13}	. . . . .	. . . .	. . . .	-0.03916	-0.03589	-0.03589	-0.03589	-0.00327	+0.020057	a_{13}
a_{14}	. . . . .	. . . .	. . . .	-0.05249	-0.03771	-0.03771	-0.03771	-0.01478	+0.020983	a_{14}
				+1.02177	+0.68361	+0.68361	+0.11261	+0.04416	-0.310480	
a_{15}	. . . . .	. . . .	. . . .	. . . .	+2.02800	+1.69500	+1.12400	+1.00000	-0.591098	a_{15}
a_{16}	. . . . .	. . . .	. . . .	. . . .	-0.46300	-0.46300	-0.46300	-0.46300	+0.207887	a_{16}
a_{17}	. . . . .	. . . .	. . . .	. . . .	-0.00589	-0.00589	-0.00589	-0.00589	+0.001402	a_{17}
a_{18}	. . . . .	. . . .	. . . .	. . . .	-0.03289	-0.03289	-0.03289	-0.00300	+0.018382	a_{18}
a_{19}	. . . . .	. . . .	. . . .	. . . .	-0.02709	-0.02709	-0.02709	-0.01062	+0.015073	a_{19}
a_{20}	. . . . .	. . . .	. . . .	. . . .	-0.45737	-0.45737	-0.07534	-0.02955	+0.207727	a_{20}
					+1.04176	+0.70876	+0.51979	+0.48794	-0.140627	
a_{21}	. . . . .	. . . .	. . . .	. . . .	. . .	+2.47300	+1.56900	+1.44500	-0.659183	a_{21}
a_{22}	. . . . .	. . . .	. . . .	. . . .	. . .	-0.46300	-0.46300	-0.46300	+0.207887	a_{22}
a_{23}	. . . . .	. . . .	. . . .	. . . .	. . .	-0.00589	-0.00589	-0.00589	+0.001402	a_{23}
a_{24}	. . . . .	. . . .	. . . .	. . . .	. . .	-0.03289	-0.03289	-0.00300	+0.018382	a_{24}
a_{25}	. . . . .	. . . .	. . . .	. . . .	. . .	-0.02709	-0.02709	-0.01062	+0.015073	a_{25}
a_{26}	. . . . .	. . . .	. . . .	. . . .	. . .	-0.45737	-0.07534	-0.02955	+0.207727	a_{26}
a_{27}	. . . . .	. . . .	. . . .	. . . .	. . .	-0.48220	-0.35363	-0.33197	+0.095674	a_{27}
						+1.00456	+0.61116	+0.60097	-0.113038	
a_{28}	. . . . .	. . . .	. . . .	. . . .	. . .	. . . .	+3.56900	+1.44500	-0.411369	a_{28}
a_{29}	. . . . .	. . . .	. . . .	. . . .	. . .	. . . .	-0.46300	-0.46300	+0.207887	a_{29}
a_{30}	. . . . .	. . . .	. . . .	. . . .	. . .	. . . .	-0.00589	-0.00589	+0.001402	a_{30}
a_{31}	. . . . .	. . . .	. . . .	. . . .	. . .	. . . .	-0.03289	-0.00300	+0.018382	a_{31}
a_{32}	. . . . .	. . . .	. . . .	. . . .	. . .	. . . .	-0.02700	-0.01062	+0.015073	a_{32}
a_{33}	. . . . .	. . . .	. . . .	. . . .	. . .	. . . .	-0.01241	-0.00487	+0.034218	a_{33}
a_{34}	. . . . .	. . . .	. . . .	. . . .	. . .	. . . .	-0.25925	-0.24346	+0.070166	a_{34}
a_{35}	. . . . .	. . . .	. . . .	. . . .	. . .	. . . .	-0.37184	-0.36564	+0.068773	a_{35}
							+2.39653	+0.34852	+0.004532	
a_{36}	. . . . .	. . . .	. . . .	. . . .	. . .	. . . .	. . .	+1.66700	-0.338085	a_{36}
a_{37}	. . . . .	. . . .	. . . .	. . . .	. . .	. . . .	. . .	-0.46300	+0.207887	a_{37}
a_{38}	. . . . .	. . . .	. . . .	. . . .	. . .	. . . .	. . .	-0.00589	+0.001402	a_{38}
a_{39}	. . . . .	. . . .	. . . .	. . . .	. . .	. . . .	. . .	-0.00027	+0.001675	a_{39}
a_{40}	. . . . .	. . . .	. . . .	. . . .	. . .	. . . .	. . .	-0.00357	+0.005073	a_{40}
a_{41}	. . . . .	. . . .	. . . .	. . . .	. . .	. . . .	. . .	-0.00191	+0.013419	a_{41}
a_{42}	. . . . .	. . . .	. . . .	. . . .	. . .	. . . .	. . .	-0.22854	+0.065865	a_{42}
a_{43}	. . . . .	. . . .	. . . .	. . . .	. . .	. . . .	. . .	-0.35954	+0.067627	a_{43}
a_{44}	. . . . .	. . . .	. . . .	. . . .	. . .	. . . .	. . .	-0.05069	-0.000659	a_{44}
								+0.55359	+0.024204	

(x) Solution of Normal Equations.

	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9
r_1	+0.297887	+0.018946	+0.134739	+0.100270	+0.303867	+0.134988	+0.112529	−0.001891	−0.043722
r_2	+0.020243	+0.003482	+0.000961	+0.002652	+0.001890	+0.020478	+0.026158	+0.006358	
r_3	−0.002068	−0.000356	−0.001077	−0.000805	−0.000492	−0.002229	−0.002718	+0.004467	
r_4	−0.062954	−0.010830	−0.032783	−0.024500	−0.090970	−0.092505	+0.135969		
r_5	−0.028119	−0.004837	−0.014643	−0.010943	−0.040633	+0.060732			
r_6	−0.160811	−0.027666	−0.045687	−0.043561	+0.178622				
r_7	−0.021403	−0.003682	−0.006081	+0.023113					
r_8	−0.032807	−0.029479	+0.035429						
r_9	+0.050395	−0.054422							
r_{10}	−0.029637								

Check in Normal Equation (1): (Page)

−0.064015	+0.070858
−0.108844	+0.046226
−0.043722	+0.347324
−0.449000	+0.060732
	+0.135969
	+0.004467
−0.665581	+0.665576
−0.66558	+0.66558

Excellent check.

Adjustment Quantities:

$V_1 = -0.0296$ $V_6 = +0.0607$
$V_2 = -0.0544$ $V_7 = +0.1360$
$V_3 = +0.0354$ $V_8 = +0.0045$
$V_4 = +0.0231$ $V_9 = +0.1736$
$V_5 = -0.0437$

LINE			Observed difference in elev.	correction	Adjusted difference
A	to	I	= −25.875	−0.0296	= −25.9046 ft.
I	to	M	= −107.201	−0.0544	= −107.2554 "
M	to	J	= + 13.695	+0.0354	= + 13.7304 "
J	to	H	= +73.460	+0.0231	= + 73.4831 "
H	to	B	= + 32.608	+0.1737	= + 32.7817 "
B	to	C	= + 8.734	+0.0607	= + 8.7947 "
C	to	F	= − 82.223	+0.1360	= − 82.0870 "
F	to	G	= − 80.066	+0.0045	= − 80.0615 "
G	to	D	= +177.647	−0.0437	= +177.6033 "
D	to	E	= − 24.844	−0.0357	= − 24.8797 "

Adjusted Elevation of Junction Points.
In Dawson Township, Phelps County.

A = 1082.800 ft. − 25.9046 = I 1061.8954 " − 107.2554 = M 954.7404 " + 13.7304 = J 968.4704 " + 73.4831 = H 1041.9535 ft.

H = 1041.9535 ft. + 32.7817 = B 1074.7352 " + 8.7947 = C 1083.5299 " − 82.0870 = F 1001.4429 " − 80.0615 = G 921.3814 ft.

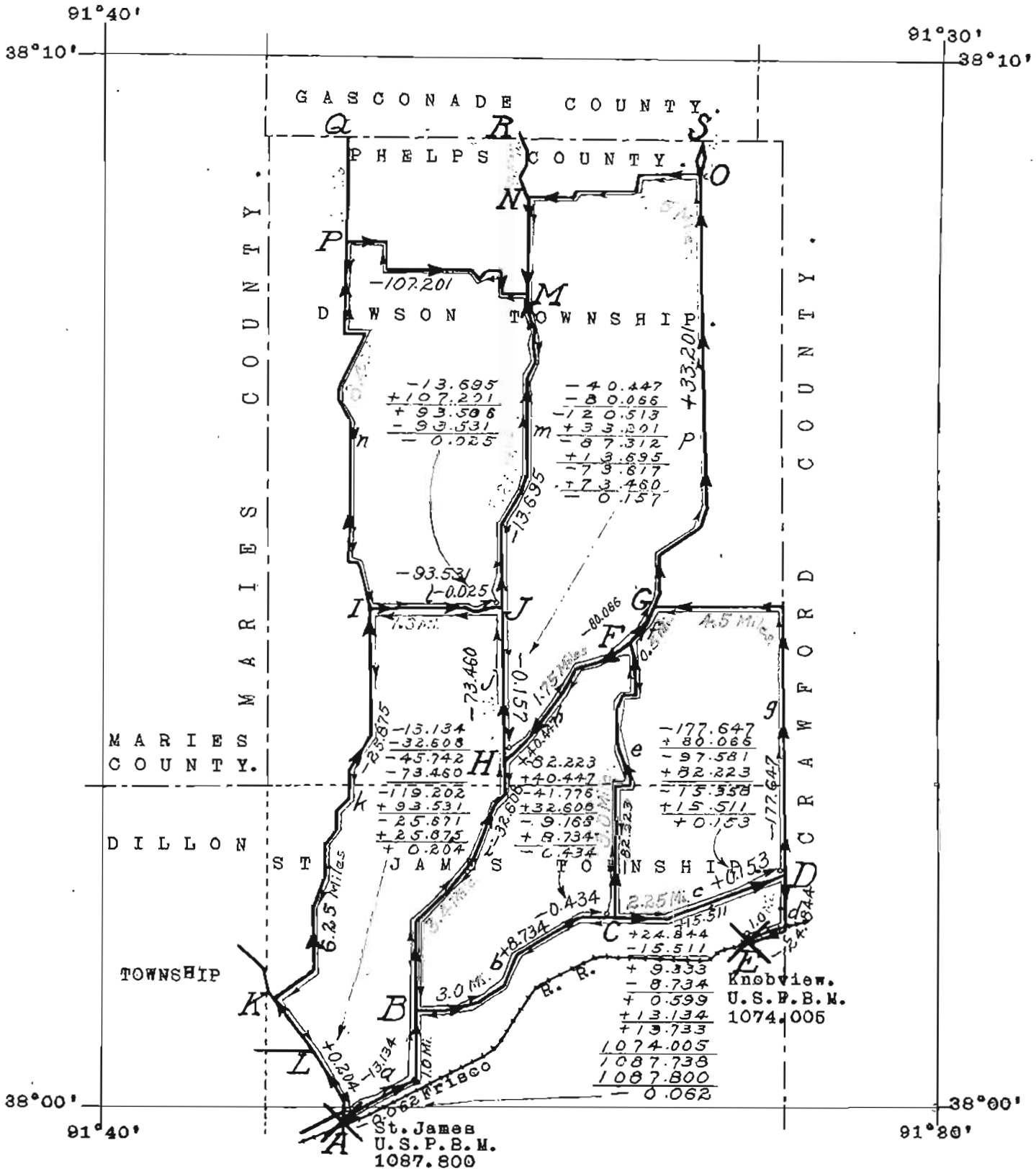
G = 921.3814 ft. + 177.6033 = D 1098.9847 " − 24.8797 = E 1074.0050 "

P A R T I V .

THE LEAST SQUARE ADJUSTMENT OF A LEVEL NET BY METHOD OF CONDITION EQUATIONS AND CORRELATIVES.

APPLIED TO THE ADJUSTMENT
OF THE
DAWSON-ST. JAMES TOWNSHIPS LEVEL NET,
PHELPS COUNTY,
MISSOURI.

(BASED ON THE LINE OF LEVELS RUN BY THE U. S. GEOLOGICAL SURVEY IN 1908 ALONG THE FRISCO R. R. WITH PERMANENT BENCHMARKS "A" AT ST. JAMES, AND "E" AT KNOBVIEW).


91°40'
91°30'
38°10'
38°10'
GASCONADE COUNTY.
PHELPS COUNTY.
MARIES COUNTY.
CRAWFORD COUNTY.
DILLON TOWNSHIP
DAWSON TOWNSHIP
ST JAMES TOWNSHIP
Q
R
S
O
N
P
M
I
J
G
F
H
e
g
k
K
B
L
C
c
D
d
E
A
-107.201
-13.695
+107.201
+93.506
-93.531
-0.025
-40.447
-80.066
-120.513
+33.201
-87.312
+13.695
-73.617
+73.460
-0.157
+33.201
-93.531
-0.025
-80.066
-73.460
-0.157
1.5 Mi.
1.75 Miles
0.5 Mi.
4.5 Mi.
-25.875
-13.134
-32.608
-45.742
-73.460
-119.202
+93.531
-25.871
+25.875
+0.204
-32.608
+40.447
82.223
+40.447
-41.776
+32.608
-9.168
+8.734
-0.434
-177.647
+80.066
-97.581
+82.223
-15.358
+15.511
+0.153
-177.647
+0.153
6.25 Miles
3.0 Mi.
2.25 Mi.
1.0 Mi.
-0.434
+8.734
+15.511
+24.844
-15.511
+9.333
-8.734
+0.599
+13.134
+13.733
1074.005
1087.738
1087.800
-0.062
+0.204
-13.134
1.0 Mi.
-0.062
St. James
U.S.P.B.M.
1087.800
Knobview.
U.S.P.B.M.
1074.005
R. R.

Least Square Adjustment of Level Net.

Link From	To	Observed Difference	Name	Miles Distance D.	P=Weight $=\frac{1}{D}$	Derived Correction	Corrected Difference
A	B	- 13.134	a	1.0	1.000	-0.0306	- 13.1646
B	C	+ 8.734	b	3.0	0.333	+0.0608	+ 8.7948
C	D	+ 15.511	c	2.25	0.444	-0.0569	+ 15.4541
D	E	- 24.844	d	1.0	1.000	-0.0354	- 24.8794
C	F	-82.223	e	3.0	0.333	+0.1367	- 82.0863
F	G	- 80.066	f	0.5	2.000	+0.0047	- 80.0613
D	G	-177.647	g	4.5	0.222	+0.0453	-177.6017
F	H	+ 40.447	h	1.75	0.571	+0.0633	+ 40.5103
B	H	- 32.608	i	3.4	0.294	-0.1731	- 32.7811
H	J	- 73.460	j	1.6	0.625	-0.0238	- 73.4838
A	I	- 25.875	k	6.25	0.160	-0.0294	- 25.9044
I	J	- 93.531	l	1.6	0.625	+0.0076	- 93.5234
J	M	- 13.695	m	3.21	0.312	-0.0353	- 13.7303
I	M	-107.201	n	6.4	0.156	-0.0543	-107.2553
G	M	+ 33.201	p	8.05	0.124	+0.1568	+ 33.3578

Condition Equations.

1) $0 = -0.062 - d - c - b - a$
2) $0 = +0.153 + g - f - e + c$
3) $0 = -0.434 + e + h - i + b$
4) $0 = -0.157 - h + f + p - m - j$
5) $0 = +0.204 + a + i + j - l - k$
6) $0 = -0.025 + m - n + l$

Each complete circuit furnishes the condition that the sum of its adjusted differences of elevation shall equal zero when given the proper signs as if run continuously around the circuit, clockwise or counter-clockwise.

Number of Conditions = (No. of lines) - (No. of Jnct. points) + 1.
" " " " = +15 - 10 + 1 = 6.

The minus signs applied to the small case letters in the above condition equations result from changing the direction of the arrows (when red arrow goes in opposite direction from the black arrows in in diagram on preceeding page) so as to be continuous around each circuit. It is not necessary that all the circuits be traversed in the same direction in a given problem, but for clearness the work will appear more consistant if all the circuits are traversed in one direction, either clockwise or counter-clockwise.

TABLE OF CORRELATES.

Lines	1/10 p	1	2	3	4	5	6	Σ
a	.10	-1	..	..	..	+1	..	0
b	.30	-1	..	+1	..	..	..	0
c	.225	-1	+1	..	..	..	..	0
d	.10	-1	..	..	..	..	..	-1
e	.30	..	-1	+1	..	..	..	0
f	.05	..	-1	..	+1	..	..	0
g	.45	..	+1	..	..	..	..	+1
h	.175	..	..	+1	-1	..	..	0
i	.34	..	..	-1	..	+1	..	0
j	.16	..	..	..	-1	+1	..	0
k	.625	..	..	..	..	-1	..	-1
l	.16	..	..	..	..	-1	+1	0
m	.321	..	..	..	-1	..	+1	0
n	.64	..	..	..	..	..	-1	-1
p	.805	..	..	..	+1	..	..	+1

Product of Cs with Correlates.

1	2	3	4	5	6	Sum	(Sum × 1/10 p)	Lines
+.3535	+.1006	+.5563	+.1948	+.0471	+.0848			
-.3535	.....	.....	.....	+.0471	.....	-.3064	-.0306	a
-.3535	.....	+.5563	.....	.....	.....	+.2028	+.0608	b
-.3535	+.1006	.....	.....	.....	.....	-.2529	-.0569	c
-.3535	.....	.....	.....	.....	.....	-.3535	-.0354	d
.....	-.1006	+.5563	.....	.....	.....	+.4557	+.1367	e
.....	-.1006	.....	+.1948	.....	.....	+.0942	+.0047	f
.....	+.1006	.....	.....	.....	.....	+.1006	+.0453	g
.....	.....	+.5563	-.1948	.....	.....	+.3615	+.0633	h
.....	.....	-.5563	.....	+.0471	.....	-.5092	-.1731	i
.....	.....	.....	-.1948	+.0471	.....	-.1477	-.0236	j
.....	.....	.....	.....	-.0471	.....	-.0471	-.0294	k
.....	.....	.....	.....	-.0471	+.0848	+.0477	+.0076	l
.....	.....	.....	-.1948	.....	+.0848	-.1100	-.0353	m
.....	.....	.....	.....	.....	-.0848	-.0848	-.0543	n
.....	.....	.....	+.1948	.....	.....	+.1948	+.1568	p

NORMAL EQUATIONS.

The normal equations may be written by inspection as follows owing to the simplicity of the condition equations.

To form the Normal Equations:

1st. Square each quantity in column 1 left hand side table of correlates and multiply by its 1/10 p, and add the results= +0.7250;

2nd. Multiply each quantity in column 1 by quantities in column 2 on same line, multiply by their 1/10 p, and add results= -0.2250;

3d. Multiply each quantity in column 1 by quantities in column 3 on same line, multiply by their 1/10 p, and add results= -0.3000;

4th. Multiply each quantity in column 1 by quantities in column 4 on same line, multiply by their 1/10 p, and add results= +0.0000;

5th. Multiply each quantity in column 1 by quantities in column 5 on same line, multiply by their 1/10 p, and add results= -0.1000;

6th. Multiply each quantity in column 1 by quantities in column 6 on same line, multiply by their 1/10 p, and add results= +0.0000.

This gives the first normal equation.

7th. Square each quantity in column 2 and multiply by its 1/10 p, and add results=+1.0250;

8th. Multiply each quantity in column 2 by each adjacent quantity in turn in the other columns by their 1/10 p as above and add results;

Nth. Proceed as above until all terms have been multiplied, and the 6th normal equation has been completed.

NORMAL EQUATIONS.

1	2	3	4	5	6	(q)	Sum	Check
+0.7250	-0.2250	-0.3000	+0.0000	-0.1000	+0.0000	-0.062	+0.1000	+0.0380
-0.2250	+1.0250	-0.3000	-0.0500	+0.0000	+0.0000	+0.153	+0.4500	+0.6030
-0.3000	-0.3000	+1.1150	-0.1750	-0.3400	+0.0000	-0.434	+0.0000	-0.4340
+0.0000	-0.0500	-0.1750	+1.5100	-0.1600	-0.3210	-0.157	+0.8040	+0.6470
-0.1000	+0.0000	-0.3400	-0.1600	+1.3850	-0.1600	+0.204	+0.6250	+0.8290
+0.0000	+0.0000	+0.0000	-0.3210	-0.1600	+1.1210	-0.025	+0.6400	+0.6150

Note: Sum and Check columns will not be used in the following solution of the above normal equations; but will be used later in a subsequent solution of normal equations. The (q) is the absolute term.

(P) Solution of Normal Equations.

	1	2	3	4	5	6	ABSOLUTE TERM
P_1	+0.7250	-0.2250	-0.3000	+0.0000	-0.1000	+0.0000	-0.0620000
P_2	(-1.3793)	+0.3103	+0.4138	+0.0000	+0.1379	-0.0000	+0.0855166
P_3	. . . .	+0.9552	-0.3931	-0.0500	-0.0310	+0.0000	+0.1337614
P_4	. . . .	(-1.0469)	+0.4115	+0.0523	+0.0325	-0.0000	-0.1400348
P_5	. . . . .	+0.8291	-0.1956	-0.3942	+0.0000	-0.4046128	
P_6	. . . . .	(-1.2061)	+0.2359	+0.4754	-0.0000	+0.4880035	
P_7	. . . . .	+1.4613	-0.2546	-0.3210	-0.2454525		
P_8	. . . . .	(-0.6843)	+0.1742	+0.2197	+0.1679631		
P_9	. . . . .	+1.1384	-0.2159	-0.0353133			
P_{10}	. . . . .	(-0.8784)	+0.1896	+0.0310192			
P_{11}	. . . . .	+1.0096	-0.0856213				
P_{12}	. . . . .	(-0.9905)	+0.0848079				

Notes on
The P,Q,R Solution of Normal Equations.

Opposite page *118* of this thesis will be found the Least Square figure adjustment of a Geodetic Quadralateral on the folded plate. The problem on the plate was taken from the Topographic Instructions of the United States Geological Survey page 61 to page 71, for the instruction of students in my class in Geodetic Computations. On comparison with the Geological Survey publication in the description of the method for solving normal equations, it will be found that the encluded plate goes much more into the details of the method.

As the P,Q,R, solution of normal equations used here in this level net adjustment is identical with that used in the adjustment of the triangulation scheme, by referring to the plate opposite page *118*, it will be found very easy to follow the process and steps in the given computation and also in other computations in this thesis where the P,Q,R solution of normal equations has been employed.

(g) Solution of Normal Equations.

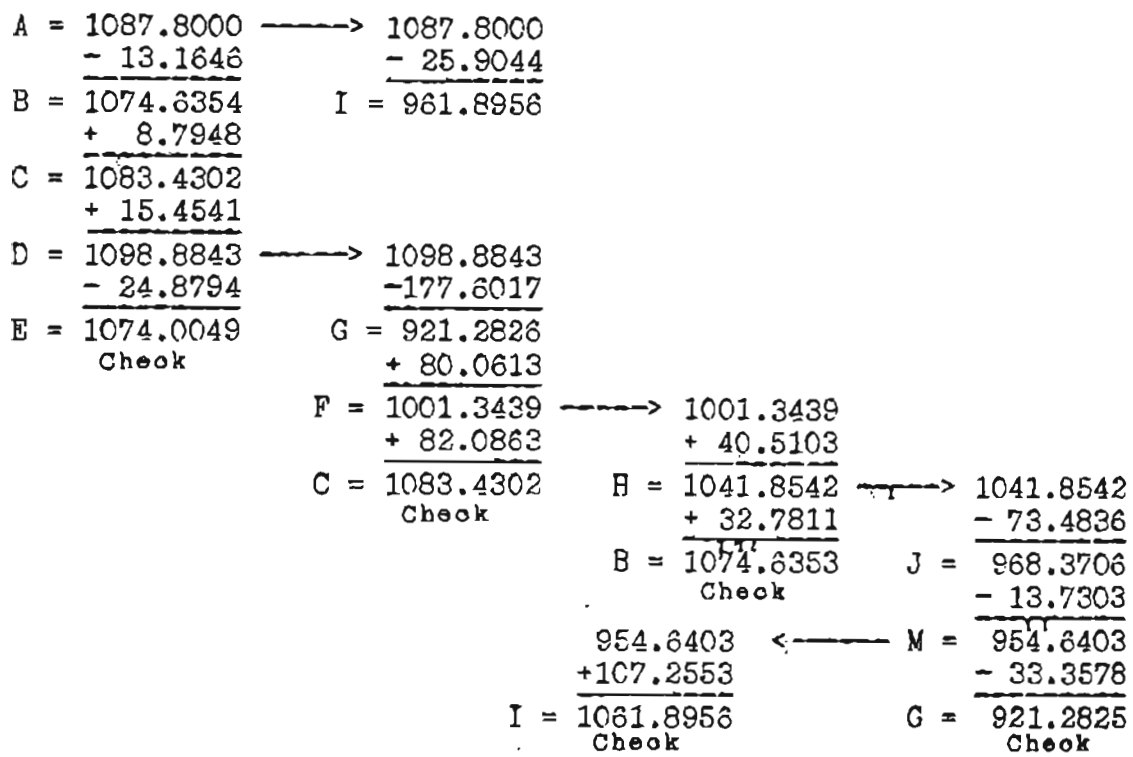
	2	3	4	5	6	ABSOLUTE TERM
q_1	+1.0250	-0.3000	-0.0500	+0.0000	+0.0000	+0.1530000
q_2	-0.0698	-0.0931	+0.0000	-0.0310	+0.0000	-0.0192386
	+0.9552	-0.3931	-0.0500	-0.0310	+0.0000	+0.1337614
q_3		+1.1150	-0.1750	-0.3400	+0.0000	-0.4340000
q_4		-0.1241	+0.0000	-0.0414	+0.0000	-0.0256556
q_5		-0.1618	-0.0206	-0.0128	+0.0000	+0.0550428
		+0.8291	-0.1956	-0.3942	+0.0000	-0.4046128
q_6			+1.5100	-0.1600	-0.3210	-0.1570000
q_7			-0.0000	+0.0000	-0.0000	-0.0000000
q_8			-0.0026	-0.0016	+0.0000	+0.0069957
q_9			-0.0461	-0.0930	+0.0000	-0.0954482
			+1.4613	-0.2546	-0.3210	-0.2454525
q_{10}				+1.3850	-0.1600	+0.2040000
q_{11}				-0.0138	+0.0000	-0.0085498
q_{12}				-0.0010	+0.0000	+0.0043472
q_{13}				-0.1874	+0.0000	-0.1923529
q_{14}				-0.0444	-0.0559	-0.0427578
				+1.1384	-0.2159	-0.0353133
q_{15}					+1.1210	-0.0250000
q_{16}					-0.0000	+0.0000000
q_{17}					-0.0000	-0.0000000
q_{18}					-0.0000	+0.0000000
q_{19}					-0.0705	-0.0539259
q_{20}					-0.0409	-0.0066954
					+1.0096	-0.0856213

(r) Solution of Normal Equations.

	1	2	3	4	5	6
r_1	+0.0855166	-0.1400348	+0.4880035	+0.1679631	+0.0310192	+0.0848079
r_2	+0.0000000	-0.0000000	+0.0000000	+0.0186323	+0.0160796	
r_3	+0.0064949	+0.0015307	+0.0223908	+0.0082046	+0.0470988	
r_4	+0.0000000	+0.0101880	+0.0459533	+0.1948000		
r_5	+0.2302166	+0.2289370	+0.5563476			
r_6	+0.0312227	+0.1006209				
r_7	+0.3534508					

Some authorities on Least Square, call "(r) Solution of Normal Equations" the, "Back Solution".

Summary of Junction Elevations in Dawson Township,
Phelps County, Missouri.



A D J U S T E D E L E V A T I O N S .

A = 1087.8000	F = 1001.3439
B = 1074.6354	G = 921.2826
C = 1083.4302	H = 1041.8542
D = 1098.8843	I = 1061.8956
E = 1074.0050	J = 968.3706

M = 954.6403.

Points K, L, N, O, P, are adjusted as intermediate points and take corrections proportionate to their distances from adjacent junction points.

ADJUSTED ELEVATIONS.

K =	N =
L =	O =

P =

Points Q, R, S, are at the ends of spur lines and will take the same corrections as P, N, O, respectively.

ADJUSTED ELEVATIONS.

Q = R = S =

Abridged Method A.

NORMAL EQUATIONS. Cont'd.

	A 1	B 2	C 3	D 4	E 5	F 6	Const. N	Check. Σ
(I)	+0.7250	−0.2250	−0.3000	+0.0000	−0.1000	+0.0000	−0.0620	+0.0308
(2)	· · · ·	+1.0250	−0.3000	−0.0500	+0.0000	+0.0000	+0.1530	+0.8030
(3)	· · · · · · · ·	+1.1150	−0.1750	−0.3400	+0.0000	−0.4340	−0.4340	
(4)	· · · · · · · · · ·	+1.5100	−0.1600	−0.3210	−0.1570	+0.6470		
(5)	· · · · · · · · · · · ·	+1.3850	−0.1600	+0.2040	+0.8290			
(6)	· · · · · · · · · · · · · ·	+1.1210	−0.0250	+0.6150				

Note.

It must be remembered that the (Check Σ) column includes all of the coefficients of an equation, whether written or not, so that, when the abridged form is used, as just to the left, the coefficients must be read down and to the right; i.e: Σ line(2); add column 2 with line(2)=+.8030.

"P" solution

Solution of Normal Equations by Abridged Method.

	A	B	C	D	E	F	Const.	Check.	Process.
(2)	· · · ·	+1.0250	−0.3000	−0.0500	+0.0000	+0.0000	+0.1530	+0.8030	(Normal Equation (2).
(7)	· · · ·	−0.0698	−0.0931	+0.0000	−0.0310	+0.0000	−0.0192	+0.0118	(I)×(+225/7250) or × 0.3103.
(II)	· · · ·	+0.9552	−0.3931	−0.0500	−0.0310	+0.0000	+0.1338	+0.6148	✓(Σ hor.0.8148) (2)+(7).
(3)	· · · · · · · ·	+1.1150	−0.1750	−0.3400	+0.0000	−0.4340	−0.4340		(Normal Equation (3).
(8)	· · · · · · · ·	−0.1241	+0.0000	−0.0414	+0.0000	−0.0257	+0.0157		(I)×(+3000/7250) or ×0.4138.
(9)	· · · · · · · ·	−0.1618	−0.0206	−0.0128	+0.0000	+0.0551	+0.2530		(II)×(+38331/93552) or × 0.4115
(III)	· · · · · · · ·	+0.8291	−0.1956	−0.3942	+0.0000	−0.4046	−0.1653		✓(Σ hor.0.1652) (3)+(8)+(9).
(4)	· · · · · · · ·	· · · ·	+1.5100	−0.1600	−0.3210	−0.1570	+0.6470		(Normal Equation (4).
(10)	· · · · · · · ·	· · · ·	+0.0000	+0.0000	+0.0000	+0.0000	+0.0000		(I) × (+0/7250) or × 0.0000.
(11)	· · · · · · · ·	· · · ·	−0.0026	−0.0016	+0.0000	+0.0070	+0.0322		(II)×(+2500/93552) or × 0.0523.
(12)	· · · · · · · ·	· · · ·	−0.0461	−0.0930	+0.0000	−0.0954	−0.0390		(III)×(+19564/82981) or× 0.2359.
(IV)	· · · · · · · ·	· · · ·	+1.4613	−0.2546	−0.3210	−0.2454	+0.6402		✓(Σ hor.0.6403) (4)+(10)+(11)+(12).
(5)	· · · · · · · ·	· · · ·	· · · ·	+1.3850	−0.1600	+0.2040	+0.8290		(Normal Equation (5).
(13)	· · · · · · · ·	· · · ·	· · · ·	−0.0138	+0.0000	−0.0085	+0.0052		(I)×(+10000/72500) or × 0.1379.
(14)	· · · · · · · ·	· · · ·	· · · ·	−0.0010	+0.0000	+0.0043	+0.0200		(II)×(+3019/93552) or× 0.0325.
(15)	· · · · · · · ·	· · · ·	· · · ·	−0.1874	+0.0000	−0.1924	−0.0786		(III)×(+38743/82981)or× 0.4755.
(16)	· · · · · · · ·	· · · ·	· · · ·	−0.0444	−0.0559	−0.0427	+0.1115		(IV)×(+2544/14613)or× 0.1742.
(V)	· · · · · · · ·	· · · ·	· · · ·	+1.1384	−0.2159	−0.0353	+0.8871		✓(Σ hor.0.8872) (5)+(13)+(14)+(15)+(16).
(6)	· · · · · · · ·	· · · ·	· · · ·	· · · ·	+1.1210	−0.0250	+0.6150		(Normal Equation (6).
(17)	· · · · · · · ·	· · · ·	· · · ·	· · · ·	+0.0000	−0.0000	+0.0000		(I)×(+0/7250) or × 0.0000.
(18)	· · · · · · · ·	· · · ·	· · · ·	· · · ·	+0.0000	−0.0000	+0.0000		(II)×(+0/93552)or × 0.0000.
(19)	· · · · · · · ·	· · · ·	· · · ·	· · · ·	+0.0000	−0.0000	+0.0000		(III)×(+0/82981)or× 0.0000.
(20)	· · · · · · · ·	· · · ·	· · · ·	· · · ·	−0.0705	−0.0539	+0.1407		(IV)×(+3219/14613)or× 0.2197.
(21)	· · · · · · · ·	· · · ·	· · · ·	· · · ·	−0.0410	−0.0067	+0.1683		(V)×(+2158/11384)or × 0.1897.
(VI)	· · · · · · · ·	· · · ·	· · · ·	· · · ·	+1.0095	−0.0856	+0.9240		✓(Σ hor.0.9239)(6)+(17)+(18)+(19)+(20)+(21).

Note: Thruout the process that the numerator of each fraction used as a multiplier has its sign changed.

ABRIDGED METHOD A.

"R" CORRELATES.

	A	B	C	D	E	F	PROCESS.
Constants	+0.0620	−0.1338	+0.4046	+0.2454	+0.0353	+0.0856	Copy (VI)(V)(IV)(III)(II)(I)Const Column.
						+1.0095	Divide(VI)ConstColumn by(VI)F Column=+0.0848
F-terms	+0.0000	+0.0000	+0.0000	+0.0272	+0.0183		Multiply(V)(IV)(III)(II)(I)F Col.×+0.0848
						+0.0848=F	Place at head of Col.6 Table of Correlates.
					+0.0536		Σ column E.
					+1.1384		Divide Σ Col.E by(V)Col.E=+0.04708
E-terms	+0.0047	+0.0015	+0.0157	+0.0120			Multiply(IV)(III)(II)(I)Col.E × +0.0471.
						+0.04708=E	Place at head of Col.5 Table of Correlates.
				+0.2846			Σ column D.
				+1.4613			Divide Σ Col.D by(IV)Col.D=+0.1948.
D-terms	+0.0000	+0.0097	+0.0381				Multiply(III)(II)(I)Col.D × +0.1948.
						+0.1948=D	Place at head of Col.4 Table of Correlates.
			+0.4584				Σ column C.
			+0.8291				Divide Σ Col.C by(III)Col.C= +0.5529.
							Multiply(II)(I)Column C × +0.5529.
C-terms	+0.1659	+0.2173				+0.5529=C	Place at head of Col.3 Table of Correlates.
		+0.0947					Σ column B.
		+0.9552					Divide Σ Col.B by(II)Col.B = +0.0991.
B-terms	+0.0223						Multiply (I)Col.B × +0.0991=+0.0223.
						+0.0991=B	Place at head of Col.2 Table of Correlates.
	+0.2549						Σ column A.
	+0.7250						Divide Σ Col.A by (I)Col.A = +0.3516.
	+0.3516=A						Place at head of Col.1 Table of Correlates.

(Reference is made above between this page and the page preceeding this.)

The method given above is a substitute for the (P),(q),(r) Solution of Normal Equations; all other steps in the process of level net adjustment are identical with those given in Part of this Thesis, and will not be given again here.

Note: the ✓ (Check mark) at the ends of rows (II),(III),(IV),(V),and(VI), are written to indicate that the Σ of Column and Row are exactly satisfied. Unavoidable discrepancies in the last figure of the Check✓term,due to remainders,should be removed by arbitrarily correcting the check-term before preceeding with the next step in the elimination;this is best done by drawing a line thru the erroneous figure and writing the correct one above it.

A B R I D G E D M E T H O D B.

N O R M A L E Q U A T I O N S.

	1	2	3	4	5	6	N	Σ
1)	+0.7250	-0.2250	-0.3000	+0.0000	-0.1000	+0.0000	-0.0620	+0.0380
2)	. . . .	+1.0250	-0.3000	-0.0500	+0.0000	+0.0000	+0.1530	+0.6030
3)	. . . . .	. . . .	+1.1150	-0.1750	-0.3400	+0.0000	-0.4340	-0.4340
4)	. . . . .	. . . .	. . . .	+1.5100	-0.1600	-0.3210	-0.1570	+0.6470
5)	. . . . .	. . . .	. . . .	. . . .	+1.3850	-0.1600	+0.2040	+0.8290
6)	. . . . .	. . . .	. . . .	. . . .	. . . .	+1.1210	-0.0250	+0.6150

Note.
It must be remembered that the(Check Σ) column includes all of the coeffioients of an equation,whether written or not,so that, when the abridged form is used,the coefficients must be read down and to the right; i.e=Σ Form of check oposite line (14).

Solution of Normal Equations by Abridged Method.

	1	2	3	4	5	6	N	Σ	Process.
1)	+0.7250	-0.2250	-0.3000	+0.0000	-0.1000	+0.0000	-0.0620	+0.0380	Normal Equation(1).
I)	. . . .	+0.3103	+0.4138	-0.0000	+0.1379	-0.0000	+0.0855	-0.0524	Divide Normal Equation (1) by -0.7250.
2)	. . . .	+1.0250	-0.3000	-0.0500	+0.0000	+0.0000	+0.1530	+0.6030	Normal Equation (2).
7)	. . . .	-0.0698	-0.0931	+0.0000	-0.0310	+0.0000	-0.0182	+0.0118	Mult.Normal Eq.(1) × +0.3103.
8)	. . . .	+0.9552	-0.3931	-0.0500	-0.0310	+0.0000	+0.1338	+0.6148 ✓	Σ (2) +(8). (Σ horizontally +0.6149).
II)	. . . . .	. . . .	+0.41154	+0.05235	+0.03245	+0.0000	-0.14008	-0.64363	Divide (8) by -0.9552.
3)	. . . . .	. . . .	+1.1150	-0.1750	-0.3400	+0.0000	-0.4340	-0.4340	Normal Equation (3).
9)	. . . . .	. . . .	-0.1241	+0.0000	-0.0414	+0.0000	-0.0257	+0.0157	Mult.Normal Eq.(1) × +0.4138.
10)	. . . . .	. . . .	-0.1618	-0.0206	-0.0128	+0.0000	+0.0551	+0.2530	Mult.(8) × +0.41154.
11)	. . . . .	. . . .	+0.8291	-0.1956	-0.3942	+0.0000	-0.4046	-0.1653 ✓	Σ (3)+(9)+(10).(Σ horizontally -0.1653).
III)	. . . . .	. . . .	. . . .	+0.23591	+0.47546	+0.0000	+0.48800	+0.19937	Divide (11) by -0.8291.
4)	. . . . .	. . . .	. . . .	+1.5100	-0.1600	-0.3210	-0.1570	+0.6470	Normal Equation (4).
12)	. . . . .	. . . .	. . . .	-0.0000	+0.0000	-0.0000	+0.0000	-0.0000	Mult.Normal Eq.(1) × +0.0000.
13)	. . . . .	. . . .	. . . .	-0.0026	-0.0016	+0.0000	+0.0070	+0.0322	Mult.(8) × +0.05235.
14)	. Form of Check. . . .	. . . .	. . . .	-0.0461	-0.0930	+0.0000	-0.0854	-0.0380	Mult.(11) × +0.23591.
15)	. +1.4613 ——— >	. . . .	. . . .	+1.4613	-0.2546	-0.3210	-0.2454	+0.6402 ✓	Σ (4)+(12)+(13)+(14).(Σ hor.+0.6403).
IV)	. -0.2546	. . . .	. . . .	. . . .	+0.17423	+0.21967	+0.16793	-0.43810	Divide (15) by -1.4613.
	-0.3210								
5)	. -0.2454	. . . .	. . . .	. . . .	+1.3850	-0.1600	+0.2040	+0.8290	Normal Equation (5).
16)	. +0.6403 ✓Horizontally	. . .	. . . .	. . . .	-0.0138	+0.0000	-0.0085	+0.0052	Mult. Normal Eq.(1) × +0.1379.
17)	. +0.6402 ΣColumn	. . .	. . . .	. . . .	-0.0010	+0.0000	+0.0043	+0.0200	Mult.(8) × +0.03245.
18)	. 0.0001 error.	. . . .	. . . .	. . . .	-0.1874	+0.0000	-0.1924	-0.0786	Mult.(11) × +0.47546.
19)	. . . . .	. . . .	. . . .	. . . .	-0.0444	-0.0559	-0.0428	+0.1115	Mult.(15) × +0.17423.
20)	. . . . .	. . . .	. . . .	. . . .	+1.1384	-0.2159	-0.0354	+0.8871 ✓	Σ (5)+(16)+(17)+(18)+(19).(Σ hor.+0.8871).
V)	. . . . .	. . . .	. . . .	. . . .	. . . .	+0.18965	+0.03110	-0.77925	Divide (20) by -1.1384.
6)	. . . . .	. . . .	. . . .	. . . .	. . . .	+1.1210	-0.0250	+0.6150	Normal Equation (6).
21)	. . . . .	. . . .	. . . .	. . . .	. . . .	+0.0000	+0.0000	+0.0000	Mult. Normal Eq.(1) × +0.0000.
22)	. . . . .	. . . .	. . . .	. . . .	. . . .	+0.0000	+0.0000	+0.0000	Mult. (8) × +0.0000.
23)	. . . . .	. . . .	. . . .	. . . .	. . . .	+0.0000	+0.0000	+0.0000	Mult.(11) × +0.0000.
24)	. . . . .	. . . .	. . . .	. . . .	. . . .	-0.0705	-0.0539	+0.1406	Mult.(15) × +0.21967.
25)	. . . . .	. . . .	. . . .	. . . .	. . . .	-0.0409	-0.0067	+0.1682	Mult.(20) × +0.18965.
26)	. . . . .	. . . .	. . . .	. . . .	. . . .	+1.0096	-0.0856	+0.9238 ✓	Σ (6)+(21)+(22)+(23)+(24)+(25)(Σ=+0.9240).
VI)	. . . . .	. . . .	. . . .	. . . .	. . . .	. . . .	+0.08479	-0.91502	Divide (26) by -1.0096.

B A C K S O L U T I O N.

1	2	3	4	5	6	
+0.08550	-0.14008	+0.48800	+0.16793	+0.03110	+0.08479.	Copy (VI),(V),(IV),(III),(II),(I) Col.N.
+0.00000	-0.00000	+0.00000	+0.01863	+0.01608.		Mult.(V),(IV),(III),(II),(I)Col(6)×+0.08479.
+0.00649	+0.00153	+0.02239	+0.00800	+0.04718.		Mult.(IV),(III),(II),(I)Col(5)×+0.04718.
+0.00000	+0.01019	+0.04595	+0.19476.			Mult.(III),(II),(I)Column(4) × +0.19476.
+0.23022	+0.22894	+0.55634.				Mult.(II),(I) Column (3) × +0.55634.
+0.03122	+0.10058.					Mult.(I) Column (2) × +0.10058.
+0.35343.						Σ Column (1).

ADJUSTMENT BY THE METHOD OF CORRELATES.
T H E O R Y .

The final adjusted values of the observed quantities must satisfy the conditions of the problem in hand, and must be the most probable values, according to the Theory of Least Squares, that will satisfy all the conditions. Therefore, the sum of the weighted squares of the corrections (residuals, v) must be a minimum.

Then:

$$U = w_1 a^2 + w_2 b^2 + w_3 c^2 + w_4 d^2 \cdots\cdots + w_n z^2 = \text{a minimum} \qquad (1)$$

which must be satisfied simultaneously with the following conditions. Multiplying the condition equations on page 50 in succession by the factors, $-2C_1$, $-2C_2$, $-2C_3$, etc., respectively,

$$\begin{aligned}
&+2C_1 q_1 + 2C_1 d + 2C_1 c + 2C_1 b + 2C_1 a = 0 \\
&-2C_2 q_2 - 2C_2 g + 2C_2 f + 2C_2 e - 2C_2 c = 0 \\
&+2C_3 q_3 - 2C_3 e - 2C_3 h + 2C_3 i - 2C_3 b = 0 \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&-2C_5 q_5 - 2C_5 i - 2C_5 j + 2C_5 l + 2C_5 k - 2C_5 a = 0 \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots
\end{aligned} \qquad (2)$$

Adding these equations to (1) and collecting the coefficients of the separate a, b, c, etc., we have the requirement that,

$$\begin{aligned}
&w_1 a^2 + 2a(C_1 - C_5) + \\
+\ &w_2 b^2 + 2b(C_1 - C_3) + \\
+\ &w_3 c^2 - 2c(C_2 + C_1) + \\
+\ &\cdots\cdots\cdots + \\
+\ &2(C_1 q_1 - C_2 q_2 + C_3 q_3 + \ldots) = \text{a minimum.}
\end{aligned} \qquad (3)$$

For the minimum, the first derivative of this expression with respect to each a, b, c, etc., must be placed equal to zero. Therefore,

$\frac{dU}{da} = 2w_1 a + 2(C_1 - C_5) = 0$, dividing by 2 and solving for a,

$$a = \frac{1}{w_1}(-C_1 + C_5),$$

$\frac{dU}{db} = 2w_2 b + 2(C_1 - C_3) = 0$, dividing by 2 and solving for b,

$$b = \frac{1}{w_2}(-C_1 + C_3),$$

$\frac{dU}{dc} = 2w_3 c - 2(C_2 + C_1) = 0$, dividing by 2 and solving for c,

$$c = \frac{1}{w_3}(-C_1 + C_2)$$

$\frac{dU}{dz} = ... = 0$, etc.

Where w_1, w_2, w_3, etc.$= \frac{1}{d}$ are the weights respectively, d= distances
U = Σ weighted residuals squared, desired to be a minimum,
a, b, c, etc. = corrections for the links,
C_1, C_2, C_3, etc.= the adjustment factors solved for in the normal equations.
q_1, q_2, q_3, etc.=constants of observed closure respectively in the several circuits.
$\frac{dU}{da}$ = the differential of U with respect to a, or the first derivative.
The constant 2 comes from the differentiation of the expressions in each equation equated to 0.

Substituting these values of the a, b, c, etc. in the condition equations on page 50, and combining the coefficients of C_1, C_2, C_3, etc., we obtain the *Normal Equations:* Which partake of the following form:

$$\begin{aligned}
[\tfrac{aa}{w}]C_1 + [\tfrac{ab}{w}]C_2 + [\tfrac{ac}{w}]C_3 + ... + q_1 &= 0 \\
[\tfrac{ab}{w}]C_1 + [\tfrac{bb}{w}]C_2 + [\tfrac{bc}{w}]C_3 + ... + q_2 &= 0 \qquad (4) \\
[\tfrac{ac}{w}]C_1 + [\tfrac{bc}{w}]C_2 + [\tfrac{cc}{w}]C_3 + ... + q_3 &= 0
\end{aligned}$$

If the foregoing substitutions and and like Cs combined, we arrive at the normal equations as appear on the top of page 52.

In forming the normal equations, it may be found somewhat simpler to follow the rules of thumb formulated from the foregoing theory. which will be found on page 51 for the formation of normal equations from the condition equations, and on pages 10 and 41, by a similar demonstration, for the formation of normal equations from observation equations.

APPLICATION OF THE THEORY.

Least Squares computation consists in rendering the value of U in the following expression a minimum.

$$U = a^2 + .333b^2 + .444c^2 + d^2 + .333e^2 + 2f^2 + .222g^2 + .571h^2 + .294i^2 + .625j^2 + .16k^2 + .625l^2 + .312m^2 + .156n^2 + .124p^2 .$$

Condition Terms.

$-2C_1 [-.062 - d - c - b - a]$
$-2C_2 [+.153 + g - f - e + c]$
$-2C_3 [-.434 + e + h - i + b]$
$-2C_4 [-.157 - h + f + p - m - j]$
$-2C_5 [+.204 + a + i + j - l - k]$
$-2C_6 [-.025 + m - n + l]$

The coefficients of the above condition terms is taken arbitrarily as −2 in order to simplify the expressions for the "V" corrections (ie. a, b, c, etc.) for the links.

To make the above expression for U a minimum requires that its differentials with respect to a, b, c, etc. be each equated to zero, as follows:

$2a + 2C_1 - 2C_3 = 0,$
then
$a = -C_1 + C_3 \ldots\ldots = 1 \text{ times}(-C_1 + C_3)$

$.666b + 2C_1 - 2C_3 = 0,$
then
$b = -3C_1 + 3C_3 \ldots\ldots = 3 \text{ " } (-C_1 + C_3)$

$.888c + 2C_1 - 2C_2 = 0,$
then
$c = -2.25C_1 + 2.25C_2 = 2.25 \text{ " } (-C_1 + C_2)$

These expressions when simplified and with the Cs transposed to the right-hand side of the equations, will then appear as extended to the left, and form the correlates as they are tabulated on page 51.

$2d + 2C_1 = 0,$
then
$d = -C_1 \ldots\ldots\ldots\ldots = 1 \text{ " } -C_1$

$.666e + 2C_2 - 2C_3 = 0,$
then
$e = -3C_2 + 3C_3 \ldots\ldots = 3 \text{ " } (-C_2 + C_3)$

$4f + 2C_2 - 2C_4 = 0,$
then
$f = -.5C_2 + .5C_4 \ldots\ldots = 0.5$ times $(-C_2 + C_4)$

$.444g - 2C_2 = 0,$
then
$g = +4.5C_2 \ldots\ldots\ldots\ldots = 4.5$ " $(-C_2)$

$1.142h - 2C_3 + 2C_4 = 0,$
then
$h = +1.75C_3 - 1.75C_4 \ldots = 1.75$ " $(+C_3 - C_4)$

$.588i + 2C_3 - 2C_5 = 0,$
then
$i = -3.4C_3 + 3.4C_5 \ldots\ldots = 3.4$ " $(-C_3 + C_5)$

$1.250j + 2C_4 - 2C_5 = 0,$
then
$j = -1.6C_4 + 1.6C_5 \ldots\ldots = 1.6$ " $(-C_4 + C_5)$

$.32k + 2C_5 = 0,$
then
$k = -.625C_5 \ldots\ldots\ldots\ldots = 6.25$ " $-C_5$

$1.250l + 2C_5 - 2C_6 = 0,$
then
$l = -1.6C_5 + 1.6C_6 \ldots\ldots = 1.6$ " $(-C_5 + C_6)$

$.624m + 2C_4 - 2C_6 = 0,$
then
$m = -.321C_4 + .321C_6 \ldots = 0.321$ " $(-C_4 + C_6)$

$.312n + 2C_6 = 0,$
then
$n = +6.4C_6 \ldots\ldots\ldots\ldots = 6.4$ " $+C_6$

$.248p - 2C_4 = 0,$
then
$p = +.805C_4 \ldots\ldots\ldots\ldots = 8.05$ " $+C_4$

P A R T V .

PROPORTIONAL ADJUSTMENT OF INTERMEDIATE BENCHMARKS FROM CORRECTED ELEVATIONS OF ADJACENT JUNCTION POINTS.

APPLIED TO THE ADJUSTMENT
OF
INTERMEDIATE BENCHMARKS
IN THE
DAWSON-ST. JAMES TOWNSHIPS LEVEL NET,
PHELPS COUNTY,
MISSOURI.

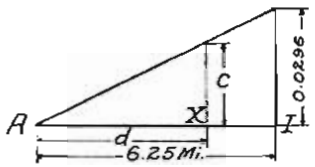
(BASED ON THE LINE OF LEVELS RUN BY THE
U. S. GEOLOGICAL SURVEY IN 1908 ALONG THE FRISCO R. R.
WITH PERMANENT BENCHMARKS "A" AT ST. JAMES,
AND "E" AT KNOBVIEW).

Adjustment of intermediate bench marks between point A at St.James and point I at Miles School on Republic Road.

The adjustment of these intermediate bench marks can be effected by scaling the adjustment corrections from the diagram below and subtracting each correction from the observed elevation as it appears in the level notes;or, if desired, the adjustment corrections can be computed by the following formula for this section,(A to I).

DERIVATION OF ADJUSTMENT FORMULA.

Elev. A = 1087.800
Cor. = 0.000



Elev I. = 1061.9250 Obser.
" I. = 1061.8954 Adjus.
= 0.0296 Cor.

Correction c. for any intermediate point at distance d. from A and between points A and I:

$$C = - \frac{0.0296 \times d}{6.25} \text{ or } = - 0.004736d.$$

GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINT BETWEEN A AND I.

Merimec Springs Quadrangle.
(Latitude 37 45'-38;longitude 91 30'-91 45')

(St.James Township)

Line from St. James,northwest via Jefferson road to the intersection of Republic road, thence north via Republic road,9.5 miles to the Phelps-Gasconade County Line.

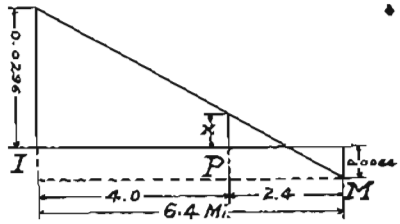
A.	St.James,corner of Meramec and Washington Sts.,about 70 ft.N. of,center line of Frisco R.R. track,25 ft.from corner of Commercial Hotel,in angle between cross walk and side walk;iron post stamped "1098".	1087.800	0.000	1087.800
L.	St.James,0.5 miles N.W.of,on Jefferson road at intersection of Parker Lane road,concrete culvert,north end wall,top of; chiseled square.	1078.427	-0.002	1078.425
K.	St.James,1.75 miles N.W.of,on Jefferson road at intersection of Republic road,on south side of road,10" black jack tree,in base of;copper nail and washer.	1092.694	-0.008	1092.686
1	St.James,2.25 miles N.of,on Republic road,Jefferson road,0.5 miles N.of,culvert,west end of, sandstone slab,top of;painted square.	1034.345	-0.010	1034.335
2	Jefferson road,1.0 mile N.of,on Republic road,T road east at Cox's house,100 ft. E. of,sand stone boulder in northeast corner of T road;top corner broken off and painted white.	1024.715	-0.013	1024.702
3	Jefferson road,1.5 miles north of, Robinsons Creek,0.75 miles south of,Fitzenrieder's residence,100 feet west of,wagon gate post,in base of;copper nail and washer.	998.469	-0.016	998.453

4	Robinson's Creek,0.25 miles south of,on Republic road,on east side of road,10" post oak tree,near Kenndy's mail box,in base of tree; nail.	923.575	-0.018	923.557
5	Jefferson road,2.25 miles north of, on Republic road,Robinsons Creek, west bank of,24" white oak tree, in root of;copper nail and washer.	917.604	-0.019	917.585
	(Dawson Township)			
6	Robinsons Creek,0.25 miles north of, 15 feet west of road,sandstone boulder on St.James-Dawson Township line, opposite wire fence corner on east side of road,point is on south hill side 300 feet south of wood chopper's cabin;painted cross on top of boulder.	1022.394	-0.020	1022.374
7	Robinsons Creek,075 miles north of, on Republic road,township line,0.5 miles north of,at north end of ridge, 30 feet west of road,sandstone boulder, top of;chiseled square.	1039.423	-0.021	1039.402
8	Robinsons Creek,1.0 mile north of, on Republic road,at foot of hill, concrete culvert across road,west end wall, top of;chiseled square.	934.325	-0.023	934.302
9	Robinsons Creek,1.25 miles north of, Miles School House,1.0 mile south of, T road west,center of roads;road elevation.	976.005	-0.024	975.981
10	Miles School House,0.5 miles south of, on Republic road,Marrs Cemetary,400 feet north of,east side of road,sand stone boulder at rail fence corner, top of;chiseled square.	1059.077	-0.025	1059.052
I'	St.James,6.0 miles N.of,on Republic Rd.,Safe,2.5 miles E.of,cross Rds.at Miles School,N.E.corner of School House,limestone wheel guard, top of; painted square.	1061.925	-0.030	1061.895

Adjustment of point P on Republic Road 0.5 miles south of Harrison School, and the determination of intermediate points between I and P only.

Elev. I = 1061.9250 Observ.
" I = 1061.8954 Adjus.
= 0.0296 Cor.

Elev M = 954.7240 Arranged
" M = 954.7304 Adjusted
+ 0.0064 Cor.



CALCULATION OF THE CORRECTION FOR POINT P.

$$\frac{x + .0064}{.0360} = \frac{2.4}{6.4}$$

$$x = \frac{.0360 \times 2.4}{6.4} - .0064$$

$$x = .0135 - .0064$$

$$x = -0.0071 \text{ Cor. for P.}$$

To be applied to observed elevation.

Elev P = 928.5520 Observed
− 0.0071 Cor.
Elev P = 928.5449 Adjusted.

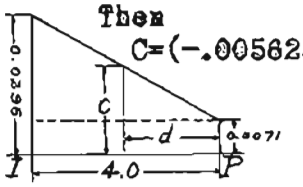
Correction c. for any intermediate point at distance d. from P and between points I and P.

DERIVATION OF ADJUSTMENT FORMULA.

$$\frac{C - .0071}{.0225} = \frac{d}{4.0}$$

$$C = \frac{.0225\ d}{4.0} + .0071$$

Then
C=(-.005625d-.0071)



GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN I AND P.

I.	St.James,6.0 miles north of,on Republic road,Safe,2.5 miles east of,Miles School,25 feet west of,S.W.Cor.S.E.Qtr.of the S.W.Qtr.,Sec.20,T 39 N.,R.6 W., 60 feet southeast of,concrete well curb,top of;bronze tablet.	1059.528	-0.030	1059.498
12	Miles School,0.75 miles north of, on Republic road,W.L.Martin's house,150 feet southeast of,on top of hill,on east side of road, 12" post oak tree, in base of; copper nail and washer. ---------	1023.857	-0.025	1023.832
13	Miles School,1.2 miles north of, on Republic road,Bourbeuse River, 0.75 miles south of ford,at dim cross road,southeast corner of; painted square on sandstone boulder. ----------	924.251	-0.023	924.228
14	Miles School,1.75 miles north of, on Republic road,Bourbeuse River, 0.2 miles south of,farm house,600 feet east of,on east side of road, 15" walnut tree,in base of;copper nail and washer. ----------	873.567	-0.020	873.547
15	Wash School House,0.75 miles south of,Bourbeuse river ford,100 ft.S. of,east bank of river,30" sycamore tree,in base of;copper nail and washer. ----------	839.998	-0.017	839.981
15a	Bourbeuse River,at ford on Republic road,center of creek;road elevation. Road Elevation. ----------	834.000	-0.017	833.983
16	Bourbeuse river,0.25 miles north of, on Republic road,at T road east,N.E. corner of,in base of fence post; copper nail and washer. ---------	840.106	-0.016	840.090
17	Bourbeuse Riv.,0.75 miles N.of,Wash Sch.,400 ft.S.W.of,J.W.Crossner's house,300 ft.S.E. of,8 ft.W.of three mail boxes,on W.side of road,14" scaley bark hickory tree,in root of; copper nail and washer. ---------	947.596	-0.013	947.583

Adjustment of intermediate bench marks between the points P and Q on the Republic Road near the Phelps-Gasconade County Line.

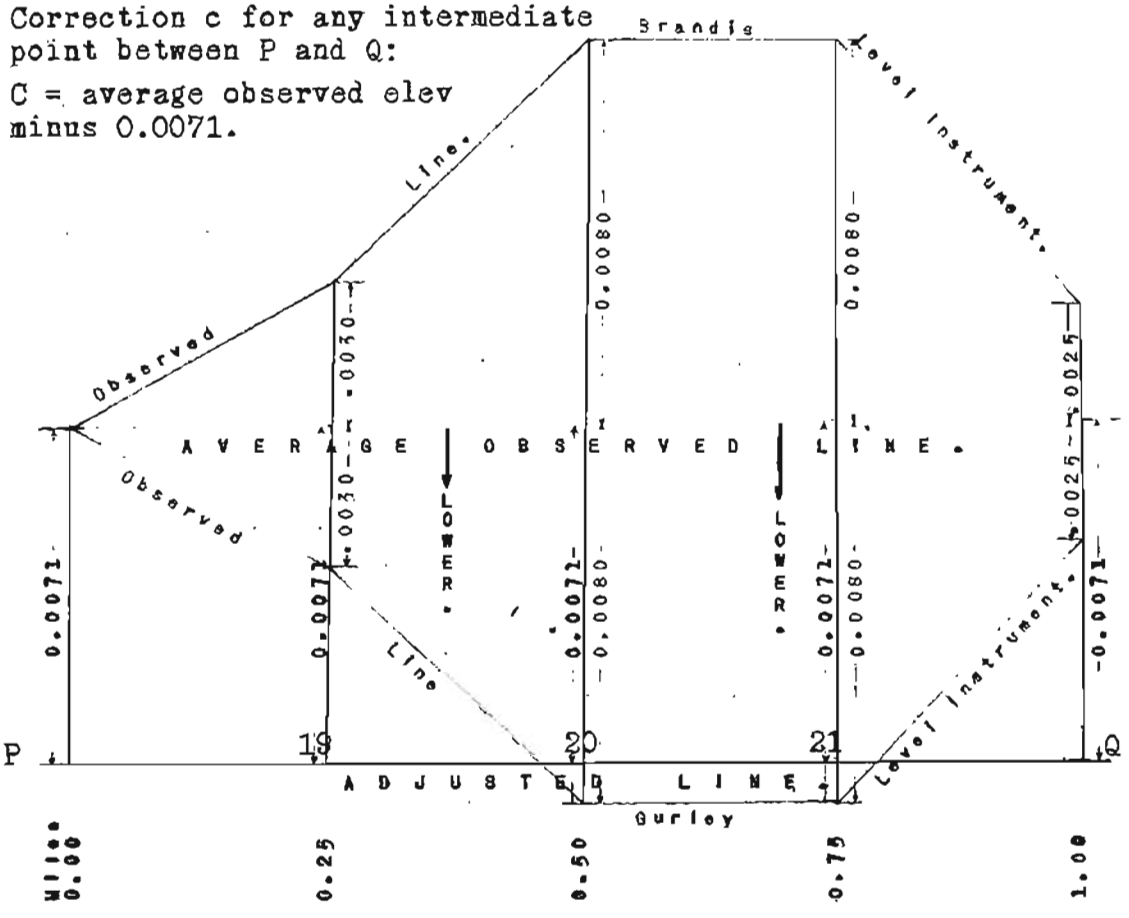
It will be remembered that the line P to Q is a spur line of levels in our scheme of adjustment. The line P to Q was run as a parallel duplicate line of levels, and the most probable values for the elevations of the points on such a line is the average of the several determinations. The diagram below shows lines of levels run by Brandis and Gurley instruments; as these instruments were of equal refinement, equal weight was applied to the work done by each instrument. The green line represents the average of the instrumental determinations. In the level net adjustment proceeding, it was found that the point P was 0.0071 ft. high. As the spur line P to Q hinges directly on the point P, then the whole line P to Q will have to be lowered 0.0071 ft. In the diagram the adjusted line is shown in black.

Elev. P = 928.5520 Observed
" P = 928.5449 Adjusted
- 0.0071 Cor.

Elev. Q = 997.2400 Observed
" Q = 997.2329 Adjusted
- 0.0071 Cor.

Correction c for any intermediate point between P and Q:

C = average observed elev minus 0.0071.



GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN P AND Q.

18.	Wash School,0.5 miles north of, on Republic road,Little Bourbeuse Creek,at ford,on south bank,on west side of road,10" scaley bark hickory tree,in base of;copper nail and washer. -----	835.927	-0.010	835.917
P.	Wash School,1.0 miles north of, Little Bourbeuse Creek,0.5 miles north of,Harrison School,0.5 miles south of,Phelps-Gasconade County Line,1.0mile south of,Austin E. Bruer's house,200 feet west of, at T road east from Republic road, on west side of road,2 feet inside of fence,10" post oak tree, in base of;copper nail and washer. ----------	928.552	-0.007	928.545
19	Wash School,1.25 north of,Little Bourbeuse Creek,0.75 miles north of,Harrison School,0.25 miles south of,Phelps-Gasconade County Line,0.75 miles south of,Steve Lort's house,70 feet north of,T road west(Mail road to High Gate), northwest corner of roads,24" oak stump in fence corner,top of; ten penny nail.----------	962.732 Brandis, 962.726 Gurley.		
	Average	962.729	-0.007	962.722
20	Harrison School House,on St.James-Republic road,west side of School Building,north concrete door step, top of;chiseled square.--------	965.674 Brandis, 965.658 Gurley.		
	Average	965.666	-0.007	965.659
21	Harrison School House,0.25 miles north of,on Republic road,Phelps-Gasconade County Line,0.25 miles south of,private T road west,south west corner of,16" blazed black oak tree;bent nail in base of.--	959.594 Brandis, 959.578 Gurley.		
	Average	959.586	-0.007	959.579

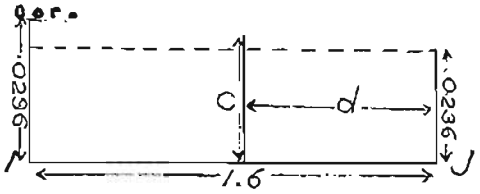
Q.	Harrison School House, 0.5 miles north of, on Republic road, intersection of St. James-Republic road and High Gate-Red Bird road, 0.5 miles south of, Phelps-Gasconade County Line, 28 feet south of, center of road, 30 feet west of, 5 feet inside wire fence, in line with center of road north into Gasconade County, dim T road west, 50 feet south of; iron post stamped "997". ----------	P.B. M. 997.242 Brandis, 997.237 Gurley. -------		
		Average 997.240	-0.007	997.223

Adjustment of intermediate temporary bench marks between point I at Miles School of Republic Road and Point J two miles due east on the Red Bird Road.

DERIVATION OF ADJUSTMENT FORMULA.

Elev. I = 1061.9250 Observ.
" I = 1061.8954 Adjust.
- 0.0296 Cor.

Elev. J = 968.3940 Observ.
" J = 968.3704 Adjust.
- 0.0236 Cor.



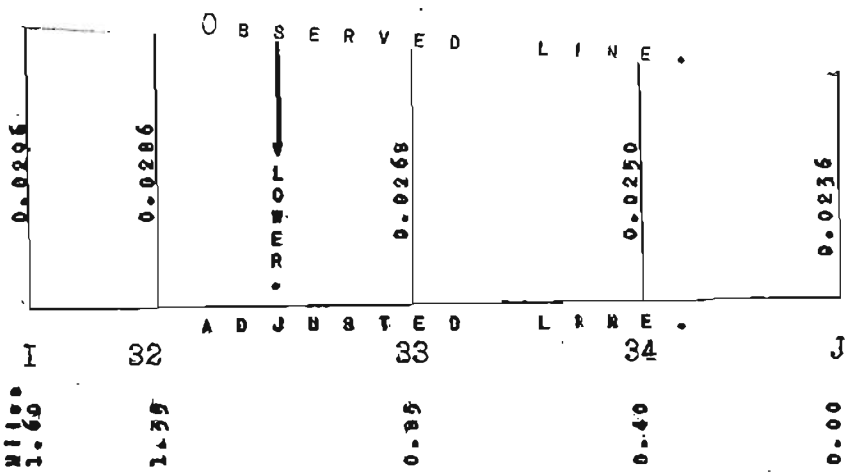
Correction c. for any intermediate point at distance d. from J and between I and J:

$$\frac{C - .0236}{.0060} = \frac{d}{1.6}$$

$$C = \frac{.0060\ d}{1.6} + .0236$$

$$C = -\ (\ .00375\ d\ +\ .0236\ )$$

$$C = (\ -\ .00375\ d\ -\ .0236\ )$$



GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN I AND J.

Red Bird Quadrangle.
(Latitude 38 -38 15';longitude 91 30'-91 45')

(Dawson Township)

Line from Miles School House on Republic Road,east via road 2.0 miles,to point on Red Bird Road,two miles north of St.James-Dawson Township Line.

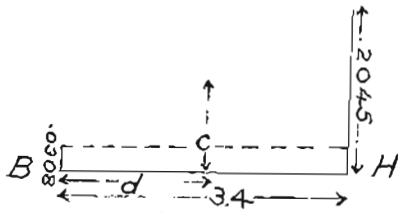
I.	St.James,6.0 miles north of,on Republic Road,Safe,2.5 miles east of,cross roads at Miles School,northeast corner of school house,sandstone wheel guard,top of;painted square.---	1061.925	-0.030	1061.895
32	Miles School,0.25 miles east of, on north and south center line and between sections 20 and 29, R.6 W.,T.39 N., at dim T road north,center of road;road elev.	1052.867	-0.029	1054.838
33	Miles School, 0.75 miles east of,Rinehart's house,200 feet north of,at cross roads,southwest corner of,sandstone boulber,top of;dob of.paint. ------	1045.151	-0.027	1045.124
34	Miles School, 1.2 miles east of, via crooked road through woods, Red Bird Road,0.4 miles west of, on south side of road,16" post oak tree,in root of;copper nail and washer. ----------	1037.012	-0.025	1036.987
J.	Robinsons Branch,2.5 miles north of,St.James-Dawson Township Line, 30 miles north of,on top of hill, Red Bird Road at T road west to Miles School,northwest corner of roads,15" white oak tree,in root of;copper nail and washer. ----	968.394	-0.024	968.370

Adjustment of intermediate temporary bench marks between point B at the intersection of Ozark Highway and Red Bird Road, and point H at the intersection of Jakes Prairie and Red Bird Roads.

DERIVATION OF ADJUSTMENT FORMULA.

Elev.B = 1074.6660 Obser.
" B = 1074.6352 Adjus.
- 0.0308 Cor.

Elev.H = 1042.0580 Obser.
" H = 1041.8535 Adjus.
- 0.2045 Cor.



Correction c. for any intermediate point at distance d. from B and between B and H.

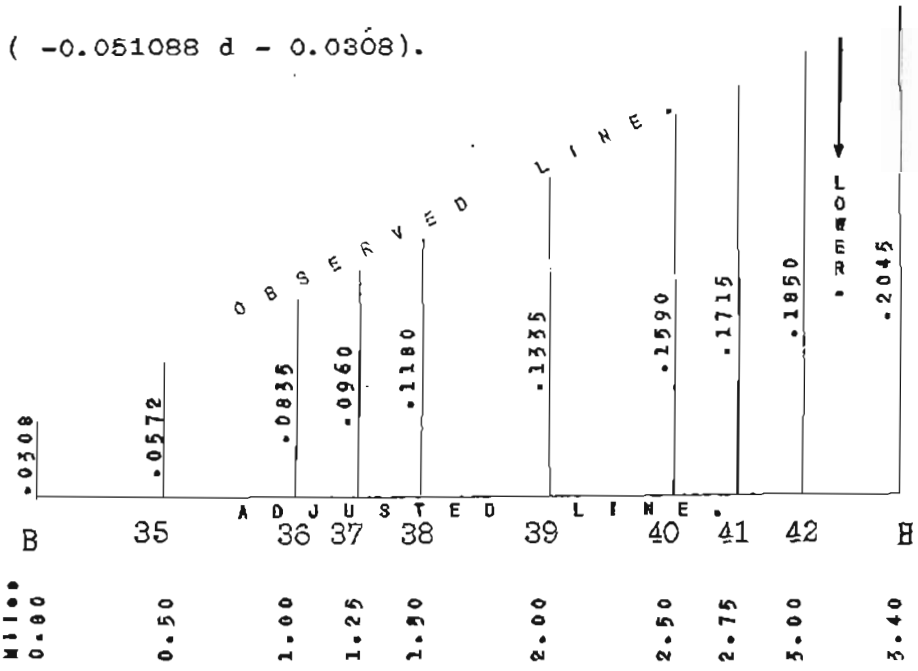
$\frac{C - 0.0308}{0.1737} = -\frac{d}{3.4}$ and $C = -\frac{0.1737\,d}{3.4} + 0.0308$

simplifying:

$C = -(0.051088\,d + 0.0308)$

and finally

$C = (-0.051088\,d - 0.0308)$.



GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN B AND H.

Red Bird Quadrangle.
(Latitude 38 -38 15';longitude 91 30' -91 45')

(St.James Township)

Line from intersection of Ozark Highway and Red Bird Road,which is 1.0 mile northeast of St.James,thence north via Red Bird Road,to Royal P.O.,and Phelps-Gasconade County Line.

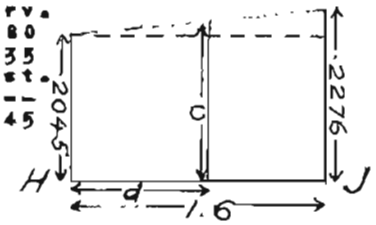
B	St.James,1.0 mile northeast of, on Ozark Highway,T road east, (intersection Ozark Highway and Red Bird Road),400 feet south of, Dr.E.A.Scott's house,300 feet west of,on west side of road,10" elm tree,in root of;copper nail and washer. ----------	1074.666	-0.031	1074.635
35	St.James, 1.5 miles north of, intersection of Red Bird road and Ozark Highway,0.5 miles north of,N.E.Johnson's house, 75 feet east of,on east side of road,20 feet inside of fence,12" hickory tree,in root of;copper nail and washer. ----	1069.326	-0.057	1069.269
36	Flag Spring School,0.25 miles east of, Britton's Store,0.25 miles west of,on Red Bird road, turn in road to east,30 feet south of,sandstone culvert across road,west end wall,top of;chiseled square. ----------	1029.621	-0.084	1029.537
37	Flag Spring School,0.5 miles east of,Britton's Store,in front of, center of road at turn to north;road elevation.--	1039.320	-0.096	1039.224
38	Flag Spring School,0.75 miles northeast of,St.James,2.5 miles north of,at Y road to north,in center of Y,10" post oak tree, in base of;copper nail and washer. ----------	1023.293	-0.118	1023.175

39	Flag Spring School, 1.25 miles northeast of, Brittons Store, 0.75 miles north of, dim cross roads, southeast corner of, 10" post oak tree, in base of; copper nail and washer.	1016.547	-0.134	1016.413
40	Brittons Store, 1.25 miles north of, St. James 3.5 miles north of, on Red Bird Road, at ford across Robinson's Branch, 50 feet south of, projecting Cotton Rock Stone in bluff, on west side of road; chiseled square. (Bench Mark obliterated by dynamite)	952.626	-0.159	952.467
41	Robinson's Branch, 025 miles north of, on Red Bird Road, at dim cross roads, tile culvert on west side of road, south end wall, top of; chiseled square.	1025.576	-0.172	1025.404
	(Dawson Township)			
42	Robinson's Branch, 0.5 miles north of, on Red Bird Road, on St. James-Dawson Township Line, on east side of road, opposite T.-Snow's Mail Box, #12A, gate post, in base of; copper nail and washer.	1054.905	-0.185	1054.720
H'	Robinson's Branch, 0.9 miles north of, on Red Bird Road, at intersection of Red Bird and Jakes Prairie roads, (Y road to northeast), F. Berries house, 300 feet south of, southeast corner of roads, 14" post oak tree, root of; copper nail and washer.	1042.947	-0.205	1042.742
H.	Robinsons Branch, 09 miles north of, on Red Bird road, at intersection of Red Bird and Jakes Prairie roads, (Y road northeast), F. Berries house, 230 feet south of, crotch in Y roads, opposite east end of wagon gate, five feet south of P.B.M. fence; iron poststamped "1042"	1042.058	-0.205	1041.853

Adjustment of intermediate temporary bench marks between point H at intersection of Jakes Prairie and Red Bird Roads, and point J on Red Bird Road two miles north of St. James–Dawson Township Line.

DERIVATION OF ADJUSTMENT FORMULA.

Elev. H = 1042.0580 Observ.
" H = 1041.8535 Adjust.
Cor. = 0.2045



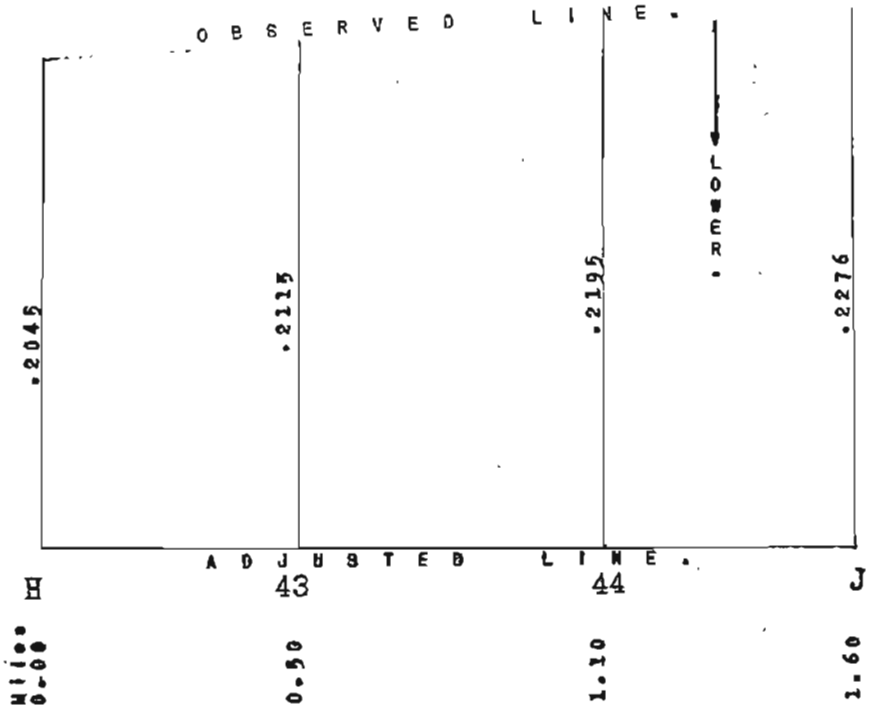
Elev. J = 968.5980 Observ.
" J = 968.3704 Adjust.
- 0.2276 Cor.

Correction c. for any intermediate point at distance d. from H and between H and J.

$$\frac{C - 0.2045}{0.0281} = \frac{d}{1.6} \quad \text{and} \quad C = \frac{0.0281\,d}{1.6} + 0.2045$$

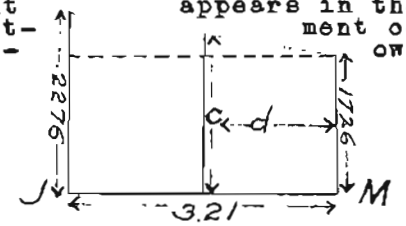
Then:

$C = -(0.0144375\,d + 0.2045)$ and $C = (-0.0144375\,d - 0.2045)$.



Adjustment of intermediate bench marks between point J on Red Bird Road two miles north of St. James- Dawson Twp. Line, and point M 500 feet north of Royal Post Office.

The adjustment of these intermediate bench marks can be effected by scaling the adjustment corrections from the large diagram below and subtracting each correction from the observed elevation as it appears in the level notes; or, if desired, the adjustment corrections can be computed by the following formula for this section, J to M.



DERIVATION OF ADJUSTMENT FORMULA.

Elev. J = 968.598 Observed.
" J = 968.3704 Adjusted.
- 0.2276 Cor.

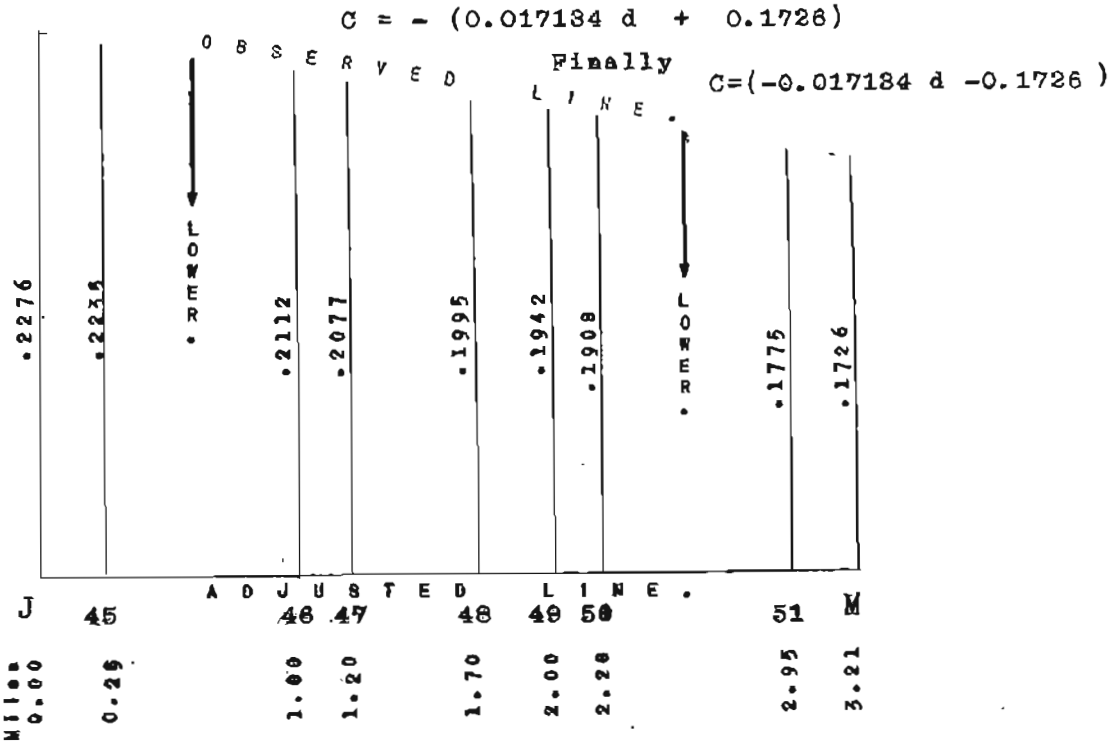
Elev. M = 954.9030 Observed.
" M = 954.7304 Adjusted.
- 0.1726 Cor.

Correction c. for any intermediate point at distance d. from M and between the points J and M.

$$\frac{C}{0.0550} = \frac{-0.1726}{3.21} = \frac{d}{3.21} \quad \text{and} \quad C = \frac{.0550\ d}{3.21} + .1726$$

$$C = -(0.017134\ d + 0.1726)$$

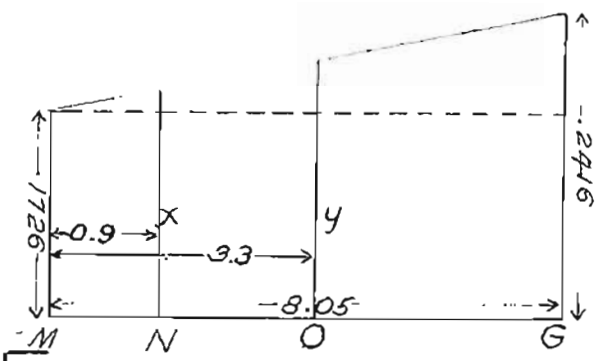
Finally $C = (-0.017134\ d - 0.1726)$



GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN J AND M.

43	Robinson's Branch, 1.4 miles north of, Township line, 09 miles north of, on Red Bird road, on top of hill, at T road west, northwest corner of, 12" white oak tree base of; copper nail and washer.	1036.078	-0.212	1035.866
44	Robinson's Branch, 2.0 miles north of, Dawson Township Line 1.5 miles north of, on Red Bird. Road, Eikman's house, 50 feet east of, on west side of road at gate, sandstone boulder, top of; chiseled square.	996.226	-0.220	996.006
J.	Robinson's Branch 2.5 miles north of, Dawson St. James Township Line, 2.0 miles north of, on top of hill, at T road west to Mile School, northwest corner of roads, 15" white oak tree, root of; copper nail and washer	968.598	-0.228	968.370
45	Robinson's Branch, 2.75 miles north of, on Red Bird Road, on top of hill, N.D. Putnam's house, 40" southeast of, on west side of road, five feet inside of yard fence, 12" walnut tree, root of; copper nail and washer.	967.970	-0.224	967.746
46	Robinson's Branch, 3.5 miles north of, intersection of Red Bird and Jakes Prairie Road, 2.6 miles north of, ford at Clear-creek, 07 miles south of, on Red Bird Road, sandstone boulder at side of road, top of painted square.	975.188	-0.211	974.977
47	Clear Creek, 0.5 miles south of, on Red Bird Road, intersection of Red Bird Road with Jakes prairie Road, 2.8 miles north of, 18" Black Oak tree at side of road, in root of; copper nail and washer	894.860	-0.208	894.652

Adjustment of point N on Red Bird Road 0.9 miles north of Royal P.O., and point O on Jakes Prairie road 0.25 miles south of Phelps-Gasconade County Line. N and O are intermediate junction points on the line M via N and O to G.



M via N & O to G.

Elev.M = 954.7240 Arranged
-79.6530

Elev.N = 875.0710 Arranged
+94.6650

Elev.O = 969.7360 Arranged
-48.2180

Elev.G = 921.5230 Arranged
Elev.G = 921.2814 Adjusted
- 0.2416 Correction at G.

Elev.M = 954.9030 Observ.
" M = 954.7304 Adjust.
- 0.1726 Cor.at M.

Let x = cor.to N, then:

$\frac{x}{0.0690} = \frac{-.1726}{8.05}$

$x = \frac{.0690 \times 0.9}{8.05} + .1726$

x = -(.0077 + .1726)

Then:

x = -0.1803 Cor.to N.

Elev.N=875.0710 Arrangd
-0.1803 Cor.
" N=864.8907 Adjustd

Let Y = Cor.to O, then:

$\frac{Y}{0.0780} = \frac{-.1726}{8.05}$

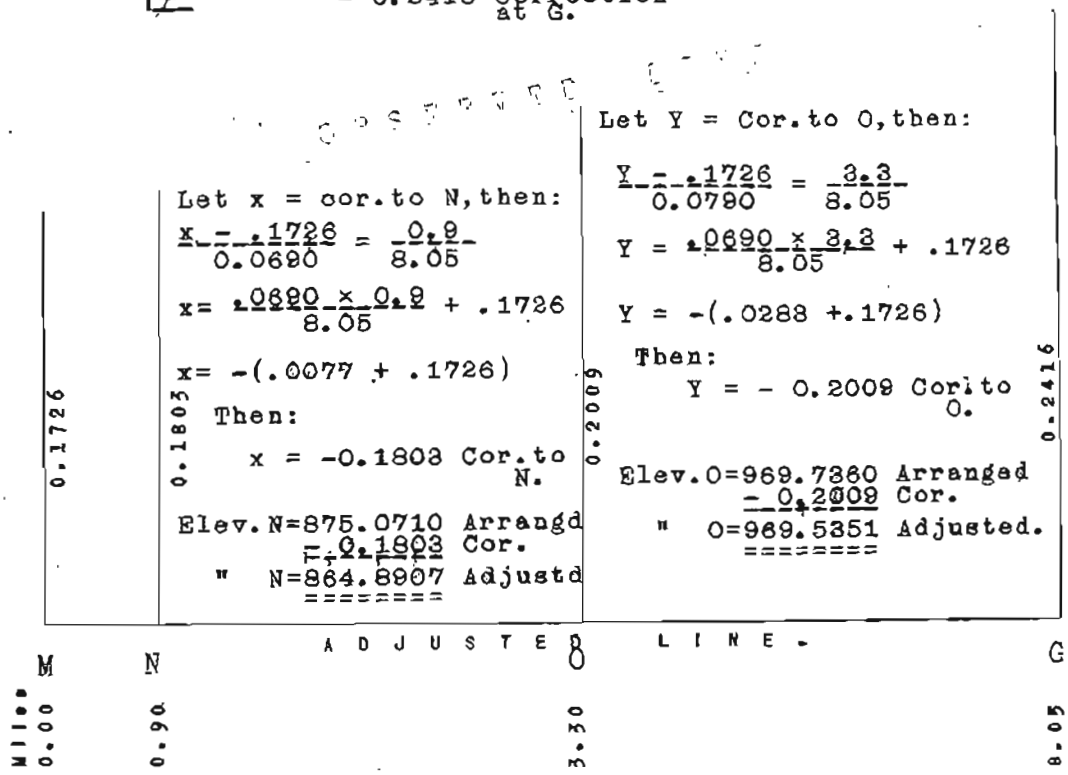
$Y = \frac{.0690 \times 3.3}{8.05} + .1726$

Y = -(.0288 +.1726)

Then:

Y = - 0.2009 Cor.to O.

Elev.O=969.7360 Arranged
- 0.2009 Cor.
" O=969.5351 Adjusted.

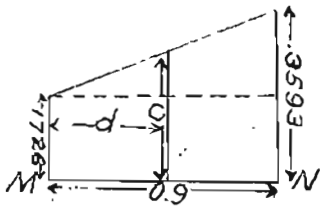


GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE JUNCTION POINTS N AND O.

Adjustment of intermediate bench marks between point M 500 feet north of Royal Post Office, and point N 1100 feet south of Red Bird Road Ford across Big Bourbeuse River.

The adjustment of these intermediate bench marks can be effected by scaling the adjustment corrections from the large diagram below and subtracting each correction from the observed elevation as it appears in the level notes; or, if desired, the adjustment corrections can be computed by the following formula for this section of the net, M to N.

DERIVATION OF ADJUSTMENT FORMULA.



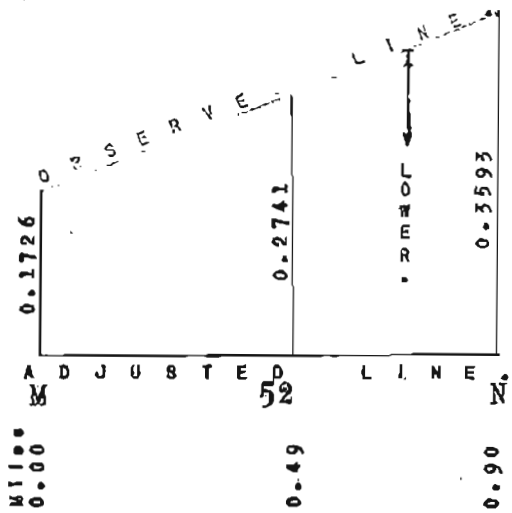
Elev. M = 954.9030 Observ.
" M = 954.7304 Adjust-
~ 0.1726 Cor.

Elev. N = 875.2500 Observ.
" N = 874.8907 Adjust.
- 0.3593 Cor.

Correction c. for any intermediate point at distance d. from M and between points M and N.

$$\frac{c - 0.1726}{0.1867} = -\frac{d}{0.9} \qquad \text{and} \qquad c = \frac{0.1867\,d}{0.9} + 0.1726$$

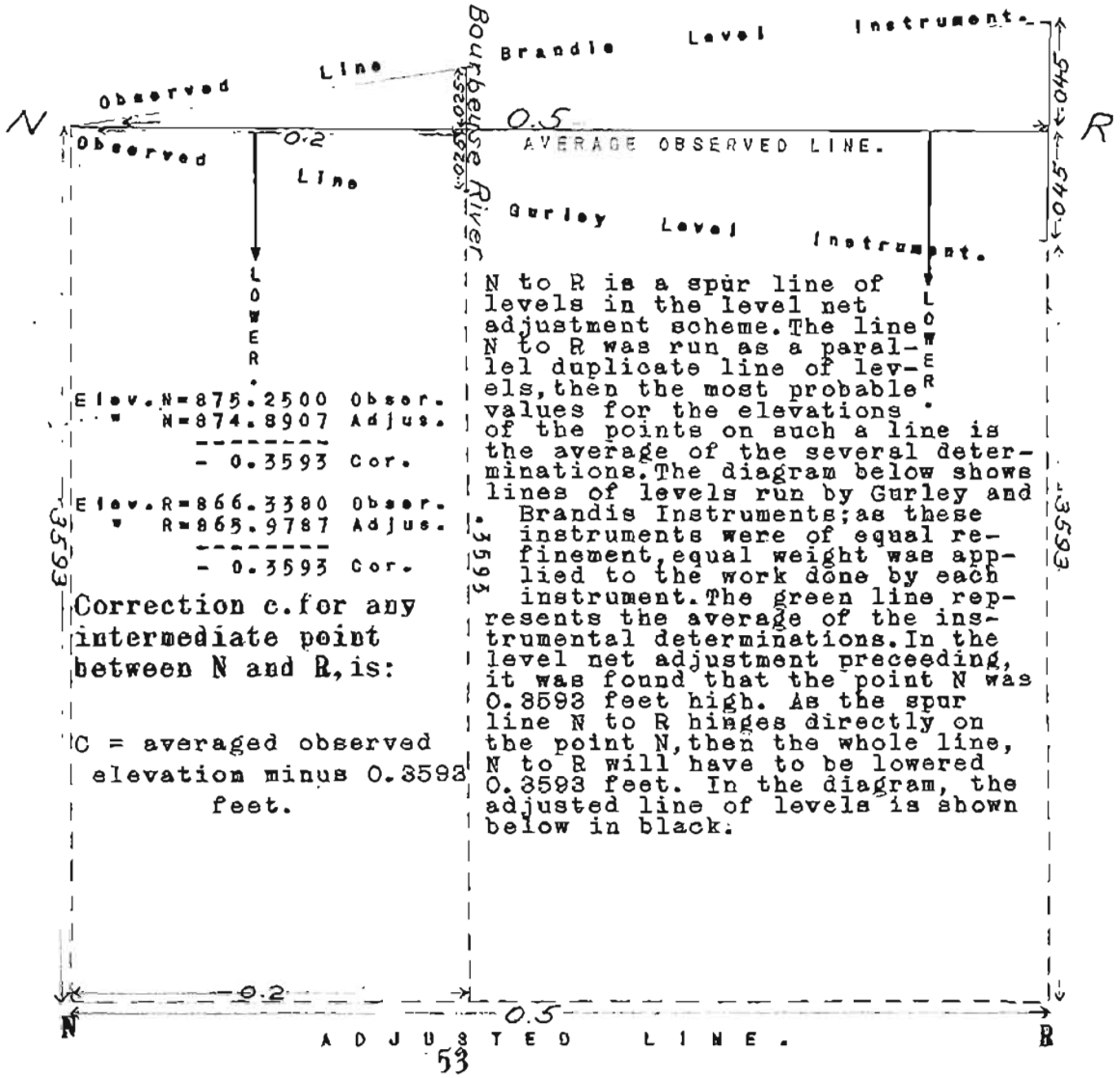
$$c = -(0.20744\ d + 0.1726) \quad \text{or} \quad c = (-0.20744\ d - 0.1726).$$



GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN M AND N.

48	Royal Post Office, 1.5 miles south of, on Red Bird Road, Clear Creek, bridge over, south east end of concrete bridge, elevation on hand rail.------	872.690	-0.200	872.490
49	Royal Post Office, 1.2 miles south of, Clear Creek bridge, 0.3 miles north of, T road east, 100" south of, concrete culvert across road, center of; road elevation. ----------	924.914	-0.194	924.720
50	Royal Post Office, 1.0 mile south of, at T road northwest, 24" Black Oak tree on west side of road, in base of; copper nail and washer.-------	983.208	-0.191	983.017
51	Royal Post Office, 0.25 miles south of, on Red Bird Road, on east side of road, 30" White Oak tree, in root of; copper nail and washer.-------	985.210	-0.178	985.032
M'	Royal Post Office, on Red Bird Road, Steen's Store, north end of concrete porch, one foot out from Building, six inches from north end of porch; bronze tablet stamped "961." ----------	P.B.M. 960.631	-0.173	960.458
M.	Royal Post Office, 500 feet north of, Royal School House, 1000 feet north of, at T road west to Republic Road, southwest corner of, telephone pole, in base of; copper nail and washer.-------	954.903	-0.173	954.730
52	Royal Post Office, 0.6 miles north of, on Red Bird Road, Huntsman's House, 200 feet north west of, on east side of road, 16" Red Oak Tree, in root of; copper nail and washer.---------	935.969	-0.274	935.695

Adjustment of intermediate bench marks between the points N and R on the Red Bird Road near the Phelps-Gasconade County Line.



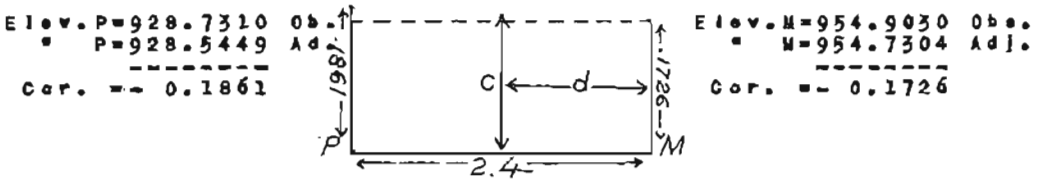
53

GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN N AND R.

N.	Royal Post Office,1.25 miles north of,on Red Bird Road,at T road east to Jakes Prairie Road, 50" feet south of T road,on east side of road,14" Red Oak Tree, in base of;copper nail and washer.----------	875.250	-0.359	874.891
53	Royal Post Office,1.45 miles north of,on Red Bird Road,Ford across Big Bourbuese River,125 feet southwest of,center of road,30 feet west of,main channel of River,80 feet south of,leaning 12" slippery Elm Tree in south bank of River,inbase of; ten penny nail.----------	812.023	-0.359	811.664
R.	Royal Post Office,1.75 miles north of,on Red Bird Road,Red Bird Post Office,2.00 miles south of,intersection Red Bird - St. James and Red Bird - High Gate roads,1.50 miles south of,Ford across Big Bourbuese River,0.3 miles north of,large Red Barn on east side of road,100 feet southwest of,center of road, 25 feet west of,(Primary Traverse Station#87),worm fence,4 feet inside of,in lot;Iron Post set on Phelps Gasconade County Line, P.B.M. stamped "866."----------	866.338	-0.359	865.979

Adjustment of intermediate bench marks between point P on Republic Road 0.5 miles south of Harrison School, and point M on Red Bird Road 500 feet north of Royal Post Office.

DERIVATION OF ADJUSTMENT FORMULA.



Correction c. for any intermediate point at distance d. from M and between P and M.

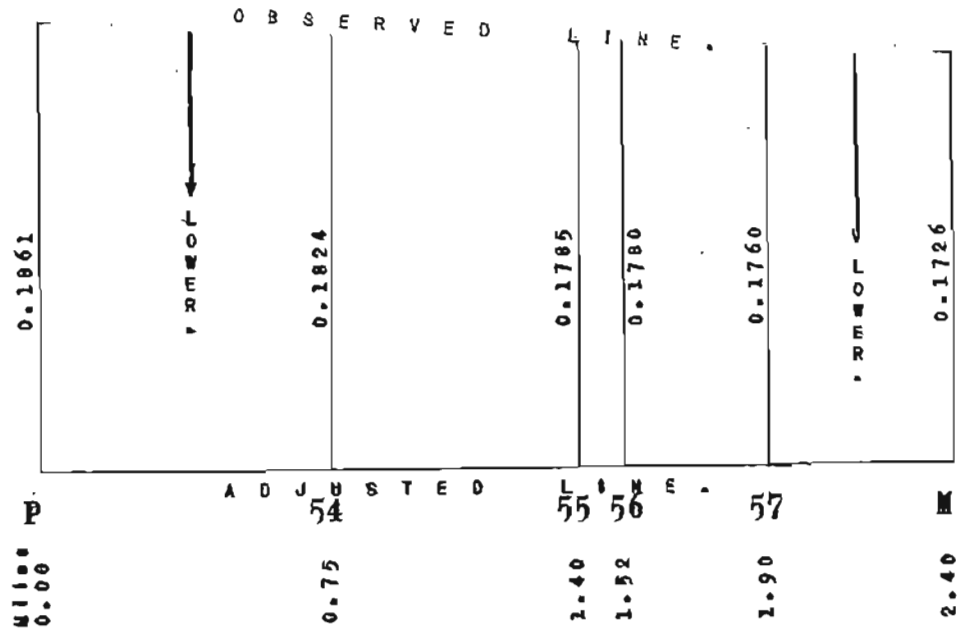
$$\frac{C - 0.1726}{0.0135} = \frac{d}{2.4}$$

$$C = \frac{0.0135\ d}{2.4} + 0.1726$$

$$C = -(0.005625\ d + 0.1726)$$

and finally

$$C = (-0.005625\ d - 0.1726).$$



GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN P AND M.

Red Bird Quadrangle.
(Latitude 38 -38 15';longitude 91 30'-91 45')

(Dawson Township)

Line from point on Red Bird Road 500 feet north of Royal Post Office, thence west 2.5 miles via road to point on Republic Road 1.0 mile north of Wash School.

M	Royal Post Office, 500 feet north of, Royal School House, 1000 feet north of, at T road west to Republic Road, southwest corner of, telephone pole, in base of; copper nail and washer. ----------	954.903	-0.173	954.730
57	Royal Post Office, 0.5 miles west of, at foot of hill, at private road southwest to farm house, on north side of road, infence corner, twin white oak tree, in base of; copper nail and washer. -----	863.080	-0.176	862.904
56	Bourbeuse River Ford, at mouth of Little Bourbeuse, middle of ford; road elevation. ----------	813.000	-0.178	812.822
55	Royal Post Office, 1.0 west of, S.T.Mitchel's house, 600 feet east of, Bourbeuse River ford, 700 feet west of, on north side of road, at corner of woods, 20" white oak tree, in root of; copper nail and washer. ----------	831.792	-0.179	831.613
54	Royal Post Office, 1.65 miles west of, Bourbeuse River ford, 0.75 miles west of, Republic Rd., 0.75 miles east of, Phelps-Gasconade County Line, 1.0 mile south of, south side of road, at T road south, 12" white oak tree, in base of; copper nail and washer. -----	924.692	-0.182	924.510

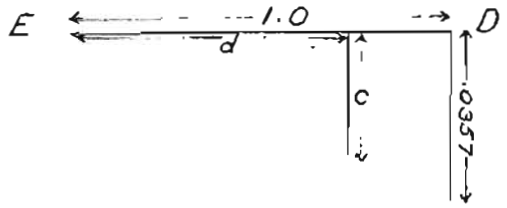
P.	Wash School,1.0 mile north of, Little Bourbeuse Creek,0.5 miles north of,Harrison School, 0.5 miles south of,Phelps-Gasconade County Line,1.0 mile south of,Austin E.Breur's house 200 feet west of,at T road east from Republic Road,on west side of road,2 feet inside of fence, 10" post oak tree,in base of; copper nail and washer.--------	928.731	-0.186	928.545

Adjustment of intermediate bench marks between point E at Knobview Depot, and point D at the intersection of the Ozark Highway and the Phelps-Crawford Line.

DERIVATION OF ADJUSTMENT FORMULA.

Elev.E = 1074.605 Adjust.
Cor. = 0.000 ft.

Elev.D = 1098.8490 Observ.
" D = 1098.8847 Adjust.
+ 0.0357 Cor.

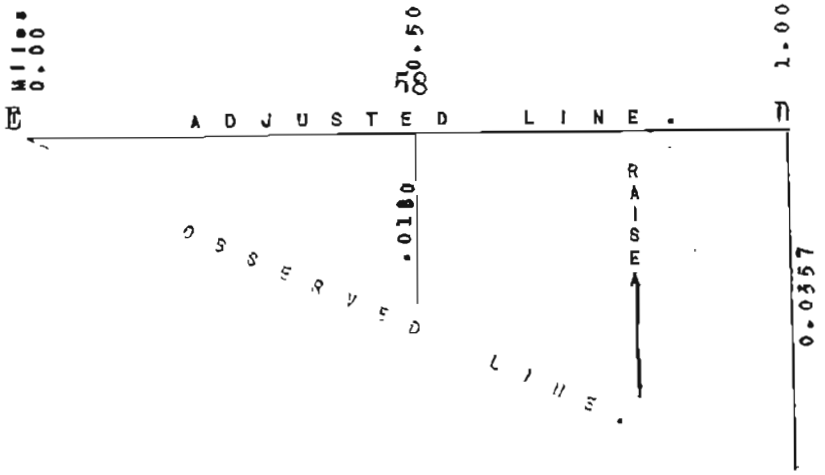


Correction c. for any intermediate point at distance d. from E, and between the points E and D.

$$\frac{c}{0.0357} = \frac{d}{1.0}$$

Then:-

$$c = (+0.0357\ d).$$



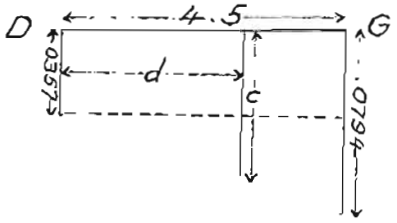
GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN E AND D.

Adjustment of intermediate bench marks between point D at the intersection of the Ozark Highway and the Phelps-Crawford County Line, and point G at the intersection of Jakes Prairie and Tribby roads, 0.5 miles south of Rinkle P.O.

DERIVATION OF ADJUSTMENT FORMULA.

Elev. D = 1098.849 Observ.
" D = 1098.8847 Adjust.
" 0.0357 Cor.

Elev. G = 921.2020 Observ.
" G = 921.2814 Adjust.
" 0.0794 Cor.

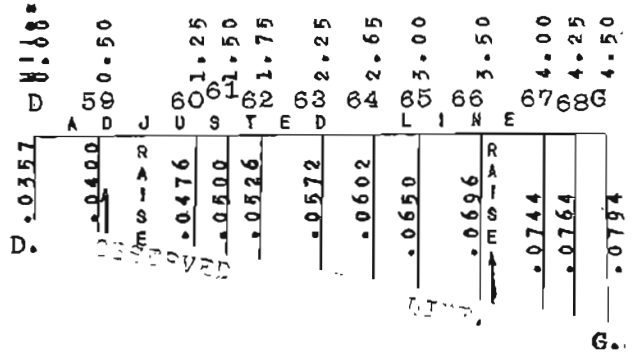


Correction c. for any intermediate point at distance d. from point D and between points D and G.

$$\frac{c-0.0357}{0.0437} = \frac{d}{4.5}$$

$$c=\frac{0.0437\,d}{4.5} + 0.0357$$

$$c=(+0.009711\ d\ +\ 0.0357)$$



GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN D AND G.

Red Bird Quadrangle.
(Latitude 38-38 15';longitude 91 30'-91 45')

(St.James Township)

Line from Knobview Depot,east 0.5 miles via Frisco Railroad to Phelps-Crawford County Line,thence north via Pheps-Crawford County Line road,3.5 miles to T road west, to Jakes Prairie road,thence west via T road,1.5 miles to Jakes Prairie road at a point 0.5 miles south of Hinkle Post Office.

E.	Knobview Depot,90 feet east of, 45 feet south of center of track, 12 feet east of R.R. crossing, 2 feet north of R.R.Property line, iron post stamped "1084".	1074.005	0.000	1074.005
58	Knobview Depot,0.5 miles east of, via R.R.,concrete semaphore base, southwest corner, top of;sigmal 949 on Phelps-Crawford County line;painted square.	1081.484	0.018	1081.502
D.	Friendship School,2.0 miles east of, on Ozark Highway,at cross roads on Phelps-Crawford County Line,south-west corner of roads,14" hickory tree, in base of;copper nail and washer.	1098.849	0.036	1098.885
59	Knobview Depot,1.5 miles north of, on Phelps-Crawford County line road, Ozark Highway,0.5 miles north of, Cucarolo's house,75 feet northeast of,on west side of road,in fence line,12" locust tree,in base of; copper nail and washer.	1071.411	0.040	1071.451

(Dawson Township)

60	Knobview Depot,2.25 miles north of, on Phelps- Crawford County line road, Ozark Highway,1.25 miles north of, drain to northwest,100 feet south of, on west side of road,20" white oak tree,in base of; copper nail and washer.	1064.348	0.048	1064.396

61	Knobview Depot,2.5 miles north of,on Phelps-Crawford County Line Road,Ozark Highway,1.5 miles north of,on top of hill, opposite mail box on east side of road,center of road;road elevation. ----------	1087.589	0.050	1087.639
62	Knobview Depot,2.75 miles north of,on Phelps-Crawford County Line Road,Ozark Highway,1.75 miles north of,farm house,350 feet northwest of,on rock projecting out of ground in center of road,15 feet northwest of fence corner,top of rock; painted square. ----------	1012.794	0.053	1012.847
63	Ozark Highway,2.25 miles north of,on Phelps-Crawford County Line Road,at dim cross roads, on top of hill,15 feet southwest of roads,on stump; painted square. ----------	1071.623	0.057	1071.680
64	Ozark Highway,2.65 miles north of,on Phelps-Crawford County Line Road,Hinkle P.O.,225 miles southeast of,on top of hill,in old abandoned field,west side of road,conglomerate boulder,top of;painted square. ----------	1040.245	0.060	1040.305
65	Ozark Highway,3.00 miles north of,on Phelps-Crawford County Line road,right angled turn in road east,75 feet south of,T road west to Hinkle P.O.,just opposite,on east side of road, sandstone boulder,top of; painted square. ----------	994.501	0.065	994.566
66	Phelps-Crawford County Line, 0.5 miles west of,on Tribby road,Hinkle P.O.,1.5 miles south east of,30 feet east of drain, on south side of road,sandstone boulder,top of;painted square.	994.905	0.070	994.975

67	Phelps-Crawford County Line, 1.0 mile west of,on Tribby road, Hinkle P.O.,1.0 mile southeast of,at dim cross roads,10feet south of center of roads,sand-stone boulder projecting out of ground,top of;painted square.--	950.010	0.074	950.084
68	Phelps-Crawford County Line, 1.25 miles west of,on Tribby road,southeast corner,S.W.Qtr. of the S.E.Qtr.Sec.25,T.39N., R.6 W.,at private road north, mail box on north side of road, center of road just opposite; road elevation. ----------	949.773	0.076	949.849
G.	Hinkle Post Office,0.5 miles south of,intersection Jakes Prairie and Tribby roads,(T road east)(sign board 10 miles to Cuba),east side of Jakes Prairie road,30 feet north of road intersection,8" walnut tree, in base of;copper nail and washer. ----------	921.202	0.079	921.281

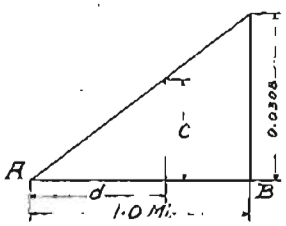
Adjustment of intermediate temporary bench marks between point A at St. James and point B at the intersection of Red Bird Road and Ozark Highway.

DERIVATION OF ADJUSTMENT FORMULA.

Elev. A = 1067.800
Cor. = 0.000

Elev. B = 1074.6660 Observ.
" B = 1074.6352 Adjust.

= 0.0308 Cor.

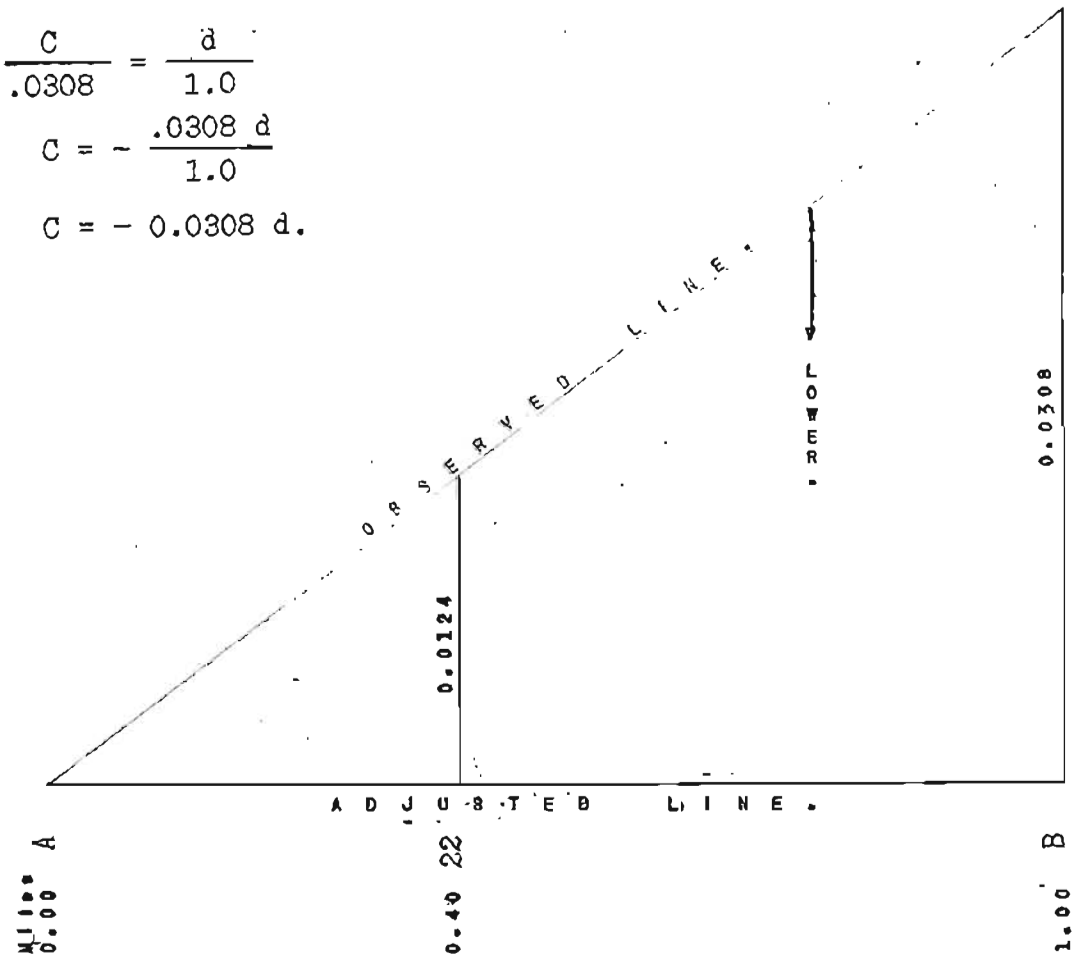


Correction c. for any intermediate point at distance d. from A and between points A and B.

$$\frac{C}{.0308} = \frac{d}{1.0}$$

$$C = -\frac{.0308\ d}{1.0}$$

$$C = -\ 0.0308\ d.$$



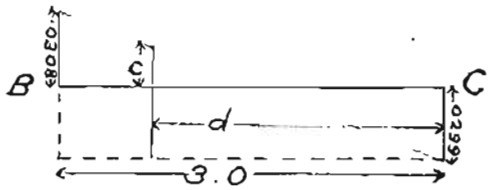
GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN A AND B.

Adjustment of intermediate bench marks between point B at the intersection of Red Bird Road and the Ozark Highway, and point C on the Ozark Highway at Friendship School.

The adjustment of these intermediate bench marks can be effected by scaling the adjustment corrections from the large diagram below and combining each correction algebraically with the observed elevation as it appears in the level field notes; or, if desired, the adjustment corrections for the intermediate points can be computed by the following formula for this section of the level net, B to C.

DERIVATION OF ADJUSTMENT FORMULA.

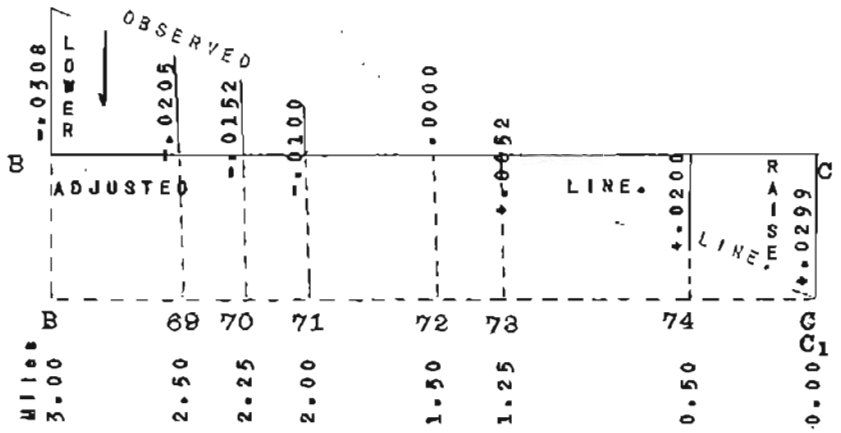
Elev. B	= 1074.6660	Observ.	Elev. C	= 1083.4000	Observ.
" B	= 1074.6352	Adjust.	" C	= 1083.4299	Adjust.
	- 0.0308	Cor.		+ 0.0299	Cor.



Correction c. for any intermediate point at distance d. from C and between the points B and C.

$$\frac{C + 0.0299}{0.0607} = \frac{d}{3.0} \qquad \text{and} \qquad C = \frac{0.0607\ d}{3.0} - 0.0299$$

$$C = -(0.020233\ d - 0.0299) \quad \text{then} \quad C = (+0.0299 - 0.020233\ d).$$



GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN B AND C.

MERAMEC SPRINGS QUADRANGLE.

(Latitude 37°45' -38°; longitude 91°30' -91°45')

(St. James Township)

LINE FROM ST. JAMES EAST VIA OZARK HIGHWAY TO PHELPS-CRAWFORD COUNTY LINE, SAID POINT ONE MILE NORTHEAST OF KNOB-VIEW RAILROAD DEPOT.

	Description.	Observed Elevation	COR.	Adjusted Elevation
A.	(See description on page 64)	1087.800	0.000	1087.800
22	St.James 0.4 miles east of, on St.James-Cuba road(Ozark Highway), Frisco R.R.,50ft. north of,turn in road to north, 500 ft.west of,concrete culvert, acrossroad,north end wall, top of;chiseled square. ----------	1065.363	-0.0124	1065.351

RED BIRD QUADRANGLE.

(Latitude 38°-38°15'; longitude 91°30' -91°45')

(St.James Township)

B.	St.James,1.0 miles N.E. of, Ozark Highway, T road east (intersection of Ozark Highway and Red Bird road),400 ft. S.of, Dr.E.A.Scott's house, 300 ft. W. of, on W. side of road, 10" elm tree, in root of;copper nail and washer. ----------	1074.666	-0.031	1074.635
69	St.James,1.5 miles N.E. of, on Ozark Highway,0.5 miles east of Red Bird road intersection, on N.side of road,12" post oak tree, in base of; copper nail and washer. ----------	1062.669	-0.021	1062.648
70	St. James,1.75 miles northeast of, on Ozark Highway, concrete culvert across road, south end wall, top of; chiseled triangle.	1034.437	-0.015	1034.422
71	St. James, 0.2 miles N.E. of, on Ozark Highway, on top of hill, center of road; road elevation.	1054.446	-0.010	1054.436

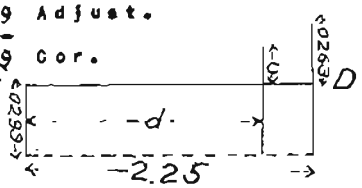
Adjustment of intermediate bench marks between point C on the Ozark Highway at Friendship School, and point D at the intersection of the Ozark Highway and the Phelps-Crawford County Line.

The adjustment of these intermediate temporary bench marks can be effected by scaling the adjustment corrections from the large diagram below and combining each correction according to its sign with the observed elevation as it appears in the level field notes; or, if desired, the adjustment corrections can be computed by the following formula for this section of the level net, C to D.

D E R I V A T I O N O F
A D J U S T M E T F O R M U L A .

Elev. C = 1083.4000 Observ.
" C = 1083.4299 Adjust.
+ 0.0299 Cor.

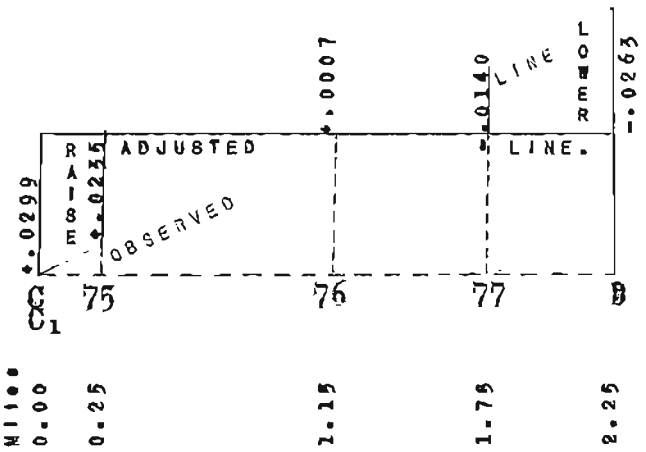
Elev. D = 1098.9110 Observ.
" D = 1098.8847 Adjust.
- 0.0263 Cor.



Correction c. for any intermediate point at distance d. from C and between the points C and D.

$$\frac{C + 0.0299}{0.0562} = \frac{d}{2.25} \quad \text{and} \quad C = \frac{0.0562\,d}{2.25} - 0.0299$$

$C = -(0.024978\,d - 0.0299)$ then $C = (+0.0299 - 0.024978\,d)$.



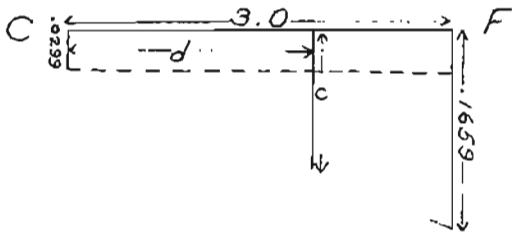
GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN C AND D.

72	St. James, 2.5 miles northeast of, on Ozark Highway, Friendship School, 1.5 miles west of, concrete culvert(errected 8-13-20 by T. Ball), north end wall, top of; chiseled square.	1010.057	0.000	1010.057
73	Friendship School, 1.25 miles west of, on Ozark Highway, at T. road N. to Hinkle P.O., Corrugated iron culvert, rock on east end of culvert, top of; chiseled triangle.	1018.926	+0.005	1018.931
74	Friendship School, 0.5 miles W. of, on Ozark Highway, Gerber's Store, 600 feet W. of, concrete culvert, north end wall, top of; chiseled triangle.	1062.045	+0.020	1062.065
C.	Friendship School, on Ozark Highway, southwest cornerof concrete porch of school house, top of; chiseled triangle, painted "1083.3".	1083.400	+0.030	1083.430
C1.	Friend School, on Ozark Highway, 50feet S.W. of, in School Yard, 10 ft. from fence corner, 30 ft. N. of center of Ozark Highway, 30 ft. E. of center of T. road north; iron post stamped"1983".	1083.458	+0.030	1083.488
75	Friendship School, 0.25 miles E. of, private T. road N. to Louis Ball's residence, 75 ft. W. of, concrete culvert, S. end wall top of; chiseled square,	1037.041	+0.024	1037.065
76	Friendship School, 0.9 miles E. of, on Ozark Highway, Phelps-Crawford County Line, 1.1 miles west of, concrete culvert, south end end wall, top of; chiseled Square.	1042.297	+0.001	1042.298

77	Friendship School, 1.5 miles east of, on Ozark Highway, Phelps Crawford County Line, 0.5 miles west of, Knowview, 0.5 miles north of across field, concrete culvert under Ozark Highway, north end wall, top of; chiseled square.-----	1028.412	-0.014	1028.398
D.	Friendship School, 2.0 miles east of, on Ozark Highway, at cross roads on Phelps - Crawford county Line, southwest corner of roads, 14" hickory tree, in base of ; copper nail and washer.----------	1098.911	-0.026	1098.885

This completes the description and adjustment of the line of levels from St. James east via Ozark Highway to the Phelps-Crawford County Line, said point one mile northeast of Knobview Railroad Depot.

Adjustment of intermediate bench marks between point C on the Ozark Highway at Friendship School, and point F on the Jakes Prairie Road 1.0 mile south of Hinkle Post Office.

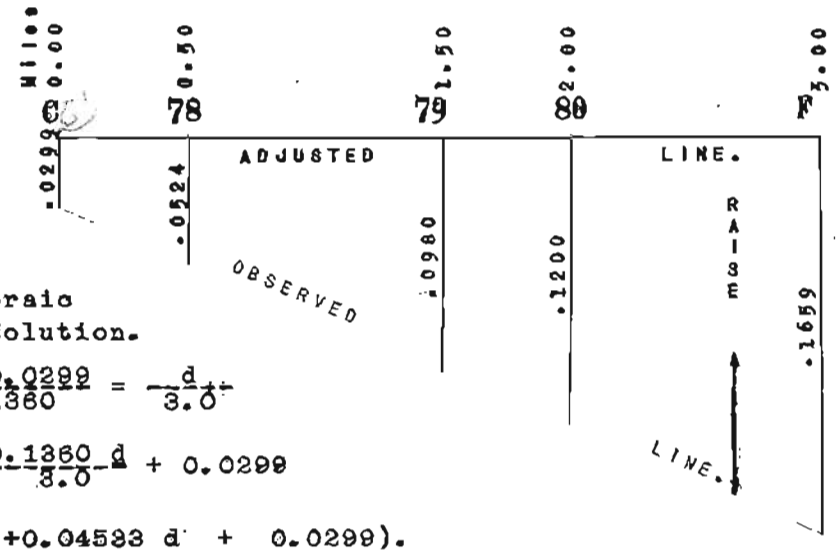


DERIVATION OF ADJUSTMENT FORMULA.

Elev.C =	1083.4000 Observ.	Elev.F =	1001.1770 Observ.
" C =	1083.4299 Adjust.	" F =	1001.3429 Adjust.
	+ 0.0299 Cor.		+ 0.1659 Cor.

Correction c. for any intermediate point at distance d. from C and between the points C and F.

GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN C AND F.



Algebraic Solution.

$$\frac{C - 0.0299}{0.1360} = \frac{d}{3.0}$$

$$C = \frac{0.1360\ d}{3.0} + 0.0299$$

$$C = (+0.04533\ d + 0.0299).$$

Red Bird Quadrangle.
(Latitude 38-38 15';longitude 91 30'-91 45')

(St.James Township)

Line from Friendship School on Ozark Highway,in center of section 11,T. 38 N., R.6 W., north via road 3.0 miles to point on St.James-Jakes Prairie road,1.0 mile south of Hinkle Post Office,at a T road south east.

C.	Friendship School,onOzark highway,in center of sec.11,T.38 N., R.6 W.,southwest corner of concrete porch of School House,top of;chiseled triangle.----------	1083.400	0.030	1083.430
C'.	Friendship School,on Ozark Highway,in center of sec.11,T.38 N., R.6 W.,50 feet southwest of,in school yard,10 feet from fence corner,30 feet north of center of Ozark Highway,30 feet east of center of T road north;iron post P. B. M. stamped.---------- "1083".	1083.458	0.030	1083.488
78	Friendship School,0.5 miles north of,Hinkle P.O.,3.5 miles south of, T road west,east side of road,8" black oak tree,in root of;copper nail and washer. ----------	1018.238	0.052	1018.290

(Dawson Township)

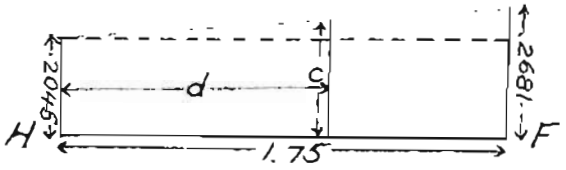
79	Friendship School,1.5 miles north of, Hinkle P. O.,2.5 miles south of, on township line between St.James and Dawson Townships,bridge on township line road over south fork of Clear Creek,northwest corner of bridge; copper nail and washer. -------	951.418	0.098	951.516
80	Friendship School,2.0miles north of,Hinkle P.O.,2.0 miles south of, east side of road,inrock ledge; chiseled triangle.----------	945.415	0.120	945.535

F.	Friendship School, 3.0 miles north of, Hinkle P.O., 1.0 mile southwest of, on Jakes Prairie road, at T road southeast (road to Friendship Sch.) southeast corner of roads, 15" white oak tree, in base of; copper nail and washer. ------	1001.177	0.166	1001.343

Adjustment of intermediate bench marks between point H at the intersection of Red Bird and Jakes Prairie roads, and point F on Jakes Prairie Road 1.0 miles south of Hinkle P.O. and 3.0 miles north of Friendship School.

DERIVATION OF ADJUSTMENT FORMULA.

Elev.H =	1042.0580 Observed.	Elev.F=	1001.6110 Observed.
" H =	1041.8535 Adjusted.	" F=	1001.3429 Adjusted.
	- 0.2045 Cor.		- 0.2681 Cor.



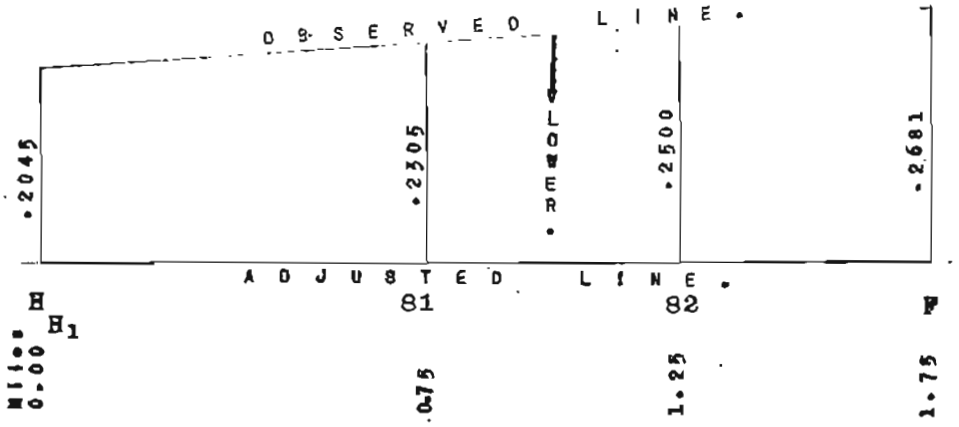
Correction c.for any intermediate point at distance d.from H and between the points H and F.

$$\frac{C}{0.2045} = \frac{d}{1.75}$$

$$C = \frac{0.0636\ d}{1.75} + 0.2045$$

$$C = -(0.036343\ d + 0.2045)$$

$$C = (-0.036343\ d - 0.2045).$$



GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN H AND F.

Red Bird Quadrangle.
(Latitude 38 -38 15';longitude 91 30'-91 45')

(Dawson Township)

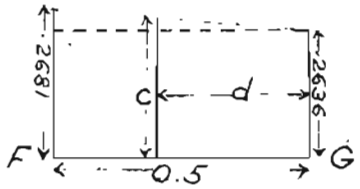
Line from intersection of Red Bird and Jakes Prairie roads, thence northeast 2.75 miles via Jakes Prairie road to Collins School and Hinkle P.O., thence north 4.5 miles via Jakes Prairie road to Phelps-Gasconade County Line, 2.0 miles west of Jakes Prairie Post Office.

H	Robinsons Branch, 0.9 miles north of, on Red Bird Road, at intersection of Red Bird and Jakes Prairie roads, (Y road Northeast), F.Berries house, 230 feet south of, in crotch in Y roads, opposite east end of wagon gate, 5 feet south of fence; iron post stamped "1042".	P.B.M. 1042.058	-0.205	1041.853
81	Intersection Red Bird and Jakes Prairie roads, 0.75 miles northeast of, on Jakes Prairie road, main fork of Clear Creek, 0.5 miles southwest of, via way of road, H.Eikman's house, 100 feet east of, on east side of road, 14" post oak tree, in base of; copper nail and washer.	964.540	-0.231	964.309
82	Intersection of Red Bird and Jakes Prairie roads, 1.25 miles northeast of, on Jakes Prairie road, Hinkle P.O., 1.5 miles southwest of, Clear Creek Ford, (Main fork), 50 feet east of, at angle in road to north, southeast corner of road, in fence line, 20"hickory tree, in base of; copper nail and washer.	916.091	-0.250	915.841
F	Friendship School, 3.0 miles north of, Hinkle P.O., 1.0 mile southwest of on Jakes Prairie road, at T road southeast(road to Friendship Sch.), southeast corner of roads, 15"white oak tree, in base of; copper nail and washer.	1001.611	-0.268	1001.343

Adjustment of intermediate temporary bench marks between point F on Jakes Prairie Road 1.0 Mile south of Hinkle P.O. and 3.0 miles north of Friendship School, and point G 0.5 miles south of Hinkle P.O. at the intersection of Jakes Prairie and Tribby roads.

Derivation of Formula for the Adjustment of Intermediate Points.

Elev. F =	1001.6110	Observed.	Elev. G = 921.5450 Observed.
" F =	1001.3429	Adjusted.	" G = 921.2814 Adjusted.
	- 0.2681	Cor.	- 0.2636 Cor.

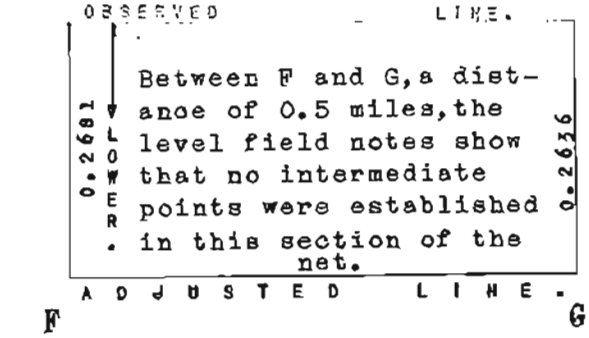


$$\frac{C - 0.2636}{0.0045} = \frac{d}{0.5}$$

$$C = \frac{0.0045\ d}{0.5} + 0.2636$$

$$C = -(0.009\ d + 0.2636)$$

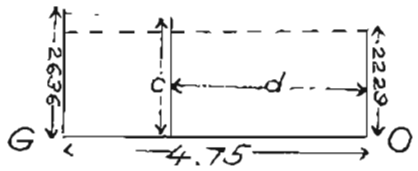
$$C = (-0.009\ d - 0.2636)$$



GRAPH SHOWING ADJUSTMENT OF POINTS F AND G.

Adjustment of intermediate bench marks between point G 0.5 miles south of Hinkle P.O. at the intersection of Jakes Prairie and Tribby roads, and point O on Jakes Prairie Road 0.25 miles south of the Phelps-Gasconade County Line.

The adjustment of these intermediate temporary bench marks can be effected by scaling the adjustment corrections from the large diagram below and subtracting each correction from the observed elevation as it appears in the field level notes; or, if desired, the adjustment corrections can be computed by the following formula for this section of the level net, G to O.



DERIVATION OF ADJUSTMENT FORMULA.

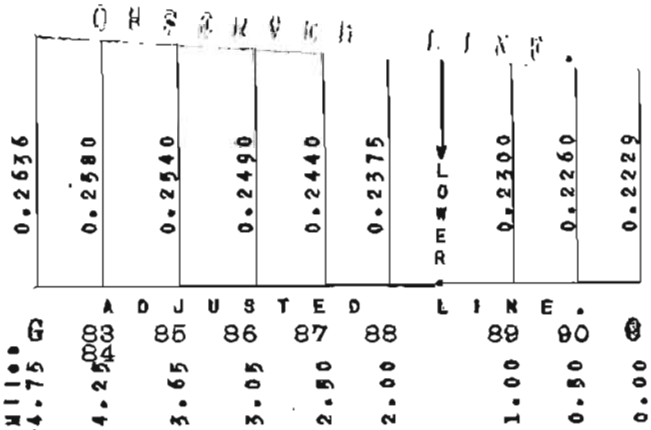
Elev. G =	921.5450	Observed.	
" G =	921.2814	Adjusted.	
	- 0.2636	Cor.	

Elev. O =	969.7580	Observed.	
" O =	969.5351	Adjusted.	
	- 0.2229	Cor.	

Correction c. for any intermediate point at distance d. from G and between G and O.

$$\frac{C - 0.2229}{0.0407} = \frac{d}{4.75} \qquad \text{and} \qquad C = \frac{0.0407\,d}{4.75} + 0.2229$$

$$C = -(0.008568\,d + 0.2229) \quad \text{or} \quad C = (-0.008568\,d - 0.2229).$$



GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN G AND O.

G	Hinkle P.O.,0.5 miles south of, intersection Jakes Prairie and Tribby roads,(T road east)(sign board 10 miles to Cuba),east side Jakes Prairie road,30 feet north of road intersection,8"walnut tree,in base;copper nail and washer.----------	921.545	-0.264	921.281
83	Collins School,800 feet north of, Hinkle Store and P.O.,40 feet southeast of,40 feet west of center of road,in line with tangent of road to northeast;iron post stamped "1039." ---------- P. B. M. 1038.938		-0.258	1038.680
84	Collins School,800 feet north of, Hinkle Store and P.O.,southeast corner of porch,top of floor; copper nail and washer. --------	1040.617	-0.258	1040.359
85	Hinkle PO.,0.6 miles north of, on Jakes Prairie road,southwest corner sec.13,R.6W,T.39 N.,350 feet north of,on west side of road, opposite private T road east to white house,16" black jack tree, in base of;copper nail and washer. ----------	984.964	-0.254	984.710
86	Hinkle P.O.,1.2 miles north of, on Jakes Prairie road,at private T road east,southeast corner of road,10" post oak tree,in base of; copper nail and washer. --------	1057.863	-0.249	1057.614
87	Hinkle P.O.,1.75 miles north of, Oak Grove School,2.0 miles south of,old deserted farm houseon west side of road,125 feet southeast of, on top of hill, east side of road, 10" black jack tree,in root of; copper nail and washer,---------	1086.932	-0.244	1086.688

Adjustment of Permanent Bench Mark S on Jakes Prairie Road at the Phelps-Gasconade County Line. The adjustment is to be made between the points O and S with the line tied back at O.

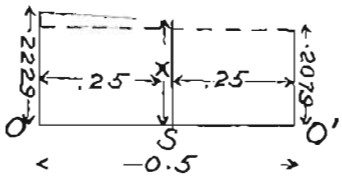
This section, O to S, is a spur line to the level net; it was run as a duplicate line of levels, that is: the line was run over the same route twice, but in opposite directions: i.e. O to S and from S back to O. See page of this thesis for a discussion on duplicate lines of levels.

A N A L Y T I C A D J U S T M E N T

O F P O I N T S.

Elev. O = 969.7580 Observ. in circuit O via O and N to N.
" O = 969.5351 Adjusted.
- 0.2229 Cor.

Elev. O = 969.7430 Observ. on spur return.
" O = 969.5351 Adjust.
- 0.2079 Cor.



Now for the adjustment of S.

Let X = the correction desired.

Then:

$$\frac{X - 0.2079}{0.0150} = \frac{0.25}{0.50} \quad \text{and} \quad X = \frac{0.0150}{2} + 0.2079$$

and:

$$X = -(0.0075 + 0.2079)$$

and finally: X = - 0.2154 ft. the correction to be applied to the observed elevation of S.

Then: Elev. S = 942.5160 Observed.
" - 0.2154 Correction.
" S = 942.3006 Adjusted.

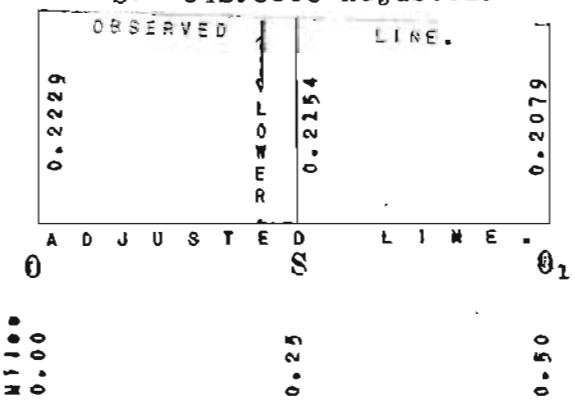
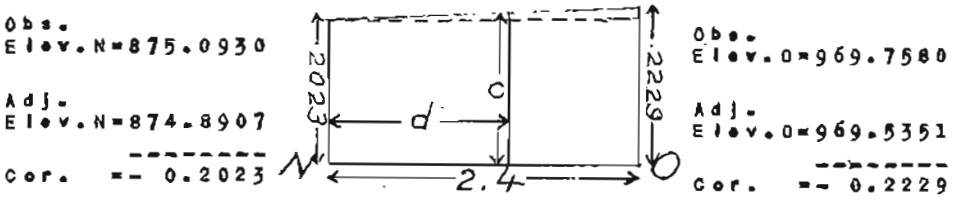


DIAGRAM FOR THE GRAPHIC ADJUSTMENT OF POINT S.

88	Oak Grove School,1.5 miles south of,on Jakes Prairie road,dim T road west,200 feet north of,farm house on east side of road,200 ft west of,in fence corner,east side of road,14" red oak tree,in base of;copper nail and washer.-------	999.372	-0.238	999.134
89	Oak Grove School,0.5 miles south of,on Jakes Prairie road,W. E. Wright's house,200 feet north of, at T road east and private road west thru gate,top of hill,center of roads; road elevation. -------	967.072	-0.230	966.842
90	Oak Grove School,on Jakes Prairie road,Dawson Township,Phelps-Gasconade County Line,0.75 miles S. of,in front of School House,125 feet west of,east side of road, 14" scaley bark hickory tree,in root of;copper nail and washer.--	900.143	-0.226	899.917
O.	Oak Grove School,in Dawson Twp., 0.5 miles north of,T road east, 50 feet south of,T road west to Red Bird road,20 feet north of, northwest corner of roads,10" post oak tree,in base of;copper nail and washer. ----------	969.758	-0.223	969.535
S.	Oak Grove School,0.75 miles north of,T road east on County Line, Jakes Prairie P.O.,2.0 miles west of,at private T road west on County Line, Sewell's house,60 ft southwest of,Phelps-Gasconade County Line,10 feet north of,north east corner of roads;iron post stamped "942".----------	P. B. M. 942.516	-0.215	942.301
O.	Oak Grove School,0.5 miles north of,T road east,50 feet south of, T.road west to Red Bird road,20 ft north of,northwest corner of roads, 10" post oak tree,in base of; copper nail and washer. ---------	969.743	-0.208	969.535

Adjustment of intermediate temporary bench marks between point N on the Red Bird Road 0.9 miles north of Royal P.O., and point O on the Jakes Prairie Road 0.25 miles south of the Phelps-Gasconade County Line.

The adjustment of these intermediate temporary bench can be effected by scaling the adjustment corrections from the large diagram below and subtracting each correction from the observed elevation as it appears in the level field note book; or, if desired, the adjustment corrections can be computed by the following formula for this section, N to O.



DERIVATION OF ADJUSTMENT FORMULA.

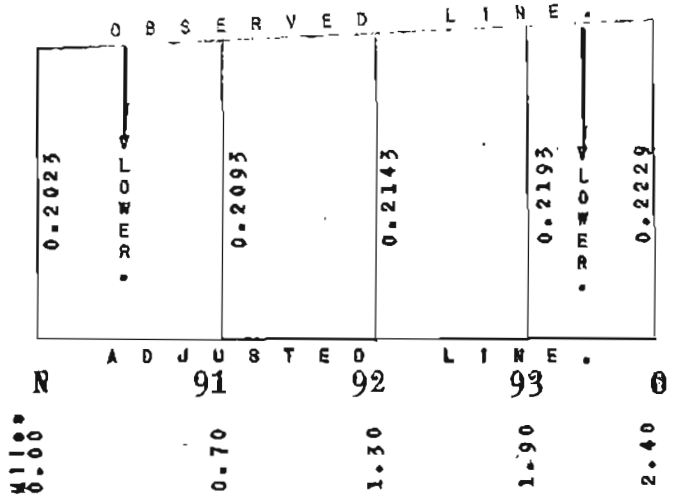
Correction c. for any intermediate point at distance d. from N and between the points N and O.

$$\frac{C - 0.2023}{0.0206} = \frac{d}{2.4}$$

$$C = \frac{0.0206\ d}{2.4} + 0.2023$$

$$C = -(0.008583\ d + 0.2023)$$

$$C = (-0.008583\ d - 0.2023).$$



GRAPH FOR THE ADJUSTMENT OF INTERMEDIATE POINTS BETWEEN N AND O.

Red Bird Quadrangle.
(Latitude 38 -38 15';longitude 91 30' -91 45')

(Dawson Township)

Line from point on Jakes Prairie Road 0.25 miles south of County Line, thence west two-one half miles to point on Red Bird Road 0.5 miles south of Phelps-Gasconade County Line and 1.25 miles north of Royal P.O.

O	Oak Grove School in Dawson Twp, 0.5 miles north of, T road east, 50 feet south of, T road west to Red Bird road, 20 feet north of, northwest corner of roads, 10" post oak tree, in base of; copper nail and washer. ----------	969.758	-0.223	969.535
93	Oak Grove Church, 800 feet north east of, J.B. Copeland's house, 800 feet west of, in bottom of branch to north, north side of road, 24" sycamore tree, in root of; copper nail and washer. ----	834.182	-0.219	833.963
92	Oak Grove Church, 0.6 miles west of, T road south on line between Secs. 2 and 3 produced north, 30 feet west, concrete culvert across road, north end wall, top of; painted square. ----------	895.903	-0.214	895.689
91	Oak Grove Church, 1.2 miles west of, Red Bird Road, 0.65 miles east of, Spurgeon's house, 300 feet southwest of, on north side of road, twin trunk walnut tree, in root of; copper nail and washer. -------	863.954	-0.209	863.745
N.	Royal P. O., 1.25 miles north of, on Red Bird Road, Oak Grove Church, 1.85 miles west of, Phelps-Gasconade County Line, 0.5 miles south of, Red Bird Ford across Bourbeuse River, 1100 feet south of, at T road east to Jakes Prairie road, 50 feet south of T road, on east side of road, 14" red oak tree, in base of; copper nail and washer.	875.093	-0.202	874.891

C O N C L U S I O N .

In conclusion, a brief outline will be given covering the main classes of problems which have been considered and the methods of solution and their appropriate applications to the problem at hand.

Direct Observations of a Single Quantity.

Adjustment. Take the mean or the weighted mean of the observed quantities.

Indirect Observations.

Adjustment. Write the observation equations and from them the normal equations; the solution of the latter gives the unknown quantities themselves or the corrections to their assumed approximate values. The number of the observations equations will be the same as that of the observations; the number of the normal equations will equal that of the unknown quantities, which must always be less than that of the observations.

Conditioned Observations.

Adjustment. Write the condition equations in their general form and then in their simple form involving the corrections. From them form the normal equations, the same in number as the conditions. The solution of the normal equations gives a set of factors, called correlates, one for each condition equation, from which the desired corrections to the observed quantities are determined.

In summarizing what has been stated above, there are in general two methods of finding the most probable values of the unknown quantities in cases involving conditioned quantities.

In the first method the *conditional* equations are avoided (or illiminated) by impressing their significance on the observation equations, which reduces the problem to the cases as considered in Parts (II) and (III).

In the second method the *observation* equations are illiminated by impressing their significance on the conditional equations, when the solution may be effected by the method of correlatives as considered in Part (IV).

The first method is the most direct in elementary problems, say with not more than three residual unknowns, but the second method by using conditional equations instead of observation equations and completing the solution by the method of correlatives in connection with the results obtained from the solution of the normal equations, greatly reduces the work of computation in the case of complicated problems with four or more unknown quantities to be solved.

The best methods of solving normal equations, as I have found them, are those given in Part (IV) pages 55, 56, and 57.

APPLICATIONS OF THE METHOD OF LEAST SQUARES
TO THE ADJUSTMENT OF LEVEL CIRCUITS.
ADJUSTMENT OF ELEVATIONS IN DAWSON
AND ST. JAMES TOWNSHIPS,
PHELPS COUNTY,
MISSOURI.

B I B L I O G R A P H Y.

B I B L I O G R A P H Y .

References on the Method of Least Squares.

Manual of Spherical and Practical Astronomy, Chauvenet.
J.B. Lippincott Co., Philadelphia, 1885.

Approximate Determination of Probable Error, Appendix No. 13, Report for 1890, U.S. Coast and Geodetic Survey.

Theory of Errors and Method of Least Squares, Johnson.
John Wiley & Sons, New York, 1893.

Practical Astronomy as Applied to Geodesy and Navigation, Doolittle,
John Wiley & Sons, New York, 1893.

Precise Surveying and Geodesy, Merriman. John Wiley & Sons,
New York, 1899.

Adjustment of Observations, Wright and Hayford. D. Van Nostrand & Co,
New York, 1904.- 1907.

Text Book on Geodesy and Least Squares, Crandall.
John Wiley & Sons, New York, 1907.

Handbuch der Vermessungskunde, I. Metzler, Stuttgart, 1910. By Jordan.

Ausgleichungsrechnung. By Helmert. Tubner, Leipzig, 1907.

Method of Least Squares, Merriman, Wiley, New York, 1913.

Method of Least Squares, Comstock. Ginn, Boston, 1895.

Method of Least Squares, Bartlett, Boston, 1915.

Theory of Errors and Least Squares, Weld. Macmillan, New York, 1916.

Theory of Errors and Method of Least Squares, Johnson. Wiley 1892.

Combinations of Observations, Brunt, Cambridge University Press, 1917.

Application of Least Squares to the Adjustment of Triangulation,
Special Publication No. 28, U.S.C.& G. Survey, 1915.

Practical Least Squares, O.M. Leland, McGraw-Hill Book Co.,
New York, 1921.

Geodetic Surveying and the Adjustment of Observations (Method of
Least Squares), E.L. Ingram, McGraw-Hill Book Co.,
New York, 1911.

Adjustment of Triangulation, E.M. Douglas, School of Mines Quarterly
No. 4 Vol. 14 Columbia College, New York.

Topographic Instructions of the U.S. Geological Survey, Washington, 1918

Topographic, Trigonometric, and Geogetic Surveying, Wilson.
John Wiley & Sons, New York, 1912.

Of the above treatise on the subject of Least Squares, the last five mentioned were consulted as well as the book further up in the list by Weld. The book by Leland is recommended as an exceptionally good book on the subject.

APPLICATIONS OF THE METHOD OF LEAST SQUARES
TO THE ADJUSTMENT OF LEVEL CIRCUITS.
ADJUSTMENT OF ELEVATIONS IN DAWSON
AND ST. JAMES TOWNSHIPS,
PHELPS COUNTY,
MISSOURI.

I N D E X .

I N D E X .

I N D E X .

I N D E X .

I N D E X .

I N D E X .

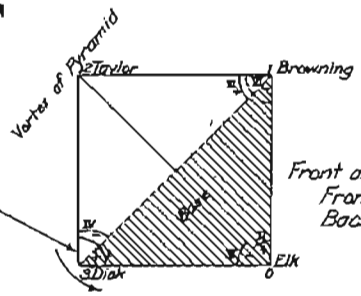
V

W

ETIC COMPUTATION

SQUARE ADJUSTMENT

Rules for sides
Stand outside angle: sides both read towards angle's vertex: left-hand side read −, right hand side next and read +. Consider Dick, then −⅝ + ¹⁄₁₆



(e) Sine Equation

Sides	Angle	Sine	Diff. for 1"	Correction in Seconds		Corr. to Sine		Corrected Sine
−⅝ + ⁹⁄₁₆	83°-48'-53.15	9.9974645	+02.2	−1.33	=	-3	→	9.9974642
−¹⁄₁₆ + ¹⁄₁₆	40°-39'-19.77	9.8130350	+24.6	+2.12	=	+.32	→	9.8130402
−³⁄₁₆ + ⁵⁄₁₆	45°-12'-37.04	9.8510731	+20.9	+2.18	=	+46	→	9.8510777
		+ 9.8613728						9.8613821
−⅝ + ¹³⁄₁₆	40°-09'-14.16	9.8094543	+24.9	−0.91	=	-23	→	9.8094520
−⅜ + ⁹⁄₁₆	45°-38'-34.90	9.8540576	+20.8	−2.97	=	-61	→	9.8540515
−⅝ + ⁵⁄₁₆	95° 29' 07.82	9.9980787	− 2.0	+0.71	=	−1	→	9.9980786
		− 9.8815906						9.8613821
		+ 9.8613728						
		Error −180						

Spherical excess gotten from preliminary computation.

Second groups have signs changed in (i) except when the diff. for 1" is minus, then the sign remains same.

Note that the diff. for 1" for angles over 90° is minus.

(i)

0 = −180 − .022°⅝ + .022°⅝ − .246°⅜ + 246⅜ − .209°¹¹⁄₁₆ + .209°¹¹⁄₁₆ + .248°⅝
−.249'⅝ + .206'⅜ − .206'⅜ − .020'¹¹⁄₁₆ + .020'¹¹⁄₁₆

Combine like terms algebraically, then

0 = 1.80 + .227°⅝ + .022°⅝ − .452°⅜ + 246'⅛ − .209'¹¹⁄₁₆ + .229'¹¹⁄₁₆ − .249'⅝ + .206'⅜ − .020'¹¹⁄₁₆

Whole equations divided by 100.

Can use any 3 of 4 angle equations
In this problem leave out (d)

Equations of Condition

(f) 0 = −3.45 −⅜ + ¹⁄₁₆ − ⁹⁄₁₆ + ⁵⁄₁₆ − ⅝ + ⁹⁄₁₆ (from a)
(g) 0 = +4.70 −⅜ + ⁵⁄₁₆ − ⅝ + ⁵⁄₁₆ − ⅝ + ⁹⁄₁₆ (from b)
(h) 0 = +2.74 −⅜ + ¹⁄₁₆ − ⁹⁄₁₆ + ¹¹⁄₁₆ − ⅝ + ⁹⁄₁₆ (from c)

Table of Correlates

(k)	(k')	(l)	(m)	(n)	Correlates, computed values substituted				Corr. for sides in seconds
sides	for (k)	for (q)	for (h)	for (l)	+0275	-0.083	-0.589	+3.007	
⅜	+1		+1	+0246	+0275		-0.589	+0.740 =	+0.426
⅝	−1	+1		−0452	−0275	−0.083		−1.359 =	−1.697

1	Title Page
6	Contents
2	Introduction
4	Front Plates in Folder
118	Body of Thesis
1	Rear Plate
132	Pages Total.

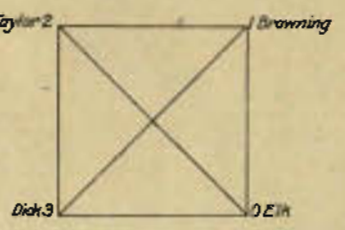
Thesis
C. E. S. Bardsley,
Candidate for the Degree
of
Civil Engineer.

Designed and Computed by C. E. Bardsley

Chart lettered by Mrs. T. Raney. 1922.

GEODETIC COMPUTATION

LEAST SQUARE ADJUSTMENT

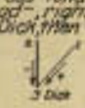


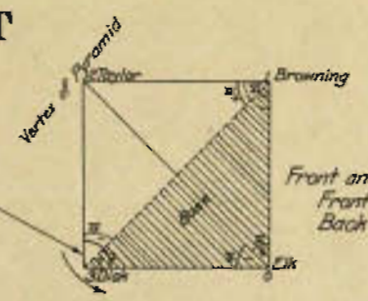
L-S+1 = No. of angle equations
L = No. of lines in Figure = 6.
S = No. of stations = 4
Then 6-4+1 = 3 angle equations

L-2S+3 = No. of sine equations
6-(2X4)+3 = 1 sine equations

Rules for sides

Stand outside angle; sides both read towards angle's vertex: left hand side read −, right hand side next and read +. Consider Dick, then − ⅛ + ⅛





Front and back angles
Front angles +
Back angles −

Angle Equations

Station	Side	Observed Angle	Cor. for Angle	Corrected Sp. Angle
a	Elk	40° 33' 19.77	+2.12	40°-33'-21.89
	Browning	56° 23' 07.62	+0.71	56°-23'-08.33
	Taylor	44° 03' 30.68	+0.68	44°-03'-31.14
		179° 59' 57.31	+3.45	180°-00'-00.76
		Spher. Excess 0.76		Sp ex. 0.76
b	Elk	45° 36' 34.80	-2.97	45°-36'-31.83
	Taylor	50° 34' 37.57	-0.40	50°-34'-37.17
	Dick	83° 48' 53.16	-1.33	83°-48'-51.82
		180° 00' 05.53	-4.70	180°-00'-00.88
		Spher. Excess 0.88		Sp ex. 0.88
c	Elk	66° 09' 54.07	-0.84	66°-09'-53.23
	Browning	50° 10' 30.56	-1.47	50°-10'-29.11
	Dick	43° 39' 38.99	-0.43	43°-39'-38.56
		180° 00' 03.64	-2.74	180°-00'-00.90
		Spher. Excess 0.90		Sp ex. 0.90
d	Dick	40° 09' 14.16	-0.90	40°-09'-13.26
	Browning	46° 12' 37.04	+2.18	46°-12'-39.22
	Taylor	94° 38' 08.09	+0.21	94°-38'-08.30
		179° 59' 59.29	+1.49	180°-00'-00.78
		Spher. Excess 0.78		Sp ex. 0.78

Spherical excess gotten from preliminary computation

This second groups have signs changed in (i) except when the diff. for 1" is minus, then the sign remains same

Note that the diff. for 1" for angles over 90° is minus.

Can use any 3 of 4 angle equations
This problem leave out (d)

(e) Sine Equation

Sides	Angle	Sine	Diff. for 1"	Correction in Seconds		Sin. to Sine	Corrected Sine
− ⅛ + ⅛	83°-48'-53.16	9.9974642	+02.2	-1.33	=	-3	→ 9.9974642
− ⅛ + ⅛	40°-33'-19.77	9.8130360	+24.6	+2.12	=	+52	→ 9.8130402
− ⅛ + ⅛	46°-12'-37.04	9.8510731	+20.8	+2.18	=	+46	→ 9.8510777
		+ 9.8615726					9.8615821
− ⅛ + ⅛	40°-09'-14.16	9.8094643	+24.9	-0.91	=	-23	→ 9.8094520
− ⅛ + ⅛	46°-36'-34.90	9.8540576	+20.6	-2.97	=	-61	→ 9.8540515
− ⅛ + ⅛	56°-23'-07.62	9.8980787	-2.0	+0.71	=	-1	→ 9.8980786
		− 9.8615908					9.8615821
		+ 9.8615726					
		Error -180					

Whole equations divided by 100.

(i)

0 = -180 - .022⅝ + .022⅝ - .246⅝ + .246⅝ - .209⅜ + .209⅜ + .249⅛
-.249⅛ + .206⅝ - .206⅝ - .020⅜ + .020⅜

Combine like terms algebraically, then

0 = -180 + .227⅜ + .022⅛ - .452⅜ + .246⅛ - .209⅜ + .229⅜ - .249⅛ + .206⅜ - .020⅜

(a) Normal Equations

1	2	3	4	Absolute Terms
+8.000	-2.000	+2.000	+0.947	-3.450
-2.000	+6.000	+2.000	-0.863	+4.700
+2.000	+2.000	+6.000	+0.122	+2.740
+0.947	-0.863	+0.122	+0.517792	-1.800

(P) Solution of Normal Equations

	1	2	3	4	Absolute Terms
P1	+8.000	-2.000	+2.000	+0.947	-3.450
P2	(-0.1667)	+0.333	-0.333	-0.1578	+0.575
P2		+5.333	+2.667	-0.8413	+3.550
P4		(-0.1875)	-0.500	+0.1026	-0.666
P3			+4.000	+0.0800	+2.115
P5			(-0.250)	(-0.02001)	-0.529
P7				+03.080	-0.834
				-32.(98)	+3.007

Equations of Condition

(f) 0 = -3.45 - ⅝ + ⅛ - ⅝ + ⅝ - ⅛ + ⅛ (from a)
(g) 0 = +4.70 - ⅝ + ⅛ - ⅝ + ⅛ - ⅛ + ⅛ (from b)
(h) 0 = +2.74 - ⅛ + ⅛ - ⅝ + ⅝ - ⅛ + ⅛ (from c)

The above terms are put in the table of correlates, with their same sign, but can be given any weight desired. In this case given a weight of 1.

Table of Correlates

(i)	(h)	(g)	(f)	Correlates, computed values substituted				
Sides	for (f)	for (g)	for (h)	for (i)	+0.275	-0.063	-0.589	+.3007
⅛	+1		+1	+0.246	+0.275		-0.589	+0.740 = +0.426
⅝	-1	+1		-0.452	-0.275	-0.063		-1.359 = -1.697
⅛		-1	-1	+0.206		+0.063	+0.589	+0.669 = +1.271
⅝	-1		-1	-0.020	-0.275		+0.589	-0.060 = +0.254
⅝	+1			+0.229	+0.275			+0.669 = +0.964
⅛			+1	-0.209			-0.589	-0.628 = -1.817
⅛	+1	-1			+0.275	+0.063		+0.338
⅝	-1				-0.275			-0.275
⅝		+1				-0.063		-0.063
⅛		+1	+1	+0.022		-0.063	-0.589	+0.056 = -0.580
⅛			-1	-0.249			+0.589	-0.749 = -0.160
⅝		-1		+0.227		+0.063		+0.663 = +0.746

Add horizontally

(q)

	2	3	4	Absolute Terms
q1	+8.000	+2.000	-0.863	+4.700
q2	-0.667	+0.667	+0.357	-1.150
	+5.333	+2.667	-0.6473	+3.550
q3		+6.000	+0.122	+2.740
q4		-0.667	-0.3157	+1.150
q5		-1.333	+0.2737	-1.775
q6		+4.000	+0.0800	+2.115
q8			+0.517792	-1.800
q7			-0.14944	+0.644
q9			-0.05615	+0.364
q8			-0.00160	-0.042
			+03.1080	-0.834

(r)

	1	2	3	4
r1	+0.575	-0.666	-0.529	+3.007
r2	-0.475	+0.309	-0.050	
r3	+0.196	+0.294	-0.569	
r4	-0.021	-0.053		
r5	+0.275			

(S) Angle Corrections

Angle at Elk	Angle at Browning	Angle at Taylor
- Cor ⅛ = +1.697	- Cor ⅛ = -0.254	- Cor ⅛ = +0.275
+ Cor ⅛ = +0.426	+ Cor ⅛ = +0.964	+ Cor ⅛ = +0.338
+2.123	+0.710	+0.613

Angle at Elk	Angle at Taylor	Angle at Dick
- Cor ⅛ = -1.271	- Cor ⅛ = -0.336	Cor ⅛ = -0.746
+ Cor ⅛ = -1.697	+ Cor ⅛ = -0.063	Cor ⅛ = -0.586
-2.968	-0.401	-1.332

Angle at Elk	Angle at Browning	Angle at Dick
- Cor ⅛ = -1.271	- Cor ⅛ = -0.254	- Cor ⅛ = +0.160
+ Cor ⅛ = +0.426	+ Cor ⅛ = -1.217	+ Cor ⅛ = -0.586
-0.845	-1.471	-0.426

Angle at Dick	Angle at Browning	Angle at Taylor
- Cor ⅛ = -0.746	- Cor ⅛ = +1.217	- Cor ⅛ = +0.275
+ Cor ⅛ = -0.160	+ Cor ⅛ = +0.964	+ Cor ⅛ = -0.063
-0.906	+2.181	+0.212

Note: The correction for any angle is the difference between the corrections for the two sides bounding it (Table of correlates)

In calculating the angle corrections, change sign of top correction, leaving bottom as it is, then add algebraically.

Copy above corrections in their proper places in column of corrections in both angle equations and sine equations.

DIRECTIONS

Follow major operations alphabetically and the details numerically.

Computation by
R. L. YEAGER
Vocational Student
MISSOURI SCHOOL OF MINES AND METALLURGY