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Design of automotive structural components using high strength sheet steels preliminary study of members consisting of flat and curved elements

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DESIGN OF AUTOMOTIVE STRUCTURAL COMPONENTS USING HIGH STRENGTH SHEET STEELS

PRELIMINARY STUDY OF MEMBERS CONSISTING OF FLAT AND CURVED ELEMENTS

by

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I. INTRODUCTION

A. GENERAL

In recent years, a rapid increase in the use of high strength sheet steel (HSSS) in automobiles has been brought about by the increased safety requirements of the Motor Vehicle Safety Standards (MVSS) and by a demand for improved fuel economy. Typical automotive applications of HSSS can be divided into two broad categories: 1) structural and safety related parts, such as side impact bars, roof pillars, rails, and brackets, and 2) body panels, such as doors, hoods, deck-lids, fenders, and quarter panels as well as for unexposed parts such as door-,hood-, and deck-lid inners.[1,2] Many of these HSSS applications consist either entirely of curved elements, such as outer body panels, or of curved elements used in combination with flat elements, such as the structural components shown in Figure 1.1.[3]

Curved elements are normally employed in automotive structures for: 1) increased aerodynamic performance, 2) improved aesthetic quality, 3) space and layout limitations, and 4) increased compressive strength over that of flat elements with similar dimensions.[50]

Because curved elements form such vital parts of modern automobiles and because existing design procedures either lack the desired accuracy, are extremely complex, or are of uncertain applicability to HSSS, the present investigation was initiated to determine any possible methods of improving the current design procedures for curved elements. In the early stages of the investigation, all of the structural components considered will be straight with uniform cross sections throughout their lengths. This report is presented as an initial study of the compressive behavior of curved elements and contains a summary of the available literature on the subject. It is hoped that as a result of this initial study and the proposed future research, simplified and accurate design equations for the prediction of the compressive behavior of curved elements made of HSSS can be developed.

The present study was performed as a part of the proposed tasks to be conducted in Phase II of a three-phase research project entitled "Structural Design of Automotive Structural Components Using High Strength Sheet Steels". The project was started in early 1982 at the University of Missouri-Rolla (UMR) under the sponsorship of the American Iron and Steel Institute (AISI).

B. SCOPE OF INVESTIGATION

In Section II of this report, the literature on members with curved cross sections is reviewed. Section II.A contains a discussion of the behavior of individual curved plates when subjected to either elastic or inelastic buckling. For each type of buckling, the possible equations, which could be of some value for the prediction of the initial buckling stress of curved plates, are given, and the expected post-buckling behavior normally associated with each buckling type is discussed.

In Section II.B, the approximate design procedures, which have been employed by the aerospace industry for compression members consisting of both curved and flat elements, are discussed. After a brief discussion of the two basic design procedures (the Air Force Method and Crockett's Method), the two methods are compared. A few other related articles, which might be useful for future studies, are also considered.

The effective width concept normally used for flat elements is reviewed in Section II.C to provide background information for the possible development of a similar effective width concept for curved elements. Also, the sparse amount of literature available on the existing effective width equations for curved elements is summarized.

In Section II.D, the available procedures for predicting the buckling behavior of curved plates when subjected to shear loading are analyzed. This information should be of some value for future studies of beams with curved webs.

Section III contains a general summary of the future research needs for the prediction of the compressive strength of curved plates and sections composed of flat and curved elements. The information gained through the literature review is summarized in Section IV.

II. PREDICTION OF CRITICAL BUCKLING STRESS OF CURVED ELEMENTS

A. COMPRESSION OF CURVED PLATES

The accurate prediction of the compressive strength of curved plates is extremely complex. It seems that classical stability equations based on linear theory are insufficient because they consistently overestimate the critical buckling stress, f_{cr}, of curved plates. The major cause of this overestimation is the fact that buckling of curved plates is accompanied by compressive transverse membrane stresses, which result in a deflected geometry that is unstable. For this reason, large deflection theory is essential for reasonably accurate prediction of f_{cr}. It has been observed that when compressive membrane stresses are produced transverse to the direction of buckling, such as for the compressive buckling of curved plates or cylinders, large deflection theory is required. However, when tensile membrane stresses are produced perpendicular to the direction of buckling, such as for buckling caused by lateral pressure on a relatively short, closed cylinder, torsion on a cylinder, or compression on flat plates, linear theory is sufficient to predict f_{cr}.[4]

It is generally accepted that the buckling of a curved plate can be properly described by a geometric parameter, Z_b, and a buckling coefficient, k_c, as follows:

$$Z_b = (b^2/Rt)(1 - \mu_e^2)^{\frac{1}{2}} \qquad (2.1)$$

and

$$k_c = \frac{12(1 - \mu_e^2)f_{cr}}{\pi^2 E}(b/t)^2 \qquad (2.2)$$

in which

E = modulus of elasticity

t = curved plate thickness

R = radius of curved plate

b = circumference of curved plate

μ_e = elastic Poisson's ratio

Figure 2.1 shows the relationship between k_c and Z_b for a series of compression tests made by Jackson and Hall[5] on curved panels made of an aluminum alloy.[4] Other similar test results are listed in References 6 through 11.

At values of $Z_b < 10$, the behavior of curved plates is approximately the same as that of flat plates with similar boundary conditions, and thus the buckling coefficient, k_c, approaches that of a flat plate. The boundary conditions applied by Jackson and Hall[5] were between simple support and clamped conditions. Therefore, an average of the buckling coefficients for flat plates of these two limiting cases, $k_c = 5.7$, was used to plot the portion of the $k_c - Z_b$ curve for $Z_b < 10$.[4]

For values of $Z_b > 1000$, long cylinder behavior dominates, and the effects of boundary conditions are negligible. By observing the relationships in Equations (2.1) and (2.2), it can be seen that for large values of Z_b, the buckling coefficient appears to be linearly related to Z_b. Thus, the resulting equation takes on the form of the classical buckling equation for cylinders:

$$f_{cr} = C\,E\,t/R \tag{2.3}$$

in which C represents the slope of the relationship between f_{cr} and $E(t/r)$.

It is within the intermediate range of $10 < Z_b < 1000$, where boundary conditions still exert considerable influence on f_{cr},[12] that extreme difficulty is experienced in predicting the critical buckling stress. It seems obvious that some sort of transition curve must exist between the two limiting cases described above. A few of the more successful attempts to develop such a curve are described in Section II.A.1.

The range of the various possible geometries of curved plates is illustrated in Figure 2.2.[4] Of the possibilities shown in Figure 2.2, only the extreme combinations of a long curved plate with large curvature (e.g., a closed cylinder) and a short curved plate with small curvature (e.g., a flat plate) are well researched and defined. All other combinations fall into the previously described transition range.

In the following sections, the elastic and inelastic buckling and post-buckling behaviors of curved plates are discussed, and a brief summary of some of the methods proposed for the prediction of buckling is included.

1. <u>Elastic Buckling</u>.

a. <u>Transition Equations</u>. Many attempts have been made over the years to develop a transition equation that would accurately predict the critical stress of curved plates when the geometric parameters of these plates lie somewhere between those of flat plates and complete cylinders. One of the first such attempts was performed by Redshaw[13,14] who developed the following relationship based on the classical energy approach:[4]

$$f_{cr} = \frac{E}{6(1-\mu_e^2)} \sqrt{12(1-\mu_e^2)\frac{t^2}{R^2} + \frac{\pi^4 t^4}{b^4}} + \frac{\pi^2 t^2}{b^2} \qquad (2.4)$$

It can be seen that this equation reduces to the classical buckling equation for cylinders when $(b/t)^2$ is large compared to R/t. Also, when $(b/t)^2$ is small compared to R/t, Redshaw's equation reduces to that of a flat plate.

Sechler and Dunn[15] later showed that Equation (2.4) could be expressed in terms of the critical buckling stresses for complete cylinders and for flat plates as follows:

$$(f_{cr}/E)_p = (f_{cr}/E)_c + 1/4\ (f_{cr}/E)_f$$
$$+ 1/2\ (f_{cr}/E)_f \qquad (2.5)$$

in which

$(f_{cr}/E)_p$ = buckling stress ratio of a simply supported curved plate

$(f_{cr}/E)_c$ = buckling stress ratio of a complete cylinder with an R/t ratio equal to that of the curved plate,

$(f_{cr}/E)_f$ = buckling stress ratio of a simply supported flat plate with the same t/b ratio as the curved plate.

Because it is well known that the following classical expression for the buckling of cylinders,

$$(f_{cr}/E)_c = 0.6\ t/R, \qquad (2.6)$$

consistently predicts f_{cr} values, which are as much as twice the actual f_{cr} values, Sechler and Dunn suggested replacing the classical value of $(f_{cr}/E)_c$ with the following empirical relationship:

$$(f_{cr}/E)_c = 0.3\ t/R. \qquad (2.7)$$

Several other investigations into the development of transition equations for curved plates have been performed. Among the more noteworthy are the semi-empirical investigations conducted by Stowell,[16] Wenzek,[17] and Lindquists.[10] Levy[18] on the basis of large deflection theory developed the equations required to predict f_{cr}.

b. Post-Buckling Behavior. The post-buckling behavior for the elastic buckling of curved plates depends on the geometry of the plates and the magnitude of initial imperfections. It should come as no surprise that just as for the initial elastic buckling, the post-buckling behavior of curved plates also varies between the extremes of a flat plate and a cylinder.

For $Z_b < 10$, a curved plate acts much the same as a flat plate with similar dimensions. Thus, as shown in Figure 2.3, the effects of initial imperfections are insignificant, and the compressive load increases well past f_{cr}. The ultimate load is reached when the effects of plasticity become predominant.

At values of $Z_b > 1000$, the post-buckling behavior of a curved plate should be similar to that of a cylinder. From Figure 2.3,[12] it can be seen that for cylinders, the load-carrying capacity drops off sharply after initial buckling and never regains the original buckling stress in the post-buckling range. Thus, the initial buckling stress, f_{cr}, and failure are coincident.

In the intermediate range of $10 < Z_b < 1000$, there is obviously a transition between the post-buckling effects of flat plates and cylinders. However, there are no known studies of the exact post-buckling behavior of curved plates in this range.

2. Inelastic Buckling. If the various parameters described in Section II.A are such that the critical buckling stress is greater than the proportional limit of a given material, the buckling is said to be inelastic, and an adjustment in the elastic buckling equations must be made. (It is important to note that this type of buckling only occurs for materials with gradual - yielding stress-strain curves. An example of a typical gradual - yielding stress-strain curve is shown in Figure 2.4 (a). For sharp - yielding materials with stress-strain curves similar to Figure 2.4 (b), elastic buckling prevails until f_{cr} reaches the yield point of the material.) This adjustment is necessary because the elastic buckling equations were developed under the assumption that the stress and strain were linearly related. However, for stresses above the proportional limit, the relationship between stress and strain is, by definition, nonlinear. In order to account for the nonlinear stress-strain relationship, the value of the modulus of elasticy, E, is altered in the elastic buckling equations by means of a plasticity reduction factor, η. Several different values of η have been studied by various investigators. In the following sections, the available literature on the different plasticity reduction factors is reviewed.

a. Plasticity Reduction Factors.

i. Tangent Modulus Method. In 1895, Engesser[19] proposed that the modulus of elasticity, which is the slope of the stress-strain curve in the elastic range, should be replaced by the instantaneous slope of the stress-strain diagram in the inelastic range. This instantaneous slope is defined as the tangent modulus, E_t, as shown in Figure 2.5. Thus, in the elastic range, $E_t = E$. In the inelastic range, the value of E_t is substantially reduced. The value of the plasticity reduction factor for this method is simply

$$n = \frac{E_t}{E} . \tag{2.8}$$

ii. Secant Modulus Method. This method is quite similar to the tangent modulus method. The only difference is in the definition of the secant modulus, E_s. The secant modulus is defined as the slope of a line from the origin of the stress-strain diagram to the critical stress. The value of E_s is illustrated in Figure 2.5. The plasticity reduction factor for this case is

$$n = \frac{E_s}{E} . \tag{2.9}$$

In the mid 1940's, Schuette,[9] who used this method for curved plates constructed of magnesium alloy materials, reported fair agreement between the predicted and test results.

iii. Reduced Modulus Method. This method was originally proposed by Engesser[19] and later revised by von Karman.[20] The reduced modulus, E_r, (also known as the double modulus) is a function of the original modulus, E, the tangent modulus, E_t, and the shape of the cross section. This modulus is derived from the equilibrium equations in the cross section at the onset of buckling and, thus, is technically more correct than the tangent modulus method.[21] The reduced modulus is defined as

$$E_r = \frac{EI_1}{I} + \frac{E_t I_2}{I} \qquad (2.10)$$

in which I_1 and I_2 are the moments of inertia with respect to the neutral axis of the tensile and compressive stresses caused by column instability. For a more detailed description of the reduced modulus, the reader is referred to the work of Bleich.[22]

According to Fischel,[21] the reduced modulus for compression members with rectangular cross sections, such as flat plates, may be expressed as

$$E_r = \frac{4EE_t}{(\sqrt{E} + \sqrt{E_t})^2} \; . \qquad (2.11)$$

Fischel reports good correlation between the test results of curved plates made of aluminum alloy and the predicted values of f_{cr} when Equation (2.11) is used for the calculation of E_r.

iv. Gerard's Method. Another method for reducing the modulus of elasticity for inelastic buckling by means of a plasticity reduction factor is given by Gerard[4] as

$$n = \frac{E_s}{E}\left[\frac{(1-\mu_e^2)}{(1-\mu^2)}\frac{E_t}{E_s}\right]^{\frac{1}{2}} \qquad (2.12)$$

in which $\mu = \mu_p - (\mu_p - \mu_e)(E_s/E)$ and μ_p = plastic Poisson's ratio. The remaining terms have been previously defined.

In checking the test data published by Schuette[9] on curved plates made of magnesium alloy, good agreement was obtained between the test results and those predicted by using the above value of n. It is interesting to note that in using this method, the accuracy of the predicted results was better than the accuracy obtained when the secant modulus method was used with the same data.[4]

b. Post-Buckling Behavior. The approximate buckling and post-buckling behavior of flat plates and columns that buckle inelastically is shown in Figure 2.3 (b). Again, depending on the value of Z_b, the behavior of curved plates would be expected to be somewhere between that of a flat plate and a cylinder.

B. COMPRESSION MEMBERS CONSISTING OF FLAT AND CURVED ELEMENTS

Structural engineers are often faced with the problem of predicting the local buckling stress of compression members composed of both flat and curved elements. This problem is particularly evident for relatively "short" columns for which the critical buckling load is normally governed by local buckling or yielding of the individual elements of the cross section. If test results are not readily available, the engineer usually determines the strength of the given cross section based on the summation of the local buckling strengths of the individual flat and curved plate elements.[15,23-25] This procedure is desirable because the buckling stress of each of the curved and flat elements may be predicted by using existing equations. The boundary conditions of the elements are assumed to be either simply supported, if they are bounded by other elements, or free. Figure 2.6 illustrates the assumed boundary conditions for some typical cross sections.

Because it has been noted that, unlike flat elements, curved elements typically exhibit very little post-buckling strength, the cross section is assumed to have failed when the cricical stress is reached in a curved element of a given cross section. Two methods found in the literature for predicting the critical stress of cross sections composed of flat and curved elements are reviewed in the following discussion.

1. Air Force Method. This method was originally published by Newell and Sechler[24] and can best be described by the following example:

If, in the cross section shown in Figure 2.6(b), $f_{cr3} < f_{cr1}$ and $f_{cr3} < f_{cr2}$, then the critical stress will be

$$f_{cr} = \frac{f_{cr3}(2A_1 + 2A_2 + A_3)}{2A_1 + 2A_2 + A_3} = f_{cr3} \tag{2.13}$$

If $f_{cr1} < f_{cr3}$ and $f_{cr2} > f_{cr3}$, the critical stress will be

$$f_{cr} = \frac{f_{cr1}(2A_1) + f_{cr3}(2A_2 + A_3)}{2A_1 + 2A_2 + A_3} \tag{2.14}$$

If $f_{cr1} < f_{cr3}$ and $f_{cr2} < f_{cr3}$, the critical stress will be

$$f_{cr} = \frac{f_{cr1}\,(2A_1) + f_{cr2}\,(2A_2) + f_{cr3}\,(A_3)}{2A_1 + 2A_2 + A_3} \tag{2.15}$$

The maximum value of any of the above stresses is limited to the yield strength of the material.

2. Crockett's Method. A slightly different approach for predicting the critical stress of this type of cross section has been introduced by Crockett.[25] Crockett's method is based on a series of tests on aluminum sheet stiffeners when used alone or in combination with aluminum sheets. The test results obtained with this method for the most part are within 15 percent of those predicted. The basic equation used to predict the critical stress is as follows:

$$F_{cc} = K \frac{\Sigma\, b_n t_n f_{cn}}{\Sigma\, b_n t_n} = K\, F_{ccl} \qquad (2.16)$$

in which

F_{cc} = final predicted crippling stress, psi for $L/\rho \leq 20$

F_{ccl} = uncorrected predicted stress = $\Sigma\, b_n t_n f_{cn} / \Sigma\, b_n t_n$

K = the stability shape factor given in Table 1

$b_n t_n$ = area of individual element, sq. in.

f_{cn} = average ultimate stress of the individual element, given empirically by Figures 2 and 3 of Ref. 25 for flat and curved elements, respectively, psi

ρ = the radius of gyration of the stiffener along about an axis parallel to the sheet in a stiffener-sheet combination, in.

L = length of stiffener or panel, in.

The variations of K for the various cross sections are shown in Table 2.1.[25]

3. Comparison of the Air Force Method and Crockett's Method. There are two basic differences between Crockett's method and the Air Force method. The first is the introduction of the stability shape factor, K, by Crockett, which accounts for the differences in cross sectional shapes. The other is that Crockett's method does not limit the critical stress in the cross section to that of the curved elements.

Because there is only a limited amount of published test data on the compression of cross sections with flat and curved elements, it is difficult to make any broad assumptions about the accuracy of either method. It does seem that the stability shape factor suggested by Crockett would be desirable because it is obvious that cross sections with sloped elements would be less stable than those composed of straight elements. However, the fact that Crockett does not limit the critical stress of the cross section to that of the curved elements appears undesirable because curved elements are noted for their small post-buckling strengths.

In any event, the authors of both methods suggest that these procedures be used only to determine preliminary designs. The adequacy of the final designs should be proven by tests.

4. Additional Literature. Other procedures, which may be useful for computing the compressive strength of members composed of flat and curved elements, consist of 1) an equation for the prediction of the compressive buckling stress of a curved flange by Buchert,[26] 2) a method developed by Needham[27] for compression members composed entirely of flat elements in which he divided the cross section into a series of angles in order to account for the cold work effect in the cold-formed corners, 3) an empirical approach used by Gerard[12] who presented the critical buckling stress in terms of the number of corners in the cross section, and 4) the design criteria given by the Aluminum Association in the "Specification for Aluminum Structures"[28] for aluminum curved plates and elements.

C. EFFECTIVE WIDTH OF COMPRESSION ELEMENTS

The concept of an "effective width" was orginally introduced by von Karman *et al.*[29] to simplify the calculations needed to predict the ultimate strength of flat plates. Since that time, there has been a considerable amount of research performed in this area for flat plates; however, the research data for curved plates are quite limited. Therefore, in the following sections, the effective width concept for flat plates is discussed to provide background information for future studies of the effective width of curved plates. Also the available studies on the effective width of curved plates are briefly reviewed.

1. Flat Plates. For flat plates, which are supported on four sides, such as the upper flange of a hat section, the stress distribution after buckling becomes nonuniform with the maximum stress occurring along the supported edges. With the application of more load, the maximum edge stress increases until the yield strength of the material is reached. At this point, the maximum post-buckling strength of the plate is normally assumed to be reached.[30] Figure 2.7 illustrates the different stress distributions in the plate as the load is progressively increased.

The effective width is defined as an imaginary width of plate, b_e, (as shown in Figure 2.8)[30], which, when loaded with the maximum edge stress, f_{max}, resists the same ultimate load as the full width plate described above. In other words,

$$\int_o^w f dx = b_e f_{max}. \qquad (2.17)$$

Because the actual stress distribution, f, across the full width of the buckled plate is not easily determined, approximate methods are employed to determine the effective width.

In 1932, von Karman[29] suggested that the effective width, b_e, can be approximated as the width of plate, which buckles just when the compressive stress reaches the yield point of the material. Therefore, based on this assumption, the effective width may be derived from the theoretical equation for the buckling of flat plates by setting $f_{cr} = F_y$, i.e.,

$$f_{cr} = F_y = \frac{\pi^2 E}{3(1-\mu_e^2)(b_e/t)^2}. \tag{2.18}$$

Thus,

$$b_e = Ct\sqrt{\frac{E}{F_y}} = 1.9t\sqrt{\frac{E}{F_y}} \tag{2.19}$$

in which

$$C = \frac{\pi}{\sqrt{3(1-\mu^2)}} = 1.9$$

μ_e = 0.3

t = flat plate thickness

Based on an experimental investigation conducted by Winter[31] and much experience in the design of flat plates,[30,32] the constant, C, given in Equation (2.19) has been modified such that the revised effective width equation is as follows:

$$b_e = 1.9t\sqrt{\frac{E}{f_{max}}}\left[1-0.415\left(\frac{t}{b}\right)\sqrt{\frac{E}{f_{max}}}\right] \tag{2.20}$$

Equation (2.20) is currently used in the American Iron and Steel Institute (AISI) Specification[33] for the design of cold-formed, flat compression elements with both unloaded edges supported.

An equation similar to Equation (2.20) was developed by Winter[31] for the effective width of cold-formed, flat compression elements with only one of the unloaded edges supported and the other unloaded edge free. This equation is

$$b_e = 0.8t \sqrt{\frac{E}{f_{max}}} [\, 1 - 0.202 (\frac{t}{b}) \sqrt{\frac{E}{f_{max}}} \,] . \tag{2.21}$$

Additional research conducted at Cornell University[34-36] has shown good agreement with Equation (2.21).

It should be noted that the current AISI Specification for buildings[33] does not use the effective width concept for compression elements with one unloaded edge supported and the other free. Instead, the present Specification has chosen to use an allowable stress approach for flat plates with this type of boundary condition.

2. Curved Plates. As stated earlier, the available research data on the effective width of curved plates are limited. For values of $Z_b < 10$, Levy[18] showed that on the basis of a theoretical analysis, the effective width of curved plates is not appreciably different than for flat plates. This is not surprising because for buckling considerations, it has been shown in Section II.A that for $Z_b < 10$ the behavior of flat and curved plates is practically identical.

Based on the test data collected by Ramberg et al.[37] for aluminum alloy curved plates, the effective width is approximately given by

$$\frac{b_e}{b} = K_c^{\frac{1}{2}} \frac{t}{b} (\frac{E}{f_e})^{\frac{1}{2}} \tag{2.22}$$

in which

b_e = effective width of curved plate

b = circumference of curved plate

t = thickness of curved plate

f_e = edge stress

$K_c = \dfrac{\pi^2 k_c}{12(1-\mu^2)}$; k_c as determined in Ref. 21

It should be noted that the above equation is good for Z_b ranges of 0 to 10 and 24 to 32 and for effective width ratios, b_e/b, in the range of 0.45 to 1.0.[22]

For effective width ratios less than approximately 0.45, the test data obtained by Jackson and Hall[5] for aluminum alloy curved plates seem to exhibit the following relationship for effective width:

$$\frac{b_e}{b} = K_p^{\ 0.43}\left[\frac{t}{b}\left(\frac{E}{f_e}\right)^{\frac{1}{2}}\right]^{0.85} \qquad (2.23)$$

in which

K_p = buckling coefficient for flat plates, for a long plate with clamped edges $K_p = 6.3$

The data that form the basis for Equation (2.23) are obtained for $0 \leq Z_b \leq 125$.

Gerard[12] warned that Equations (2.22) and (2.23) should be used with caution for $Z_b > 30$ because of the limited range of Z_b in the tested specimens.

Another method for using the effective width concept to predict the ultimate strength of curved panels is given by Sechler and Dunn[15] and is applied in similar form by Barton.[38] For this method, the effective width is defined in exactly the same manner as previously described for flat plates with the same developed width. However, unlike flat plates, curved panels are assumed to carry the critical buckling stress of a circular cylinder (with the same thickness and radius as the panel) over the width of panel between the assumed boundaries of the effective width. The assumed post-buckled stress distribution is shown in Figure 2.9.[38] Thus the ultimate load carrying capacity, P_{total}, is given by

$$P_{total} = b_e t f_{max} + (b - b_e) t \cdot f_c \qquad (2.24)$$

in which

b_e = effective width of curved plate determined in the same manner as a flat plate with the same dimensions

f_{max} = maximum edge stress along the supported edge

f_c = the critical buckling stress of a complete cylinder with the same thickness and radius as a curved panel

t = thickness

In Table 8.2 provided by Sechler,[15] the results of tests performed on aluminum curved panels at the Massachusetts Institute of Technology are compared to the values predicted by Equation (2.24) with f_{max} set equal to the yield stress of the material. The range of P_{total}/P_{test} was found to vary from 0.77 to 1.37; however, in most cases, the values of P_{total} and P_{test} did not differ by more than 10 percent.

By using the data presented in this Table[15], the range of Z_b was calculated to be 8.4 to 687. For values of Z_b appreciably greater than this range (i.e. $Z_b > 1000$), there seems to be little use for the effective width concept because initial buckling and failure are coincident. According to Levy,[18] other studies of the post-buckling strength of curved plates are given by von Karmon and Tsien,[39,40] Cox and Clenshaw,[41] Newell,[42] Ebner,[43] and Wenzek.[17]

D. CURVED PLATES SUBJECT TO SHEAR LOADING

1. Unstiffened Curved Plates. The buckling stress for an unstiffened curved plate loaded primarily in shear, such as the curved web of a beam, is considerably greater than the buckling stress for a flat plate of the same dimensions.[44] Just as for the axial compression of curved plates, the theoretical buckling stresses are usually greater than those obtained experimentally. The following theoretical buckling stress equation was derived by Batdorf *et al.*[45] for the theoretical shear buckling stress, f_{cr}, for curved plates:

$$f_{cr} = K_s E (t/b)^2 \qquad (2.25)$$

in which K_s is a function of the length, circumference, radius, and thickness of the curved plate.

An empirical equation has been proposed in ANC-5[46] as:

$$f_{cr} = K E (t/b)^2 + K_1 E (t/R) \qquad (2.26)$$

in which the first term represents the buckling stress for a flat plate, and the last term the additional shear stress that the curved plate can resist because of its curvature. A value of $K_1 = 0.10$ is recommended.[46]

2. Longitudinally Stiffened Curved Plates. There has been some study of curved plates with longitudinal stiffeners in which the "tension field"[44] concept is employed in a similar fashion to the tension field analysis of thin flat webs in straight girders. In the pure tension field concept, as proposed by Wagner and Ballerstedt,[47] the curved plate is assumed to be completely flexible. Thus, its compressive strength is considered negligible, and the curved plate is assumed to buckle freely at an angle of 45° to the shear stress (i.e., the direction of maximum compressive stress caused by pure shear). Because even very thin, curved webs have appreciable in-plane stiffness, this assumption is generally considered invalid. Thus, a "semitension field" analysis is normally employed in which the compressive stiffness of curved webs is taken into consideration. Semiempirical methods of analysis and design for longitudinally stiffened curved webs are given by Kuhn and Griffin.[48]

III. FUTURE RESEARCH NEEDS

Since curved elements are used throughout a modern automobile, accurate, yet relatively simple, design equations for these elements are essential. Because of the mathematical complexities involved in the accurate analytical prediction of the critical buckling stress of curved elements, it is believed that any practical design expressions must be empirical or at least semi-empirical in nature. Since there is a limited amount of available test data for curved elements, several additional tests are proposed in this Section.

The proposed test specimens (Figures 3.1 through 3.4) represent typical, simplified cross sections (or profiles) that are normally utilized in automobiles. These specimens will be formed from the same high strength sheet steels used in Phase I of the present research project. In Phase I, the yield strengths of these (as-received) sheet steels were found to vary from 55.8 to 141.2 ksi. The yield strength, F_y, ultimate tensile strength, F_u, and thickness, t, of each HSSS, as determined in Phase I, are listed in Table 3.1.[51]

From Figures 3.1 through 3.4, it can be seen that for profiles A, B, C and D, all of the dimensions are held constant except for the radius of the curved element. For profile E, the radius is held fixed and the angle, θ, between the centerline and tangent varies. The selected values for the curved element radius, R, and the resulting arc length, b, are presented in Table 3.2. As shown in this table, three different curved element radii have been selected for each basic profile except for profile E for which two different θ values were chosen. The curved element radii were selected such that the resulting Z_b values would lie somewhere in the range between those of flat plates and cylinders (i.e., $10 < Z_b < 1000$). The Z_b values (assuming $\mu_e = 0.3$) for each test specimen are presented in Table 3.3.

Because of the difficulty in performing axial compression tests on sections that are not doubly-symmetric, profiles A and E will be tested in bending such that the curved element is subjected to compressive stresses. Profiles C and D represent the same cross section with the only difference being that profile C will be tested in bending, whereas profile D will be subjected to axial compression. It is hoped that the effect of the stress gradient caused by bending may be obtained by comparing the test results of the C and D profiles. The B profile is proposed for shear testing in order to study the effects of shear on curved webs. The load type and overall length of each of the proposed profiles are listed in Table 3.2.

As stated earlier, each of the proposed tests will be formed from the HSSS used in Phase I of the present research project. However, because there is a limited amount of this material, it will not be possible to form each type of specimen from each of the six previously tested HSSS. The number of proposed tests from each HSSS is given in Table 3.4. As shown in this table, a total of 106 tests are proposed at this time.

IV. CONCLUSIONS

Because it has been shown through the preceding review of the literature that the initial buckling stress of curved plates substantially increases with increasing curvature and because curved elements are commonly used in the modern automobile, it is essential for automotive engineers to be able to design cold-formed automotive components that contain curved elements accurately and efficiently. However, it has been learned from this review that it is difficult to obtain an accurate analytical prediction of the buckling stress of curved elements over a wide range of curvatures. This difficulty arises primarily because: 1) large deflection theory, which is much more complex than linear theory, must be used to analyze curve plate buckling caused by axial stresses, 2) curved plates with appreciable curvature are quite sensitive to initial imperfections, 3) curved plates with small curvature are particularly sensitive to the edge restraint at their boundaries, and 4) the effects of residual stresses and cold work are difficult to predict. Because of the complexities involved in predicting critical buckling stresses of curved plates, it is deemed essential that design equations for the compression of such plates be empirical or semiempirical in nature. It is hoped that it will be possible to develop a semiempirical transition equation of the same form as Redshaw's equation[13,14] (Eq. 2.2) for curved plates made of high strength sheet steel.

For cross sections composed of both flat and curved elements, it is hoped that the above transition equation may be used to predict the critical buckling stress of the curved elements. The load carrying capacity of the entire cross section can then be obtained with the methods outlined in Section II.B.

The effective width concept, as originally developed for the analysis of the post-buckling strength of flat plates, is also rather complicated for curved plates. There obviously is a transition in the post-buckling behavior of curved plates that extends from flat plates ($Z_b = 0$), which have considerable post-buckling strength, to cylinders, which normally exhibit little post-buckling strength. Thus, some sort of transition relationship must exist between the two extreme behaviors.

Because there is little specific information in the literature on the behavior of curved webs, the literature on curved plates subject to pure shear loading was reviewed in Section II.D. It is hoped that this review will provide background material that will be beneficial for future studies of curved webs. Other loading conditions, which must be considered in future studies, are combined shear and bending stresses, web crippling, and combined bending and web crippling.

In conclusion, it appears that the use of curved plates has significant potential, but a considerable amount of research must be preformed before this potential can be realized.

ACKNOWLEDGMENTS

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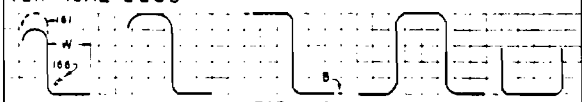
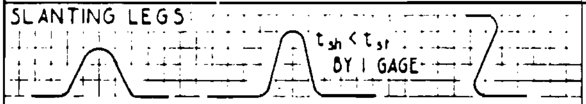
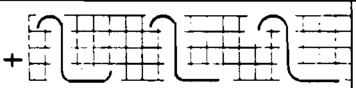
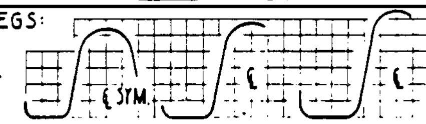
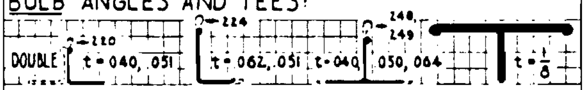
TABLE 2.1 STABILITY SHAPE FACTORS (REF. 25) $F_{cc} = K F_{cci}$	
STIFFENERS *	K **
24 ST ALCLAD STIFFENERS, 24 ST ALCLAD SHEET	
VERTICAL LEGS:  LS 160, 161, 166 XLS165 X-785-A, X-785-B X-785-D LS 140	1.07
SLANTING LEGS:  LS 173 LS 174 X-785-C	1.00
24 SRT ALCLAD STIFFENERS, 24 ST ALCLAD SHEET	
VERTICAL LEGS: LS160, LS161, XLS165 X-785-A, X-785-B, X-785-D +  LS160 MOD LS 156 LS 157	1.08
SLANTING LEGS: LS 174, X-785-C +  LS 152 LS 153 LS 154	.92
24 ST EXTRUDED SHAPES, 24 ST ALCLAD SHEET	
BULB ANGLES AND TEES:  LS 222, 220 LS 223, 224 LS 247, 248, 249 LS 234	1.08
ANGLES, TEES, AND CHANNELS:  LS 219, 208 LS 209, 200, 210 LS 236, 232 LS 264	1.00

NOTES:

*(a) The stiffeners shown are those tested. K applies to any stiffeners judged to lie within the range of the method given. (b) One division represents $^1/_4$ in. (c) The underlined identification numbers indicate that those stiffeners were tested with, and in some cases also without, sheet attached.

** K is given for stiffeners of various materials and angularity of legs assuming that thickness of sheet is less than, or equal to, thickness of stiffener unless otherwise indicated.

TABLE 3.1

Material Properties and Thicknesses of Six Sheet Steels to Be Used for Future Study[51]

Material Designation	F_y (ksi)	F_y (ksi)	t (in.)
80SK	82.2	88.8	0.061
80DF	55.8	88.8	0.114
80DK	58.2	87.6	0.048
80XF	88.3	98.7	0.082
100XF	113.1	113.1	0.062
140XF	141.2	141.2	0.043

TABLE 3.2

Dimensions of Proposed Test Specimens Consisting of Curved Elements

Specimen No.	R(in.)	b(in.)	Length (in.)	Load Type
A-1	15	4.01	60	Bending
A-2	3.5	4.26	60	Bending
A-3	2	6.29	60	Bending
B-1	15	4.01	30	Shear
B-2	3.5	4.26	30	Shear
B-3	2	6.29	30	Shear
C-1	4	2.02	60	Bending
C-2	1.25	2.32	60	Bending
C-3	1	3.14	60	Bending
D-1	4	2.02	12	Compression
D-2	1.25	2.32	12	Compression
D-3	1	3.14	12	Compression
E-1	2	3.14	60	Bending
E-2	2	4.19	60	Bending

TABLE 3.3

Z_b Values for Each Specimen and Material

Specimen No.	Material Designation 80SK	80DF	80DK	80XF	100XF	140XF
A-1	17	-	21	12	16	-
A-2	81	-	103	60	80	-
A-3	309	-	392	230	304	-
B-1	-	-	21	12	16	-
B-2	-	-	103	60	80	-
B-3	-	-	392	230	304	-
C-1	-	9	20	12	16	-
C-2	-	36	85	50	66	-
C-3	-	83	196	115	152	-
D-1	-	9	20	12	16	-
D-2	-	36	85	50	66	-
D-3	-	83	196	115	152	-
E-1	-	41	98	57	76	-
E-2	-	73	174	102	135	-

TABLE 3.4

Proposed Number of Tests for Each Material

Specimen No.	Material Designation 80SK	80DF	80DK	80XF	100XF	140XF	Total
A-1	2	-	2	2	2	-	8
A-2	2	-	2	2	2	-	8
A-3	2	-	2	2	2	-	8
B-1	-	-	2	2	2	-	6
B-2	-	-	2	2	2	-	6
B-3	-	-	2	2	2	-	6
C-1	-	2	2	2	2	-	8
C-2	-	2	2	2	2	-	8
C-3	-	2	2	2	2	-	8
D-1	-	2	2	2	2	-	8
D-2	-	2	2	2	2	-	8
D-3	-	2	2	2	2	-	8
E-1	-	2	2	2	2	-	8
E-2	-	2	2	2	2	-	8
Total	6	16	28	28	28	0	106

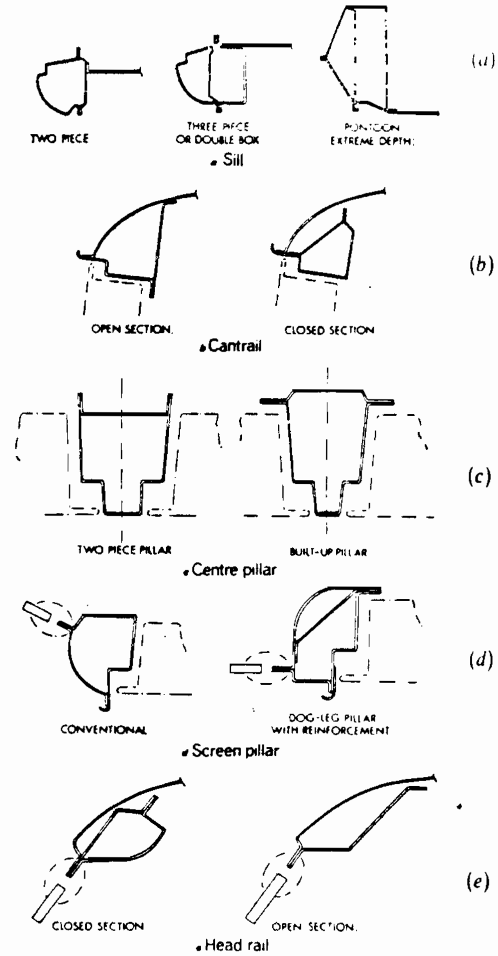


Fig. 1.1 Typical Car Body Member Sections[3]

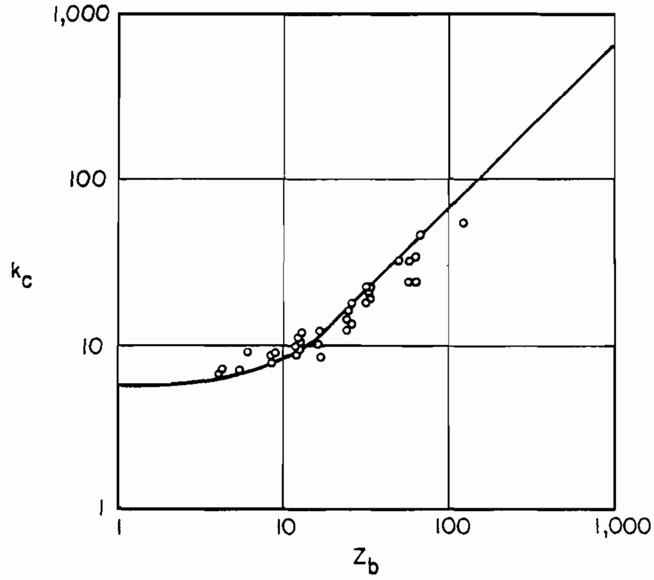


Fig. 2.1 Comparison of Test Data With Theory for Axially Compressed Curved Plates.[21]

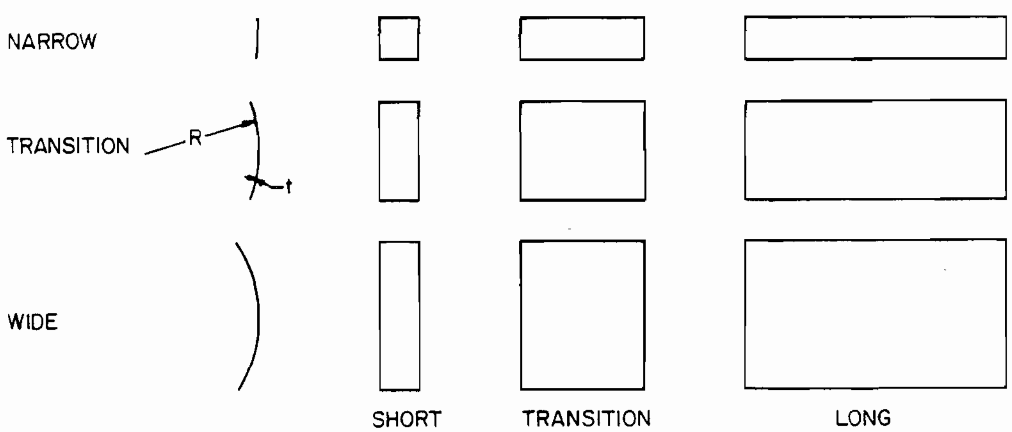
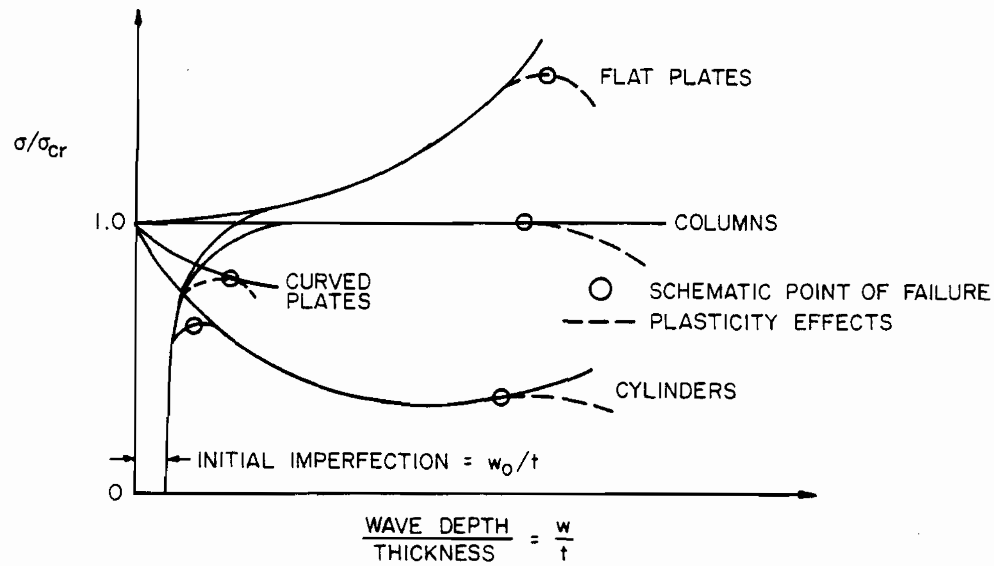
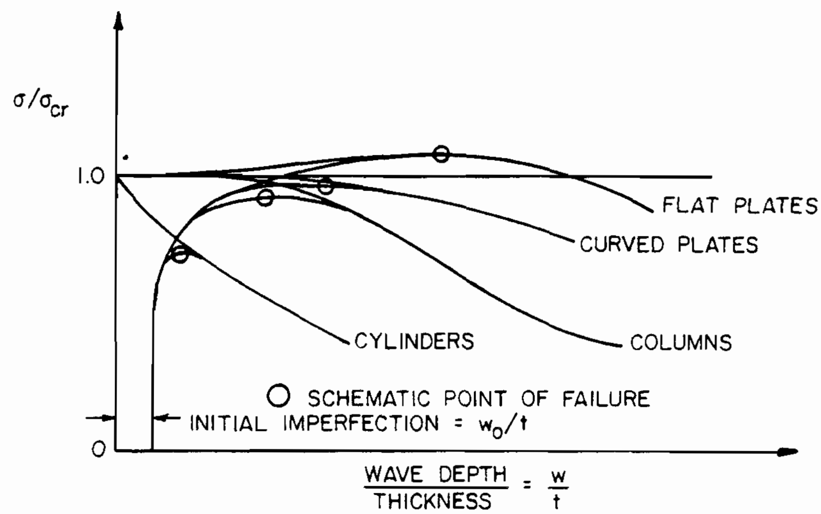


Fig. 2.2 Ranges of Possible Curved Plate Geometries[21]

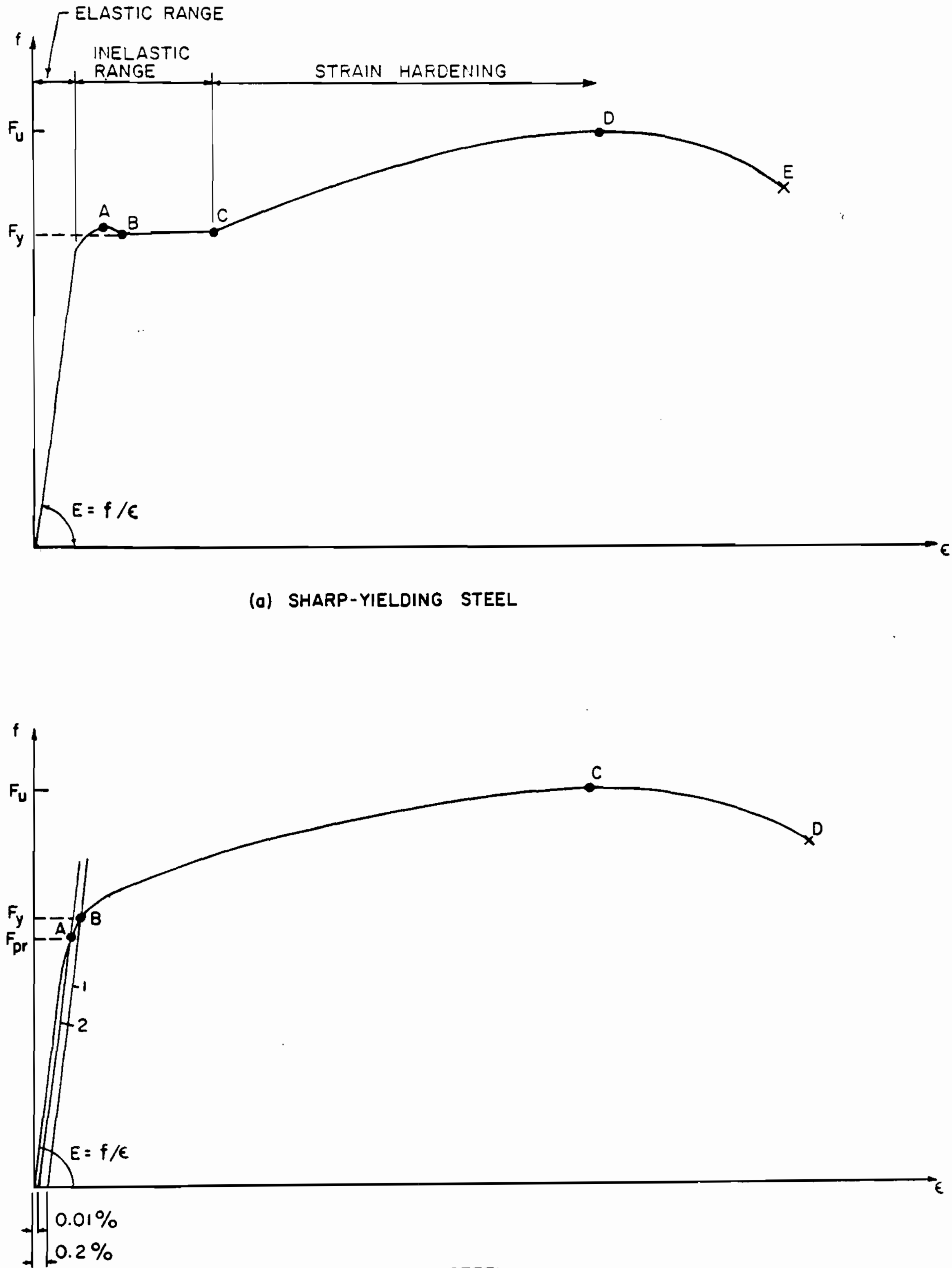


(a) Elastic Buckling



(b) Plastic Buckling

Fig. 2.3 Schematic Postbuckling Behavior of Various Axially Compressed Elements.[12]



(b) GRADUAL-YIELDING STEEL

Fig. 2.4 Stress-Strain Curves of Carbon Steel Sheets[30]

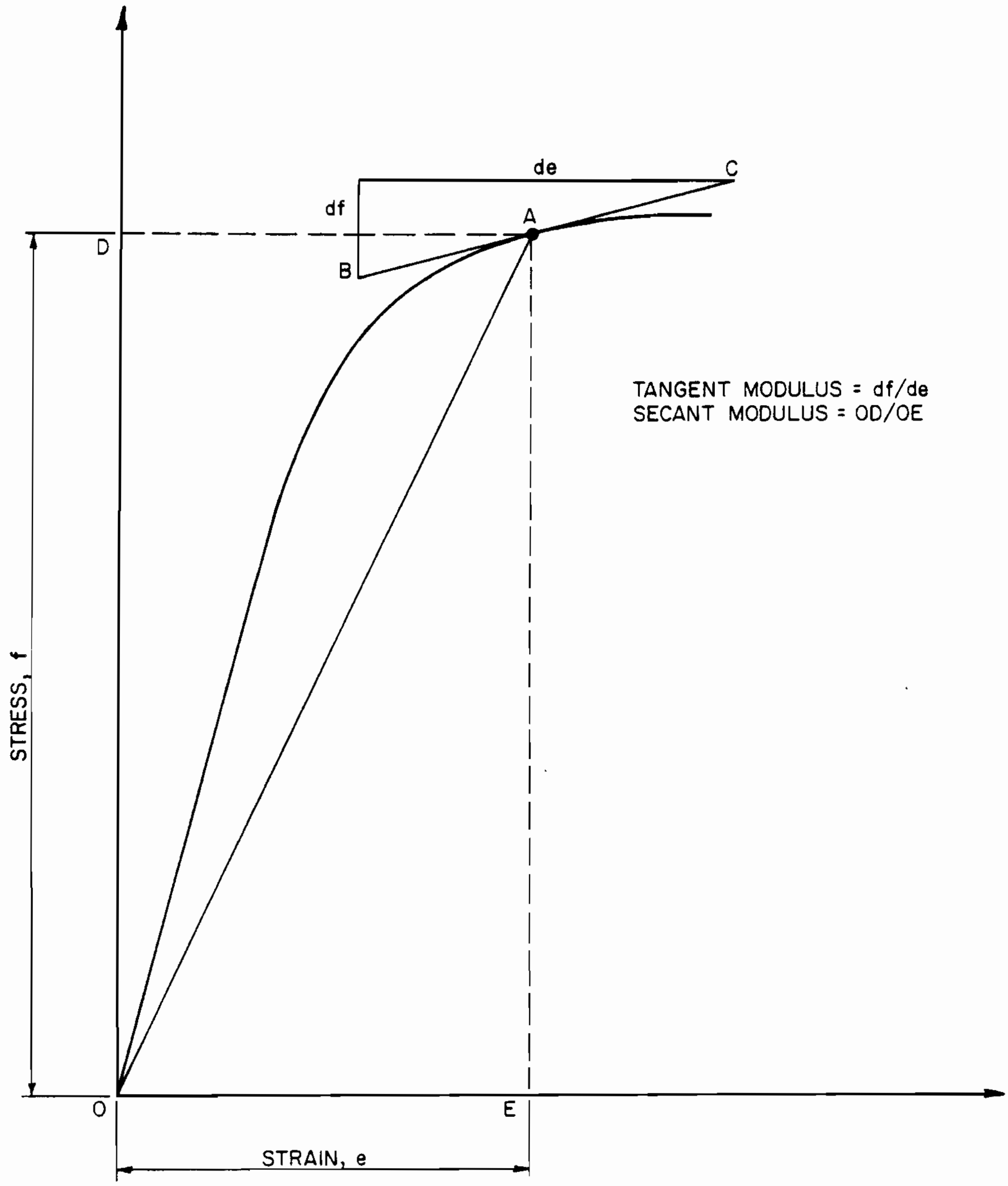


Fig. 2.5 Graphical Representation of the Tangent and Secant Moduli.

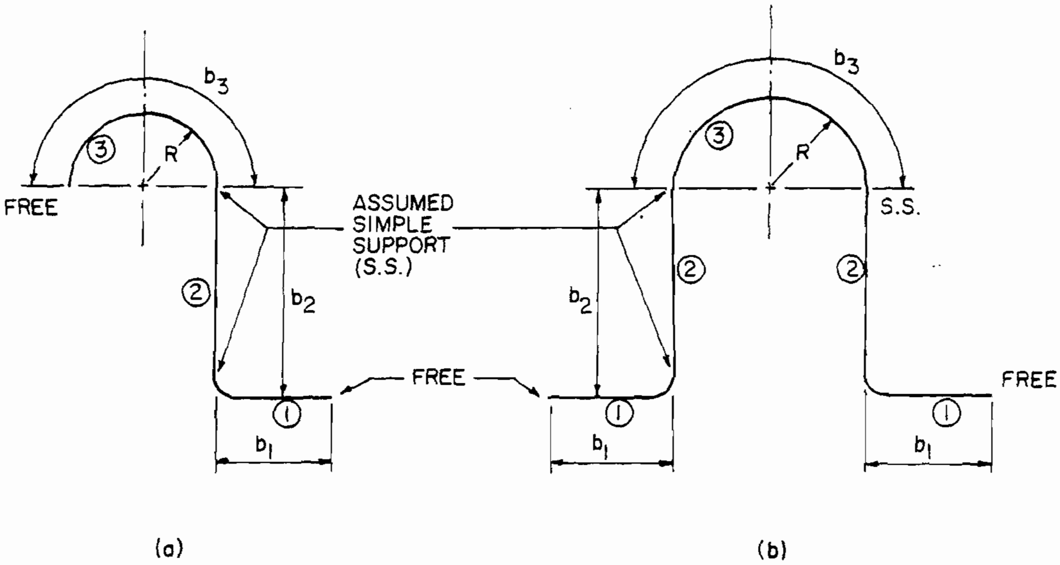


Fig. 2.6 Typical Cross-Sections Consisting of Flat and Curved Elements.

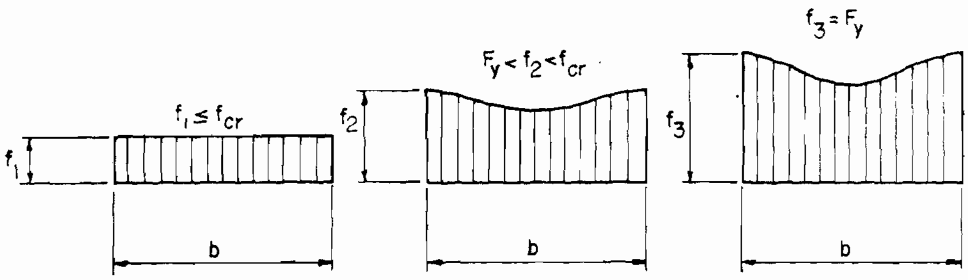


Fig. 2.7 Consecutive Stages of Stress Distribution in Stiffened Compression Elements.[30]

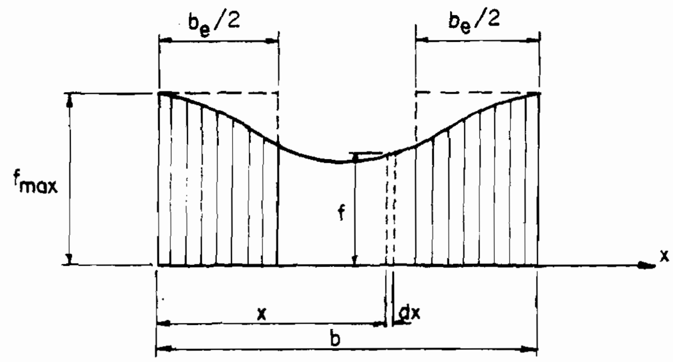


Fig. 2.8 Effective Width of a Stiffened Compression Element[30]

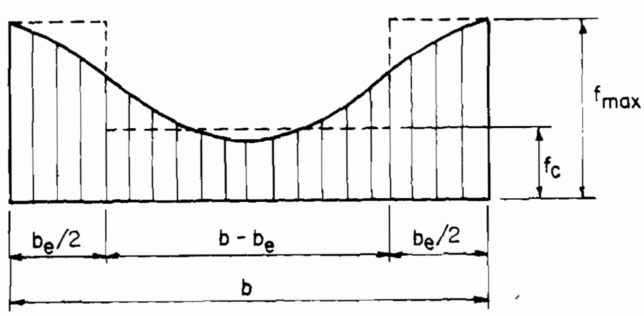


Fig. 2.9 Effective Width of a Curved Compression Element[38]

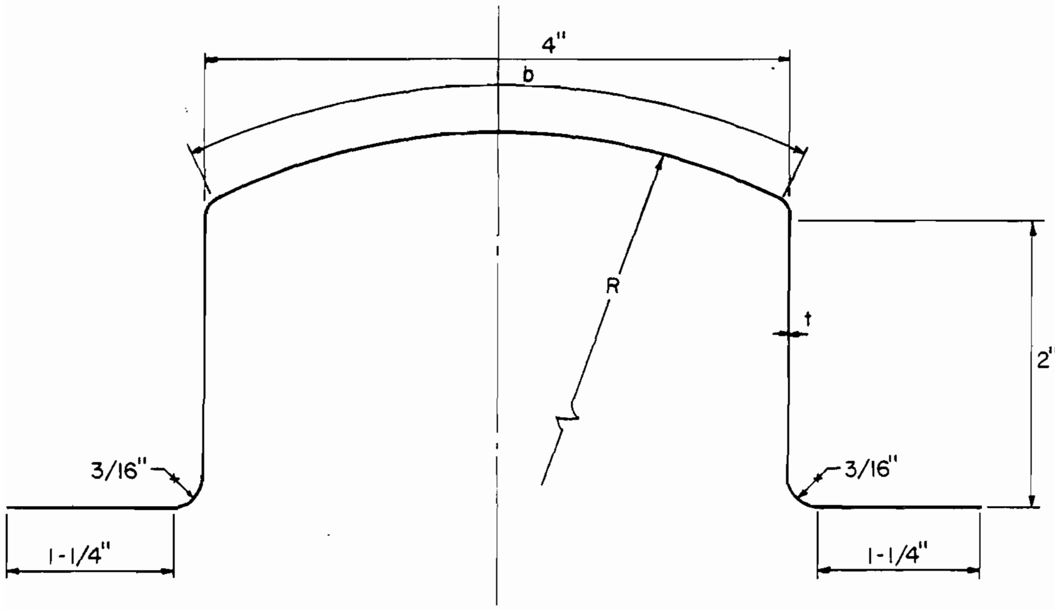


Fig. 3.1 Proposed A Profile

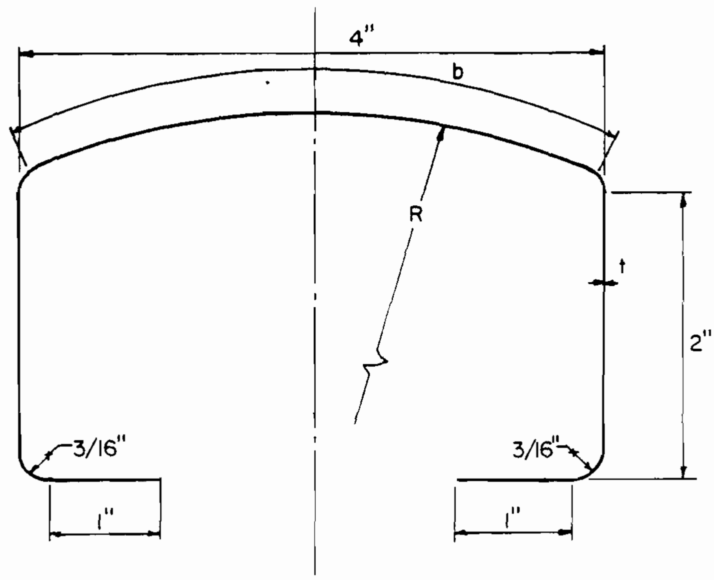


Fig. 3.2 Proposed B Profile

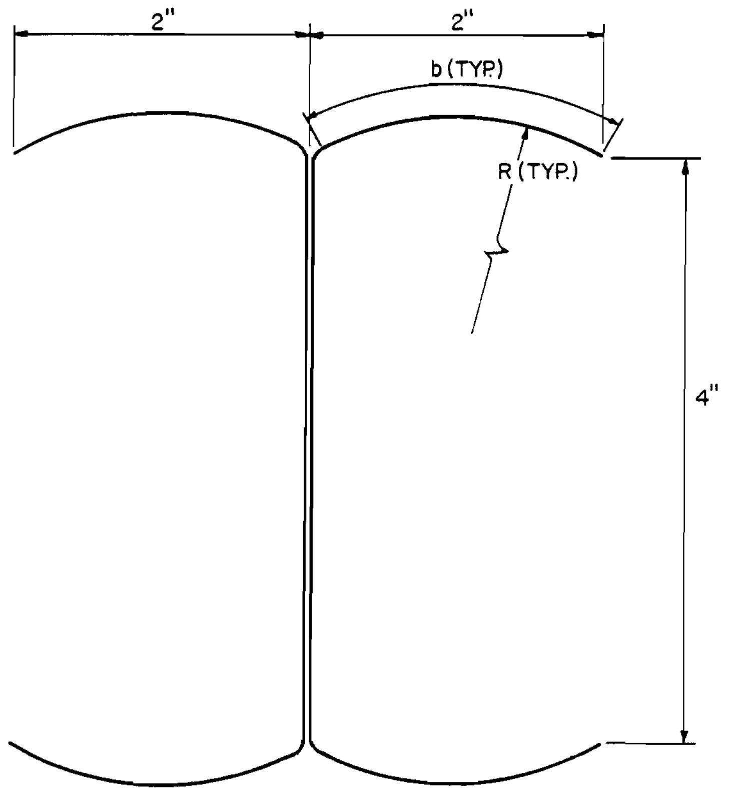


Fig. 3.3 Proposed C and D Profiles

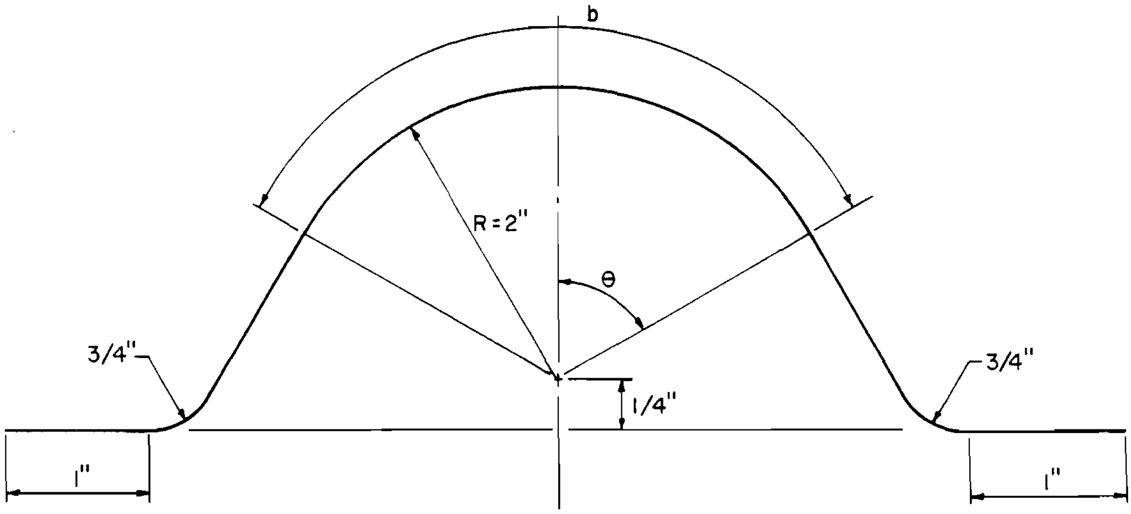


Fig. 3.4 Proposed E Profile