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A RE-EXAMINATION OF THE ENERGY-GNP HYPOTHESIS

by

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I. Introduction

This paper re-examines the hypothesis that there is a constant relationship between U.S. energy consumption and gross national product (GNP). If this ratio is constant then providing incentives for energy conservation may be an unacceptable policy since economic activity in some sectors of the economy may be adversely affected. Further, if the energy-GNP ratio has begun to rise over time, as some researchers have suggested, then expectations of the future demand for various energy resources must be revised accordingly. Berndt and Wood (1974) have shown that the energy-GNP ratio is consistent with standard economic theory only under highly restrictive conditions, and argue that changes in the ratio may not represent structural shifts in the economy but may be due to rational responses of firms to changes in the relative price of energy. Regardless of the possible economic reasons for shifts in the energy-GNP ratio, the question of its stability still remains. This paper presents evidence that the rate of change in the energy-GNP ratio has been linear over the 1947-1976 period, and that the ratio itself has not undergone a shift, but is in fact decreasing at a constant decreasing rate and will continue to do so for approximately the next several hundred years.

II. Testing the Structural Stability of the Energy-GNP Ratio

A reasonably straight forward statement of the energy-GNP hypothesis can be given in terms of a classical regression model such as equation (1).

$$y_t = x_t \beta_t + u_t \qquad (1)$$

where:

y_t = the vector of observations on the dependent variable.

x_t = the column vector of observations on k regressors.

β_t = vector of coefficients for the k regressors subscripted to denote that they may have different values in different time periods.

u_t = stochastic error term assumed independent normally distributed with mean zero and variance σ_t^2.

If y_t is the ratio of gross energy input to gross national product in constant dollars at period t, accepting the energy-GNP hypothesis calls for the coefficients of a time trend variable to remain constant over the sample period. This is comparable to a test of the null hypothesis:

$$H_o: \quad \beta_1 = \beta_2 = \ldots \beta_T \qquad (2)$$

where β_t is the coefficient of the time trend variable over the sub-period k + 1 (k = the number of regressors) to t; (t = 1, 2, . . . T).

This is in fact a test of the structural stability of the regression model at each time period in the sample. The alternative against which the hypothesis is tested is:

$$H_a: \quad \beta_1 = \beta_2 = \ldots \beta_T \qquad (3)$$

No restrictions have been placed on the alternative hypothesis; therefore the null hypothesis will not be accepted if there has been a structural shift in any time period.

Tests for the stability of a regression model, in the sense that the estimated coefficients remain constant over the time period in question, have in general been limited to specific alternative hypotheses that are generated by *a priori* notions of when precisely one or more of these coefficients might have changed. By specifying dummy variables or applying the Chow test to the hypothesized sub-samples, statistical tests of significance can be performed to determine of the coefficients shifted at precisely that point in time. An alternative approach is the use of spline functions to detect non-linearities and thus structural shifts in the equation, but this also requires specifying the degree of the spline function *a priori*.

Recently, statistical tests for a less restrictive alternative hypothesis have been developed requiring no prior information as to the timing of the shift. The simplest of these methods to apply is that developed by Brown and Durbin. In essence, the Brown-Durbin technique tests to determine if the percentage of residual variance in each period is within prescribed confidence limits of that percentage which would be expected if the relationship had remained stable throughout the time period. However, the OLS residuals cannot be used because even though the error term is assumed classically well behaved, the OLS residuals will be heteroscedastic with a covariance structure related to the regressors. Brown and Durbin show that a Helmert Orthogonal Transformation of the regression model produces residuals that are independent $N(0, \sigma^2)$. Let $\hat{\beta}_r$ be the OLS estimate of β based on the first r observations ($r \geq k + 1$), then the orthogonal residuals are given by:

$$w_r = \frac{y_r - x_r' \hat{\beta}_{r-1}}{\left[1+x_r' \, (X_{r-1}' \, X_{r-1})^{-1} x_r\right]^{1/2}} \qquad (4)$$

where $X_{r-1} = [x_1, \ldots x_{r-1}]$.

If the null hypothesis is true up to period r, but rejected thereafter, that is, if the relationship becomes unstable at r, w_r will have a mean of zero up to period r and a non-zero mean thereafter.

For a visual test of the null hypothesis Brown and Durbin suggest plotting the variable S_r against time, where:

$$S_r = \sum_{k+1}^{r} w_r^2 \Big/ \sum_{k+1}^{T} w_r^2 \qquad (5)$$

Thus defined, S_r is the percentage of the transformed residual variance in each time period. The expected value of S_r is:

$$E(S_r) = \frac{r-k}{T-k} \qquad (6)$$

The value of S_r will thus lie along this mean value line if the β's are constant, i.e., if the parameters are stable. Confidence limits may be calculated for preassigned significance levels to test for significant changes in the underlying structure. For a two-sided test these limits are defined by:

$$\pm c_a + \frac{r-k}{T-k} \quad (7)$$

which defines two lines parallel to $E(S_r)$. If the path of S_r crosses either limiting line the null hypothesis is rejected and it can be stated with the pre-assigned confidence that at the period where S_r crossed the confidence line, and for all periods in which S_r lies outside the confidence band, the underlying structure made a significant shift. The statistic c_a is distributed as Pyke's modified Kolmogorov-Smirnov statistic. Durbin (1969) has computed values of c_a for various significance levels.

Summarizing the Brown-Durbin test for structural stability:

(1) normalized sum of squared residuals are calculated for models estimated with increasing sample sizes,

(2) S_r, the cumulative values of the normalized sum of squared residuals, is calculated and plotted against time,

(3) $E(S_r)$ is calculated and plotted,

(4) confidence bands are drawn parallel to the mean value line, $E(S_r)$, at a distance $\pm c_a$,

(5) for periods where S_r crosses or lies outside the confidence limits it can be said that the underlying structure made a significant shift, and thus the null hypothesis is rejected. This does not necessarily mean that the shift occurred at these specific times, but that they became significant then. There may be underlying lags and leads that are just then emerging as a significant change.

III. Results and Conclusions

The ordinary least squares results of regressing the ratio of gross energy input in BTUs to gross national product in constant 1958 dollars from 1947 to 1976 on time and time squared are presented below, where the numbers in parentheses are the absolute values of the t-statistics.*

$$GEI/GNP = \underset{(4.75)}{133.3} - \underset{(4.73)}{.135t} + \underset{(4.73)}{.00003t^2} \quad (8)$$
$$R^2 = .71, \ D\text{-}W = .62$$

The coefficients are statistically significant and indicate the overall downward trend in energy consumption to GNP noted by others. However, the Durbin-Watson statistic indicates significant autocorrelation.

Solving equation (8) for the minimum yields the implication that the energy-GNP ratio would be expected to continue to decline until the year 2250. This result is significantly different from previous predictions that the energy-GNP ratio hit a minimum in the middle or late 1960's. In order to confirm this relationship, however, it is necessary to test that the equation is a stable over time, i.e., that the parameters have not made significant shifts in any of the time periods under investigation.

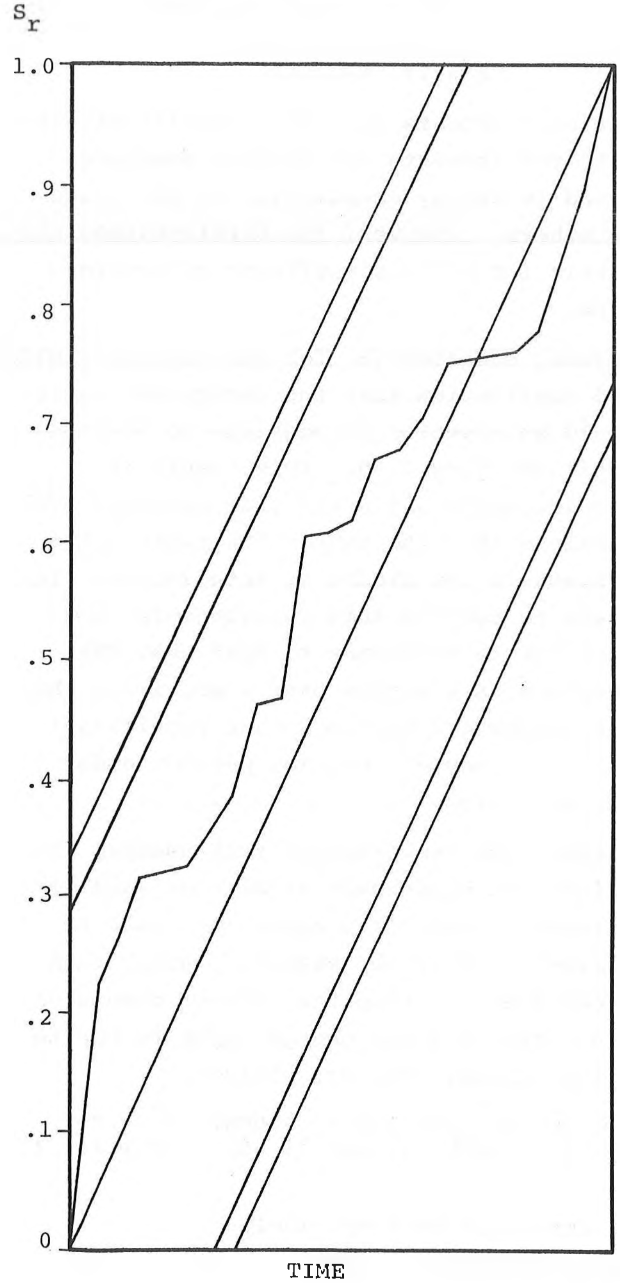
Because the Brown-Durbin test assumes the maintained hypothesis of non-serially correlated errors, this equation cannot be tested directly for stability using this technique. Fitting the rate of change of the ratio of gross energy input to GNP to a time trend, however, yields:

$$\Delta\left(GEI/GNP\right) = \underset{(1.66)}{-.150} + \underset{(1.65)}{.00008t} \quad R^2 = .09 \quad D\text{-}W = 1.74 \quad (9)$$

*1947-1975 data from Bureau of Mines, Bureau of Economic Analysis; 1976 estimates from Chase Manhattan Bank projections.

The coefficients are all significant at the 95 percent level of confidence on a one-tailed test and the hypothesis of non-serially correlated errors cannot be rejected. The interpretation of equations (8) and (9) in terms of the energy-GNP hypothesis is that the ratio has been decreasing at a decreasing rate and will until 2250 and that the rate of change of the decline is small.

Figure 1
Trace of Statistic S_r



It is important to test the hypothesis that the rate of decline has been stable over the time period analyzed. If there have been shifts in the rate of decline of the energy-GNP ratio, the prediction of a continuing decline is suspect. In fact, no predictions can be based upon parameters of an equation estimated over peroids where there have been underlying structural shifts. Thus, the test of the stability of equation (9) are interpreted as an indirect test of the stability of energy-GNP ratio over time.

Figure 1 shows the trace of the statistic S_r obtained by applying the Brown-Durbin procedure to equation (9). The diagonal line is the plot of the expected value of S_r. The lines parallel to the expected value line are the 90 to 95 percent confidence limits; the 90 percent confidence limit lies closest to the expected value line on both sides. Because the trace of S_r remains inside the confidence bands, the hypothesis that there has been no shift in the rate of change in the energy-GNP ratio over time cannot be rejected. By accepting this hypothesis, one is forced to accept that the ratio of gross energy input to GNP has declining at a decreasing rate and that this rate has been stable over the 1947 to 1976 period.

Netschert (1972) has suggested that the historically declining ratio of energy to GNP has reversed and dates the structural shift as 1967. He further concludes that this trend reversal will persist, however, at a declining rate. Similar findings have been made by Kraft and Kraft (1975) using piece-wise regression to locate the mimimum of a cubic spline. They date the minimum as 1966, concluding that their results would agree with Netschert that the shift was not a one-period occurrence, that is, the ratio has continued to rise after 1966. Barnett (1974), on the other hand, has concluded that while the BTU/GNP ratio turned upward in 1967, it represents only a short-run phenomenon and that it will most probably decline or remain constant over the next generation, although he provide little statistical evidence for this conclusion.

The results using the Brown-Durbin technique to detect structural changes in the rate of change in the energy-GNP ratio provides statistical support for Barnett's conclusion that the BTU/GNP ratio has not undergone a significant reversal, but will continue to decline at a declining rate for an extensive period in the future. The usual caveat concerning predictions of distant future periods is in order. There is nothing in the above analysis that implies that significant structural shifts in the energy-GNP ratio may not occur at some future point. The evidence to date, however, does not support the hypothesis that the U.S. economy has already undergone such a shift.

Further work on energy usage by sector and estimates of the elasticity of substitution among inputs is necessary before truely confident estimates of future energy consumption can be made. This research indicates, however, that radical reevaluation of future energy consumption is not necessarily in order.

REFERENCES

1. Berndt, Ernst R. and David O. Wood. "An Economic Interpretation of the Energy-GNP Ratio," in Energy: Demand, Conservation and Institutional Problems, Michael S. Macrakis, ed. (The MIT Press. Cambridge), 1974.

2. Barnett, Harold J. "Energy, Resources, and Growth" Statements before the Joint Economic Committee of the Congress of the United States, December 19-21, p. 171-190, 1973, GPO: Washington, D.C. 1974.

3. Brown, R.L., J. Durbin, and J.M. Evans. "Techniques for Testing the Constancy of Regression Relationships Over Time." Journal of the Royal Statistical Society, Series B, Vol. 37, No. 2, 1975: p. 149-163.

4. Darmstadter, Joel, with Perry D. Teitelbaum, and Jaroslav G. Polach. Energy in the World Economy, Resources for the Future, Inc., (The John Hopkins Press: Baltimore), 1971.

5. Durbin, J., "Tests for Serial Correlation in Regression Analysis Based on the Periodogram of Least Squares Residuals". Biometrika 56 (March 1969): p. 1-15.

6. Netschert, Bruce. "Fuels for the Electric Utility Industry, 1971-1985," a report of National Economic Research Associates, Inc., Washington, D.C., to the Edison Electric Institute, 1972.

7. Kraft, Arthur, and John Kraft. "A Test of the Energy/GNP Hypothesis Using Cubic Splines." Paper presented at the Econometric Society Meetings, Dallas, 1975.

8. Schurr, Sam H. et al. Energy in the American Economy, 1850-1955: An Economic Analysis of its History and Prospects (The John Hopkins Press: Baltimore, 1960.

9. Smith, V. Kerry. "Re-examination of the Trends in the Prices of Natural Resources Commodities, 1870-1972." Paper presented at the Econometric Society Meetings, San Francisco, 1974.