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Stresses in suspension bridges

William M. Claypool

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Stresses in Suspension Bridges.

T.159.

W. M. CLAYPOOL.

 $\mathbf{r} = \mathbf{r} \mathbf{r}_1 \mathbf{r}_2 \mathbf{r}_3 \mathbf{r}_4$ **MSM** HISTORICAL COLLECTION Stresses Suskinsion Bridges. 7674

Stresses —in — **Suspension Bridges**

Suspinsion Bridger a surprision bridge in the live, which the roadway over the attenum on about to be cruised in suspended from chainnesse aim ropes. a suspension bidge concriste of the towne or fine over which the main chain. or caller part ; the anchorages to which the ends of the cable are attached; the main chain on cables from which the wadway in ruskended; the suspending rade or chaine which canned the roadway with the cable and the roadway. The pub-structure consists of the boundations, piers se. The super-structure consists of the roadway and the chaine on cables.

Towns -The towns, frequently timed fiers, an generally made of marving.

Suspension Bridges

1

A suspension bridge is one in which the roadway over the stream or span to be crossed is suspended from chains or wire ropes.

A suspension bridge consists of the towers or piers over which the main chains or cables pass; the anchorages to which the ends of the cables are attached; the main chains or cables from which the roadway is suspended; the suspending rods or chains which connect the roadway with the cable and the roadway.

The sub-structure consists of the foundations, piers &c. The super-structure consists of the roadway and the chains or cables.

T owers-

The towers, frequently termed piers, are generally made of masonry, although iron has sometimes been

used. The particular trial som france of towns will defend to some replied upon the locality and characters of survivindings. This dimensions will different upon Their hight and the amount of strain they will have to runt. Where the cable fare over the town are saddler. Au construction of sadden in which the calle passes over friction rollen rigidly attached to the top of the pier, account the caller to reip backworde and forwards over it with camparatively little prietion, so that the stress on the cable may be taken as equal on hath sides of the saddle. In another construction who chain in recent to the sadder, which, however, in fruits moon harryontally on the Top of the firm. du unter finish farmer af radder unter not be vertical unless the chain

used. The particular kind or form of towers will depend to some extent upon the locality and character of surroundings. Their dimensions will depend upon their height and the amount of strains they will haves to resist. Where the cables pass over the towers are saddles. Our construction of saddles in which the cable passed our friction rollers rigidly attached to the top of the pier, allows the cable to slip backwards and forwards over it with comparatively little friction, so that the stress on the cable may be taken as equal on both sides of the saddle. In another construction the chain is secured to its saddle, which, however, is free to move horizontally on the top of the piers. In the first form of saddles the resultant pressure on the pier will

not be vertical unless the chain

2

leavier the pier at me equal includion on each side, and was when the bridge in designed with an equal Maps of claim in hath side of the fin, a change in the distribution of weight due to any passing load, wie cause some departum from ite equal alogu of the chaine, and therefore from the truly virtical pressure and the piece. This departum is easily allumed for in the design of the bridge frim. The friction on the saddle render it assumption of equal sturry on each side slightly incornect, and with This ligke of saddle, can must be taken to provide against the wave froduced by the motion of the chain. In the second type, the war of rollers under the solid sadde have the mation of the saddle very free; the recultant present ou the town in always vertical, and the chains may

leaves the pier at an equal inclination on each side, and even when the bridge is designed with an equal slope of chain on both sides of the pier, a change in the distribution of weight due to any passing load, will cause some departure from the equal slope of the chains, and therefore from the truly vertical pressure of the piers. This departure is easily allowed for in the design of the bridge piers. The friction on the saddle renders the assumption of equal stresses on each side slightly incorrect, and with this type of saddle, care must be taken to provide against the wear produced by the motion of the chain. In the second type, the use of rollers under the solid saddle leaves the motion of the saddle very free; its resultant pressure on the tower is always vertical, and the chains may

leave the town at any angle, equal or mugual. The chain must in no case be rigidly attached to the pier, unless the pier, or rache support in this instance, is free to rock on its have, as for example, when the place of the pier is taken by inn struts marking on a larigantal ajir. Auchange - If the show or haute be of rock, a virtical passage slaved be exercited and a strong mon plate placed in the hollow and finnly imbadded in the side of the passage. Though this flate the tuck of the caller an passed and seemed on the under side. after the cables are fut in flow, the and maring, If the bank in not suitable for the

leave the tower at any angle, equal or unequal.

The chain must in no case be rigidly attached to the pier, unless the pier, or rather support in this instance, is free to rock on its base, or for example, when the place of the pier is taken by iron struts working on a horizontal axis.

Anchorage

If the shore or bank be of rock, a vertical passage should be excavated and a strong iron plate placed in the bottom and firmly imbedded in the side of the passage. Through this plate the ends of the cables are passed and secured on the under side.

After the cables are put in place, the passage should be filled with concrete and masonry.

If the bank is not suitable for the

auchanger, a heavy mass of massing should be huich of large blocks of cut store are londed together for the purpose. In this can it is well to construct a passage way so that the chaine and facturing a may at any Time be examined. The mass of masonry or the natural rock to which the ends of the cables are fastured is brequently called the abutument Ste stability must be greater than the terrior of the cables. It wight and thekness must be refficient to frement its bring vousturing, and its cuter of resistance must be in safe himits, Thi calentations in regard to the aucharage, when it is artificial, properly belong with the purposicion bridge, but I have left them out, since they can be very appropriately included in maronny.

anchorage, a heavy mass of masonry should be built of large blocks of cutstone well bonded together for this purpose. In this case it is well to construct a passage way so that the chains and fastenings may at any time be examined. The mass of masonry or the natural rock to which the ends of the cables are fastened is frequently called the abutment. Its stability must be greater than the tension of the cables. Its weight and thickness must be sufficient to prevent its being over turned, and its center of resistance must be in safe limits. The calculations in regard to the anchorage, when it is artificial, properly belong with the suspension bridge but I have left them out, since they can be very appropriately included in masonry.

Cables - These may be made of iron bars
connected by eye bar and fin pints, of connected by eye bar and fin junte, of inne linke se, but the customs now is to use wire roper or caller. The smachet munter of calier in two, one to support each side of the roadway. Generally intime them two are used, since, boi the same amount of material, They offer at least the rain réstance, au mon décessation manufactured, an liable to less danger of accident, and can be more easily put in place and replaced than a single cable of an equal amount of material. bread care is taken to give each win the parme degree of territor. To ensure this it und to be ilought necessary to strain each win reparately our the actual piece, or piere similarly placed, and lind them Together when hanging, strained by their own weight with the dip proposed for the bridge. If was also thought essential that each rope

Cables

These may be made of iron bars connected by eye bar and pin joints, of iron links &c, but the custom now is to use wire ropes or cables. The smallest number of cables is two, one to support each side of the roadway. Generally more than two are used, since, for the same amount of material, they offer at least the same resistance, are more accurately manufactured are liable to less danger of accident, and can be more easily put in place and replaced than a single cable of an equal amount of material. Great care is taken to give each wire the same degree of tension. To ensure this, it used to be thought necessary to strain each wire separately over the actual piers, or piers similarly placed, and bind them together when hanging, strained by their own

weight with the dip proposed for the bridge. It was also thought essential that each rope

should be an aggregate of paralle wires, not spen, as in a rope. Experiment. however, has shown that wise rapes spine with a machine which does not feet a twist in each wire, but lays it helically and untwisted, and with no straight cintral win, are as strong as win ropes of equal weight made with straight wires. It is the custom now to make the cable of wire 16 to 15" in diameter, and bring them to a cylindrical shake by a spiral wrapping of win. The wine are coated with varnish bafon bring hanned rep, and the caber itself in ruitably protected from almosphine influmens.

Surpension Rads -

When the cable in campared of links or have, then are attached directly to them. If of rope, the surpension rod in attached to a collar of iron of suitable shape but around the cable, or to a sadder piece resting on it. When there are two caller,

should be an aggregate of parallel wires, not spun, as in a rope. Experiment, however, has shown that wire ropes spun with a machine which does not put a twist in each wire, but lays it helically and untwisted, and with no straight central wire, are as strong as wire ropes of equal weight made with straight wires. It is the custom now to make the cable of wire $\frac{1}{6}$ " to $\frac{1}{5}$ " in diameter, and bring them to a cylindrical shape by a spiral wrapping of wire. The wires are coated with varnish before being bound up, and the cable itself is suitably protected from atmospheric influences.

Suspension Rods

When the cable is composed of links or bars, they are attached directly to them. If of rope, the suspension rod is attached to a collar of iron of suitable shape bent around the cable, or to a saddle piece resting on it. When there are two cables,

can must be taken to distribute the load upon the cablen according to their degree of strunget. Rudway -The wadway branch an aupported by the surpension rude. On the beauer are laid longitudinal jointe, and on there the planking, on the planking is laid directly on the roadway branch. The latter are stiffered by diagonal the of iron placed harizontally bitween each pain of roadway brann. roadway brann.
Isuural farmeifle - The great ment of a ruspension bridge in its cheapmen, arising from the camparaturely amall quantity of material required to carry a given passing had across a given span. This cheapment may be run more clearly by considering an example. a man might Cross a charm of 100 by hanging to a steel were 21" in diameter, dipping 10

care must be taken to distribute the load upon the cables according to their degree of strength.

Roadway

The roadway bearers are supported by the suspension rods. On the bearers are laid longitudinal joists, and on them the planking, or the planking is laid directly on the roadway bearers. The latter are stiffened by diagonal ties of iron placed horizontally between each pair of roadway bearers.

General Principles

The great merit of a suspension bridge is its cheapness, arising from the comparatively small quantity of material required to carry a given passing load across a given span. This cheapness may be seen more clearly by considering an example. A man might cross a chasm of 100' by hanging to a steel wire .21" in diameter, dipping 10';

The wight of the inne would be 12.75 lbs. a wringht iron bram of reclangular crase section than time an deep as it in howard, would have to be about 27" dues and 9" broad to carry him and it own weight. It would be wiegh 87,500 lbs. an iron I bram of but construction 10' dup would weigh about 120 lbs. In each case 4' have been alluned for brasings at the ende of the opane. The enouncie difference would not exist if the beam and wire had only to carry the man, wen then there waved be a great difference in favor of the min. The main difference arise from the fact that the bidge hats carry its view winght. The chief merit of a suspension bridge done not, therefore, came into play, until the weight of the rope or brain is considerable when campaid with the platform and racing load; for attach

the weight of the iron would be 12.75 lbs. A wrought iron beam of rectangular cross-section three times as deep as it is broad, would have to be about 27" deep and 9" broad to carry him and its own weight. It would be weigh 87,500 lbs. An iron I beam of best construction 10' deep would weigh about 120 lbs. In each case 4' have been allowed for bearings at the end of the spans. The enormous differences would not exist if the beam and wire had only to carry the man, even then there would be a great difference in favor of the wire. The main difference arises from the fact that the bridge has to carry its own weight. The chief merit of a suspension bridge does not, therefore, come into play, until the weight of the rope or beam is considerable when compared with the platform and rolling load; for although

the chain will for any given load be

lighter than a beam , the paving in this respect will, for muall space, be more than campennated by the expense of the anchorages. The disadvantage of the suspension bridge are numerous, a change in the distribution of the load causes à very succider deformation of the structure, for the cable of the suspension lindge must adopt its form to the new position of The load, whereas in the brain the deformation in hardly runiter, equilibrium being attained by a new distribution of the stresser through the material. This physiciality of the suspension budge rendere it unimitable por the passage of a railway train at any canciderable speed. The pealfume rise up a a wave in front of any rapidly advancing load, and the masser in mation produce strenge much greater than there which waved nealt from

lighter than a beam, the saving in this respect will, for small spans, be more than compensated by the expense of the anchorages.

The disadvantages of the suspension bridge are numerous. A change in the distribution of the load causes a very sensible deformation of the structure, for the cable of the suspension bridge must adapt is form to the new position of the load, whereas in the beam the deformation is hardly sensible, equilibrium being attained by a new distribution of the stresses through the material. This flexibility of the suspension bridge renders it unsuitable for the passage of a railway train at any considerable speed. The platform rises up as a wave in front of any rapidly advancing load, and the masses in motion produce stresses much greater than those which would result from

the same wingths when at not.
The kinter effect of the asciteations froduced by hadin of men marching,
on by impulse due to wind, jury in mich strains which cannot be foreseen.

the same weights when at rest. The kinetic effect of the oscillations produced by bodies of men marching, or by impulses due to wind, give rise to strains which cannot be foreseen.

Lit Est 'C be cable of a suspension bidge carrying a load which extends over the whole space. In practice the load carried by a suspension bridge cable in uniforms in interesting in reference to a harizontal line. Theontically this accumption would not do, don't the load in as nearly insparrer few foot of span atof it is taken to be exactly so. Let $CD + BC = 2 = p$ plan BH'= h, = hight of highest lower $\mathfrak{D}f'=\mathcal{K}_2=-\cdot\cdot\cdot$. lower town. w = load for haringwith foot $x =$ distance measured haringentally from If , the lowest paint in the cable.

Fig. 1

Let EH'C be cable of a suspension bridge carrying a load which extends over the whole span. In practice the load carried by a suspension bridge cable is uniform in intensity in reference to a horizontal line. Theoretically this assumption would not do, but the load is so nearly uniform per foot of span that it is taken to be exactly so.

- Let $ED + BC = I = span$
	- $BH' = L_1$ = height of highest tower $DH' = L_2 =$ " " lower tower
	- w = load for horizontal foot
	- $x =$ distance measured horizontally from
		- H', the lowest point in the cable.

The ardivate of any paint of inx, Than The load on H'M'in W = wx, since the total load in equal to the number of minter of lingth into the load on one mint of fingth. Draw PK tungent to the energy of P, then, River the resultant of the load between P and H' acts though the paint of intersection of the taugust at P and If', and the load and turning on the claim at Pand of are ourpretivily proportional to the side of a triangle paraell to their directions, the cable Territor at Paud H'and the direction of W must interest in our point. Since we is uniform along x, the resultant direction of W passis though It, half way between It and M. Therefore, It'= HK, or, since HK in the publingent, the absence, sub-taugust, have the curve in the

The ordinate of any point P is x, then the load on H'M is

 $W = wx$, since the total load is equal to the number of units of length into the load on one unit of length. Draw PK tangent to the curve at P, then, since the resultant of the load between P and H' acts through the point of intersection of the tangents at P and H', and the load and tensions on the chain at P and H' are respectively proportional to the sides of a triangle parallel to their directions, the cable tension at P and H' and the direction of W must intersect in one point. Since w is uniform along x, the resultant direction of W passes through N, half way between H' and M. Therefore $FH' = H'K$, or, since H'K is the sub-tangent, the abscissa, FH', of the curve is equal to the sub-tangent, hence, the curve is the

ordinary paralala. also -It is known that the horizontal component of the tension of a cable wice ba a constant quantity if the loading, vertical; let that camponent be directed μ y H. Let the right triangh SNP be taken for NP represents the cable tension at P, GN the load $W= w_{\mathcal{F}}$, and Is & the cantail harizontal camponent of. PH bring normal to the curve at P, The SS APF and GRP will be similar, and we have the proportion - $\frac{\frac{1}{d} \frac{1}{d \theta}}{\frac{1}{d \theta}} = \frac{\frac{1}{d \theta}}{\frac{1}{d \theta}} = \frac{x}{\frac{1}{d \theta}} = \frac{1}{\omega} = \alpha$ candaut. IF in the subnound of the curve of the cable, and since it is constant, the surer must be the common parabola. If the load placed on a cable 'be a

ordinary parabola. $Also -$

It is known that the horizontal component of the tension of a cable wire be a constant quantity if—the loading, as is assumed in this case, be vertical; let that component be denoted by H.

Let the right triangle GNP be taken for the triangle of forces at P, in which NP represents the cable tension at P, GN the load $W = wx$, and GP the constant horizontal component H. PH being normal to the curve at P, the Δ 's HPF and GNP will be similar, and we have the proportion: HF FP x $\frac{m}{GP} = \frac{F}{GN} = \frac{x}{wx} = \frac{1}{w} = a$ constant HF is the sub-normal of the curve of the

cable, and since it is constant, the curve must be the common parabola. If the load placed on a cable be a

direct function of its lingth, The curva assumed by the mean fibre of the cable mil be a calmary. If it be a direct punction of els spain it will be a parabola. But the weight resting on the main chains is mutter a direct function of the lingth of the cable now of the span, but a function of bath. The curve in therefore, muther a calmary now a farabola. But since the roadway, which forms the principal part of the load, in distributed very rearly uniformly over the span, The curve approaches nious the parabola, and in paractice, in usually regarded ai such a curve. Mour if any two points, P and Q, ba considered fixed, and the portion PG of the cable carries the same interesting of load as before, we have - a cable carrying a load unhose interesty along a straight lim and direction and uniform. Have - if a burkethy flyible cable

direct function of its length, the curve assumed by the mean fibre of the cable will be a catenary. If it be a direct function of its span it will be a parabola. But the weight resting on the main chains is neither a direct function of the length of the cable nor of the span, but a function of both. The curve is, therefore, neither a catenary nor a parabola. But since the roadway, which forms the principal part of the load, is distributed very nearly uniformly over the span, the curve approaches nearer the parabola, and in practice, is usually regarded as such a curve. Now if any two points, P and Q, be considered fixed, and the portion PQ of the cable carries the same intensity of load as before, we have - a cable carrying a load whose intensity along a straight line and direction are uniform. Hence $-$ if - a perfectly flexible cable

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carry a load imigarme in direction and the cable will assume the fame of an be parallel to the loading.

Equation 1.
$$
Q_{\mu\nu}
$$

\nEquation 2. $Q_{\mu\nu}$

\nEquation 3. $Q_{\mu\nu}$

\nEquation 4. $Q_{\mu\nu}$

\nEquation 5. $Q_{\mu\nu}$

\nEquation 6. $Q_{\mu\nu}$

\nEquation 7. $Q_{\mu\nu}$

\nEquation 8. $Q_{\mu\nu}$

\nEquation 9. $Q_{\mu\nu}$

\nEquation 1. $Q_{\mu\nu}$

\nEquation 2. $Q_{\mu\nu}$

\nEquation 3. $Q_{\mu\nu}$

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\nEquation 7. $Q_{\mu\nu}$

\nEquation 8. $Q_{\mu\nu}$

\nEquation 9. $Q_{\mu\nu}$

\nEquation 1. $Q_{\mu\nu}$

carry a load uniform in direction and intensity in reference to a straight line, the cable will assume the form of an ordinary parabola whose axis will be parallel to the loading.

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∴.

Parameter of Curve From Fg. 1 we have the equation of the curve $x^2 = 2py$, (1) in which 2p is the parameter.

Let $BC = x_1$, $ED = X_2$, then $x_1^2 = 2ph_1$ (h₁=H[']B), therefore $x_1 = \sqrt{2ph_1}$ (2) $x_2^2 = 2ph_2$ (h2=DH'), therefore $x_2 = \sqrt{2ph_2}$ (3) Then, multiplying together equations (2) and (3), $x_2 x_1 = 2p\sqrt{h_1h_2}$ = and $2x_2 x_1 = 4p(\sqrt{h_1} + \sqrt{h_2})^2$ (4) Hence $(x_1 + x_2)^2 = x_1^2 + 2x_1x_2 + x_2^2 = l^2 = 2p(\sqrt{h_1} + \sqrt{h_2})^2(5)$ $l^2 = 2p(h_1 + 2\sqrt{h_1h_2} + h_2)$ (6)

$$
p = \frac{l^2}{2(\sqrt{h_1} + \sqrt{h_2})^2} = \frac{l^2}{2(h_1 + 2\sqrt{h_1 h_2} + h_2}
$$
 (6)

If the towers* of the same height, the $h_1 = h_2 = h$, and equation (6) becomes:

$$
p=\frac{l^2}{8h}(7)
$$

Horizontal distances from lowest point of curve to points of support.

The horizontal distance from the lowest point of the cable to the highest tower is, Fig. 1, BC = x_1 so also ED, the lower tower, $= x_2$ $x_1=\sqrt{2ph_1}$ ---(2) But $p = \frac{l^2}{2(\sqrt{h_1} + \sqrt{h_2})^2}$ (6). Substituting this value of p in (2) above- $x_1 = \sqrt{2 p h_1} = \sqrt{\frac{2 h_1 l^2}{2 (h_1 + h_2)^2}} = \sqrt{\frac{l^2 h_1}{(h_1 + h_2)^2}}$ $-\frac{l\sqrt{h_1}}{l}$ (8)

$$
\sqrt{h_1} + \sqrt{h_2}
$$
\nIn a similar, we have from (3):

$$
x_2 = \frac{l\sqrt{h_2}}{\sqrt{h_1} + \sqrt{h_2}} - (9)
$$

3f
$$
h_1 = h_2
$$
 \cdot $x_1 = x_2 = \frac{L}{2} \cdot -10$
\ndrdeinacine of echect at any point -
\n $k + \cdot$ $k + \cdot$ $k + \cdot$ *in the initial limit*
\n $k + \cdot$ $k + \cdot$ *in the initial limit*
\na being marked, $k + \cdot$ *in the initial limit*
\n P_1 *then we know from the a Ff.K.* (5K = 24),
\n $k \cdot$ *in the* $l = \frac{1}{2} \cdot \frac{1}{2}$, *in the* $i = \sqrt{1 + \frac{2V}{2}} \cdot \cdot \cdot$ (11)
\n $k + \cdot$ *in the* $i = \frac{2V}{2}$, *in the* $i = \frac{2V_{2}}{2} \cdot \cdot \cdot$ (11)
\n $l + \cdot$ *in the* $i = \frac{2V_{2}}{2}$ *in the* $i = \frac{2V_{2}}{2} \cdot \cdot \cdot$ (12)
\n $l + \cdot$ *in the* $i = \frac{2V_{2}}{2} \cdot \cdot \cdot$ (13)
\n $l + \cdot$ *in the* $i = \frac{2V_{2}}{2}$ *in the* $i = \frac{2V_{2}}{2}$ *in the* $i = \frac{2V_{2}}{2}$
\n $l + \cdot$ *in the* $i = \frac{2V_{2}}{2}$ *in the* $i = \frac{2V_{2}}{2}$
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\n $l + \cdot$ *in the* $i = \frac{2V_{2}}{2}$ *in the* $i = \frac{2V_{2}}{2}$
\n $l + \cdot$ *in the*

18

If
$$
h_1 = h_2
$$
; $x_1 = x_2 = \frac{l}{2}$ ----(10)

Inclination of cable at any point Since $KH' = H'F = y$, if i is the inclination to a horizontal line of the curve at any point P, then we have from the Δ FPK, (FK = 2y), $x \tan(i)=2y$,

$$
tan(i) = \frac{2y}{x}
$$
 therefore $sec(i) = \sqrt{1 + \frac{4y^2}{x^2}}$ ---(11)

At the tops of the towers

$$
tan(i_1) = \frac{2h_1}{x_1} - (12). \quad tan(i_2) = \frac{2h_2}{x_2} - (13)
$$

If $h_1 = h_2, tan(i_1) = tan(i_2) = \frac{4h}{l} - (14)$

Resultant tension at any point of cable It has

been shown that if the loading on a cable is uniform in direction, the component of cable tension normal to that direction will be constant at all points of the cable. Let the resultant tension at the lowest point of the cable be this constant component, denoted by H.

The flow
$$
l
$$
 and l is the flow l and l is the new value of the curve, and we have shown from 15 and 10. The sum of l is the value of l and $$

We have seen that $H = wAF$. But AF is the sub-normal of the curve, and we know from General geometry, that the subnormal to the parabola is equal to one half the parmeter 2p, or equalt to p. Hence, H=wp----(15)

Substituting in the above the value of p as found in equation (6), we have:

$$
H = \frac{wl^2}{2(h_1 + h_2)^2} = \frac{wl^2}{2(h_1 + 2\sqrt{h_1 h_2} + h_2)} \quad \text{(16)}
$$

Let R represent the resultant tension at any point, then from the triangle of forces, GNP,

PN=GPsec(i), or
\n
$$
R = Hsec(i) = H \sqrt{1 + \frac{4y^2}{x^2}}
$$
-(17) but

substituting the value of sec(i) in (11). At the tops of the towers the tensions

are:
\n
$$
R_1 = H \sqrt{1 + \frac{4h_1^2}{x_1^2}}
$$
\n
$$
R_2 = H \sqrt{1 + \frac{4h_2^2}{x_2^2}}
$$
\n(19)

$$
2f h_1 = h_2
$$
, Thus $x_1 = x_2 = \frac{1}{2}$, and from
Equation (16) -

$$
f_1 = \frac{w}{8h}
$$
---(20)
Also $h_1 = h_2 = \sqrt[3]{\sqrt{1 + \frac{16h^2}{1.2}} - \dots (21)}$
Multiply g_1 and g_2 are defined as h_1 and g_3 are defined as h_1 and h_2 are defined as g_1 and h_3 are defined as h_1 and h_2 are defined as h_1 and h_3 are defined as h_1 and h_2 are defined as h_1 and h_3 are defined as h_1 and h_4 are defined as h_1 and h_2 are defined as h_1 and h_3 are defined as h_1 and h_4 are defined as h_1 and h_4 are defined as h_1 and h_4 are defined as h_1 and h_1 are defined as h_1 and h_1 are defined as h_1 and h_1 are defined as h_1 and h_2 are defined as h_1 and h_1 are defined as h_1 and h_2 are defined as h_1 and h_3 are defined as h_1 and h_4 are defined as h_1 and h_2 are defined as

 20.00

20

If $L_1 = L_2$, then $x_1 = x_2 = \frac{l}{2}$, and from Equation (16) $H = \frac{wl^2}{2h}$ ----(20) *'*___________ Also, $R_1 = R_2 = H\sqrt{1 + [illegible]}$

Length of cables between a known point and the vertex, or between vertex and a point at which the inclination to a horizontal line in "i".

From the

calculus we have the formula for the rectification of plane curves

 $dz = \sqrt{dx^2 + dy^2}$, in which z represents

the length of the curve, and x and y the general coordinates.

From the equation of the curve

$$
x2 = 2py, we have\n
$$
dy2 = \frac{x2 dx2}{pz}, \text{ and}
$$
\n
$$
dz = \frac{1}{p}\sqrt{p2 + x2} dx
$$
$$

To integrate this expression, apply formula C of reduction.

$$
y = \int x^{m}(\omega + bx^{u})^{p} dy = \frac{x^{m+1}(\omega + bx^{v})^{p} + \omega x + \omega x^{m}(\omega + bx^{v})^{p-1} dx}{n p + n x + 1}
$$
\n
$$
Z = \frac{x \sqrt{x^{2} + p^{2}}}{2 p} + \frac{p}{2} \int \frac{dy}{(\rho^{2} + x^{2}} - (2z)
$$
\n
$$
T \cdot \text{int}y = dx + \frac{dy}{\rho^{2} + x^{2}} + \frac{p}{2} \int \frac{dy}{(\rho^{2} + x^{2}} - (2z)
$$
\n
$$
T \cdot \text{int}y = dx + \frac{x \sqrt{x}}{\rho^{2} + x^{2}} dy
$$
\n
$$
= \frac{x + \sqrt{p^{2} + x^{2}}}{\sqrt{p^{2} + x^{2}}} dy
$$
\n
$$
= \frac{x + \sqrt{p^{2} + x^{2}}}{\sqrt{p^{2} + x^{2}}} dy
$$
\n
$$
\frac{dx}{dx} = \frac{y + \sqrt{p^{2} + x^{2}}}{\sqrt{p^{2} + x^{2}}} dy
$$
\n
$$
\frac{dx}{dx} = \frac{y - \sqrt{p^{2} + x^{2}}}{\sqrt{p^{2} + x^{2}}} = \frac{y - \sqrt{p^{2} + x^{2}}}{\sqrt{p^{2} + x^{2}}} - (2y)
$$
\n
$$
\int \frac{dx}{dx} = \int \frac{dx}{\sqrt{p^{2} + x^{2}}} = \log x \quad \text{Reluning } u = u = u
$$
\n
$$
\int \frac{dx}{\sqrt{p^{2} + x^{2}}} = \log (x + \sqrt{p^{2} + x^{2}}) - \frac{1}{2} (2x - x^{2})
$$

$$
\frac{1}{2}k\pi\sqrt{2\pi\left(\frac{1}{2}x^{2}+x^{2}\right)}+\frac{1}{2}log\left(x+\sqrt{2^{2}+x^{2}}\right)+C
$$

$$
y = \int x^m (a + bx^n)^p dx = \frac{x^{m+1} (a + bx^n)^p + a_n p \int x^m (a + bx^n)^{p-1} dx}{n p + m + 1}
$$
 (C)

$$
z = \frac{x\sqrt{x^2 + p^2}}{2p} + \frac{p}{2} \int \frac{dx}{\sqrt{p^2 + x^2}} \dots (22)
$$

To integrate
$$
\frac{dx}{\sqrt{p^2 + x^2}}
$$
, put $z = x + \sqrt{p^2 + x^2}$ ---(23)
then $dz = \frac{x + \sqrt{p^2 + x^2}}{\sqrt{p^2 + x^2}} dx$

Now we have

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$$
\frac{dz}{z} = \frac{\frac{x + \sqrt{p^2 + x^2}}{\sqrt{p^2 + x^2}} dx}{x + \sqrt{p^2 + x^2}} = \frac{dx}{\sqrt{p^2 + x^2}} - (24)
$$
\n
$$
\int \frac{dz}{z} = \int \frac{dx}{\sqrt{p^2 + x^2}} = \log(z) \text{ Restoring the value}
$$
\nof z:\n
$$
\int \frac{dx}{\sqrt{p^2 + x^2}} = \log(x + \sqrt{p^2 + x^2} - (25)
$$

Therefore

$$
z = \frac{x\sqrt{p^2 + x^2}}{2p} + \frac{p}{2} log(x + \sqrt{p^2 + x^2} + C
$$

Estimating the arc from the vertex, it
being the origin $C = -\frac{p}{2} log p$

Thus the carrieded integral is:

\n
$$
z = \frac{xy + y + x}{2} + \frac{p}{2} \log \left[\frac{x + y - z}{2} \right] - (26)
$$
\nNow by putatitative, in the above, for 600 and 600.

\n
$$
z = \frac{x^2}{4y} \left[\frac{2y}{x} \right] + \frac{y - z}{x^2} + \log \left(\frac{2y}{x} + \sqrt{1 + \frac{y - z}{x^2}} \right) - \frac{2y}{x} \log \left(\frac{2y}{x} + \sqrt{1 + \frac{y - z}{x^2}} \right) - \frac{2y}{x} \log \left(\frac{2y}{x} + \sqrt{1 + \frac{y - z}{x^2}} \right) - \frac{2y}{x} \log \left(\frac{2y}{x} + \sqrt{1 + \frac{y - z}{x^2}} \right) - \frac{2y}{x} \log \left(\frac{2y}{x} + \sqrt{1 + \frac{y - z}{x^2}} \right) - \frac{2y}{x} \log \left(\frac{2y}{x} + \sqrt{1 + \frac{y - z}{x^2}} \right) - \frac{2y}{x} \log \left(\frac{2y}{x} + \sqrt{1 + \frac{y - z}{x^2}} \right) - \frac{2y}{x} \log \left(\frac{2y}{x} + \sqrt{1 + \frac{y - z}{x^2}} \right) - \frac{2y}{x} \log \left(\frac{2y}{x} + \sqrt{1 + \frac{y - z}{x^2}} \right) - \frac{2y}{x} \log \left(\frac{2y}{x} + \sqrt{1 + \frac{y - z}{x^2}} \right) - \frac{2y}{x} \log \left(\frac{2y}{x} + \sqrt{1 + \frac{y - z}{x^2}} \right) - \frac{2y}{x} \log \left(\frac{2y}{x} + \sqrt{1 + \frac{y - z}{x^2}} \right) - \frac{2y}{x} \log \left(\frac{2y}{x} + \sqrt{1 + \frac{y - z}{x^2}} \right) - \frac{2y}{x} \log \left(\frac{2y}{x} + \sqrt{1 + \frac{y - z}{x^2}} \right) - \frac{2y}{x} \log \left(\frac{2y}{x} + \sqrt{1 + \frac{y - z}{x^2}} \right) - \frac{2z}{x}
$$

22

 $z = \frac{x\sqrt{p^2 + x^2}}{2p} + \frac{p}{2} log(\frac{x + \sqrt{p^2 + x^2}}{p}) - \cdots$ (26) Thus the corrected integral is Now by substituting in the above for p its value $\frac{x^2}{2y}$, the equation can be put in the form- $z = \frac{x^2}{4y} \left(\frac{2y}{x} \sqrt{1 + \frac{4y^2}{x^2}} + log\left[\frac{2y}{x} + \sqrt{1 + \frac{4y^2}{x^2}}\right] \right)$ ----- (27) Now we have seen that $\frac{x^2}{4y} = \frac{p}{2}$; $\frac{2y}{x} = \tan(i)$; $\sqrt{1+\frac{4y^2}{x^2}}$ = sec(i), and by substituting these values, eq. (27) becomes $z = \frac{p}{2}[tan(i)sec(i) + log(tan(i) + sec(i))]$ ----(28) In the above formulas, the Naperian logarith is used, since the modulus is 1. Since the above formulae were deduced for the distance from the vertex to any particular point, the total length of cable will be found by substituting for $y_1 h_1$; and for $x_1 x_1$ in equation (27); or i_1 for i in Eq. (28); then x_2 and h_2

for x and y in (27), or $i₂$ for i in (28) and adding the results. Denoting these

results by l_1 and l_2 , then total length will be:

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 $l_1 + l_2$ ----(29) A formula, which is close enough for practical purposes, and which is frequently used, is deduced as follows. In fig. 1 suppose H'P is an arc of a circle whose radius is R. The coordinates x and y are the same as before. The expression for a circular arc in the integral calculus is:

$$
\frac{dx}{\sqrt{1-\frac{x^2}{R^2}}} = \int \frac{dx}{1-\frac{x^2}{2R^2}}, \text{ approximately,}
$$

considering R very large as compared with x.

Suspension rods -It is usually assumed in the calculations relating to purposeion rods, that the cable lies in a vertical frame, and that the surferision rade are virtical. Since in are causante suspension rode an fourable to each other, The above accumption does not affect the generality of the results. If the rods are inclined, the true lengths can be found by multiplying the value abtained by the search of the inclination of the rode to a virtical line. a fat parabala nearly coincide with a circle and we may suppose the camber to be found by a parabolic are.

Suspension Rods

It is usually assumed in the calculations relating to suspension rods, that the cable lies in a vertical beam, and that the suspension rods are vertical. Since in all cases the suspension rods are parallel to each other, the above assumption does not affect the generality of the results. If the rods are inclined, the true lengths can be found by multiplying the values obtained by the secant of the inclination of the rods to a vertical line.

A flat parabola nearly coincides with a circle and we may suppose the camber to be found by a parabolic arc.

Figure 2.

25

Let the ordinate AD=x be measured from A towards B, and DC=y perpendicular to it. Let AB= x_1 = the half span. The curve of the cables is the parabola as before.

From Eq. (1)
$$
p = \frac{x^2}{2p}
$$

\n
$$
y = \frac{x^2}{2p}
$$
\nFrom Eq. (1) $p = \frac{l^2}{8h}$. Substituting:
\n
$$
y = \frac{4hx^2}{l^2}
$$
\n
$$
AB = x_1 = \frac{1}{2}l, \therefore l=2x, \text{ and we have}
$$
\n
$$
y = \frac{4hx^2}{l^2} = \frac{4hx^2}{4x_1^2} = L\frac{x^2}{x_1^2}
$$

Now L=y, in Fig II, or calling y, the ordinate and x the abscissa of any point in the curve above AB, we have the general equation:

$$
y' = y_1 \frac{x^2}{x_1^2} \text{---}(33)
$$

In the same mariner for the lower curver or the camber $y''=z\frac{x^2}{x^2}-\frac{1}{3}\pi$, in which #10 in taken as the axis of alreisar and I in the ordinate Representing the lingth of any suspender as CC by I, we have $L = \mathcal{C} \mathcal{C}' = \mathbb{C} \mathbb{D} + \mathbb{D} \mathbb{D}' + \mathbb{D}' \mathcal{C}' - (35)$ Now DD'= $\mathbf{b_0} = \mathbf{c}$, and taking the value of CD and D'C'as given in Ega. (33) and (34) , $h = y' + y'' + 2 - (36)$ From this we are that each purpouder is campaigned of the constant length a and the two variable once y'and y". adding equations (33) and (34), and representing the sum of the variable lugthe $\frac{q}{q}$ $y = (y' + y'') = (y_1 + z) \frac{x^2}{x^2}$ - (3) Now let the suspenders are be the same distance apart, and represent this constant distance by d, then

In the same manner for the lower curve or the camber

 $y'' = z \frac{x^2}{x^2}$ ----(34) in which

A'B' is taken as the axis of abscissae and z is the ordinate Representing the length of any suspender as CC' by L, we have -

 $L=CC' = C'D+DD'+D'C'---(35)$ Now $DD' = h_0 = C$, and taking the value of CD and D'C' as given in Eqs. (33) and (34) , h = y'+y''+ C --- (36) From this we see that each suspender is composed of the constant length c and the two variable ones y' and y". Adding equations (33) and (34), and representing the sum of the variable lengths by y --

$$
y = (y' + y'') = (y_1 + 2)\frac{x^2}{x_1^2}
$$
---(37)

Now let the suspenders all be the same distance apart, and represent this constant distance by d, then

$$
h_{2} = e + \frac{4 \lambda^{2}}{\lambda^{2}} (y_{1} + z)
$$

\n
$$
h_{3} = e + \frac{9 \lambda^{2}}{\lambda^{2}} (y_{1} + z)
$$

\n
$$
h_{n-1} = e + \frac{(n-1)^{2}}{\lambda^{2}} (y_{1} + z)
$$

\n
$$
h_{n} = e + \left(\frac{n^{2} \lambda^{2}}{\lambda^{2}} - 1\right) (y_{1} + z) = e + y_{1} + z - (38)
$$

27

$$
h_2 = c + \frac{4d^2}{x_1^2} (y_1 + z)
$$

\n
$$
h_3 = c + \frac{9d^2}{x_1^2} (y_1 + z)
$$

\n
$$
h_{n-1} = c + \frac{(n-1)^2}{x_1^2} (y_1 + z)
$$

\n
$$
h_n = c + \left(\frac{n^2 d^2}{x_1^2}\right) (y_1 + z) = c + y_1 + z
$$
---(38)

Since h_1 was assumed equal to h_2 in the above calculations, or the towers of the same height and equal to h, the lengths of the suspenders on each side of the lowest point in the cable will be equal, and having computed one side we use these values for the other. The vertical load which any rod carries multiplied by the secant of its inclination to a vertical line gives the stress on such rod.

Definition of cable for change in length,

\nat the below running at the same.

\n1.
$$
\frac{24}{3}x^{2}
$$
, and we have $-x$, $\frac{h^{2}}{2}$, and we have $-x$, $\frac{h^{2}}{2}$, and h^{2} , and we have $-x$, $\frac{h^{2}}{2}$, and h^{2} .

\n2. $\frac{h^{2}}{2}$, and h^{2} , and h^{2} , and h^{2} .

\n3. $\frac{h^{2}}{2}$, h^{2} , and h^{2} , and h^{2} .

\n4. $\frac{h^{2}}{2}$, and h^{2} , and h^{2} , and h^{2} .

\n4. $\frac{h^{2}}{2}$, and h^{2} , and h^{2} , and h^{2} .

\n5. $\frac{h^{2}}{h}$, h^{2} , and h^{2} , and h^{2} .

\n6. $\frac{h^{2}}{h}$, and h^{2} , and h^{2} , and h^{2} .

\n7. $\frac{h^{2}}{h}$, and h^{2} , and h^{2} .

\n8. $\frac{h^{2}}{h}$, h^{2} , and h^{2} , and h^{2} .

\n9. h^{2} , h^{2} , and h^{2} , and h^{2} .

\n10. h^{2} , h^{2} , and h^{2} , and h^{2} .

\n2. h^{2} , and h^{2} , and h^{2} .

\n3. h^{2} , and h^{2} , and h^{2} , and h^{2}

Deflection of a cable for change in length, the span remaining the same In Eq. (3) $x(1+\frac{2y^2}{3x-2})$, substitute x_1 for x and h_1 for y, and we have $x_1 (1 + \frac{h_1^2}{3x_1^2})$ -----(39)

Also substituting x_2 and h_2 for x and h in the same equation, it becomes

$$
x_2(1+\frac{h_2^2}{3x_2^2})\text{---}(40)
$$

Adding the above equations and denoting the two segments of the parabola by c_1 and c_2 , we have the total length of cable --

$$
c_1 + c_2 = x_1 + x_2 + \frac{2}{3} \left(\frac{h_1^2}{x_1} + \frac{h_2^2}{x_2} \right) \dots (41)
$$

Differentiating:

 28

$$
d(c_1 + c_2) = \frac{4}{3} \left(\frac{h_1}{x_1} + \frac{h_2}{x_2} \right) dx \cdots (42)
$$

Now $h_1 - h_2$ being equal to a constant,
 $dh_1 = dh_2 = dh$

$$
\therefore dh = \frac{3d(c_1+c_2)}{4(\frac{h_1}{x_1} + \frac{h_2}{x_2})} \qquad (43)
$$

From whatever cause the cable may vary in lingth, this variation in to be fut for d(c, +c2) in equations (42) and (43), and the db will be the comparating deflection of the lawest faint of the cable. If the towns an of the pursu height $e_1 = e_2$, $w_1 = w_1$, $x_1 = x_2 = \frac{p}{2}$ Then we have from (42) -
2 de, = $\frac{16}{3}$ b, --- (44) $dL = \frac{3k_1}{16}$ 2 de_1 --- (45) In equation (42) and (43) the assumption, though not strictly true, in that the lowest paint of the caber remaine at the same having antal dictance from the lowers.

From whatever cause the cable may vary in length, this variation is to be put for $d(c_1 + c_2)$ in equations (42) and (43), and then *dh* will be the corresponding deflection of the lowest point of the cable. If the towers are of the same height-

$$
c_1 = c_2, h_1 = h_2, x_1 = x_2 = \frac{l}{2}
$$

Then we have from (42)-

29

$$
2dc_1 = \frac{16 h_1}{3 l} \cdot \cdots \cdot (44)
$$

$$
dh = \frac{3h_1}{16 l} 2dc_1 \cdot \cdots \cdot (45)
$$

In equations (42) and (43) the assumption, though not strictly true, is that the lowest point of the cable remains at the same horizontal distance from the towers.

To ablain the true lingth of the curve since the choos relations were deduced from the approximate formula(), we unnel tolle itte true equation for the lingth of the curve. as before, bet (C, +e2) by the known hugt of cause before variation taken place ; but h, and h_{2,} x, and the last the original highle of towns shes requeste of span also known. Lit y, and you be the leight of the towns about the lawest fait in the cable, efter variation in ite lingth here taken place. X, and X2 are state constants. Let the variation in length of the cable be referented by v Thus $r = -(C_1 + C_2) + (C_1 + C_2 + V)$ -- (46) $V = \frac{x^2}{4y} \left(\frac{2y}{x_1} \sqrt{1 + \frac{4y_1^2}{x_1^2}} + \log \left(\frac{2y_1}{x_1} + \sqrt{1 + \frac{4y_1^2}{x_1^2}} \right) \right)$ $+\frac{\chi_{2}^{2}}{4\gamma_{2}^{2}}\left[\frac{2\gamma_{2}}{\chi_{2}}\sqrt{1+\frac{4\gamma_{2}^{2}}{\chi_{2}}}}+\log\left(\frac{2\gamma_{2}}{\chi_{2}}+\sqrt{1+\frac{4\gamma_{2}}{\chi_{2}^{2}}}\right)\right]-\left(0,+0_{2}\right)-$

To obtain the true length of the curve since the above relations were deduced from the approximate formula (), we must take the true equation for the length of the curve. As before, let $(c_1 + c_2)$ be the known length of curve before variation takes places; let h_1 and h_2 , x_1 and x_2 be the original heights of the towers also segments of span also known. Let y_1 and y_2 be the heights of the towers above the lowest point in the cable, after variation in its length has taken place. x_1 and x_2 are still constants. Let the variation in length of the cable be represented by v.

Thus
$$
v = -(c_1 + c_2) + (c_1 + c_2 + v)
$$
---(46)
\n
$$
v = \frac{x_1^2}{4y_1} \left[\frac{2y_1}{x_1} \sqrt{1 + \frac{4y_1^2}{x_1^2} + \log \left(\frac{2y_1}{x_1} + \sqrt{1 + \frac{4y_1^2}{x_1^2}} \right) \right] + \frac{x_2^2}{4y_2^2} \left[\frac{2y_2}{x_2} \sqrt{1 + \frac{4y_2^2}{x_2^2} + \log \left(\frac{2y_2}{x_2} + \sqrt{1 + \frac{4y_2^2}{x_2^2}} \right) \right] - (c_1 + c_2)
$$
---(47)

But since $y_1 - y_2 = h_1 - h_2 = a$ constant,
we can toke the value of y_1 or y_2 and subilitate in Equation (47), and we wice then have only one unknown quantity in the right member, and this inknown in determined by trial. (y, or y2, h, or h2 may be taken as the) The first value of y, or ye taken may be h, or h = mariared or decreased, as the case may by by dh. If I is taken from Eg, (45) y, -h, = y2- h2 in the diffection Aunight, The variation of lugt can be abtained at once from equation (47) when the neur highte y, and ye are given. height, $x_1 = x_2 = \frac{1}{2}, \, c_1 = e_2, \, and \, y_1 = y_2 = h$ making autoitation of the value in equation (47), then recenter, after adding-

But since $y_1 - y_2 = h_1 - h_2 = a constant$, we can take the value of y_1 or y_2 and substitute in equation (47), and we will then have only one unknown quantity in the right member, and this unknown is determined by trial $(y_1$ or y_2 , h_1 or h_2 may be taken as the). The first value of y_1 or y_2 taken may be h_1 or h_2 increased or decreased, as the case* may be, by *dh* is taken from Eq. (45).

 $y_1 - h_1 = y_2 - h_2$ is the deflection sought.

The variation of length can be obtained at once from equation (47) where the new heights y_1 and y_2 are given. When the towers are of the same height,

 $x_1 = x_2 = \frac{1}{2}$, $c_1 = c_2$, $y_1 = y_2 = h$ Making substitutions of the values in equation (47), then results, after adding,

 $v = \frac{2k^2}{16h} \left[\frac{4h}{3} \sqrt{1 + \frac{16h^2}{16h^2}} + \frac{1}{24} \left(\frac{4h}{16h^2} + \sqrt{1 + \frac{16h^2}{16h^2}} \right) \right] - 28 \left(4 \frac{8}{3} \right)$ If h is known, we can find a from the down the down equation. If a is known, h paint of the trues.

$$
v = \frac{2l^2}{16h} \left[\frac{4h}{l} \sqrt{1 + \frac{16h^2}{l^2}} + \log \left(\frac{4h}{l} + \sqrt{1 + \frac{16h^2}{l^2}} \right) \right] - 2c \cdots (48)
$$

If *h* is known, we can find *v* from the above equation. If *v* is known, *h* is found by trial, and v then results. $h - h_1$ = deflection of the middle

point of the truss.

Pressure on Tower

Fig. 3

Bracing to recist heavy traveling load-Varian methods have been proposed, and some of them tried, to enable a suspension bridge to recent the setime of a heavily traveling load so as to undergo no mon disfigurant than a girden. I a unsure this in a bridge of several bays, the faire must be made very strong, and the chain accurdy factured to them. The bird way of bracing in by means of auxilary girders, on a fair of straight girders of any convenient form hing from the cable by suspending rode and supporting the cross juids of the platform. These girden should be rupported at each and, and also factured down, as then are certain paintine of the rolling load which would true to vain our und of the true. The ends showed be fun to move horizontally, howevert,

Bracing to resist heavy traveling load Various

methods have been proposed, and some of them tried, to enable a suspension bridge to resists the action of a heavily traveling load so as to undergo no more disfigurement than a girder. To ensure this in a bridge of several bays, the piers must be made very strong, and the chains securely fastened to them.

The best way of bracing is by means of auxiliary girders, or a pair of straight girders of any convenient form hung from the cable by suspending rods and supporting the cross joists of the platform. These girders should be supported at each end, and also fastened down, as there are certain positions of the rolling load which would tend to raise one end of the truss. The ends should be free to move horizontally, however.

By franculing a change in the form of stiffening trues, we not only prevent very injuriour undulations, but also hessen the work of computing stresses, which would be very defficult if the cable did not retain the parece form. The cable will assume the same parabolic curve only where there is a uniform pull on the suspension rode from and to end. It I be the uniform fuel on any suspension and, and the interesting for unit of obout. Now if - I represents one paul lingth in the true. "I = pt Lit we be the fixed load per unit of span sustained by the cables, and with moving load sustained by them; Let l' be the above; R the reaction of B(Seg.); Ri the reaction A. Suppose the moving load to fass on from $\sqrt{3}$. Id x, but the distance from B to the front of the moving food. The load is supposed continuous.

 $35 -$

By preventing a change in the form of the cable, which is accomplished by the stiffening truss, we not only prevent very injurious undulations, but also lessen the work of computing stresses, which would be very difficult if the cable did not retain the same form. The cable will assume the same parabolic curve only when there is a uniform pull on the suspension rods from end to end. Let T be the uniform pull on any suspension rod, and *t* its intensity per unit of span. Now if *p* represents one panel length in the russ, *T=pt* Let *w* be the fixed load per unit of span sustained by the cables, and w' the moving load sustained by them; Let ℓ be the span; R the reaction at B (fig.); *R*' the reaction A. Suppose the moving load to pass on from B. Let x_1 be the distance from B to the front of the moving load. The load is supposed continuous.

Fig. 4

All the forces acting on the truss are vertical in their direction, and we have two equations of equilibrium, indicating that the sum of the external vertical forces and that the sum of these moments about any point must each equal zero.

 $wl + w'x_1 - tl - R - R' = 0$ -----(53) Thus by taking the moments about the head of the moving load-

 $(w + w')\frac{x_1^2}{2} - t\frac{x_1^2}{2} - Rx_1 + (t - w)\frac{(l - x_1)^2}{2} + R'(l - x_1) = 0$ --(54)

In each of the above equations are three unknown quantities, *t,* R,and *R'.* If any one of these is known, the others may found from the two conditional equations. From this we see that unless at least one condition is imposed, it

be impossible to ascertain have much the trues will carry, wither in connection with the cable, or alone. assuming a value for 2, R, or R', makes the abilitaring true all allegation in connection with the caber, and carry no load as an ordinary true. From the please it is seen that the sum of we the loads w, wire, must be equal to the own of all the uniform upward forces, $T^0 = pt$. The resultants of the two forces act in different lines, and the true is then subjided to the adien of a cauple, which must be counteracted abother couple of equal moment but opposite lines of action. These cauples must set of the extremitive of and B - They are the reaction R and R. Therefore - $R = -R'$ Substituting this value in Guation (53),

is impossible to ascertain how much the truss will carry, either in connection with the cable, or alone. Assuming a value for *t, R,* or R', makes the stiffening truss act altogether in connection with the cable, and carry no load as an ordinary truss. From the above it is seen that the sum of all the loads w , w' , and c^* , must be equal to the sum of all the uniform upward forces, *T=pt.* The resultant of the two forces act in different lines, and the truss is then subjected to the action of a couple, which must be counteracted another couple of equal moment but opposite lines of action. These couples must act at the extremities A and B. They are the reactions R and R'. Therefore- $R = -R''$ Substituting this value in Equation (53),

and solving for
$$
t
$$
 :
\n t from multiplication of t = $w + w \frac{N}{L} = -5$ (55)
\n t from multiplication of $R = -R' = \frac{w'X}{2}(1-\frac{x}{L}) = -56$
\n $\frac{N}{2}$ (56), if $x = 0$, or $w' = 0$, *lost* notation
\n W can give $R = -R' = 0$
\n $\frac{N}{2}$ is also equal, that R' and R' are
\n $\frac{N}{2}$ is a downward, that R and R' are
\n $\frac{N}{2}$ is a downward, and $\frac{N}{2}$ are given by $\frac{N}{2}$.
\n $\frac{dR'}{d\eta} = \frac{w'}{2} - \frac{w'X}{2L} - \frac{w'X}{2L}$, and $x_1 = \frac{1}{2}$.
\n $\frac{N}{2}$ is a quadratic, $W = \frac{1}{2}$ is a equation (56)
\n $R = \frac{w'Z}{8} = -1$ (57)
\n $R = \frac{w'Z}{8} = -1$ (57)
\n $\frac{d\eta}{d\eta} = \frac{1}{2}$ is a quadratic, $\frac{1}{2}$ is a quadratic, 1 (56)
\n $R' = \frac{w'Z}{8} = -1$ (57)
\n $\frac{d\eta}{d\eta} = \frac{1}{2}$ is a quadratic, $\frac{1}{2}$ is a linearly
\n $\frac{d\eta}{d\eta} = \frac{1}{2}$ is a quadratic, $\frac{1}{2}$ is a linearly
\n $\frac{1}{2}$ is a linearly independent, $\frac{1}{2}$

and solving for z, $z = w + w' \frac{x_1}{l}$ ----(55) From substituting the same in (54), $R = -R' = \frac{W'x_1}{2}(1 - \frac{x_1}{l})$ ----(56) In (56), if $x_1 = l$, or \overline{w} = 0, both reactions become zero, $R = -R' = 0$ It is also seen that *R* and *R '* are numerically equal, but have opposite directions.
R' is a downward reaction and will

show the amount of anchorage required. Differentiating (56) and finding value of $\overline{}$,

$$
\frac{dR}{dx_1} = \frac{w'}{2} - \frac{w/x_1}{2l} - \frac{w/x_1}{2l}, \text{ and } x_1 = \frac{l}{2}
$$

Now substituting $x_1 = \frac{l}{2}$ in equation (56)

$$
R = \frac{w'l}{8} \text{---}(57)
$$

Equation shows the greatest shear to be provided for at either end of the truss, and also the maximum amount of anchorage to be provided for.

The matter on Aturn fragen in very Ruflectfully Submitted -Rocea, Mo, June 7", 1884.

The matter on stresses proper is very limited, on account of space.

> Respectfully Submitted W. M. Claypool

Rolla, Mo., June 7th 1884.