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The analysis of stresses and design of multiple-arch dam

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THE ANALYSIS OF STRESSES

AND

DESIGN OF MULTIPLE-ARCH DAM

BY

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-INTRODUCTION-

The so-termed: "Multiple", or "Scallop-Arched" dam, as its name implies, is one composed of a series of arches, either semi-circular or segmental in plan. These arches were first vertical but were later inclined downstream, when the engineers recognized the value of the weight of water in increasing the resisting moment. The thrust of these arches, due to the water pressure and the weight of the arch itself, is transmitted directly to the buttresses.

There are numerous advantages to this type of dam over that of the gravity type. An engineer would hesitate to place a gravity section on clay, but by increasing the area of the footings of the buttresses, a multiple-arched dam may safely be constructed. Another advantage of the multiple-arched dam is the saving in material. The liquid pressure on the arch barrel exerts a radial pressure which is entirely carried by compression. Steel reinforcement is practically unnecessary except for temperature stresses.

There is only one advantage of this type over the single vertical arch. The single arch can be used only in a comparatively narrow gorge with rock sides and floor, while the multiple arched dam can be used for any length span.

These advantages cause one to question why more of these dams are not built; they have been built, and very few of them have failed, however, confidence is yet lacking among the engineers. There is a diversified opinion as to how the water pressure really acts. Most engineers maintain that a certain part of the water is transmitted to the foundation by cantilever action and that the remainder is carried as axial thrust to the buttresses. Consider a cylinder placed in some water and the bottom of the cylinder rigidly fastened to the bottom of the vessel containing the water. The pressure of the water on the cylinder creates compression in the wall. This compression causes a certain deformation and the circumference of the wall becomes less. When this happens the sides of the cylinder, instead of being parallel, as they were at first, are tilted in. Now, if a vertical section of this cylinder acting as a cantilever beam, is considered, it will be seen that a certain amount of the stress is transmitted to the foundation by this action; the remainder of the pressure is, of course, carried as compression in the walls of the cylinder. Some engineers doubt this method, even though a good many dams have recently been designed on this basis and after being constructed have proved successful. Their argument is based upon the fact that the base of the arch is not rigidly fixed on the foundation; also, the action of concrete under stress is not accurately known. It is known that it possesses the ability to flow or re-form itself, to a certain extent, when stressed. This flow tends to absorb this stress and lessen it. This can be illustrated by placing some reinforcing in fresh concrete. The concrete shrinks while settling and if the concrete did not have this ability to flow there would be no bond between the concrete and steel, beyond that due to friction. However, there is a bond, demonstrating that while the mass of concrete is shrinking, setting up stresses around the reinforcing, it is also reshaping itself to absorb these stresses. This can be illustrated also by repeatedly loading a concrete beam and plotting the deformation before and after each loading. Another objection that the conservative engineer has, is the comparatively slender buttresses and the danger of buckling; however, this can be remedied by cross-bracing where necessary.

One of the first dams of this type was the

Mir Alum Tank dam, in southern India. This structure was built about 1806, of coursed rubble masonry laid in lime mortar. It is a low dam, being only thirty-three feet high, and the arch is vertical. This dam is still serviceable after more than a century of duty. Another more recent achievement is the Big Bear Valley dam . It is constructed of reinforced concrete and the inclination of the arch is 36° 52' from the vertical; its vertical height is 92 feet. These are only two illustrations of the daring of the pioneer engineers who conceived these works and this type has in the last few years been recognized as a competitor of the old gravity type.

However, there has been some failures, one of the most recent being the Gleno Dam in Italy, which gave way Dec. 1, 1923 and the resulting flood destroyed numerous power stations, houses and villages. This dam, built since the War, was 143 feet high; its failure was attributed to porous concrete, unclean aggregate, and poor construction. The reinforcing used was scrap wire netting, which had been used as protection against hand grenades during the late War. Some who have examined the ruins think that tectonic disturbances were the initial causes of failure, as they were numerous cracks in the foundation which were not there prior to the construction.

As the interest in the arch is becoming so great many theoretical formulae have been derived for the solution of this problem. The Engineering Foundation has recently appointed a committee to investigate the deflections and stresses in the arch and arch dams. Some writers even advocate the construction of moderate sized models and then a test to destruction. The problem is a hard one to solve, because it is dependent upon so many conditions which are never the same for two cases; such as the material, workmanship and climatic conditions which together produce a concrete, the strength of which varies to an unknown extent. In this work, at times, approximate formulae are used; but when used, they are reasonably accurate and the reason for their use is explained at their introduction.

General Considerations Entering Into The Design of Multiple-Arch Dams:

As in all dams, the foundation must be impervious. In some cases, grouting or deep cut-off trenches must be resorted to. The unit pressure on the footings must also be a safe value so that no uneven settlement occurs. The foundation must also be well keyed to prevent sliding. The spacing between buttresses is theoretically more economical when made small. However, this is left to the judgment of the designer.

The arch should slope downstream as the additional weight caused by the vertical component of the water pressure increases the resisting moment. Some arches have been built with an angle of inclination of 45° with the vertical.

The weight of the arch is assumed to act in two directions, one component acting parallel to the axis of the arch and the other normal to the axis. The division of the water pressure between the cantilever and arch action is obtained by applying the elastic theory.

The stresses caused by these forces should be calculated at intervals sufficiently close together in order that the analysis is reasonably accurate. The stress due to temperature should also be calculated and steel provided, if necessary. It is disregarded in this work as the chief object is to obtain some workable formulae for the analyses of stresses on the arch due to water pressure,

-THE DIVISION OF WATER PRESSURE-

As it has been previously stated, the pressure due to the water is to be assumed as being carried by both arch action and cantilever action. Little is known of the accurate distribution of this load. There have been numerous lengthy formulae derived for this division, all differing, but each sufficiently accurate for designing purposes. The method used in this analysis is similar to that used in the design of the Shoshone Dam in Wyoming, constructed under the direction of the United States Reclamation Service. The formulae obtained is somewhat simpler and reasonably accurate. The distribution is determined from the deflection of the dam caused by the water pressure.

Nomenclature:

L = distance measured on the upstream face of the dam to the point where the plane of the two sides meet at "O" (See Fig. 1, Plate I).

L_1 = vertical distance from "O" to base.

h = slant height of dam.

h_1 = vertical height of dam.

b_1 = thickness of section at any point measured perpendicular to upstream face.

b = thickness of section at base measured perpendicular to upstream face.

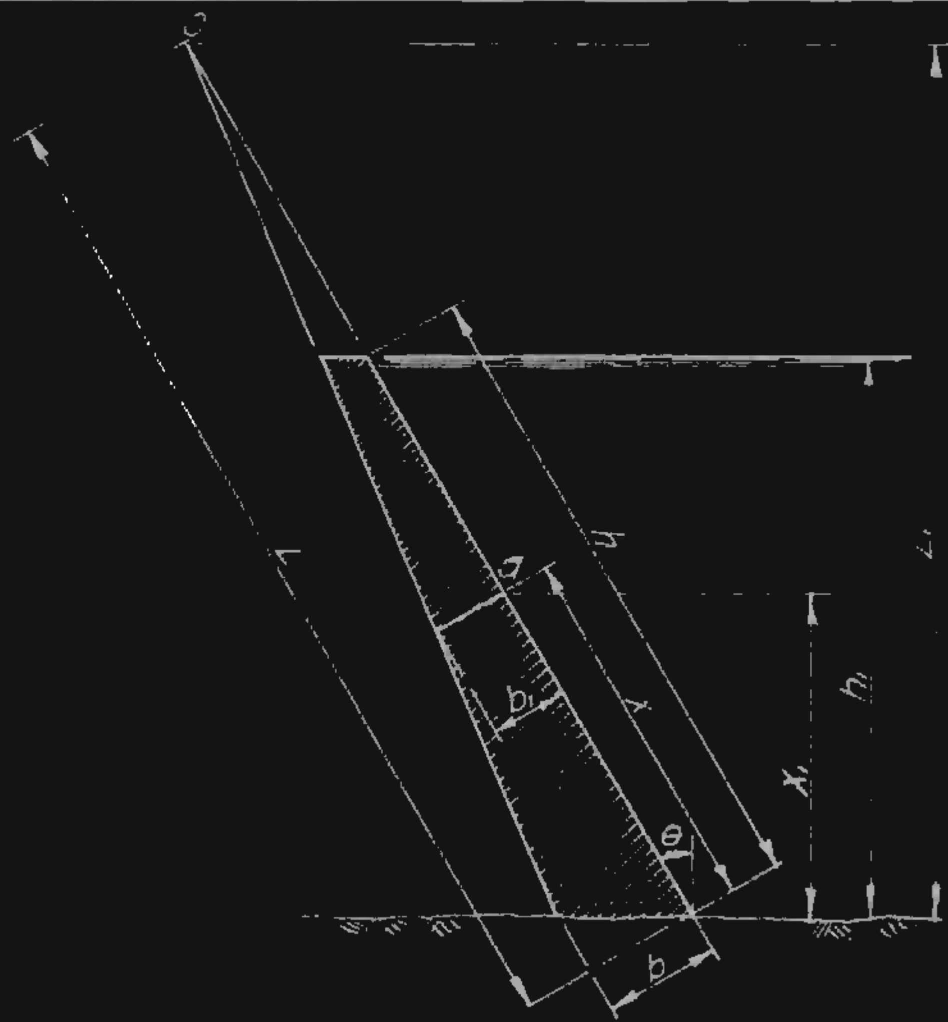


Fig. I.

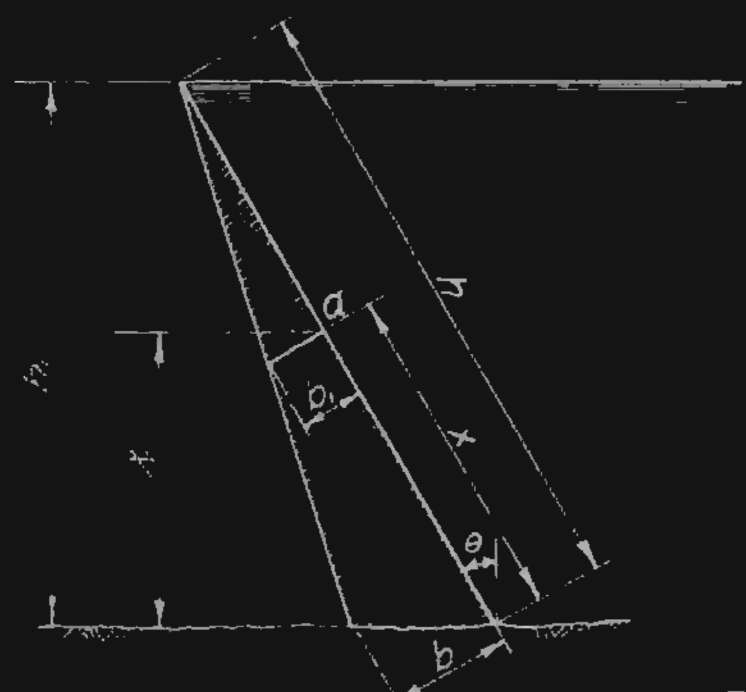


Fig. 2.

Plate I.

X = slant distance from base to any point.

X_1 = vertical distance from base to any point.

θ = angle the upstream face makes with the vertical.

ϕ = interior angle formed by the two extreme radii.

p = unit hydrostatic pressure (62.5 pounds per cubic foot).

ka = that part of p which is carried by the arch action.

kg = that part of p which is carried by the cantilever action.

P = total water pressure on section.

y = deflection at any point due to cantilever deflection.

e = shortening of one-half the arch when loaded.

T = axial thrust in arch.

d = deflection of arch, due to arch shortening, ($d = y$ for any given point)

R = radius of arch.

This nomenclature will be used throughout this work.

Cantilever Deflection:

Fig. 1, Plate I, represents the vertical section of the barrel of an inclined arch dam. Assume a vertical beam of unit width taken anywhere in the arch. This beam is assumed to be rigidly fixed at the foundation.

The well-known expression for the deflection of beams under loads will be used.

$$\frac{d^2 y}{dx^2} = \frac{M}{EI}$$

Now where the proper values for M, E and I, for any point "a", are substituted the expression becomes:

$$\frac{d^2y}{dx^2} = \frac{Kg\ L\ (L-H-x)^3 \cos\theta}{E\ b\ (L-x)^3}$$

When the integration of this expression is performed, a formulae is obtained which is entirely too long and complicated for practical purposes.

For this reason, the assumption is made that the crest of the dam is zero units thick. (see Fig. 2, Plate I). This assumption, that the thickness varies uniformly from the maximum at the base to zero at the crest, is safe because the additional material used in the construction of the dam will cause the unit stresses to be smaller than those figured.

Under this condition, $L = h$. In the investigation of any point "a"

$$P = \frac{kg\ (h_1 = x_1)\ (h-x)}{2}$$

Or, $$P = \frac{kg\ (h-x)^2 \cos\theta}{2}$$

The resulting moment M equals

$$M = \frac{kg\ (h-x)^3 \cos\theta}{6}$$

The moment of inertia of the section of the beam at "a"

$$I = \frac{\left(\frac{b}{h} \quad (h-x) \right)^3}{12}$$

Therefore, $$\frac{d^2 y}{dx^2} = \frac{2 \text{ kg } h^3 \cos \theta}{E b^3}$$

And, $$y = \frac{kg\ h^3 \cos \theta\ X^2}{E b^3} + C. \text{ (where } C = 0)$$

Or, $$y = \frac{(62.5 = Ka)\ h^2 \cos \theta\ X^2}{E b^3} \text{ - - - - - - - - -A.}$$

Arch Deflection:

In this phase of the work the triangular section of Fig. 1, Plate I, will be used.

From the theory of compression of elastic bodies,

$$e = \frac{T}{b_1 E} 1$$

Where 1 equals half the length of the arch lamina

Since $T = Ka\ (h-x) \cos \theta\ R$

and $b_1 = \frac{h}{H}\ (h-x)$

and $$e = \frac{Ka\ (h-x) \cos \theta\ R1}{\frac{b}{h}(h-x)\ E}$$

or $$e = \frac{Ka\ h \cos \theta\ R\ 1}{E b}$$

Vischer and Waggoner used a short formula, in their article "Strains In Curved Dams" published in the "Transactions of the Technical Society of the Pacific Coast" (Vol.VI Dec. 1889) which enabled them to obtain the deflection of the arch was known.

This expression is:

$$z = (1/2\ B\ B_1)\ 2V$$

which, when expressed in the nomenclature of this work,

$$e = \frac{d\ \phi}{4} \quad \text{or} \quad d = \frac{4\ e}{\phi}$$

but,

$$\frac{\phi}{4} = \frac{1}{2\ R}$$

Therefore,

$$d = \frac{ka h \cos\theta\ Rl}{E\ b} \cdot \frac{2\ R}{l}$$

$$d = \frac{2\ Ka\ h\ \cos\theta R^2}{E\ b} \text{ - - - - - - - - - - - - B.}$$

Now since y = d, equations A and B may be combined.

$$\frac{(62.5 - Ka)\ h^3\ X^2\ \cos\theta}{E\ b^3} = \frac{Ka\ h\ R^2\ \cos\theta}{E\ b}$$

from which

$$Ka = \frac{62.5\ h^2\ X^2}{h^2\ X^2 + 2\ R^2\ b^2} \text{ - - - - - - - - - - - - - - C.}$$

and,

$$Kg = 62.5 - Ka \text{ - - - - - - - - - - - - - - - - - - D.}$$

These expressions are correct for a triangular shaped section and sufficiently accurate for this work.

DETERMINATION OF THE SECTION OF THE ARCH

In this work, a section will be adopted, the dimensions of which are to be determined by standard formulae and the stresses in this section, then analyzed.

Conditions:

The section chosen will fit these conditions:

Vertical height = 80'

Angle of Inclination = 30^{o}

Radius = 40'

Total stress in masonry 16.5 tons per sq. ft.

The interior angle formed by the two outer radii will be 180^{o}.

Crest Width:

$Wc = 1/2 \sqrt{h}$ (Eq. 23, p 104; Dams & Weirs, Bligh)

$Wc = 1/2 \sqrt{80} = 4.5'$

Thickness of Base:

The thickness is found from the formula,

$$b = \frac{P\ R}{S}$$

S = unit stress due to water pressure and equal to 15 tons per sq. ft.

Then $b = \frac{80 \times 40}{31.2 \times 15} = 6.85$ use 6' - 10"

<u>Stress Due to Weight of Arch</u>:

$S_1 = R\ W\ \sin\theta$ Eq.25, page 150 Dams & Weirs, Bligh.

W = Weight per cu. ft. of masonry = 145 lbs.

$$S_1 = \frac{(40 \times 145 \times .5)}{2000} = 1.45 \text{ tons}$$

Total stress is then 15 + 1.45 = 16.45 tons per sq.ft.

(see plate IV for details)

-ANALYSIS OF STRESSES IN ARCH-

In this analysis, the arch will be divided into five laminae, each 18.476' deep. The pressure head carried by arch action and cantilever action will be found for each lamina at its center of hydrostatic pressure. This slant distance to this point, measured on the upstream face of the dam, is found by the expression:

$$h - x = 2/3 \; \frac{n_2^3 - n_1^3}{n_2^2 - n_1^2}$$

Where $h - x$ = distance from the top of the arch to the center of pressure of the section,

n_1 = distance from top of arch to top of section.

n_2 = distance from top of arch to base of section.

The values for $h - x$ and x are worked out in Table I.

-TABLE I-

Calculations for (h - x) and (x)

Lamina No.	n_1	n_2	$n_2^3 - n_1^3$	$n_2^2 - n_1^2$	$\frac{n_2^3 - n_1^3}{n_2^2 - n_1^2}$	h - x	x
1	o	18.476	6,331.63	342.25	18.43	12.32	80.06
2	18.476	36.952	46,321.37	1,026.75	45.11	30.08	62.30
3	36.952	55.428	119,378.46	1,700.16	70.22	46.81	45.57
4	55.428	73.904	233,551.96	2,392.05	97.62	65.08	27.30
5	73.904	92.38	385,305.603	3,076.55	125.24	83.49	8.89

Calculations for Ka and Kg

By substituting the known constants for this section in equation "C", we have,

$$Ka = \frac{62.5 \; X^2}{X^2 + 18}$$

-TABLE II-

Showing Calculations for Ka and Kg:

Lamina No.	X	X^2	$62.5\ X^2$	$X^2 + 18$	Ka	Kg
1	80.06	6416	401,000	6434	62.3	0.2
2	62.30	3881	242,581	3899	62.2	0.3
3	45.57	2079	129,960	2098	61.7	0.8
4	27.30	745	46,581=	763	61.0	1.5
5	8.89	81	5,051	99	51.0	10.5
base	0	0	0	0	0	62.5

Calculations for pressure

The total pressure on the section:

$$P - W A H$$

Where H = head on center of pressure = $(h-x) \cos \theta$ ($\theta = 30^\circ$)

W = Ka for arch - kg for cantilever

and $A = 18.476 \times 1 = 18.476$ sq. ft.

-TABLE III-

Showing calculations for pressures.

Lamina No.	(h-x)	(h-x)cos 30°	W(h-x)COS θ Arch	Cantilever	P= W A H Arch	Cantilever
1	12.32	10.67	665	2.	12,287	37
2	30.08	26.05	1620	8.	26,931	148
3	46.81	40.54	2501	32.	46,208	591
4	65.08	56.36	3438	85.	63,520	1520
5	83.49	72.30	3687	759	68,121	14,023

Calculations for Cantilever Moments

The equation for the cantilever moment at the base of the several laminae is $M = P_1 c_1 + + P_n c_n$

P = the cantilever pressures from Table III

c = the distance from the center of pressure of the corresponding laminae to the base of the lamina under investigation.

To this moment must be added algebraically the moment due to the component of the weight of the overlying masonry. This component is found by multiplying the weight of this masonry by sin 30°. The weight of the masonry carried by the arch is then this weight multiplied by the cos. 30°.

-TABLE IV-

Lamina No.	P	Weight Mas. x Sin 30°	Moment due to Water	Moment due to Mas.	Resulting Moment
1	37	11,370	+ 228	-1,365	- 1,137
2	148	23,800	+1,928	-5,474	- 3,547
3	591	37,550	+10,440	-12,760	- 2,320
4	1,520	52,100	+38,041	-21,980	+16,061
5	14,023	68,450	+206,639	-35,400	+171,239

The next step is to find the stress in the assumed cantilever.

$$S = \frac{M\ C}{I}$$

M = sum of the moments at that point

C = distance from center of gravity of the section to the edge of section.

I = moment of inertia of the section.

Therefore,

$$S = \frac{M.[.025\ (h-x) + 4.5]}{2}\ \frac{12}{(.025\ (h-x) + 4.5)^3}$$

Or,

$$S = \frac{6\ M}{(6.83 = .025\ x)^2}$$

-TABLE V-

Calculations for Cantilever Stress

Lamina No.	M	$(6.83-.025x)^2$	$\frac{6}{(6.83-.025x)^2}$	S_u	S_d
1	- 1,137	24.8	.24	+ 273	+ 273
2	- 3,547	29.6	.20	+ 709	- 709
3	- 2,320	34.9	.17	+ 394	- 394
4	+ 16,061	40.6	.15	- 2,410	+ 2,410
5	+171,239	46.7	.13	-22,250	+ 22,250

The center of gravity of the section is taken as one-half the thickness of the dam, which causes the stress in the upstream face (S_u) to equal that in the downstream face (S_d) but, as the signs show, the stesses are of opposite character . (minus denotes tension and plus denotes compression.)

To these stresses must be added the stesss due to the average weight of the component of the masonry weight, parallel to the face of the dam.

-TABLE VI-

Calculations for complete Stress

Lamina No.	Av. Mas. Wt.	S_d	S_u	Complete S_d	Complete S_u
1	+ 2,283	- 273	+ 273	+ 2,010	+ 2,556
2	+ 4,380	- 709	+ 709	+ 3,671	+ 5,089
3	+ 6,350	= 349	+ 349	+ 5,956	+ 6,744
4	+ 8,260	+ 2,410	- 2,410	+10,670	+ 5,850
5	+ 9,960	+ 22,250	- 22,250	+32,210	+12,290

Arch Stresses

If an arch lamina of unit width be considered, rigidly fixed at the buttresses and with no friction between the upper and lower parts of the dam, it will be found that there are stresses in this lamina due to a bending moment caused by the deformation when pressure is applied to the extrados. As the arch is not rigidly fixed in the dam considered in this work, this stress can be neglected. However, it will be investigated, just to show the effect of a fixed support. The derivation of the expression for this stress has been very elaborately derived in "Masonry Dam Design" by Morrison and Brodie. It will be used herewithout derivation as it is somewhat lengthy and the writer has previously worked through it.

The stress which will be used in this analysis is that stress due to axial thrust and is a compressive stress only. These stresses will be worked out for each lamina at their centers of pressure.

Morrison and Brodie's formula:

$$Mc = \frac{g_n\, l\, n^2}{12}\left\{\frac{\phi n - \sin\phi n}{\phi n}\right\}\frac{2\ \sin\phi n}{3\phi n + \sin\phi n\ \cos\phi n - 4\ \sin\phi n}$$

substituting the symbold used in this analysis, but retaining g_n for the time being, we have:

$$Mc = \frac{g_n\ 6_1{}^2}{12}\left\{\frac{\phi_1 - \sin\phi_1}{\phi_1}\right\}\frac{2\ \sin\phi_1}{3\phi_1 + \sin\phi_1\ \cos\phi_1 - 4\ \sin\phi_1}$$

ϕ_1 = one half the central angle subtended by the arch.

Now let:

$$\frac{\phi_1 - \sin\phi_1}{\phi_1}\quad\frac{2\ \sin\phi_1}{3\phi_1 + \sin\phi_1\ \cos\phi_1 - 4\ \sin\phi_1} = Z$$

Then $Z = \dfrac{1.57-1}{1.57} \quad \dfrac{2}{3x1.57-4 \text{ x } 1.57} = -.464$

Since $b_1 = 6.83 - .025X$

$$Mc = \frac{qn\ (6.83 - .025X)^2\ (-.464)}{12}$$

The stress due to this moment is equal to

$$Sa = \frac{M\ C}{I}$$

$$I = \frac{(6.83 - .025)^3}{12}$$

and $$C = \frac{6.83 - .025}{2.}$$

therefore,

$$Sa = -.232\ qn.$$

Since $qn = \dfrac{R}{R - \frac{b1}{2}} \left\{ Ka\ (h-x)\ \cos\theta \right\}$

$$Sa = \frac{R}{R - \frac{b1}{2}} \left\{ Ka\ (h-x)\ \cos\theta \right\} (-.232).$$

This stress S_a will be worked out to see how it will effect the main axial stress given by

$$S_t = \frac{P\ R}{b_1}$$

Since $P = Ka\ (h-x)\ \text{Cos}\ \Theta$

$$St = \frac{R}{b^1} \left\{ Ka\ (h-x)\ \text{Cos}\ \Theta \right\}$$

The values for P will be found in Table III, page 17.

TABLE VII
Calculations for Sa and St.

Lam. No.	$R-\frac{b^1}{2}$	$\frac{R}{R-b^1/2}$	$\frac{R}{b^1}$	$(Ka(h-x)\cos\theta)$	$(Ka(h-x)\cos\theta)(-0.234)$	Sa	St	TOTAL
1-	37.58	1.06	8.3	665	-153	-162	+ 5,520	+ 5,368
2-	37.22	1.07	7.6	1,620	-373	-399	+12,212	+11,813
3-	37.15	1.08	7.0	2,501	-575	-621	+17,507	+16,886
4-	36.92	1.08	6.5	3,438	-791	-854	+20,347	+19,493
5-	36.69	1.09	6.1	3,687	-848	-924	+22,490	+21,566

It will be noticed that the stresses Sa, due to the bending moment, are very small when compared with the stresses S_t, caused by the axial thrust.

Now, to the stresses S_t must be added the stress due to that component of the masonry weight which acts normal to the arch.

TABLE VIII
Calculations for St due to masonry:

Lam.No.	$\frac{R}{b^1}$	W	St
1	8.3	355	+ 2,946
2	7.6	746	+ 5,670
3	7.0	1,170	+ 8,190
4	6.5	1,625	+10,583
5	6.1	2,120	+12,932

TABLE IX

-Results-

Lam. No.	ARCH STRESSES Water	Masonry	Total	CANTILEVER STRESSES Upstream	Downstream
1-	+ 5,520	+ 2,946	+ 8,466	+ 2,556	+ 2,010
2-	+12,212	+ 5,670	+17,882	+ 5,089	+ 3,671
3-	+17,507	+ 8,190	+25,697	+ 6,744	+ 5,956
4-	+20,347	+10,583	+30,930	+ 5,850	+10,670
5-	+22,490	+12,932	+35,422	+12,290	+32,210

The resulting stresses in this section, due to arch and cantilever action, are shown in Table IX. The stresses obtained are safe stresses; however, the compressive stress in lamina number 5 is slightly greater than 16.5 tons, as was assumed. However, as the stress is less than 300 lbs. per sq. in., it is safe. A comparison of the weight of the water carried by arch and cantilever action is shown on Plate IV. It will be noticed that that part carried by cantilever is very small the upper two-thirds of the arch but increases rapidly until at the base it carries the entire stress.

-DESIGN OF BUTTRESSES-

The buttresses of a multiple arch dam, naturally, are wider at the base than a gravity section of the same height. This is because they must carry all the horizontal water pressure and that component of the arch weight that acts thru the arch itself. The method used in designing a gravity section is applicable to the design of the buttresses of the multiple arch type of dam. The buttresses are so designed that the resultant falls in the middle third. The pressure on the footing varies then, from a maximum at the toe to zero at the heel. The upward pressure is not considered in the design of the buttresses in this work, although in practice this factor would enter into the design, varying in importance with the character of the foundation. The nature of the foundation is also considered to be able to carry the weight of the dam in this case and no calculations made as to the possibility of this dam failing by crushing the foundation. These factors are omitted because this is a general design and the two factors, upward water pressure and the bearing capacity of the foundation are individual problems that must be worked out for different structures.

The overtunning force acting on the buttresses is the stress per square foot in the arch. This will be divided into two components, a vertical and horizontal component. The buttresses will be designed in horizontal section, 16 ft. deep. Some engineers design the buttresses in sections normal to the axis of the arch, but there is little difference between the results of the two methods.

From Plate II, it will be seen that in order to have the resultant pass thru the third point,

$$W_1 a + Fb + Wm = \frac{Hh}{3}$$

Where W = the weight of the masonry of that section,

W_1 = weight of overlying masonry,

F = vertical component of the sum of the external forces above the base of the section.

H = horizontal component of the sum of the external forces above the base of the section.

h = the vertical distance from the base of the section to the crest of the dam.

n = distance from the downstream toe to the center of gravity of the overlying masonry of the buttresses.

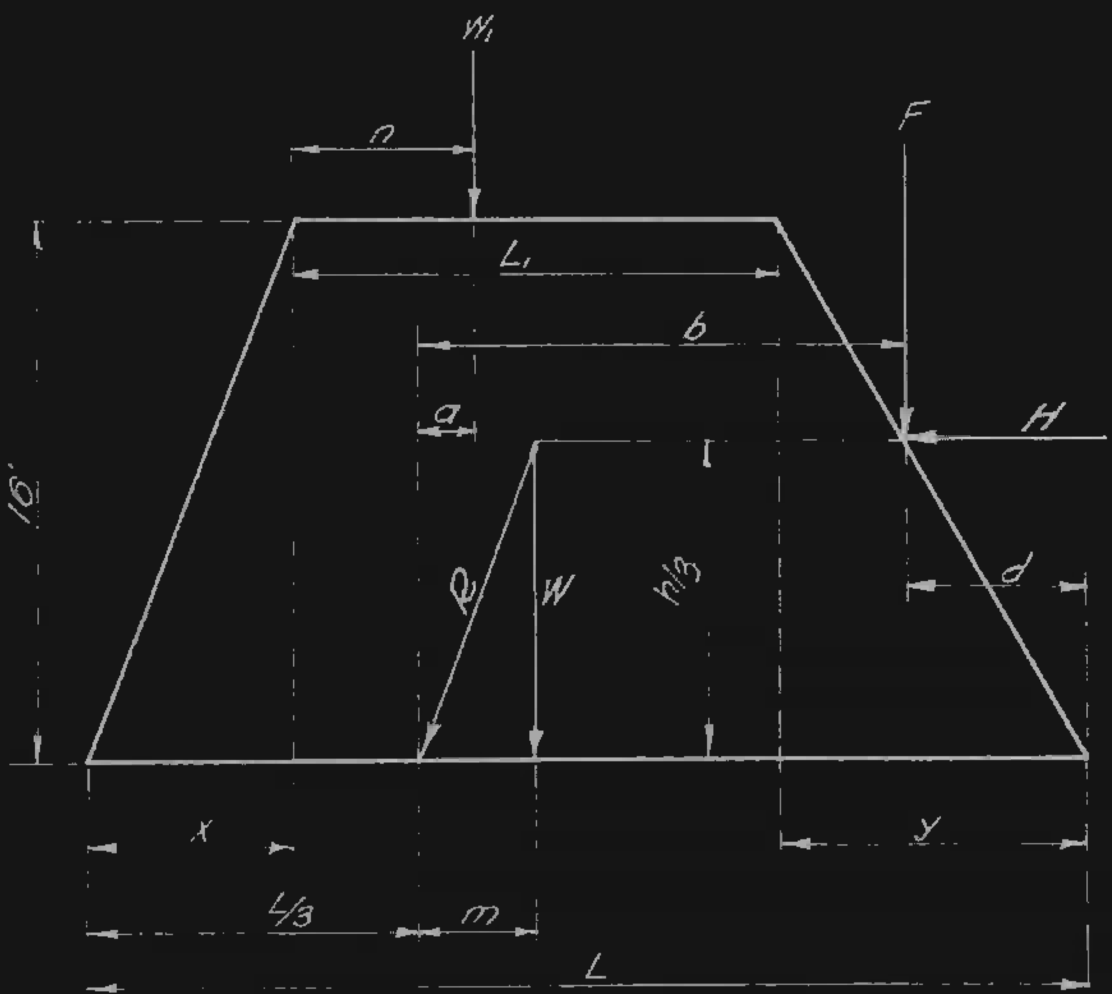


Plate II

Now $W = (8X + 16\ L_1 + 8\ y)\ 145$

$$M = \frac{16X^2 + X(48L_1 + 24y) + 24\ L_1^2 + 24\ L_1 y + 8\ y^2}{3\ (8X + 16\ L_1 + 8\ y)}$$

$$Wm = 48.33 \left\{ 16X^2 + x\ (48L_1 + 24y) + 24\ L_1 + 24\ L_1 y + 8\ y^2 \right\}$$

$$Fb = \frac{F}{3}\ (\ 2X + 2L_1 + 2\ y - 3d)$$

In which $y = 16 \tan 30^\circ$

and $d - \frac{h}{3} \tan 30^\circ$

$$W_1 a = \frac{W_1}{3}\ (2X + 3n - L_1 - y)$$

Therefore,

$$\frac{W_1}{3}\ (2X+3n - L_1 - y) + \frac{F}{3}\ (2X + 2L_1 + 2\ y - 3\ d) + 48.33 \{ 16X^2 + X(48L_1 + 24y) + 24\ L_1 + 24\ L_1\ y + 8\ y^2 = \frac{H\ h}{3} \text{ ----E}$$

TABLE X

Calculations for H and F:

Lamina No.	Total Arch Force on Lamina	Horizontal force per Lamina	Vertical force per Lamina	H	F
1-	156,420	135,460	78,210	135,460	78,210
2-	330,390	286,120	165,195	421,580	243,405
3-	474,780	411.160	237,390	832,740	480,795
4-	571,460	494,880	285,730	1,327,620	766,525
5-	654,460	566,760	327,230	1,894,380	1,093,755

TABLE XI

Showing Constants for Formula E

Sec. No.	W_1	n	L_1	$\frac{W_1}{3}(3n - L_1 - y)$	$\frac{F}{3}$	d	$\frac{F}{3}(2L_1+2y+3d)$
1-	0	0	10	0	26,070	3.08	762,030
2-	33,872	7.6	19.23	-53,070	81,135	6.15	3,120,375
3-	118,220	10.4	33.5	-453,180	160,265	9.23	9,459,860
4-	206,640	21.0	50.73	+206, 640	255,508	12.31	21,258,400
5-	360,830	30.0	67.46	+1,587,700	364,585	15.39	38,788,100

TABLE XI (CONT'D)

Sec. No.	$48.33\ (48L_1 + 24y)$	$48.33(24L_1 + 24L_1y + y^2)$	$\frac{H\,h}{3}$
1-	10,706	122,758	722,030
2-	55,310	232,320	4,498,310
3-	88,440	401,380	13,324,000
4-	128,410	606,100	28,318,350
5-	167,220	804,646	50,523,110

Equation E, becomes for:

Sec. I- $W_1 = 0$

$$52,140X + 762,030 + 773\ X^2 + 10,706\ X + 122,758 = 722,030.$$

$$773.\ X^2 + 62,846x + 162,758 = 0$$

X= (-2') Use Vertical.

Sec. II-

$$22,582X - 53,070 + 162,266x + 3,120,375 + 773X^2 + 55,310x + 232,320 = 4,498,310.$$

$$773\ X^2 + 240,158\ X - 1,198,700 = 0$$

$$X = 5.0'$$

Sec. III-

$$78,814x - 453,180 + 321,764X + 9,459,860 + 773X^2 + 88,440X + 401.380 = 13,324,000.$$

$$773X^2 + 489,018X - 3,916,800 = 0.$$

$$X = 8.0'$$

Sec. IV-

$$137,760x + 206,640 + 512,250\ X + 21,258,400 + 773X^2 + 128,410x + 606,100 = 28,318,350.$$

$$773X^2 + 778,420X - 6,247,200 = 0.$$

$$X = 7.5'$$

Sec. V-

$$240,560\ X + 1,587,700 + 730,404X + 38,788,100 + 773X^2 + 167,220X + 804,646 = 50,523,110.$$

$$773X^2 + 1,138,184X - 9,342,600 = 0.$$

$$X = 9.5'$$

Maximum Compressive Stress in Footing

Amt. Masonry in Buttresses, - 40,147 Cu. ft.

Total Weight of Buttresses, -5,821,750#

Weight per ft. = 5,821,750 ÷ 13.67 = 426,000#

Total External Pressure, = 1,093,755#

Total Pressure, 1,519,855#

Since the pressure is assumed to be the maximum at the toe and decreasing uniformly to zero at the heel, when the resultant falls at the third point.

$$P_1 = \frac{2P}{L}$$

Where P_1 = stress at downstream toe,

P = total downward pressure

L = length of buttress.

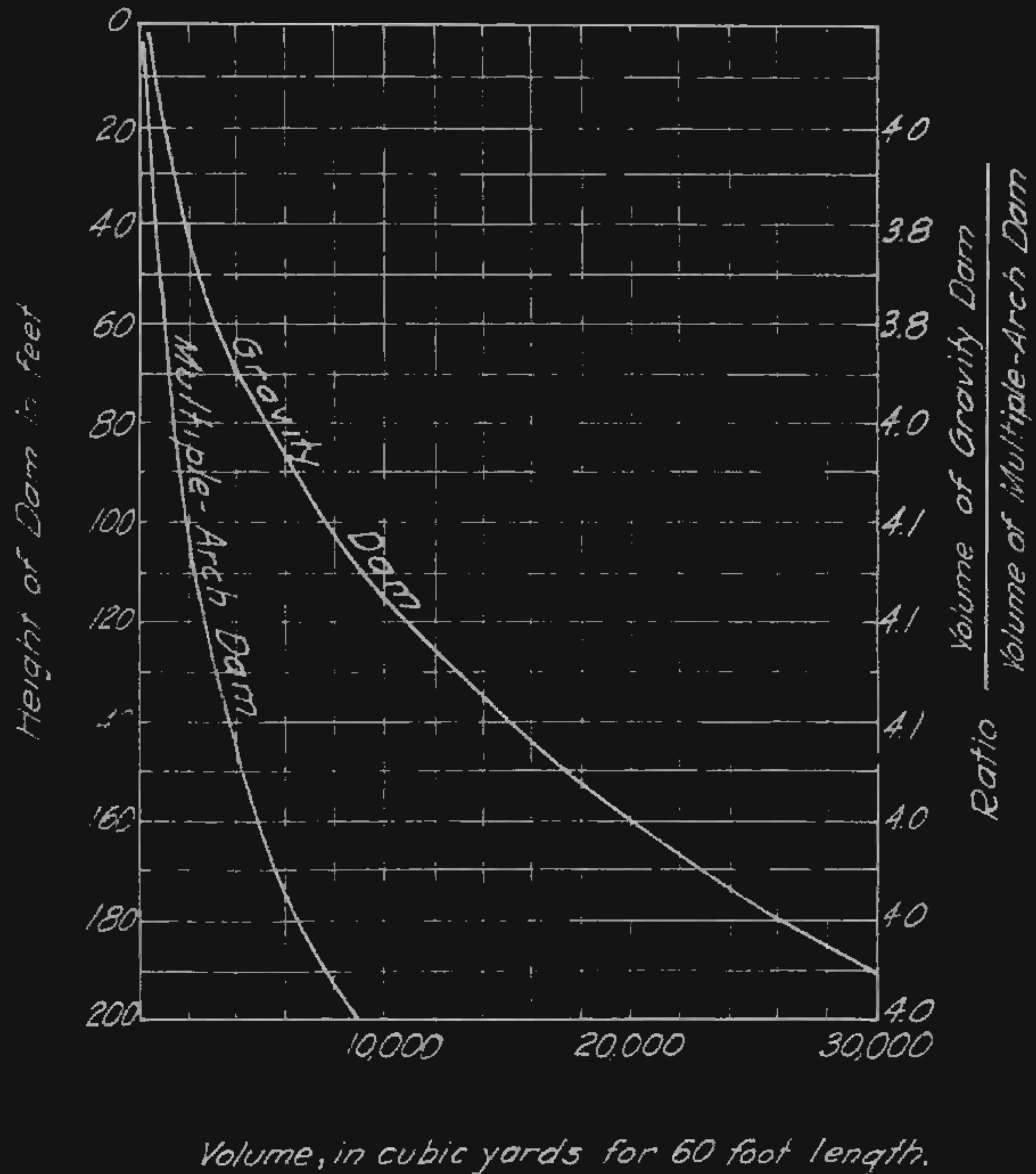
Then $P_1 = \dfrac{2 \times 1{,}519{,}855}{86.24}$ = 35,250 lbs. per sq. ft.

which is a very safe stress.

THE ECONOMIC COMPARIONS BETWEEN THE GRAVITY DAM AND THE MULTIPLE-ARCH DAM

It has been previously stated that the distance between the buttresses, controls, in a great measure, the quantity of material required for the construction of the multiple-arch dam. The smaller the spacing, the smaller the amount of masonry. This holds true only for the higher structures, as in the lower dams, a certain amount of masonry is needed in the arch, even if the radius is shortened.

Plate III, shows the comparison of materials required for a 200 ft. dam between the gravity and multiple-arch type. This chart was worked up by Mr. Noetzli, in his report in the "Transactions of the American Society of Civil Engineers Vol. XLIX Aug. 1923." The gravity dam considered is of the Wegmann's type and the multiple arch is similar to the Horseshoe dam with a 30' radius. The chart shows that the multiple arch type required only 25% of the material needed for the gravity dam. As the material required for the multiple arch will cost about twice as much as the material in the gravity dam, because of the better quality of concrete in the arch and the additional cost of construction, a saving of only 50% is accomplished.



Height of Dam in feet
0
20
40
60
80
100
120
160
180
200
10,000
20,000
30,000
Volume, in cubic yards for 60 foot length.
Gravity Dam
Multiple-Arch Dam
4.0
3.8
3.8
4.0
4.1
4.1
4.1
4.0
4.0
4.0
Ratio Volume of Gravity Dam / Volume of Multiple-Arch Dam

Plate III

Evidently, the economical radius was not chosen for this work because such favorable results were not obtained as the f ollowing figures show.

Material required per ft. of Gravity Dam, 2,430 Cu. Ft.
Avg. " " " " "Multiple-Arch Dam, 1,260 " "

From these figures it can be seen that the material in the multiple arch type is 52% of that needed in the gravity section. However, as the unit cost of the material in the multiple arch type is assumed to be twice the unit cost for the gravity dam, this type dam will cost a little more than the gravity section. Perhaps a radius of 25 or 30 feet would give more favorable results, as the buttresses would decrease in size and the amount of material required for the arch would be lessened. Another factor which caused the larger amount of material in this dam was the low working stress in the masonry. The figure chosen was 16.5 tons per sq. ft. This is only 230 lbs. per sq. in. which is only one-third the usual working stress of concrete.

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SECTION THRU CENTER LINE OF ARCH
Scale: 1"=10'

PLAN OF DAM.
Scale: 1"=20'

DETAILS OF DAM.

Plate V.