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The design of a reinforced concrete arch highway bridge

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THE DESIGN OF A REINFORCED CONCRETE ARCH
HIGHWAY BRIDGE.

Clinton Dewitt Smith.
Candidate for degree of Bachelor of Science
in
Civil Engineering.

Approved Eduard Harris April ____ 1915.

18372

The Subject of this theses is the Design of a Reinforced Concrete Arch Highway Bridge .

In making the Design the crossing of South River Street, Austin Minn., was taken as the hypothetical location of the bridge and the design made with reference to the requirements of this location .

These requirements are no more than those for an ordinary rural highway bridge. While in the city of Austin it is not in the route of travel to and from the depots and is hence not subjected to the loads due to heavy freighting. There is however a heavy rural traffic, this road being one of the main arteries leading into the city.

The present bridge on this site is a Pratt truss of one hundred feet clear span. With the exception of one year this span has always provided an ample waterway. As the floods this year were the highest on record and resultant from a cloudburst in the near vicinity it was assumed for this design that this area for waterway is sufficient. In order to make up for the reduced head room resultant from the arch construction however the span decided upon was approximately one hundred and twenty one ft in the clear.

The rise adopted is seventeen and a half feet. This is a ratio of rise to span of about one to seventeen.

The foundation available is favorable for arch construction and consists of a limestone ledge. The thickness of this ledge was not determined, it not being feasible nor necessary for the present purpose to make borings. Assuming it to be nothing but a thin ledge the allowable pressure according to the Minnesota State Highway Commissions specifications, is twenty five tons per square foot.

The method followed in making the design was that of Howe, as advanced in his text, Symmetrical Masonry Arches. Unit loads were first considered, and with this data the loading so arranged as to give the maximum stresses at the sections considered. These were three in number- at the spring line, at a point between one fourth and one third the span, and at the crown. These three sections according to Howe are those highest stressed.

SPECIFICATIONS.

Loadings-

Definition- The dead load includes the weight of the structure , floor and filling, using the following unit weights:-

Earth filling - - - 100 lb. per cu. ft.

Concrete - - - - - 140 Lb. per cu. ft.

The live load is to be one hundred pounds per square foot surface of roadway and sidewalk, and so applied to give the maximum resultant stresses, or a fifteen ton traction engine with axels at ten ft. centers and six ft. gage, two thirds of the load being carried on the rear wheels. The load giving the maximum stresses to be used.

Stresses,-

All parts of the structure shall be proportioned so that the sum of the maximum stresses shall not exceed the following amounts in pounds per square inch. Stresses induced by a temperature range of 80 degrees shall be provided for.

Steel in shear - - 10,000 lb. per sq. in.

Concrete in shear- - 50 lb per square in.

Steel in tension - - 16,000 lb. per sq.in.

Steel in tension, temp. included, 20.000 lb.

Concrete in tension - - - 0

Maximum allowable stress in extreme fibres
of the section, the steel will carry all
tension- - - - - 750 lb. per sq. in.

Concrete in compression- - 600 lb. per " ".
With temperature included- 750 lb. " " .

Steel in compression, 15 times surrounding
concrete.

Classes of concrete-

The following classes of concrete shall be used

Arch ring,- I - 2 - 4 - concrete.

Spandrel wall - - I - 2 $\frac{1}{2}$ - 5 concrete.

Abutments and piers to spring line I - 3 - 6.

Hand rails , including all concrete placed above
spandrels - I - 2 - 3.

Expansion joints- shall be left in spandrel walls,
handrails ect. and for over thirty foot span must
be five or more in number.

Drainage - Shall be provided for the back of the
arch at each abutment by pipe drains.

Spandrel walls- preferably of the gravity type and
free to move on the arch ring.

ESTIMATE.

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CONCRETE-

Arch Ring - 63 $\frac{1}{2}$ cu yd. I - 2 - 4 concrete.Spandrel walls 115 Cu. yd. I - 2 $\frac{1}{2}$ - 5 Concret.

Abutments,- 739 cu.yd I - 3 - 6 concrete.

Total number yd. concrete 1489.

STEEL -

1554 ft. $\frac{3}{4}$ " sq. rods 2973. lb.14002 ft. $\frac{1}{2}$ " sq. rods 11901. lb.15340 ft. $1\frac{1}{4}$ by 1 $1/16$ " 69330. lb.448 ft. $\frac{1}{2}$ by $2\frac{1}{4}$ " 1716. lb.

Total no. lb. steel 85920.

Fill -

834 cu yd. fill at \$1.25 -- \$1040.

Sidewalk-

177 ft at 40 cents per ft. \$ 71

Pavement,-

183 sq. yds. at \$2.25 415

Estimated cost bridge at 13.00 \$ 19300

Total \$ 20826.

TABLE IX

STRESS IN EXTREME FIBRES IN lb. per sqft.

	1	2	3	4	5	6	7
Loading	CROWN			SECTION SEVEN		SECTION ZERO	
	Top	Bottom		Top	Bottom	Top	Bottom
Dead load	-2318	+67484		+35089	+23350	+42742	+2466
Live load	-1974	+5990		+5273	-3254	-2371	+3536
" "	+3190	-732		-2470	+5183	+8110	-5983
Max Tension	-4292	0		0	0	0	-3517
Max Compress	+872	+73474		+40362	28533	50852	6002
Temp rise	40°F	40°F		40°F	40°F	40°F	40°F
" fall	40°F	40°F		40°F	40°F	40°F	40°F
Tension due to temp.	33100	46650	8350	20500	54200	49300	
comp. due to temp.	33100	46650	8350	20500	54200	49300	
Max. tens. Incl. temp.	37392	0	0	0	11458	52817	
Max. comp. Incl. temp.	33972	1,20,124	48712	49033	105,052	55302	

Extra longitudinal reinforcement for arch ring - flat bars $\frac{1}{2}$ " by $2\frac{1}{4}$ " spaced at 12" and extending for distance of 8' each side of crown - placed in bottom of arch ring.

Point	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	h	h^3	$h^3/12$	$h/2$	$h/2 - 17$.24 d ²	col. 3+7	dS/I_x				$y - \frac{\Sigma y \Delta}{\Sigma \Delta}$	$\Delta(y - \frac{\Sigma y \Delta}{\Sigma \Delta})$	y_B
			I_c	d	d^2		I_s	I_x	dS	Δ	y	y_D	$\frac{\Sigma y \Delta}{\Sigma \Delta}$	B	-
1	3.04	28.10	2.34	1.52	1.35	1.82	.44	2.78	4.80	1.80	1.96	3.53	-12.651	-22.772	-44.633
2	2.58	17.17	1.44	1.29	1.12	1.25	.30	1.74	7.76	4.46	6.07	27.07	-8.541	-38.093	-231.224
3	2.55	16.57	1.38	1.27	1.10	1.21	.29	1.67	6.95	4.16	9.00	37.49	-5.611	-23.342	-210.078
4	2.47	15.06	1.25	1.23	1.06	1.12	.27	1.52	6.75	4.44	11.4	50.62	-3.211	-14.257	-162.530
5	2.35	12.97	1.08	1.17	1.00	1.00	.24	1.32	6.68	5.06	13.4	67.80	-1.211	-6.128	-82.115
6	2.24	11.22	.93	1.12	.95	.90	.22	1.15	6.65	5.78	15.1	87.27	+4.89	+2.826	+42.672
7	2.20	10.64	.89	1.10	.93	.86	.21	1.10	6.54	5.94	16.45	97.71	+1.839	+10.923	+179.683
8	2.12	9.52	.79	1.06	.89	.79	.19	.98	6.52	6.65	17.5	116.37	+2.889	+19.212	+336.210
9	2.09	8.99	.75	1.04	.87	.76	.18	.93	6.45	6.94	18.25	126.65	+3.639	+25.255	+460.904
10	2.06	8.73	.73	1.03	.86	.74	.17	.90	6.45	7.16	18.78	134.46	+4.169	+29.850	+560.583
11.	2.00	8.00	.66	1.00	.83	.69	.16	.82	3.25	3.96	18.80	74.45	+4.189	+16.588	+311.854
Reinforcement assumed as .8% area crown section.								56.35		823.37			-104.592	-730.580	
								2		2			+104.654	+1891.906	
								112.70		1646.74			7.062	1161.326	
								$\Sigma \Delta$		$\Sigma y \Delta$			should be 0	2	
														$\Sigma y_B = 2322.652$	
															7000
															7

TABLE I - Data & Constants.

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TABLE II
HORIZONTAL THRESTS AND

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$$\Sigma Y_B = 2322.652$$

Point	1	2	3	4	5	6	7	8	9	10	11	12	13	14.
	$\frac{1}{n}$	$\frac{1}{n}$	$z-n$	z	$n^{(z-n)}$	$z(z-n)^{\Delta}$	$\frac{x-a}{x-z} z(z-n)^{\Delta}$	$\frac{x=a}{x=0} (z-n)_{2a}$	$(z-n)_{2a}$	$(z-n)^{\Delta}$	$\frac{x=a}{x=0} z(z-n)^{\Delta}$	$x=\frac{1}{2}a$	Column $B+12$	Col. 13 times $.0000328 = E$
1	39	1	-39	1	-39	-70.2	70.20	2737.80	1521	2737.80	21788.44	21788.44	24526.24	.8045
2	36	4	-36	4	-144	-692.24	712.44	25647.84	1296	5780.16	16008.28	64033.12	89680.96	2.9415
3	32	8	-32	8	-256	-1064.96	1777.40	56876.80	1024	4259.84	11748.44	93987.52	93987.52	3.0828
4	28	12	-28	12	-336	-1491.84	3269.24	91538.72	784	3480.96	8267.48	99209.76	190748.48	6.2565
5	24	16	-24	16	-384	-1943.04	5212.28	125098.72	576	2914.56	5352.92	85646.72	210741.44	6.9123
6	20	20	-20	20	-400	-2312.00	7524.28	150485.60	400	2912.00	3049.92	60818.40	211304.00	6.9308
7	16	24	-16	24	-384	-2280.96	9805.84	156883.84	256	1520.64	1520.28	36486.72	193370.56	6.3426
8	12	28	-12	28	-336	2234.40	12039.64	194475.68	744	957.60	562.68	15755.04	160230.72	5.2556
9	8	32	-8	32	-256	1776.64	13816.28	110530.24	64	444.16	118.52	3792.64	114322.88	3.7498
10	4	36	-4	36	-144	1031.04	14847.32	59989.28	16	114.56	3.96	142.56	59531.84	1.9526
11	1	39	-1	39	-39	154.44	15001.76	15001.76	-1	3.96	0	0	15001.76	.4921

$$E = \left\{ (n-k) \epsilon \sum_{i=0}^{x=a} (z-n)^i z^{\Delta} - k \epsilon \sum_{i=0}^{x=\frac{b}{2}} (z-n)^i z^{\Delta} \right\} \frac{dx}{z^{\Delta}}$$

col 8 *col 12.*

TABLE IV

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VALUES OF V_1 , V_2 , M_1 , M_2 & H
PRODUCED BY UNIT LOADS.

$P^{(n)}$	1	2	3	4	5	6	7	8	9	10	11	12
	C	D	E	M_1	M_2	$M_2 - M_1$	$\frac{M_2 - M_1}{\text{Span}}$	P_1	$\frac{c_0 + 2 + 3}{V_1}$	V_2	point	point
1	0	.805	.805	-1.610	0	+1.610	.0125	.9876	1.000	0	1'	22
2	.6925	3.143	2.941	-5.391	+4.491	+5.882	.0457	.9503	.996	.004	2'	21
3	3.159	6.005	3.083	-5.929	+2.237	+6.166	.0479	.8998	.9477	.0523	3'	20
4	6.569	8.630	6.256	-8.317	+4.195	+12.512	.0971	.8501	.9472	.0528	4'	19
5	10.557	11.001	6.912	-7.356	+6.468	+13.824	.1073	.7997	.9070	.0930	5'	18
6	14.800	13.086	6.931	-5.217	+8.645	+13.862	.1076	.7500	.8576	.1424	6'	17
7	18.832	14.836	6.343	-2.347	+10.339	+12.686	.1240	.7000	.8240	.1760	7'	16
8	22.602	16.247	5.256	+1.099	+11.611	+10.512	.0816	.6500	.7316	.2684	8'	15
9	25.509	17.276	3.750	+4.483	+11.983	+6.500	.0505	.6000	.6505	.3495	9'	14
10	27.381	17.915	1.952	+7.514	+11.418	+3.904	.0308	.5500	.5808	.4192	10'	13
11	27.934	18.083	.492	+9.359	+10.343	.984	.0076	.5125	.5201	.4799	11'	12

$$C = H \frac{\Sigma Y_A}{\Sigma A} = 14.611H$$

$$\Sigma M_1 = -36.167 + 98.185 = +62.018 = M_2$$

$$M_2 = C - D - E$$

$$E V_1 + 11 = E V_2$$

TABLE IV

STRESS IN EXTREME FIBRES
OF SECTION AT THE CROWN
CONSIDERING UNIT LOAD AS OCCUPYING
SUCCESSIVELY EACH OF THE DEVISIONS.

LOAD	1	2	3	4	5	6	7	8	9	LOAD
	$\frac{M_x}{S} = \frac{M_x}{3.35M_1}$	$\frac{N_x}{F_x} = N_x / (H \cos \phi)$	$N_x = \frac{N_x}{F_x}$	$\frac{N_x}{F_x}$	STRESS. TOP	BOTTOM.				
1	0	-0.805	0.805	0	0	0	0	0	0	1'
2	.229	-3.143	3.220	-.152	.0509	.0474	.0114	-.0395	.70623	2'
3	1.046	-6.005	6.440	.611	.2046	.2162	.0520	-.1526	.72566	3'
4	2.176	-8.630	9.66	1.146	.3839	.4496	.1080	-.2759	.74919	4'
5	3.497	-11.001	12.88	1.618	.5420	.7226	.1737	-.3683	.77157	5'
6	4.902	-13.086	16.10	1.888	.6325	1.0129	.2439	-.3886	.78764	6'
7	6.237	-14.836	19.32	1.753	.5876	1.289	.3098	-.2778	.78974	7'
8	7.486	-16.247	22.54	1.193	.3996	1.547	.372	-.0276	.7716	8'
9	8.499	-17.216	25.76	.035	.0117	1.7459	.420	.7.4317	.7.4083	9'
10	9.068	-17.915	28.98	1.997	.6690	1.8743	.451	.71.120	-.1180	10'
11	9.252	-18.083	31.40	4.065	1.3618	1.912	.460	1.8218	-.9018	11'

$$I_x = \frac{3.5}{12}^3 + 15 \times 1.25 \times 3.5 (1.50)^2 = 5.212 \quad P = \frac{N_x}{F_x} + \frac{M_x}{S} (\text{Top})$$

$$\frac{1.75}{5.212} = \frac{1}{5} = .335 \quad F_x = 3.5 + .656 = 4.156 \quad P = \frac{N_x}{F_x} - \frac{M_x}{S} (\text{Bottom})$$

TABLE VI
STRESS IN EXTREME FIBRES SECT. #7.
PRODUCED BY UNIT LOADS MOVING ACCROSS SPAN.

No.	1	2	3	4	5	6	7	8	9	10
	$-1.839H - \frac{\epsilon m_x \Delta}{\epsilon A}$	$-4E$	M_x^+	M_x	M_x/S $\frac{27.83 M_x}{27.83 M_x}$	N_x	N_x/F_x		$\frac{N_x}{F_x}$	STRESS TOP
1	0	-0.805	.322	1.127	0	0	0	0	0	0
2	-0.087	3.143	1.177	4.508	.101	.0281	.045	.0091	.0372	-.019
3	-.398	6.005	1.233	9.016	1.380	.3840	.202	.0444	.4284	-.3396
4	-.827	8.630	2.503	13.524	1.564	.4353	.432	.0951	.5304	-.3402
5	-1.329	11.001	2.765	18.032	2.937	.8174	.692	.1523	.9697	-.6651
6	-1.859	13.086	2.772	22.540	4.823	1.3422	.968	.2129	1.5551	-.1293
7	-2.370	14.836	2.537	27.048	7.305	2.0330	1.233	.2715	2.3045	-.17615
8	-2.845	16.247	2.102	25.116	3.922	1.0915	1.656	.3645	1.456	-.7270
9	-3.211	17.276	1.500	23.184	1.192	.3317	1.837	.4040	.7357	-.0723
10	-3.496	17.915	.781	21.252	.890	.2477	1.949	.4285	.1808	+.6762
11	-3.516	18.083	.197	19.803	1.993	.5546	1.974	.4350	.1296	+.9896
12	-3.516	18.083	.197	18.837	2.565	.7130	1.967	.4385	.2813	+1.1463
13	-3.446	17.915	.781	17.388	3.192	.8883	1.918	.4215	.4868	+1.3096
14	-3.216	17.276	1.500	15.456	3.536	.9841	1.780	.3916	.5925	+1.3757
15	-2.845	16.247	2.102	13.523	3.466	.9646	1.569	.3451	.6195	+1.3097
16	-2.370	14.836	2.537	11.592	3.077	.8563	1.299	.286	.5703	+1.1423
17	-1.859	13.086	2.772	9.660	2.513	.6994	1.022	.225	.4744	+.9244
18	-1.329	11.001	2.765	7.728	1.837	.5112	.787	.1733	.3379	+.6845
19	-.827	8.630	2.503	5.796	1.158	.3223	.443	.0974	.2249	+.4197
20	-.398	6.005	1.233	3.864	1.906	.3634	.213	.0469	.3165	+.4103
21	-.087	3.143	1.177	1.932	.181	.0337	.047	.0103	.0234	+.0440
22	0	.805	.322	.883	0	0	0	0	0	0

$$M_x = H \left\{ \frac{\epsilon Y \Delta}{Z \Delta} - Y \right\} - \frac{\epsilon m_x \Delta}{Z \Delta} - \frac{n-z}{n} E + m_x . \quad M_x = \int_{2n}^{2n-K} z - (z-K) \frac{dx}{4}$$

$$M_x = (14.611 - 16.45)H - \frac{\epsilon m_x \Delta}{Z \Delta} - -4E + m_x . \quad M_x = \int_{20}^{24-K} 24 - (24-K) \frac{dx}{4}$$

$$h = 3.89 \quad I_x = 6.99 \quad 1/5 = 2.783 \quad F_x = 3.89 + 6.56 = 4.546$$

$$N_x = V \sin \phi + H \cos \phi - \frac{1}{8} \frac{I_x}{\sin \phi} \sin \phi = 2.576 / 37.0 = 0.07 \quad \cos \phi = .982$$

TABLE VII

STRESS IN extreme fibres produced by unit loads,
MOVING ACCROSS THE SPAN. — SECTION - O.

R	1	2	3	4	5	6	7	8
	$M_x = 1256$	$N_x \sin \phi$	142050					
	$\frac{M_x}{S} = .1256$	N_x						
	$.816 V_x$	$.578 H$						
1	-2.022	.816	0	.816	.121	-	.0812	.3232
2	-6.771	.819	.087	.840	.1245	-	.5526	.8016
3	-6.697	.773	.125	.898	.1333	.5364	.8030	
4	-1.0446	.772	.260	1.032	.1532	.8914	.1.1970	
5	-9.239	.740	.418	1.158	.1717	.7522	.1.0956	
6	-6.552	.700	.585	1.285	.1907	.4645	.8459	
7	-2.948	.672	.745	1.417	.2100	.0848	.5048	
8	+1.1980	.596	.894	1.490	.2210	.3590	.0830	
9	+5.631	.530	1.009	1.539	.2280	.7911	.3351	
10	+9.437	.474	1.083	1.557	.2310	.1.1747	.6927	
11	+1.1755	.425	1.105	1.530	.2270	.1.4025	.9485	
12	+1.2991	.392	1.105	1.497	.2220	.1.5211	.1.0771	
13	+1.4341	.342	1.083	1.425	.2116	.1.6457	.1.2225	
14	+5.050	.285	1.009	1.299	.1920	.1.6910	.1.3130	
15	+4.583	.219	.894	1.113	.1677	.1.6260	.1.2906	
16	+2.986	.144	.745	.889	.1318	.1.4304	.1.1618	
17	+0.858	.116	.585	.701	.1040	.1.1898	.9818	
18	+8.124	.076	.418	.484	.0718	.8842	.7406	
19	+5.269	.043	.260	.303	.0450	.5719	.4819	
20	+0.298	.043	.125	.168	.0250	.0548	.0048	
21	+0.616	.003	.027	.030	.004	.0656	.0576	
22	0	0	0	0	0	0	0	

$$\begin{aligned}
 \text{stress in top fibre} &= \frac{N_x}{I_x} + \frac{M_x}{S} \quad \phi = 54^\circ 41' \\
 \text{bottom } " &= \frac{N_x}{I_x/F_x} - \frac{M_x}{S} \\
 h &= 6.09 \quad I_x = 24.20 \quad C/I = \frac{I}{S} = \frac{1}{1256}
 \end{aligned}$$

$$F_x = 6.09 + 6.556 = 6.746$$

TABLE VIII

Stress in extreme fibres produced by unit horizontal loads acting toward center of span.

X in. R	1	2	3	4	5	6	7
	M_x	γ_s	M_x/γ_s	N_x	N_x/F_x	STRESS Top	Bottom
0	14.611	.1256	1.835	.578	.086	.921	-.749
7	1.839	.2783	.512	.982	.216	.296	.728
C	4.389	.335	1.414	1	.2405	-.1173	1.655

$$M_i = H \frac{\gamma Y \Delta}{\gamma \Delta}$$

$$M_x = (\frac{\gamma Y \Delta}{\gamma \Delta} - \gamma) H$$

$$\text{For } 1^\circ \text{ Fah. } H_f = \frac{.000006(126.80)(1,500,000)/44}{2322.65}$$

$$H_f = 705$$

Stress per unit area for temp. range of $\pm 40^\circ$ Fah.
 CROWN - $705 \times 1.173 \times 40 = \pm 33100$, upper fibres
 CROWN - $705 \times 1.655 \times 40 = \pm 46650$, lower fibres.

SEVEN - $705 \times .296 \times 40 = \pm 8350$, upper fibres
 SEVEN - $705 \times .728 \times 40 = \pm 20500$, lower "

ZERO - $705 \times 1.921 \times 40 = \pm 54200$, upper fib.
 ZERO - $705 \times 1.749 \times 40 = \pm 49300$, lower "

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