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An Experimental and Theoretical Study of the Viscous Sublayer for Turbulent Tube Flow

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AN EXPERIMENTAL AND THEORETICAL STUDY OF THE VISCOUS SUBLAYER FOR TURBULENT TUBE FLOW

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ABSTRACT

Flush-mounted anemometer probes have been used to further study characteristics of the viscous sublayer for both Newtonian and drag reducing solutions, with particular emphasis given to low Reynolds number turbulent tube flow. Experimental measurements for the mean frequency of bursting or renewal within the wall region are compared with theoretical predictions obtained on the basis of the surface renewal and penetration model.

Both theory and experiment taken together suggest that the effect of the mean axial pressure gradient on the viscous sublayer becomes important for the deeper molecular penetration associated with low Reynolds number flow. Also, a pronounced lessening of the frequency of bursting within the wall region has been predicted and measured for the addition of a drag reducing agent.

INTRODUCTION

Experimental and theoretical studies of the viscous sublayer for turbulent flow have received considerable attention in the past few years. The experimental studies have involved visualization (1-4), anemometry (5-12), and electrochemical (13,14) techniques. These studies have conclusively demonstrated the dynamic nature of the viscous sublayer.

Among the analytical approaches which have been proposed, the surface renewal and penetration model has been found to be consistent with the basic experimental findings referenced above and is particularly useful in the analysis of many complex transport processes associated with turbulent flow. This model is based on the premise that an energetic exchange of fluid intermittently occurs between the wall region and the turbulent core. During the brief residency of fluid within the close vicinity of the wall, unsteady molecular transport is presumed to control. The cumulative effect of the numerous elements of fluid within the wall region on the spatial mean transport properties is then accounted for by various statistical averaging techniques.

In connection with the elementary surface renewal and penetration model, the key modeling parameter is the mean residence time, τ [i.e., the inverse of the mean frequency of renewal]. Although approximate measurements for τ have been obtained from visual observations of the mean bursting period (6, 10, 15), these data are more readily obtained by the use of flush-mounted anemometer probes (5,8). This paper reports the use of this technique and the use of the elementary surface renewal and penetration model to further study the characteristics of the viscous sublayer, especially for low Reynolds number steady turbulent tube flow.

THEORETICAL BACKGROUND

The surface renewal and penetration model has been described in detail in previous papers (5,16-19). The adaptation of this model to momentum transfer for Newtonian fluids involves a relationship for the instantaneous molecular transfer within individual elements of fluid at the wall of the form (16)

$$\frac{\partial u}{\partial \Theta} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho} \frac{dP}{dx} \tag{1}$$

with initial boundary conditions

$$u = U_i \quad \text{at } \Theta = 0 \tag{2}$$

$$u \to 0 \quad \text{at } y = 0 \tag{3}$$

$$u = \text{finite} \quad \text{as } y \to \infty \tag{4}$$

where U_i is the velocity at the first instant of renewal. Early formulations of this basic model were based on the assumption of negligible pressure gradient effect (5, 17-19).

The solution of this system of equations coupled with the use of the random contact time distribution proposed by Danckwerts (20),

$$\phi(\Theta) = \frac{1}{\tau} \exp\left(\frac{-\Theta}{\tau}\right), \tag{5}$$

leads to an expression for the mean velocity profile within the wall region of the form

$$\frac{\overline{u}}{U_i} = \int_0^\infty \frac{u}{U_i}(\Theta)\phi(\Theta)d\,\Theta$$
$$= \left(1 - \frac{\tau}{\rho U_i}\frac{dP}{dx}\right)\left(1 - \exp\left(\frac{-y}{\sqrt{\nu\tau}}\right)\right) \tag{6}$$

The mean residence time, τ, is assumed to be representative of the experimental burst period, λ, reported later. Parenthetically, the contact time distribution, $\phi(\Theta)$, is defined such that the product $\phi(\Theta)d\Theta$ represents the fraction of the surface with contact time between Θ and $\Theta + d\Theta$. Hence, a relationship can be obtained for τ in terms of the local mean friction velocity, U^*, of the form (16)

$$U^*\sqrt{\frac{\tau}{\nu}} = \frac{2\,U_i/U^*}{1 + \left[1 + \frac{4\nu}{\rho U^{*4}}\,U_i\,\frac{dP}{dx}\right]^{1/2}} \tag{7}$$

For hydrodynamically fully developed flow in a circular tube, this expression has been written as

$$U^*\sqrt{\frac{\tau}{\nu}} = \frac{2\,U_i/U^*}{1 + \left[1 - \frac{16\,U_i}{U^*\,Re}\sqrt{\frac{2}{f}}\right]^{1/2}} \tag{8}$$

With $dP/dx = 0$, Equation 7 reduces to the more familiar form

$$U^*\sqrt{\frac{\tau}{\nu}} = \frac{U_i}{U^*} \tag{9}$$

Although various assumptions have been made for the parameter U_i (5, 17-19), the substitution of $U_i = U_b$ has been found to be quite reasonable (19). Further support for this assumption is offered in a recent article by Katsibas and Gordon (43).

Theoretical formulations of the surface renewal and penetration model have been proposed for viscoelastic drag reducing solutions by several investigators (21-24) assuming negligible pressure gradient effect. Maxwell or Oldroyd constitutive equations have been utilized in each of these developments. More recently an analysis has been proposed which includes the effect of axial pressure gradient (25). This analysis gives rise to an expression which reduces to Equation 7 for small values of the relaxation and retardation times which are associated with the Oldroyd constitutive equation. Based on the work of Seyer (26), the relaxation (and retardation) time for dilute polymer solutions can be assumed to be quite small in comparison to τ. Hence, Equations 7 and 8 are expected to be applicable for the drag reducing solution tested in this investigation.

Brief mention is now made of previous applications of the underlying modeling concept to turbulent convection heat transfer. These applications generally involve the solution of the energy equation of the form

$$\frac{\partial T}{\partial \Theta} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{10}$$

for constant properties, with initial-boundary conditions of the form

$$T = T_i \quad \text{at } \theta \quad 0 \tag{11}$$
$$T = T_0 \quad \text{at } y = 0 \tag{12}$$
$$T = T_i \quad \text{at } y \to \infty \tag{13}$$

The solution of this system of equations, coupled with the contact time distribution, leads to expressions for the mean temperature profile, $\overline{T}$, within the wall region and the mean Nusselt number, Nu, in terms of τ for moderate Prandtl number, Pr, fluids. The use of the analogical relationships for τ presented in this paper then lead to predictions for $\overline{T}$ and Nu in terms of Pr and Re (19, 25, 27). Appropriate modifications of Equation 10 have also led to predictions for variable property heat transfer (28, 29), and the recovery factors for high speed flow (30). Further, the recent development of transient age distributions has led to applications of this basic model to thermal (31, 32) and hydrodynamic (33) unsteady conditions.

Although the above described form of the surface renewal and penetration model has been found to represent important aspects of the turbulent convection process, the model has required several modifications to strengthen its characterization of the mechanism and to broaden its usefulness. These modifications include considerations of a) the significance of unreplenished fluid that remains adjacent to the surface (34-37), b) molecular transport to eddies in transit from the turbulent core to the wall region (38), and c) the influence of axial convection (28). Although some of the simplicity of the elementary model is lost by the incorporation of these modifications, the resulting model proves to be a more comprehensive representation of actual turbulent convection processes. This model also provides the foundation for a physically meaningful formulation for eddy diffusivities of momentum and heat in the wall region over a wide range of Prandtl numbers (25, 39).

EXPERIMENTAL

The general experimental approach utilized in this study has been reported recently in the context of steady and pulsed flow of drag reducing solutions (8). However, the present study involves a more refined experimental technique.

The experimental arrangement is similar to the one used in Reference 8 and is represented by Figure 1. A 1/8-inch I.D. smooth Plexiglas circular test section of 4-inch length was used. All tests were conducted at room temperature using an aqueous saline solution (0.9 wt. %) with and without addition of 40 ppm Separan AP273. Flow rates were measured by a Statham SP 2202 electromagnetic flow meter. Pressure taps were located 1.5 and 3.5 inches from the front of the test section and pressure drops were measured by a Honeywell differential pressure transducer [PM 398 TC $\pm$ 0.5] and recorded by a Siemens Model MO 7633A1 ink recorder. The experimental data for friction factors are shown in Figure 2. The basic agreement between these data for saline and the standard Blasius friction factor curve indicates that fully developed flow was established. The data for the drag reducing solution lie approximately 20% below the data for saline.

A miniature hot-film flat-surface anemometer probe (Thermo Systems Inc.) was mounted 3 inches downstream from the front of the test section; a flush mount of about 0.001 inch was obtained. The sensor was maintained at a constant temperature by means of a Thermo Systems Model 1051-2 Anemometer. The instantaneous bridge voltage from the anemometer was amplified and recorded on magnetic tape; representative bridge voltage signals are shown in Figure 3.

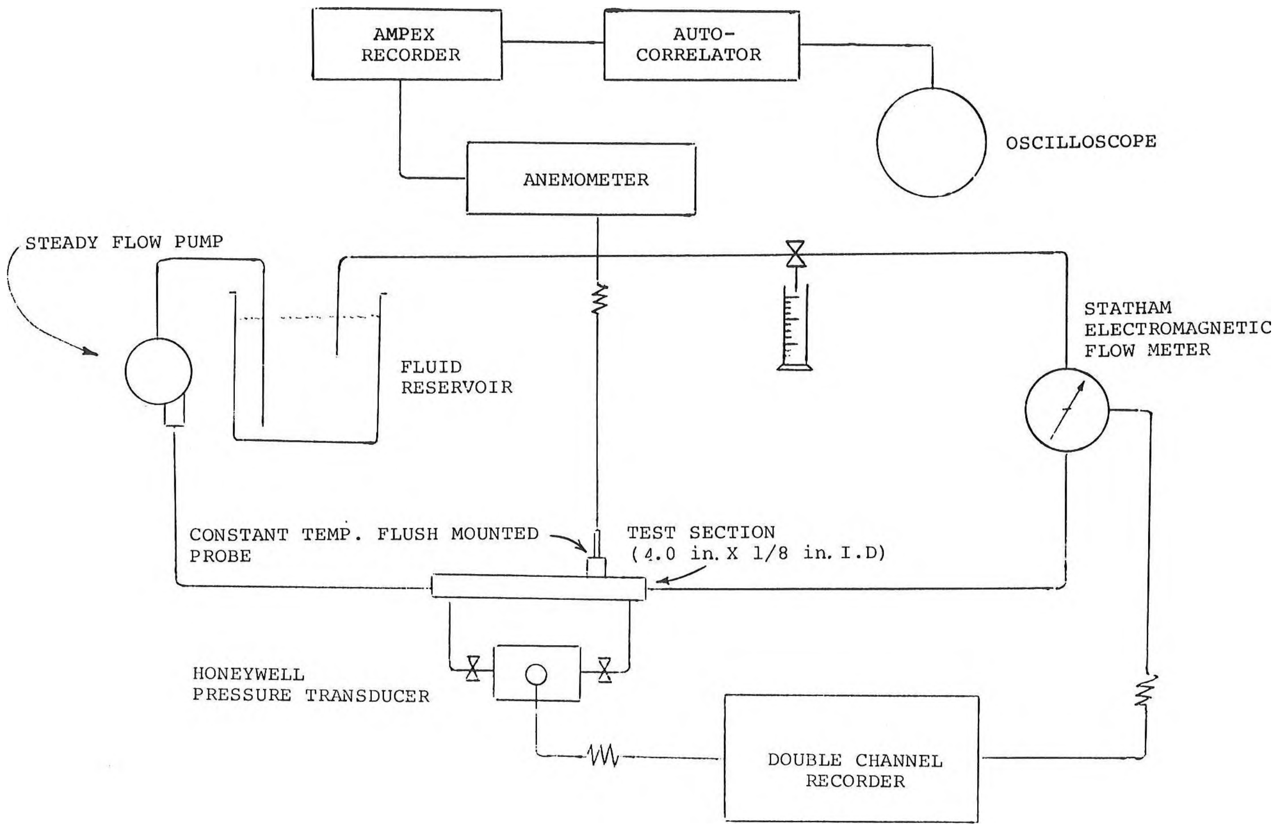

Figure 1. Experimental arrangement

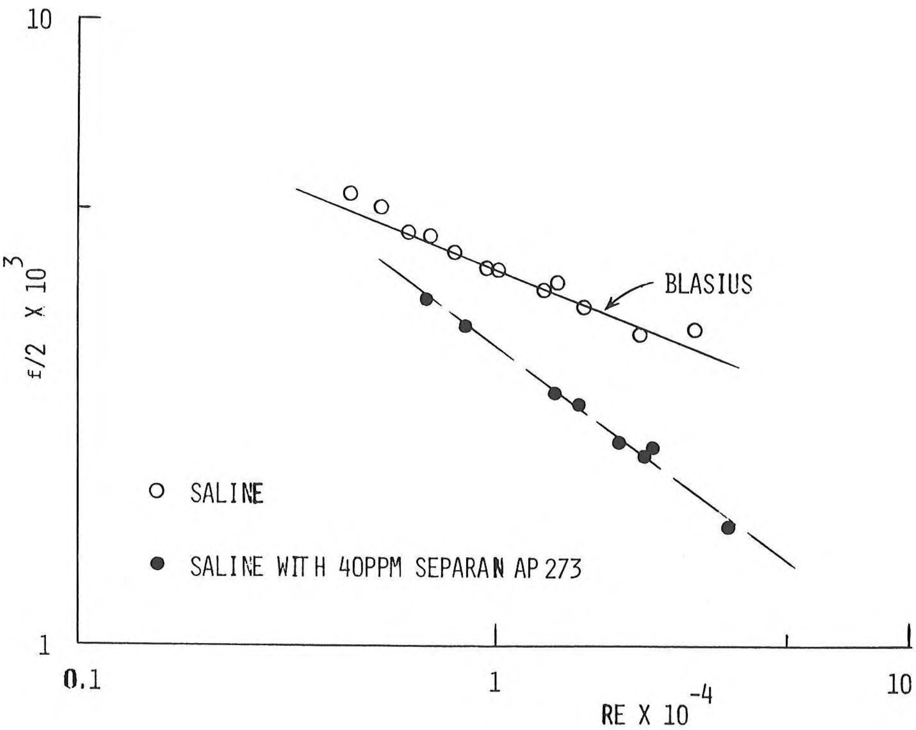

Figure 2. Friction factor data

The unsteady nature of the wall region is clearly reflected in the instantaneous bridge voltage. Accordingly, the characteristic period, λ, associated with this unsteady signal was obtained by the use of a Model 42 SAICOR Autocorrelator. A representative autocorrelation signal is shown in Figure 4 for saline. In contrast to the somewhat short correlation times, Δt, used in the earlier study [$10 < \Delta t/\lambda < 35$] (8), very long sampling periods [$\Delta t/\lambda > 1000$] were used in this study. Accordingly, the autocorrelation data reported herein can be assumed to be based on essentially ergodic samples.

The experimental data for λ obtained from autocorrelation of the instantaneous bridge voltage are shown in Figure 5.

DISCUSSION AND CONCLUSION

Equation 8, with $U_i = U_b$ and with f taken from Figure 2, is compared with the experimental data for λ in Figure 5. The theory and experiment are seen to be in basic agreement with both sets of data. On the basis of these results and a comparison between Equations 8 and 9, it appears that the influence of the pressure gradient on the mean frequency of renewal becomes negligible for values of the Reynolds number above approximately 10^4. However, both theory and experiment taken together suggest that the effect of the mean axial pressure gradient on the viscous sublayer becomes important for the deeper molecular penetration associated with low Reynolds number flow. The point at which this analysis breaks down (i.e. for $16\ U_i = U^* \mathrm{Re} \sqrt{f/2}$) lies in the vicinity of the Reynolds number at which transition from laminar to turbulent flow occurs. This breakdown has been tentatively interpreted as a prediction of relaminarization of the boundary layer for turbulent boundary layer flow with favorable pressure gradient (40). However, this interpretation becomes somewnat artificial for flow in a tube since the deep molecular penetration associated with these low Reynolds numbers violates the important constraint $d\bar{u}/dr \big|_{r=0} = 0$ and brings curvature effects into the picture. This point is more fully developed elsewhere (28).

The experimental data obtained in this study are compared with the data of Meek and Baer (5) and with Equation 8 in Figure 6. These two sets of data are seen to be in basic agreement.

Parenthetically, the coupling of Equation 6 and 7 has been found to lead to predictions for $\bar{u}$ which are in good agreement with experimental data within the wall region (25).

In connection with the drag reducing solution, theory and experiment suggest a pronounced lessening of the bursting frequency within the viscous sublayer for the addition of minute quantities of Separan AP 273. In this regard, a recent study (41) suggests the potential usefulness of drag reducing agents in combating the development of atheroma.

ACKNOWLEDGEMENT

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SYMBOLS

f	Fanning friction factor
Nu	Nusselt number
P	pressure
u	instantaneous velocity profile
$\bar{u}$	mean velocity profile
U_b	bulk mean velocity
U_i	velocity at first instant of renewal
U^*	friction velocity, $= \sqrt{\sigma_0/\rho}$
r	radial coordinate
r_o	tube radius
R_{uu}	autocorrelation coefficient
t	time
Δt	correlation time
T	instantaneous temperature profile
$\bar{T}$	mean temperature profile

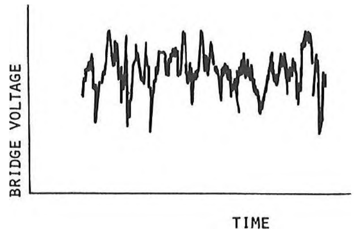

Figure 3. Representative instantaneous bridge voltage

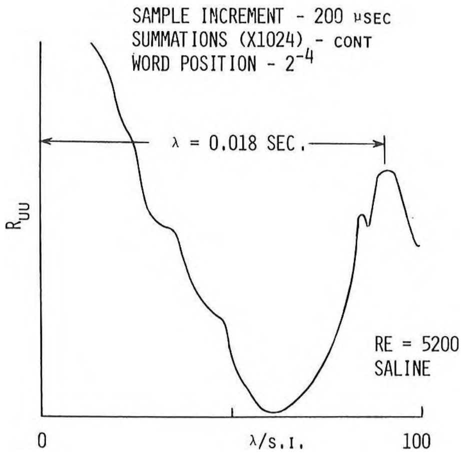

Figure 4. Representative autocorrelation

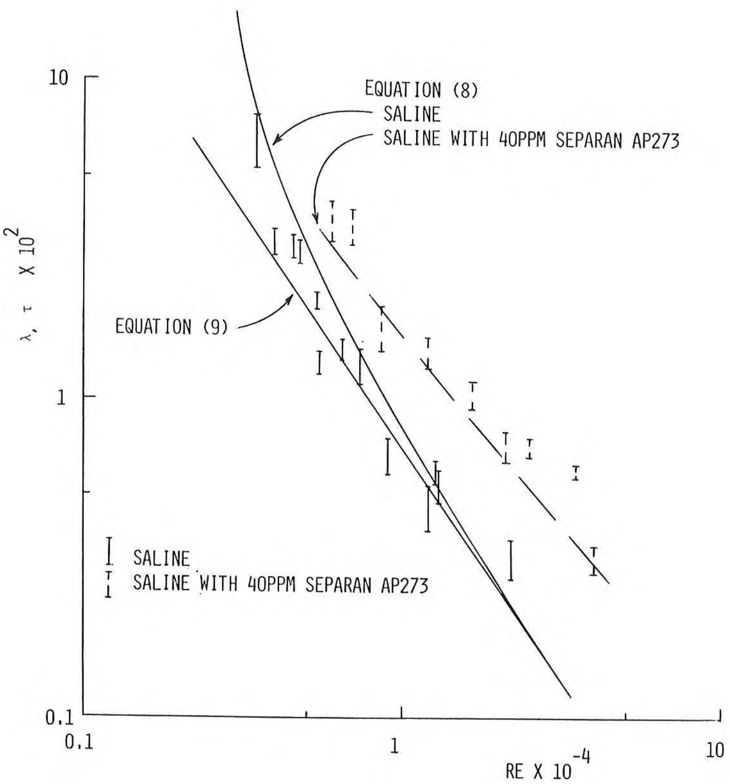

Figure 5. Comparison between experimental measurements for λ and theoretical predictions for τ

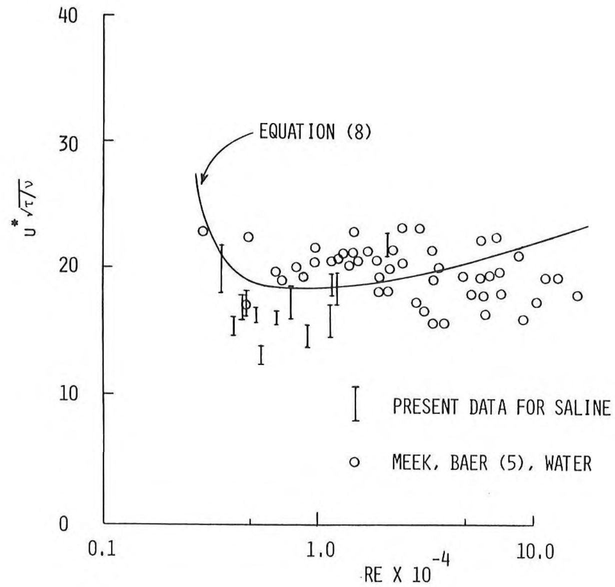

Figure 6. Predictions for $U^* \sqrt{\tau/\nu}$

SYMBOLS (cont.)

T_0	wall temperature
T_b	bulk stream temperature
T_i	temperature at first instant of renewal
x	axial coordinate
y	distance measured from wall, $= r_0 - r$
α	thermal diffusivity
σ_0	wall shear stress
ρ	density
ν	kinematic viscosity
θ	instantaneous contact time
τ	mean residence time
λ	experimental burst period - rerise time in autocorrelation

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DISCUSSION

W. R. Penney, Monsanto: How many regression parameters do you have in the model?

Thomas: For moderate Prandtl number fluids, one parameter is τ, which is measurable and physically meaningful. There are a couple of other parameters. For heat transfer you need to know what the initial temperature and velocity is as the eddy comes to the surface. U_i is the initial velocity at the first instant of renewal. However, I might mention again several approximations have been made for these parameters. For example, $U_i = U_b$ for certain flow situations at the boundary where there's no separation, it does a very nice job. I have a suspicion that there may be ways of measuring U_i, such as tying it to U' within the wall region, but I don't know how to measure it right now.

The direct approach is to predict the time period, τ, on a basis of knowledge of momentum transfer information. So, if you recall, I mentioned that we formulated a relationship for τ in terms of friction factor or shear stress. Then I took the shear stress that I obtained for either friction reducing fluid or the Newtonian fluid and I came back from that to predict τ. So I still need some input, I need to know what the shear stress is, but that's a very low grade piece of information. It's easy to measure, at least for tube flow. What I was trying to do then is to compare what we measured in terms of τ with what I predict using my model.

V. A. Sandborn, Colorado State University: τ is a time here and not a shear stress. In other words you used wall shear stress to calculate a time which is τ.

Thomas: I used sigma for shear stress. That probably causes some difficulty. We measured both σ_0 and τ at a given Reynolds number. Then with σ_0 as an input, τ was theoretically predicted.

Penney: The regression parameter that I thought you might mention is the approach distance of the eddy to the wall. Did you actually measure this or did you regress this from the data?

Thomas: I don't need that information here. The thickness of the unreplenished layer of fluid is around $Y^+ = 5$ on the basis of Popovich and Hummel's work. The resistance of that thin layer of fluid is negligible for low to moderate Prandtl number fluids. It only becomes important for the high Prandtl Number fluids, for which case one must know the mean approach distance or thickness and the statistical distribution. Both are very important.

S. J. Kline, Stanford University: You said several times something I don't quite understand - maybe you can expand on it. That is the distance of the Y^+ at which you measure the auto-correlation is significant and yet the other data that we have seen, for example Willmarth's data, Hanratty's data, Kim's data, Brodkey's data, etc., seems to give us the same auto-correlation, and we think there is a relation between the downcoming stuff at the surface with the pressure distribution we measure at the wall, that those auto-correlations ought to be the same, and yet you are telling us that there's a very distinct function of Y^+ in there. Can you expand a little bit on what you mean there?

Thomas: It goes back to the question I answered a moment ago. Popovich and Hummel's work demonstrates that this exchange of fluid occurs when the fluid is brought within varying distances of the surface into the core. And so if that's true within the wall region - it's surely true right on out.

Kline: I agree with that. All the people doing the flow modeling would agree that stronger sweeps, as Corino and Brodkey call them, get closer to the wall and when you get that you get stronger lift and a stronger burst and a bigger Reynolds stress behind it and so on. So I would agree with what you just said but what I thought I heard you say before was that in fact the auto-correlation times for these events were different and that I am not sure is right. The mean auto-correlation time, I think is independent of what you just said. Now am I misunderstanding you?

Thomas: No, I don't believe so. Let's say an eddy comes to this point within the wall region (y_1) and then it leaves. The fact that it came and left would be detected all through here ($y \geq y_1$). Okay? The fact that it has come and gone would not be detected closer to the wall than y_1, except indirectly by free molecular transport. And by the way, this point becomes very important if you want to do a prediction of eddy diffusivity based on this modeling concept.

Kline: You can't do it that way. That violates continuity and you can't be coming and going in the same place. If you look at it this way it has to come down here on that line and then it goes back up a little farther over.

Thomas: That's right. I'm not saying that the fluid eddy moves straight up. My point is that if an element of fluid comes to here (y_1), it won't affect the time (periodicity) for $y < y_1$. Hence, τ measured for $y < y_1$ will be greater than τ measured at y_1.