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Computer Visualization of Irrotational Flow Over Bluff and Streamlined Bodies

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ABSTRACT

Computer programs were developed to visualize irrotational flow over bluff bodies (circular cylinders and half-bodies) and streamlined bodies (elliptical cylinders and airfoils). The method of superposition of flows was used, with stream functions or complex velocity potentials for fundamental flows being added to obtain the flows over the bodies. For the flow over the circular cylinders, half-bodies and elliptical cylinders, the stream function was calculated directly. The flow over the airfoil was obtained by Joukowski mapping of the imaginary part of the complex velocity potential for flow over a circular cylinder.

The programs developed will be valuable learning tools for undergraduate aerospace engineering students. The students will be helped to understand the equations used to describe the flows by studying the effects of varying various parameters. Future work on these programs may include modifying them to calculate and plot the velocity and pressure distributions, and to calculate the aerodynamic forces on the bodies.

NOMENCLATURE

a	Radius of circular cylinder
c	Joukowski mapping constant
d	Source-sink separation in flow over elliptical cylinder or complex doublet flow
h	Asymptotic height of half-body
l	Length of elliptical cylinder
q	Source, sink or doublet strength in complex flow
r	Distance from origin in cylindrical coordinates
u	x-component of velocity
v	y-component of velocity
V_∞	Magnitude of freestream velocity
x	Cartesian coordinate abscissa
x_0	Source, sink, doublet or vortex x-location
x_P	Joukowski mapping parameter
xx	x-coordinate of point on streamline
y	Cartesian coordinate ordinate
y_0	Source, sink, doublet or vortex y-location
y_P	Joukowski mapping parameter
yy	y-coordinate of point on streamline
z	Complex coordinates, $x + iy$
z_0	Source, sink, doublet or vortex location in complex coordinates
z_P	$x_P + iy_P$
α	Uniform flow angle of attack
Γ	Vortex strength
Δ	Increment in quantity

θ Angle from x-axis in cylindrical coordinates
K Doublet strength
Λ Source or sink strength
ϕ Velocity potential
ψ Stream function
Ω Complex velocity potential

INTRODUCTION

The purpose of the project described in this report was to develop computer programs to visualize two-dimensional, irrotational (potential) flow over bluff and streamlined bodies. The bluff bodies were half-bodies and circular cylinders, the streamlined bodies were elliptical cylinders and airfoils. The stream function was used to describe the flow over the various bodies. The computer programs are designed as learning tools to introduce undergraduate aerospace engineering students to the principles involved in visualizing irrotational flows. The computer programs calculate and plot the streamlines for the various types oi flows.

The programs are written in the C programming language. (Refs. 1-3). C was used since it has several advantages over FORTRAN for programs of this type. The main advantages are: powerful and built-in graphics routines, ease of interaction with the user, a high degree of modularity, and ease of implementation on personal computers. When the program is compiled and linked, a DOS-based executable file is produced.

GENERAL METHODS

The general method for visualizing the flows over the various bodies consists of two main steps: finding the points on the streamlines, and plotting the streamlines. The points on the streamlines are found by: generating a grid, calculating the stream function values at points on the grid, finding the points with the same stream function values, calculating the body profile, and eliminating the points inside the body. A grid is defined by x- and y-coordinates ranging from -1 to 1. Grid divisions of 0.1 are used.

The stream function is found by the method of superposition of flows--stream functions for fundamental flows are added to obtain the stream functions for flows over various bodies. The method used to find the points with the same given stream function value is based on that described by Chow. (4) The method used is similar to subroutine SEARCH in program 2.2. The essence of the method is to approximate the location of a point (xx_k, yy_k) that has a given stream function value, ψ_a. (See figure 1 for terminology.) Its location is approximated by:

$$xx_k = x_i$$

$$yy_k = y_j - \frac{|\psi_a - \psi_{ij}|\Delta y}{|\psi_a - \psi_{ij}| + |\psi_a - \psi_{ij-1}|} \qquad (1)$$

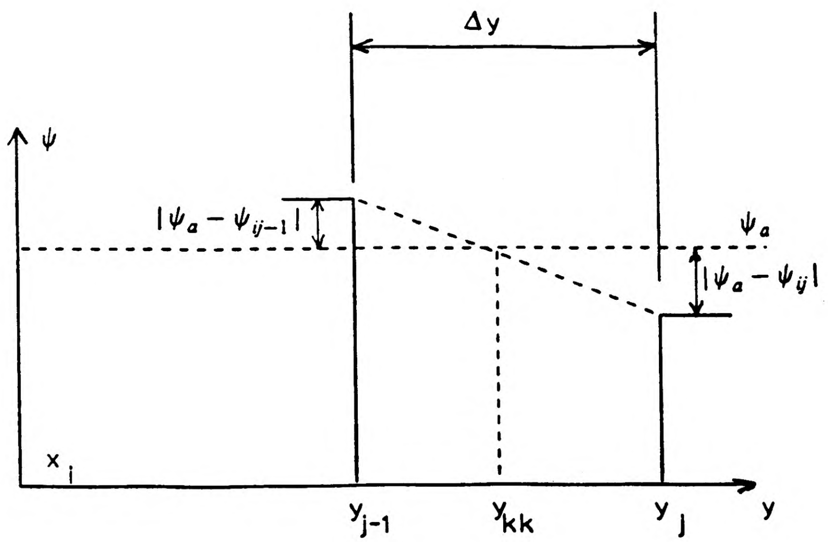

Figure 1. Approximating location of point with stream function value ψ_a

The equation for the body profile is obtained by setting the stream function equal to zero. The equation may be solved analytically or numerically to find the coordinates of the points on the body profile. Since the equations used to describe the flow are only valid for irrotational two-dimensional flow, they are not valid for points inside the body shape. Using the equation of the body profile, it is possible to determine if a point calculated as being on a streamline is inside the body shape. If it is, it is eliminated. For this project, a maximum of forty points can be plotted per streamline, and, at most, fifteen lines, including the body shape, are plotted.

FLOW OVER CIRCULAR CYLINDERS

The flow over the circular cylinder is obtained by adding uniform, doublet and vortex flows. For the vortex and the doublet located at the origin, the stream function is given by (5):

$$\psi = V_\infty y\left(\cos\alpha - \frac{a^2}{r^2}\right) - V_\infty x \sin\alpha + \frac{\Gamma}{2\pi}\ln\left(\frac{r}{a}\right) \qquad (2)$$

where $r = \sqrt{x^2 + y^2}$ and $a = \sqrt{\dfrac{K}{2\pi V_\infty}}$

The body shape is a circle of radius a. To obtain the body shape, 2π radians are swept out in increments of $\Delta\theta$ radians, and the coordinates of the points on the body shape are calculated for each θ. The increment $\Delta\theta$ is :

$$\Delta\theta = \frac{2\pi}{\text{number of points to be plotted} - 1} \qquad (3)$$

The x- and y-coordinates for the points on the circle are given by:

$$x = a\cos\theta, \quad y = a\sin\theta \qquad (4)$$

Points are eliminated if $(x^2 + y^2) < a^2$.

FLOW OVER HALF-BODIES

The flow over a half-body is obtained by adding uniform and source flows. For the uniform flow at zero angle of attack and the source on the x-axis, the stream function is given by (5):

$$\psi = V_\infty y + \frac{\Lambda}{2\pi}\tan^{-1}\left(\frac{y}{x - x_0}\right) \tag{5}$$

or

$$\psi = V_\infty\left(h\frac{\theta}{\pi} + y\right) \tag{6}$$

where $h = \frac{\Lambda}{2V_\infty}$ is the asymptotic height of the half-body.

The body shape, obtained by setting $\psi = 0$, is given by:

$$y = -\frac{\Lambda}{2\pi V_\infty}\tan^{-1}\left(\frac{y}{x - x_0}\right) \tag{7}$$

To calculate the x- and y-coordinates of the points on the body, equation (7) is solved numerically; $(2\pi - 2\beta)$ radians are swept out, where

$$\beta = \tan^{-1}\left(\frac{h}{x_{max} - x_0}\right) + \frac{\pi}{2 \times \text{number of points to be plotted}} \tag{8}$$

A point is eliminated if $y \le \frac{-\Lambda}{2\pi V_\infty}\tan^{-1}\left(\frac{y}{x - x_0}\right)$

FLOW OVER ELLIPTICAL CYLINDERS

The flow over an elliptical cylinder is obtained by adding uniform flow along the x-axis to a source and a sink. If the source and sink have the same strength and are at the same distance from the origin, the stream function for the flow is (6):

$$\psi = V_\infty y + \frac{\Lambda}{2\pi}\left[\tan^{-1}\left(\frac{y}{x - d}\right) + \tan^{-1}\left(\frac{y}{x + d}\right)\right] \tag{9}$$

The body shape is obtained by setting $\psi = 0$. The length of the body is given by:

$$l = \sqrt{\frac{\Lambda}{\pi V_\infty d} + 1}\; d \tag{10}$$

Also,from equation (9)

$$y = \frac{\Lambda}{2\pi V_\infty}\tan^{-1}\left(\frac{2dr\sin\theta}{r^2 - d^2}\right) \tag{11}$$

Equation (11) has to be solved iteratively for x and y. To do this, 2π radians are swept out in $\Delta\theta$ increments and l length units are swept out in Δr units where

$$\Delta\theta = \frac{2\pi}{2 \times \mathrm{int}\left(\dfrac{\text{number of points to be plotted} - 1}{2}\right)} \quad (12)$$

$$\Delta r = \frac{l}{\text{number of points to be plotted}} \quad (13)$$

Points are eliminated, if

$$y < \frac{\Lambda}{2\pi V_\infty}\left|\tan^{-1}\left(\frac{y}{x-d}\right) - \tan^{-1}\left(\frac{y}{x+d}\right)\right| \quad (14)$$

FLOW OVER AIRFOILS

To visualize the flow over the airfoil, the Joukowski transformation is used. Both the body shape and the streamlines of the flow over the cylinder are mapped to the flow over an airfoil.(7) The transformation parameters are shown in figure 2: the circular cylinder is in the x″-y″ plane, and the airfoil is in the x-y plane.

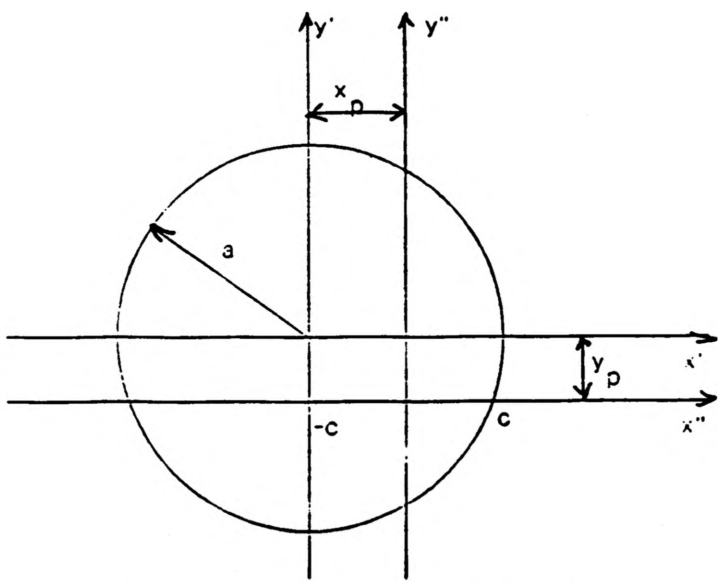

Figure 2. Mapping parameters for Joukowski transformation

The circular cylinder is first shifted to the x′-y′ plane, with its center at (x_P, y_P), with x_P always less than zero. The system of equations for the coordinate transformation from x″-y″ to x-y is:

$$z_P = x_P + iy_P \quad (15)$$

$$z' = z'' + z_P \quad (16)$$

$$z = z' + \frac{c^2}{z'} \quad (17)$$

where c is a constant. y_P determines the camber of the airfoil, and is zero for a symmetrical airfoil. A negative value of y_P will give an airfoil that is curved downward.

Since the transformation uses complex coordinates, the complex potential for flow over a circular cylinder is used (6):

$$\Omega = V_{\infty} z + V_{\infty} \frac{b^2}{z} + \frac{i\Gamma}{2\pi} \ln z \tag{18}$$

where $b^2 = \dfrac{qd}{\pi V_\infty}$

The stream function is obtained as the imaginary part of the complex velocity potential. Once the stream function for the flow over the cylinder is found, streamlines and body profile are found as for the flow over the circular cylinder, and are then mapped using the Joukowski transformation.

SPECIAL CONSIDERATIONS

Since the programs developed to visualize the flow are intended as learning tools, it is important to obtain results quickly but with reasonable accuracy. The numerical schemes used in visualizing the flow are intended to give the user a general idea of what the streamlines and body shape look like.

The method for finding the points on the streamlines works best for flows approximately parallel to the x-axis. So, there are problems in finding points on a streamline that has orientations parallel to the other coordinate axis. Because the numerical calculations for the body shape would take too long, the program cannot visualize flow over very thin elliptical cylinders.

RESULTS AND DISCUSSION

Sample results of the programs developed are shown in figures 3 through 6. Figure 3 shows the results for flow over a circular cylinder. The circulation (or vortex) in figure 3 removes the symmetry of the streamlines about the x-axis. The streamlines are closer together on top of the cylinder for clockwise circulation. Figure 4 shows the results for flow over a half-body. A larger source strength increases the height of the half-body. Figure 5 shows the results for flow over an elliptical cylinder. Decreasing the distance between the source and the sink results in a smaller, rounder body. Figure 6 shows the results for flow over an airfoil. Decreasing c increases the thickness of the body, and a non-zero value of y_p gives a cambered airfoil.

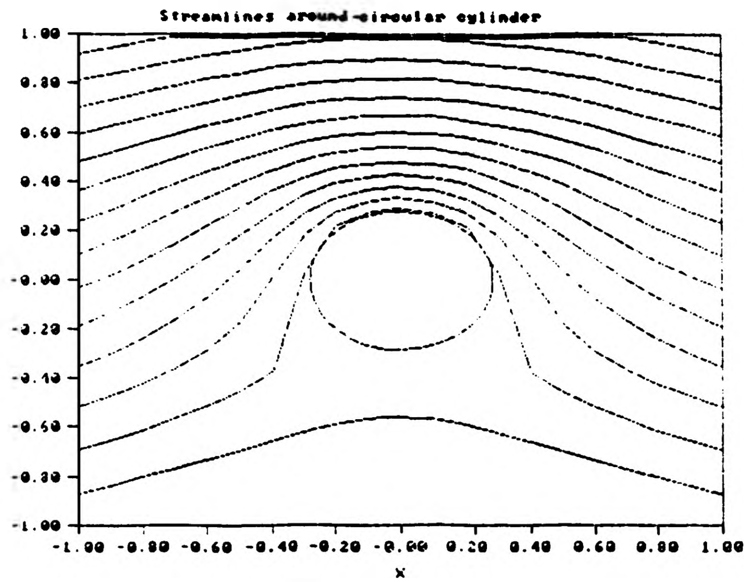

Figure 3. Flow over circular cylinder; $V_\infty = 1$, $\alpha = 0$, $K = 0.5$, $\Gamma = 3$

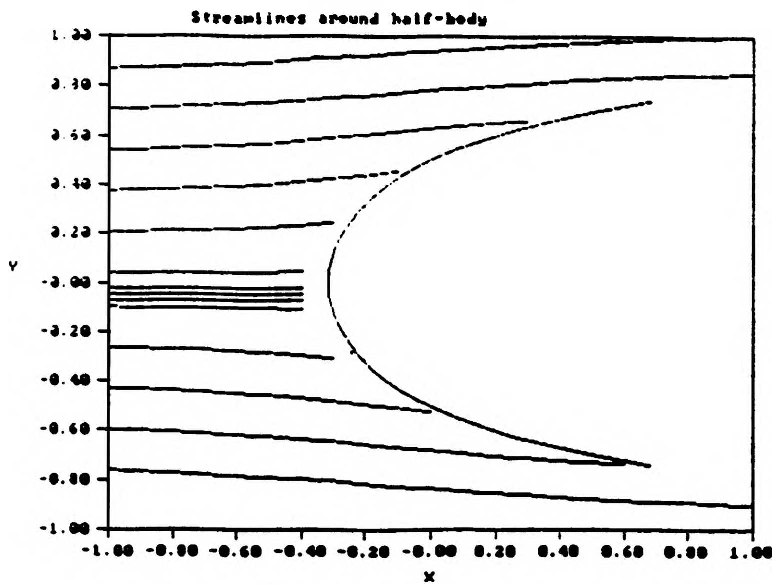

Figure 4. Flow over half-body; $V_\infty = 2$, $\Lambda = 4.0$, $x_0 = 0$

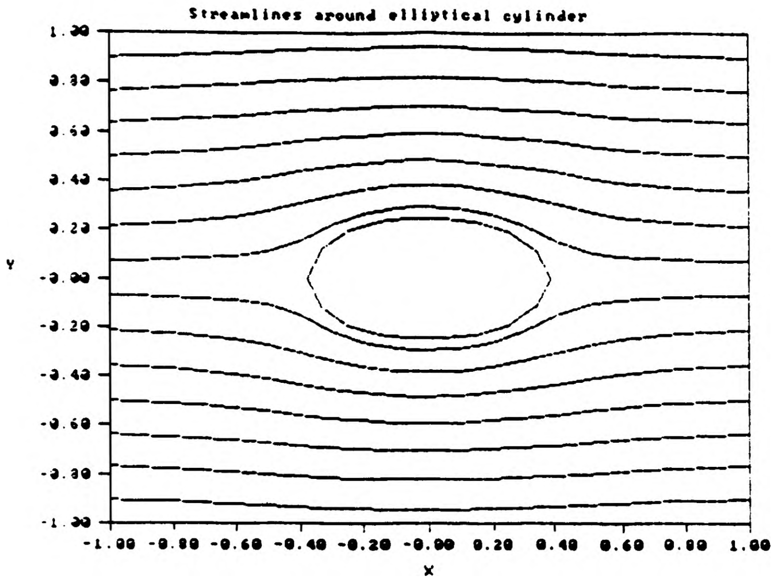

Figure 5. **Flow over elliptical cylinder;** $V_\infty = 1$, $\Lambda = 1$, $d = 0.25$

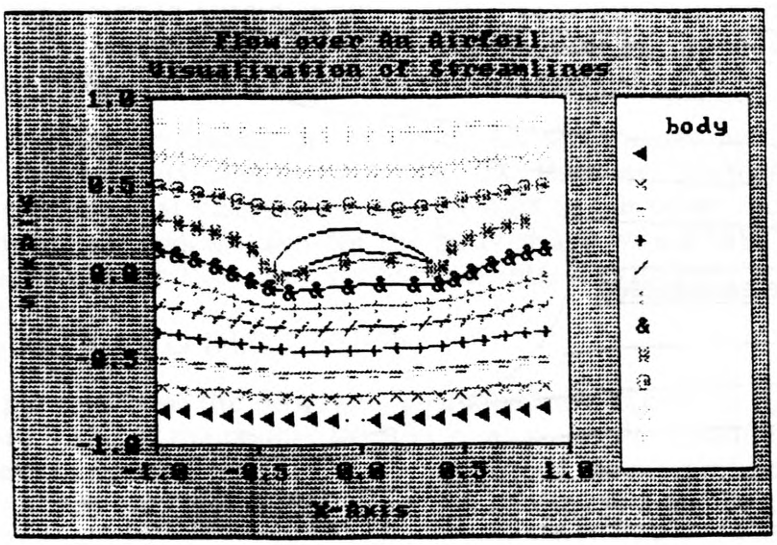

Figure 6. **Flow over airfoil;** $V_\infty = 1$, $q = 1$, $d = 0.2$, $\Gamma = 1$, $y_P = -0.1$, $c = 0.2$

CONCLUSIONS AND RECOMMENDATIONS

The program developed in this project should prove to be valuable learning tools for undergraduate aerospace engineering students. The visualization programs may be used to explore the effect of varying various parameters on the flow pattern and body shape obtained, and to give the users an idea of how the equations used to describe the flow translate into a physical flow pattern.

The programs have a few limitations, some of which may be removed by incorporating different numerical methods or optimization techniques. The effects of decreasing the grid divisions or of increasing the number of points and/or streamlines that are plotted might be investigated to optimize computational time and accuracy. Future work on the visualization software may include comparison of the computer-generated results with experimental results, and visualization of flow over additional bodies, and incorporation of routines to calculate and plot the pressure and velocity distribution, as well as to calculate the aerodynamic forces on the bodies.

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