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VERTICAL AND ANGULAR MOTION OF A WING MODEL IN STEADY FLOW: A PHYSICAL AND NUMERICAL EXPERIMENTAL STUDY

Andrew P. Johnston

Abstract

The dynamic response of a rigid wing in steady flow mounted on translational and rotational elastic supports is investigated. This study presents both a numerical and a physical experimental arrangement that can be employed to investigate the effect of a wide range of parameters on the response of the wing subjected to steady flow. An interactive computer program, which utilizes the fourth order Runge-Kutta integration scheme to solve the governing differential equations, is discussed. The program output allows for quick visualization of the change in response due to parameter changes. An experimental arrangement that utilizes the new 18" x 18" subsonic wind tunnel is also developed. The experimental setup allows for quick parameter changes as well as being adaptable to other one or two degree of freedom systems. Low frequency oscillation at low speeds for flow visualization is obtainable along with chaotic responses at higher flow speeds. Periodic, quasi-periodic, and chaotic responses are observed in both experiments. A qualitative comparison of the results from the two experimental setups is given.

Nomenclature:

A = Dimensionless aerodynamic parameter = $\rho \ell_c S/2m$
a = Distance from aerodynamic reference to axis of rotation
C_z = Dimensionless translational damping constant = $c_z/(m/t_c)$
C_α = Dimensionless rotational damping constant = $c_\alpha/(I/tc)$
$c\ell$ = Wing lift coefficient
c_z = Translational damping coefficient
c_α = Rotational damping coefficient
g = Acceleration due to gravity
I = Wing mass moment of inertia
K_z = Dimensionless translational stiffness = $k_z/(m/t_c^2)$
k_z = Translational spring stiffness
K_α = Dimensionless rotational stiffness = $k_\alpha/(m\ell_c 2/t_c^2)$
k_α = Rotational spring stiffness
L = Wing lift
ℓ_c = Characteristic length
M = Dimensionless inertia parameter = $ma\ell_c/I$
m = Wing mass
S = Wing area
t = Time
t_c = Characteristic time
T = Dimensionless time = t/tc
U_∞ = Free stream velocity
U = Dimensionless velocity in x-direction = U_∞/w_c
υ = Vertical flow (wing) velocity, $\upsilon = dz/dt$
V = Dimensionless velocity in z-direction = υ/w_c
w_c = Characteristic velocity
z = Vertical amplitude
Z = Dimensionless wing vertical displacement = z/ℓ_c

α = True angle of attack
α_o = Static angle of attack
δ_α = Induced angle of attack
ω = Angular wing velocity, $\omega = d\alpha/dt$
Ω = Dimensionless wing angular velocity = $\omega / (1/t_c)$
ρ = Air density
d/dt = First derivative with respect to time
d^2/dt^2 = Second derivative with respect to time
D/DT = Dimensionless first derivative with respect to time

Introduction

The response of dynamic systems is an important topic in the education of future engineers and is included in several college courses. However, a majority of the systems studied is not fully investigated by the students. The experimental arrangements presented will enable the user to investigate the response of a wing with two degrees of freedom through the use of parametric studies. The opportunity to perform parametric studies becomes valuable when there is a large parameter space involved and the response is no longer intuitive. The dynamic system investigated is that of a rigid wing in steady flow mounted on translational and rotational elastic supports. The objective of this paper is to present a guide on the design and construction of numerical and physical experimental arrangements that allow for exploring the dependence of the translational and rotational wing motions on the parameter space. The governing equations, the numerical experiment, the physical experiment, and the results are discussed in subsequent sections.

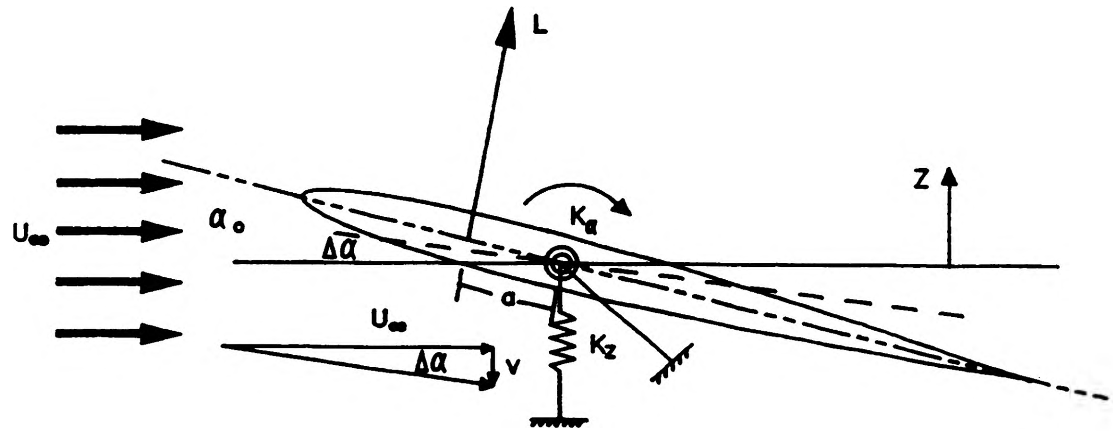


Figure 1. A wing model mounted on translational and rotational elastic supports.

Governing Equations

A schematic of the dynamic system investigated in this study is presented in Figure 1. The rigid wing is supported by translational and rotational springs with spring constants k_z and k_α respectively. It is assumed that the wing can only rotate and move vertically, and hence it is a two-degree of freedom system. Damping in the vertical and angular directions is represented by the damping coefficients c_z and c_α respectively. The length a represents the distance from the axis of rotation to the quarter chord. In the absence of flow, the wing makes and angle α_o with the horizontal, and its axis of rotation sets at a height $z = 0$. When the model is subjected to flow velocity U_∞, the wing angle of attack can be written as

$$\alpha = \alpha_o - \delta_\alpha = \alpha_o - \tan^{-1} [\upsilon/U_\infty] \tag{1}$$

where υ is the vertical flow velocity due to vertical wing motion. The wing lift normal to the flow direction is given by

$$L = 1/2\, \rho\, (U_\infty^2 + \upsilon^2) S\, c_\ell \tag{2}$$

where ρ is the air density, S is the wing planform area, and c_ℓ is the airfoil lift coefficient. The lift coefficient can be approximated as[2]

$$c = 2\pi\alpha \tag{3}$$

The assumption here is that the airfoil is thin and symmetric with a large aspect ratio and the wing angle of attack is below the static stall angle of attack. Also for this type to wing, the lift acts along the quarter-chord line, and the airfoil moment coefficient is zero. In the numerical experiment, the wing is considered to be stalled, $c_\ell = 0$, for angles of attack whose magnitudes are greater than 12 degrees. With these assumptions the equations for vertical and angular motions can be written as

$$m\, d^2z/dt^2 = -c_z\, dz/dt - k_z\, z + L \cos(\delta_\alpha) \tag{4}$$

$$I\, d^2\alpha/dt^2 = -c_\alpha\, d\alpha/dt - k_\alpha(\alpha - \alpha_0) + a\, L \cos(\delta_\alpha) \tag{5}$$

Using equations (2) and $\cos(\delta_\alpha) = U_\infty/\sqrt{U_\infty^2 + \upsilon^2}$ one can write

$$md^2z/dt^2 = -c_z\, dz/dt - k_z z + 1/2\, \rho c_\ell S\, U_\infty \sqrt{U_\infty^2 + (dz/dt)^2} \tag{6}$$

$$Id^2\alpha/dt^2 = -c_\alpha\, d\alpha/dt - k_\alpha(\alpha - \alpha_0) + 1/2 a \rho c_\ell S\, U_\infty \sqrt{U_\infty^2 + (dz/dt)^2} \tag{7}$$

Characteristic quantities for length, time, and velocity are chosen in order to write the governing equations in a dimensionless form. The period of free oscillation of the system is taken as a reference time t_c, and the deformation of the spring due to the wing weight is taken as the reference length ℓ_c. The ratio of ℓ_c/t_c is chosen as a characteristic velocity w_c. Based on these quantities, the characteristic quantity for time, length, and velocity can be written as follows:

$$t_c = 2\pi\sqrt{m/k_z} \tag{8}$$

$$_c = mg/k_z \tag{9}$$

$$w_c = g/(2\pi\sqrt{m/k_z} \tag{10}$$

The second order governing Equations (6) and (7) can be rewritten as four first order ordinary differential equations. Substituting Equations (8), (9), and (10) into Equations (6) and (7), will lead to the following dimensionless equations:

$$DZ/DT = V \tag{11}$$

$$DV/DT = -(2\pi)^2 Z - C_z V + A c_{\ell} U \sqrt{u^2 + V^2} \tag{12}$$

$$D\alpha/DT = \Omega \tag{13}$$

$$D\Omega/DT = -(2\pi)^2 K_\alpha(\alpha - \alpha_o) - C_\alpha \Omega + A M c_{\ell} U \sqrt{U^2 + V^2} \tag{14}$$

where A and M are the dimensionless aerodynamics and inertia parameters, C_z and C_α are the translational and rotational damping parameters and K_α is the rotational stiffness parameter. These dimensionless parameters are given as follows:

$$A = \rho g S/2k_z \tag{15}$$

$$M = agm^2/Ik_z \tag{16}$$

$$C_z = 2\pi c_z/\sqrt{mk_z} \tag{17}$$

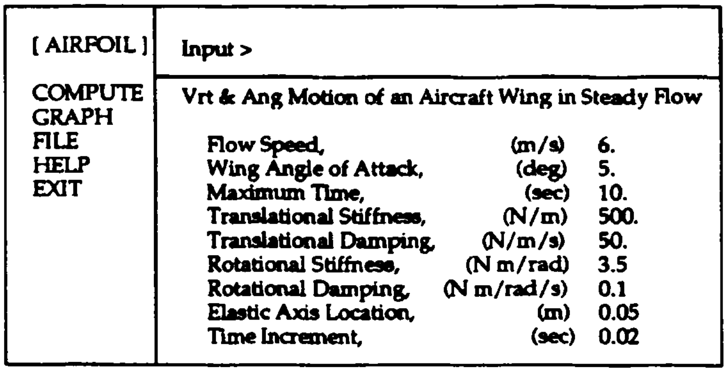
$$C_\alpha = 2\pi c_\alpha \sqrt{(mk_z)}\,/I \tag{18}$$

$$K_\alpha = mk_\alpha/Ik_z \tag{19}$$

As can be seen, the number of parameters is reduced to five dimensionless constants. Each set of values of these constants represents a large number of combinations of the dimensional parameters.

Computational Approach and Program Structure

The objective of the computer program is to integrate the governing equations in order to enable the user to perform parametric studies to investigate the dynamic response of a rigid wing with two degrees of freedom. To integrate the governing differential equations, the fourth order Runge-Kutta integration scheme is employed. This method requires the evaluation of the right hand side of Equations (12) and (14) at each time step. These functions represent the force and the moment acting on the wing respectively. The integration scheme and the results thereof are sensitive to the choice of the time step, and the program enables the user to investigate this sensitivity.



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Input >

Vrt & Ang Motion of an Aircraft Wing in Steady Flow

  Flow Speed,                      (m/s)   6.
  Wing Angle of Attack,            (deg)   5.
  Maximum Time,                    (sec)   10.
  Translational Stiffness,         (N/m)   500.
  Translational Damping,         (N/m/s)   50.
  Rotational Stiffnese,        (N m/rad)   3.5
  Rotational Damping,        (N m/rad/s)   0.1
  Elastic Axis Location,             (m)   0.05
  Time Increment,                  (sec)   0.02
```

Figure 2. User interaction environment. The main menu is on the left and the AIRFOIL submenu is on the right.

All computer routines are written in C, employing the Microsoft Quick C compiler. The language was selected because it provides direct access to most capabilities of a machine-language instruction set. It has a large variety of data types and concise syntax for effective conversions and indirections. Also, it has a very high degree of portability from one machine to another. These capabilities make C powerful, efficient, and well suited to PC applications that involve user interfaces.3 An IBM PC/2 model 30-286 running at 10 MHz with a math co-processor was used to integrate the system of equations.

The program consists of several functions that perform different tasks when called by the main driver. The program interacts with the user with the use of a main menu, shown in Figure 2 on the left side of the screen, which allows for changing the parameter values, running the program, visualizing the results in a graphical presentation, storing the results, obtaining help, and exiting the program.

Once the input parameters are set and the option to perform the calculations is entered, the program computes the required size of arrays based on the time interval requested by the user and allocates memory for these arrays. This step is followed by functions that assign the initial conditions and integrate the governing equations. Once the integration is complete, the main menu is used to enable the user to visualize and/or store the results. Parametric investigations can continue until the program is exited. As can be seen in the AIRFOIL submenu, Figure 2 on the right side of the screen, nine parameters are involved in the parametric study. Fixed variables are the wing mass, moment of inertia, and surface area. The graphical presentation of the results is in the form of four graphs: Amplitude *vs.* Time, Angle of Attack *vs.* Time, Velocity *vs.* Amplitude (phase plot), and Angular Velocity *vs.* Angle of Attack (phase plot). The results may be stored on disk as an ASCII file for future plotting by selecting the FILE option. The user interaction environment of this program allows for efficient parametric investigations.

Physical Experiment

The objective of this portion of the paper is to present an approach to the design and construction of a physical experiment used to observe the dynamic response of a wing model with two degrees of freedom and subjected to steady flow. The design goal is to create a simple experimental setup which enables the user to observe a pure two degree of freedom system and perform parameter changes easily. The parameters that may be changed are the flow velocity, static angle of attack, spring constants k_z and k_α, and the location of the axis of rotation, a. The new 18"x18" subsonic wind tunnel is used in conjunction with a removable test section with a length of 54 inches. The tunnel is driven by a 20 hp DC motor that provides feedback control for a tubular accustafoil fan. Test section flow velocities can be controlled in a range between 0 and 120 ft/sec.

As shown in Figures 3a and 3b, the majority of the wing support structure is located outside of the wind tunnel to provide for clean flow and quick changing of the parameters. A "flexible fork" was developed to transmit the lift of the wing to the translational springs located underneath the wind tunnel. A flexible tube is used to connect the support uprights and provide a self-alligning fork. This eliminated the necessity of having to construct a perfectly square fork and having to align the linear bearings exactly. A set of five linear bearings and two rotational bearings confine the system to 2 degrees of freedom. The wing itself is a semi-symmetrical airfoil constructed of a polystyrene core coated with clear epoxy. The wing is mounted by the use of a connecting rod which passes through the rotational bearings and a tube running spanwise in the wing. One rotational bearing is mounted on top of each of the uprights. There are currently two tubes placed spanwise in the wing, one at the quarter-chord and one 4.5 cm. aft of the quarter chord, which allow for the location of the rotational axis to be changed. On one side of the fork a spring holder/angle of attack adjuster is mounted to the connecting rod, and the angle of attack is adjusted by loosening the setscrews and repositioning the wing. The rotational spring is a flat spring with one end cantilevered and the other end simply supported. The spring constant is varied by repositioning the rotational spring rate adjuster, which slides along the upright. The translational spring constant is adjusted by using different sets of compression springs. The height of the translational spring housing is adjustable to accommodate a wide variety of spring sizes. The translational spring housing is mounted on a separate platform which is not connected to any portion of the tunnel in order to help eliminate any vibration caused by the running of the wind tunnel.

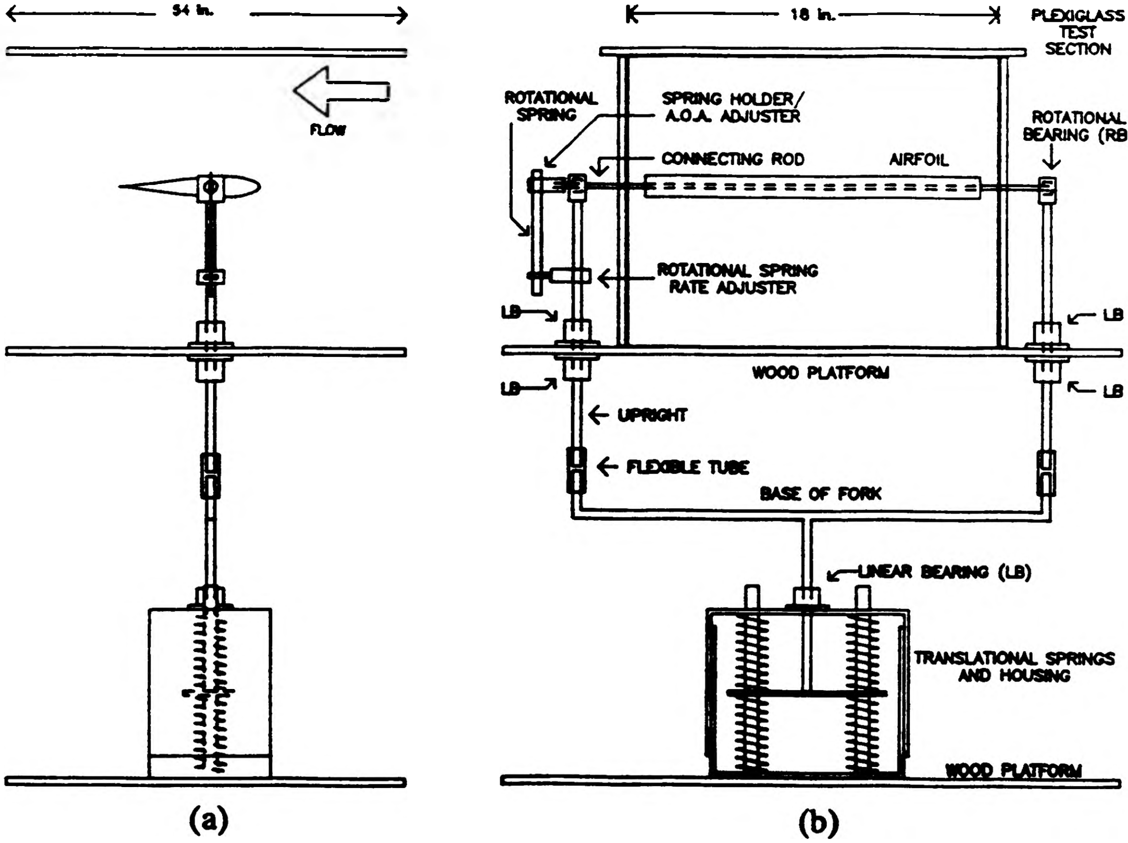

Figure 3. a) Side view of the experimental arrangement; b) Front view.

The experimental setup meets the design goals of being simple and adaptable. The entire test section is removable, and the experimental setup can be assembled or disassembled in a half hour. All of the parameters, except the location of the elastic axis, can be changed while the wind tunnel is running. Data gathering devices include an angle of attack indicator, a vertical displacement indicator, and a Pitot-static tube used to measure the flow velocity.

Results and Discussion

Computational Experiment

The program proves to be a success in the ability to observe a wide range of dynamic responses such as periodic, quasi-periodic, and chaotic oscillation. An example of the graphical presentation of the results is provided in Figures 4a through 4d. A wing reference area of 0.2 m^2 is selected. The wing mass and inertia are 2.0 kg and 0.2 kg m^2 respectively. The results show that initially the aerodynamic lift raises the wing upward from its static equilibrium position and exhibits underdamped behavior as the wing moves to a steady state height. The system is obviously stable. With a case such as this, the user can proceed to investigate the effect of different parameters on amplitude, frequency, and stability.

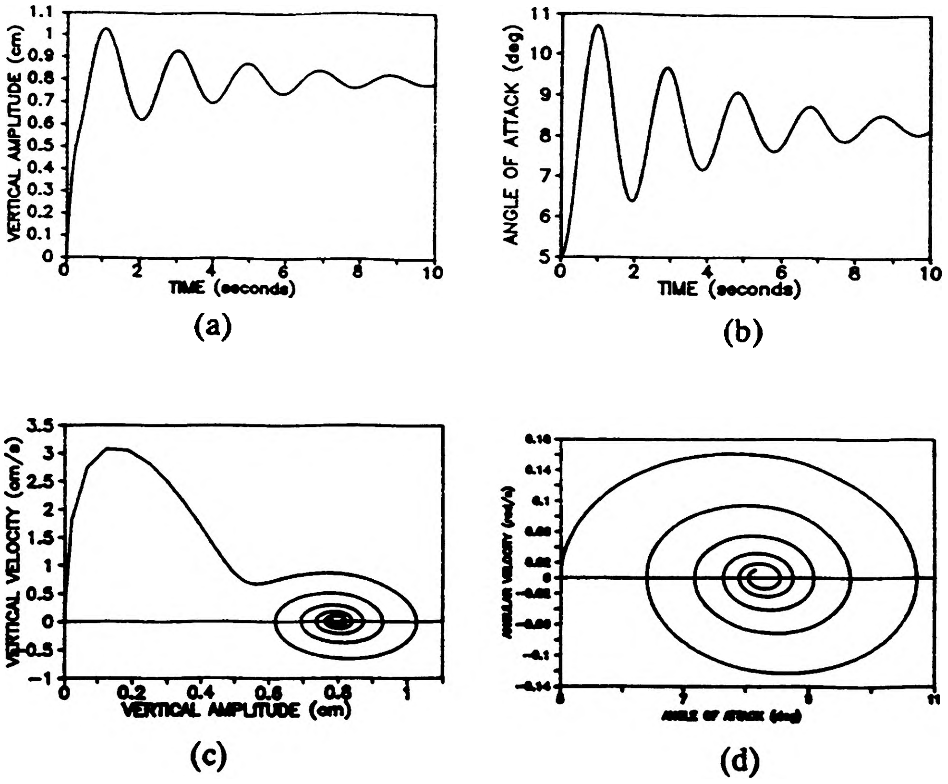

Figure 4. Results for an underdamped case (see Figure 2 for values). 4a) Amplitude *vs*. Time. 4b) Angle of Attack *vs*. Time. 4c) Vertical Velocity *vs*. Amplitude. 4d) Angular Velocity *vs*. Angle of Attack.

This dynamic system may be used to simulate the stiffness of an airplane wing in order to investigate wing flutter. Figures 5a and 5b present results for a divergent response which would be characteristic of a wing in flutter. The transition speed between stable and unstable behavior is found by a simple parametric study. The program also demonstrates that modes of flutter occur at different flow speeds.

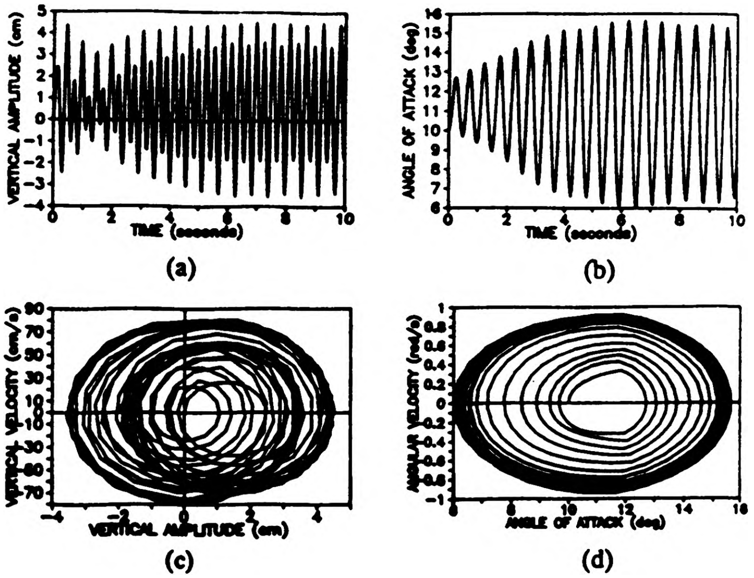

Figure 5. Results for a flutter case. 5a) Amplitude *vs*. Time. 5b) Angle of Attack *vs*. Time. 5c) Vertical Velocity *vs*. Amplitude. 5d) Angular Velocity *vs*. Angle of Attack.

Physical Experiment

The physical experimental setup meets the design goals and provides an interesting yet effective opportunity to enable the user to see the true dynamic behavior of the system. The wing used in this study has a surface area of 0.07 m^2 and a mass moment of inertia of 0.00015 kg m^2. The mass of the structure that is free to oscillate in the vertical direction is 1.0 kg. Steady oscillations begin to appear at speeds of 5 m/s using the rotational spring with the lowest rate, approximately 0.2 N m/rad. The system is usually self excited, however manual excitation is easily performed due to the experimental arrangement. The entire range of responses has been observed, including modes of flutter at different speeds. The typical ranges for the amplitudes of the vertical and rotational oscillations are 0.1 to 3.0 cm and 5° to 30° respectively. The frequency of oscillation varies from 100 cycles per minute to upwards of 500 cycles per minute. The setup can also be used to observe destructive flutter at high speeds if caution is used.

Direct quantitative comparison of the results from the two experimental setups is not easily obtained due to the current inability to measure the damping coefficients. One case has been investigated and the results of the program appear to correspond to the behavior observed by the model. The case involved low frequency oscillation (flutter) at a speed of 5.5 m/s. With the mass, surface area, spring constants and inertia known, reasonable damping coefficients were found using parametric iteration to find the calculated response that matched the behavior observed in the tunnel. Qualitative comparison is easily achieved by a general observance of the effect of the individual parameters. Both experiments are extremely sensitive to the location of the elastic axis and the rotational spring constant.

Concluding Remarks

A physical and a numerical experimental arrangement used to investigate the translational and rotational dynamic response of a rigid wing model mounted on elastic supports and subjected to steady flow is presented. Both experimental arrangements allow the user to perform parametric studies to investigate the response of the system and the effects of the parameters. These investigations enable the user to study a wide range of responses which are generally not intuitive. The two arrangements can be qualitatively compared. These experiments have been recently introduced into the aerospace laboratory course curriculum with success.

Acknowledgement

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