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DYNAMIC RESPONSE OF FORCED CONVECTIVE HEAT TRANSFER FROM CYLINDERS TO LOW PRANDTL NUMBER FLUIDS

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ABSTRACT

The present paper can be viewed as an extension of the work of Lim and Sleicher (9). They evaluated the frequency response of the heated element submerged in liquid metal by a perturbation method for Peclet numbers of up to 0.4. Velocity fluctuations were assumed small and second-order perturbations neglected. The Oseen approximation was made to the velocity field. Here the velocity configuration has been approximated to that of potential flow and the convection equation has been solved numerically with the aid of a digital computer. The potential flow approximation, as compared with the Oseen approximation, is reasonable over a larger range of Peclet numbers. Also, our scheme is valid for large amplitudes of fluctuation.

The heat response has been studied under sinusoidal variation in the free stream velocity at frequencies ranging from 1 Hz to 100 kHz for Peclet numbers of up to 1.0. The amplitude of fluctuation was 20% of the mean free-stream velocity. The Nusselt number was found to lag behind the velocity variations and the amount of lag increases with frequency and decreases as the Peclet number is increased. The amplitude of fluctuation of Nusselt number is attenuated as the frequency is increased. The attenuation is 10% at a frequency roughly given by:

$$f = .0197 \frac{\bar{U}^2}{\alpha},$$

and it is 90% at a frequency,  $f = 2.70 U^2/\alpha$ ,

where  $\bar{U}$  and  $\alpha$  are the mean free stream velocity and the thermal diffusivity of the fluid, respectively, in consistent units.

Experimental studies to verify the calculated lag and attenuation effects are in progress.

INTRODUCTION

In the last decade or so, interest has grown in measurement techniques in liquid metals primarily due to their possible application in metallurgy, nuclear reactor technology and MHD. Measurement techniques range from drag coefficient measurements to electromagnetic anemometry and thermo-anemometry.

Constant temperature thermo-anemometry has been in use with reasonable success in liquid metals for the past decade. However, one must proceed with caution in interpreting fluctuating flow measurements, such as turbulence intensity, in such low Prandtl number fluids.

This paper deals with the frequency response aspect of the constant temperature cylindrical hot-film anemometer in mercury at normal temperature. Here, we present the results of our studies of unsteady heat transfer which show that the heat transfer response of a heated element submerged in a pulsating flow is impaired because of thermal inertia of the large thermal boundary layer region (due to low Prandtl number).

Although a large number of numerical studies of the flow around a cylinder have been presented, few have incorporated heat transfer phenomena or the transient response of temperature field to velocity field. In the conventional analysis it is assumed that the temperature field adjusts itself instantaneously to variations in the velocity field. However, when the thermal boundary layer is large compared with the viscous boundary layer as occurs in fluids of low Prandtl number, the temperature field is somewhat insensitive to rapid changes in the velocity of fluid particles in the immediate vicinity of the cylinder. This thermal inertia of the surrounding fluid results in considerable phase lag and attenuation in the heat response.

Strickland and Davis (12) numerically analyzed the heat transfer response of a cylinder placed in a stream of air with small fluctuations. They also derived an empirical relationship for phase lag and frequency response.

Lighthill (8) gave an analytical solution to unsteady 2-D flow around a hot cylinder assuming the boundary layer approximation to be valid. He assumed sinusoidal fluctuations in free stream velocity and calculated lag in Nusselt number and drag coefficient along the cylinder surface. He pointed out that in the inner part of the boundary layer, lag in the local Nusselt number, with respect to its quasi-steady state value, depends upon the pressure gradient, the thicknesses of thermal and viscous boundary layers, Prandtl number and frequency. In other words, the lag depends on the Reynolds number, Prandtl number and frequency.

The publication most relevant to the present study is that of Lim and Sleicher (9). They evaluated the heat rate perturbation as a function of frequency assuming the Oseen approximation to be valid for sinusoidal fluctuation and Peclet numbers of up to 0.4. They also calculated the phase lag between heat rate and velocity.

The object of the present paper is to determine the frequency response of heat transfer phenomena for Peclet numbers up to 1.0. A

numerical scheme is presented to solve the time variant two-dimensional convection equation (energy equation) assuming potential flow. For steady flow cases, calculations with the above scheme show that error in Nusselt number is within 10% for Peclet number up to 0.4 and within 29% at Peclet number up to 1.0 when compared with experimental results of McAdams (14) (see Figure 2).

## THEORY

Mathematically, the problem is that of solving two-dimensional unsteady flow past a circular cylinder with its axis normal to the direction of flow. In other words, two time-dependent Navier-Stokes equations, the continuity equation, and the energy equation are involved. However, numerical calculations carried out using the true Navier-Stokes equations required a great deal of computer time for each Peclet number and frequency. Therefore, the only results obtained were for  $\omega = 4.71$  and  $Pe = 0.828$  and are plotted with '\*' in Figures 3, 4 and 5. The time consumed during the above computation on IBM OS/360 was about 40 minutes. This led to the decision that approximations must be sought to achieve the best trade-off between accuracy and convenience.

### Approximation

Owing to the large thermal boundary layer compared with the viscous one in low Peclet number cases such as this, the effect of velocity distribution is not so prominent in the heat transfer behavior. It is presumed that an approximation to the velocity field such as 'potential flow' can be made without much error (Figure 2).

Two Navier-Stokes equations and the continuity equation are reduced to the following two equations in terms of the vorticity and the stream function (Figure 1 shows the coordinate system):

$$\frac{2}{Pe} \cdot \frac{\partial \zeta}{\partial t} + \frac{1}{r} \left[ \frac{\partial \psi}{\partial \theta} \cdot \frac{\partial \zeta}{\partial r} - \frac{\partial \psi}{\partial r} \cdot \frac{\partial \zeta}{\partial \theta} \right] = \frac{2}{Re} \nabla^2 \zeta \quad (1)$$

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2} + \zeta = 0. \quad (2)$$

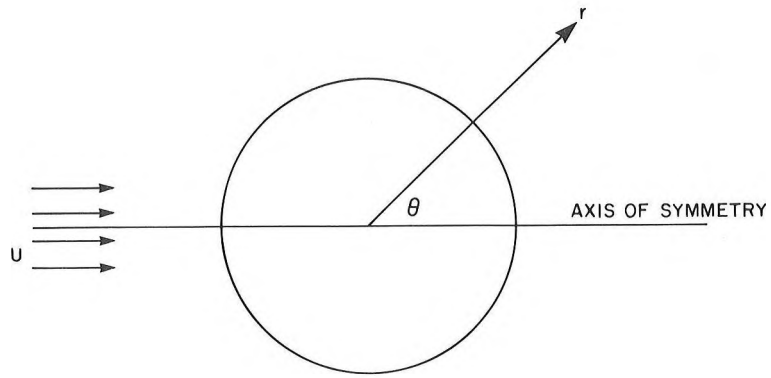


Figure 1. The Coordinate System.

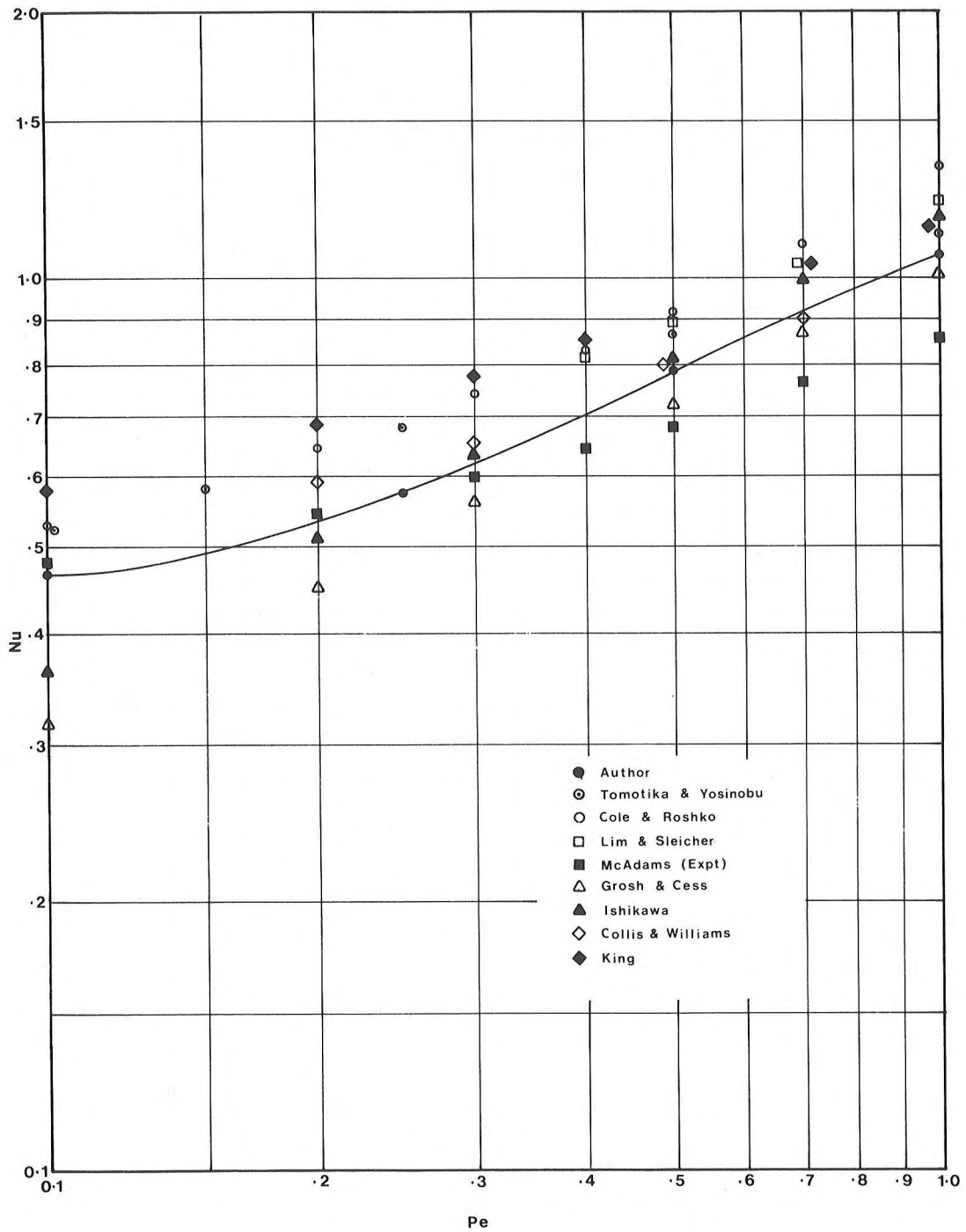


Figure 2. Nu vs. Pe for Steady Flow Past Cylinder.

The convection equation can also be written in terms of the stream function and temperature as

$$\frac{2}{Pe} \cdot \frac{\partial T}{\partial t} + \frac{1}{r} \left[ \frac{\partial \psi}{\partial \theta} \cdot \frac{\partial T}{\partial r} - \frac{\partial \psi}{\partial r} \cdot \frac{\partial T}{\partial \theta} \right] = \frac{2}{Pe} \nabla^2 T \quad (3)$$

neglecting viscous heat dissipation. For low Peclet numbers the influence of the convection term  $\frac{1}{r} \left[ \frac{\partial \psi}{\partial \theta} \cdot \frac{\partial T}{\partial r} - \frac{\partial \psi}{\partial r} \cdot \frac{\partial T}{\partial \theta} \right]$  diminishes as the diffusion term  $(2/Pe) \nabla^2 T$  becomes large. Hence an approximation to the convection term such as potential flow will affect very little the temperature field as a whole. For the sake of comparison Equation 3 was solved for steady flow for  $Pe = 0.828$  numerically by using the stream function found by solving Equations 1 and 2 in one case and by using potential flow in the other. Tsinober (13) has solved Equations 1 and 2 for  $Re = 40$  numerically. (In mercury a Reynolds number of 40 corresponds to a Peclet number of 0.828.) Nusselt numbers thus calculated from the solutions of Equation 3 indicate that Nusselt number obtained for potential flow at  $Pe = 0.828$  is only 7% higher than that for the viscous flow.

The stream function,  $\psi$ , for potential flow is given by

$$\psi(t) = \frac{U(t)}{U} r \sin \theta \left( 1 - \frac{1}{r^2} \right), \quad (4)$$

Equation 3 thus becomes

$$\frac{2}{Pe} \cdot \frac{\partial T}{\partial t} + \frac{U(t)}{U} \left[ \cos \theta \left( 1 - \frac{1}{r^2} \right) \frac{\partial T}{\partial r} - \frac{\sin \theta}{r} \left( 1 + \frac{1}{r^2} \right) \frac{\partial T}{\partial \theta} \right] = \frac{2}{Pe} \nabla^2 T. \quad (5)$$

with the boundary conditions:

$$\begin{aligned} \text{at } r = 1 & \\ & 0 < \theta < \pi \quad T = 1 \\ \text{at } r = \infty & \\ & 0 < \theta < \pi \quad T = 0 \\ \text{at } \theta = 0 \text{ and } \pi & \\ & 1 < r < \infty \quad \frac{\partial T}{\partial \theta} = 0 \end{aligned}$$

Specifically, our objective is to assign a fluctuating function to  $U(t)$  and solve Equation 5 for at least a complete cycle of fluctuation, which in turn will yield the  $Nu(t)$  function.

Let  $U(t)$  assume a sinusoidal function of the form

$$U(t) = \bar{U}(1 + a \sin \omega t). \quad (6)$$

Equation 5 was transformed to a finite difference equation and solved for  $T$  using Tsinober's numerical scheme\* (13). The Nusselt number  $Nu(t)$  was calculated from the temperature gradient near the cylinder surface for each time step. The amplitude of fluctuation,  $a$ , was assumed to be 0.2 and calculations were carried out with the help of a digital computer for mean Peclet numbers of 0.1, 0.25, 0.5 and 1.0 over a range of the non-dimensional frequency,  $\omega$ .

#### Analysis of the Results

Due to the non-linear relationship between  $Nu(t)$  and  $u(t)$ , the  $Nu(t)$  function thus obtained was not truly sinusoidal.  $Nu$  is approximately a following function of  $U$ .

$$Nu = A + BU^n \quad (7)$$

At this point, let us define  $Nu^S$  to be the Nusselt number of such a steady flow whose free stream velocity is  $U$ . It may be noted that  $Nu^S$  is not the same as  $Nu(t)$ . The latter is the instantaneous Nusselt number of an unsteady flow of which  $U(t)$  is the free stream velocity. We will refer to  $Nu^S$  as the quasi-steady Nusselt number. Our objective is to find a correlation between  $Nu^S$  and  $Nu$ . From Equations 6 and 7 we can write

$$Nu^S(t) = A + B[\bar{U}(1 + a \sin \omega t)]^n \quad (8)$$

If there were no attenuation and phase lag in  $Nu(t)$ , we would have found that  $Nu(t)$  which was numerically obtained, exactly fits Equation 8. However,  $Nu(t)$  obtained is different and is

\* A brief account of the numerical scheme is given in the appendix.

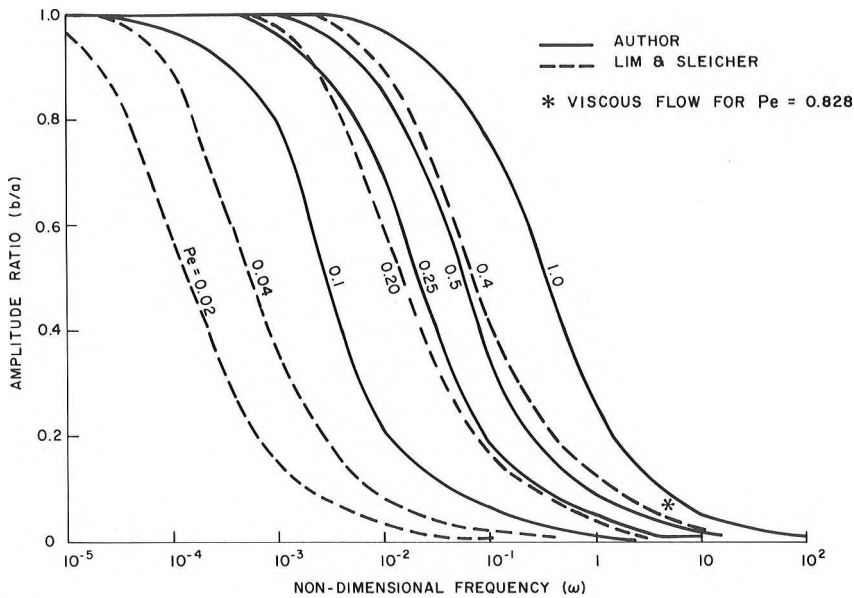


Figure 3. Amplitude Ratio vs. Non-dimensional Frequency.

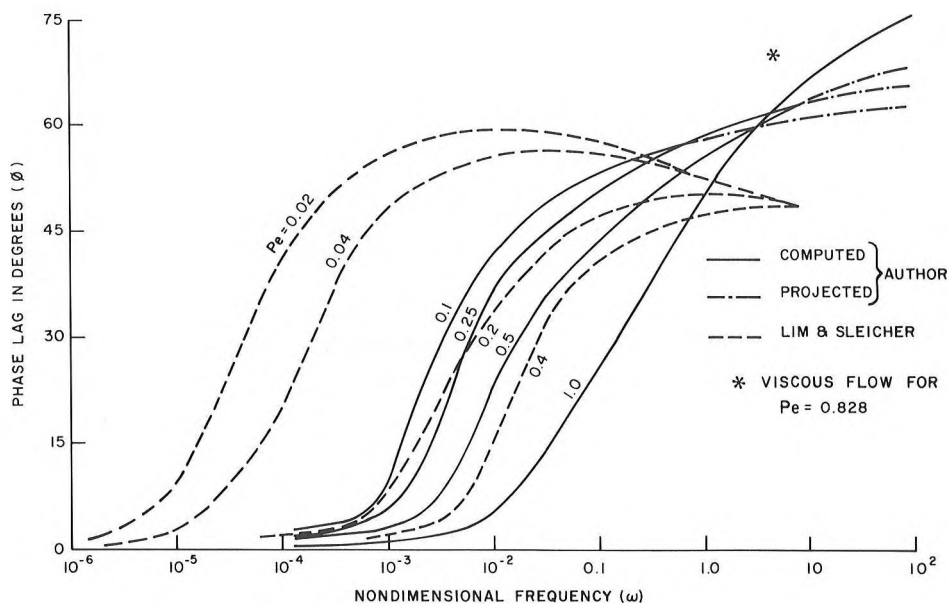


Figure 4. Phase Lag in Nu vs. Non-dimensional Frequency.

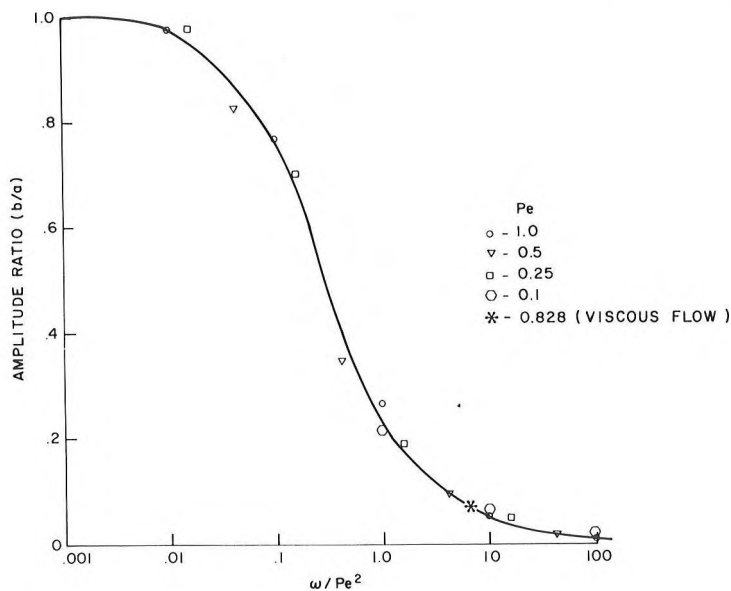


Figure 5. Amplitude Ratio vs.  $\omega/Pe^2$ .

approximately given by

$$Nu(t) = A + B [\bar{U}(1 + b \sin(\omega t + \phi))]^n \quad (9)$$

Constants  $A$ ,  $B$ ,  $b$ ,  $\phi$  and  $n$  are established from the numerical solution obtained.

The results of particular interest are the amplitude attenuation  $(1 - b/a)$  or the amplitude ratio  $(b/a)$  and phase lag  $\phi$ . Furthermore, if  $a$  and  $b$  are small

$$\begin{aligned} \frac{b}{a} &\approx \frac{(1+b)^n - (1-b)^n}{(1+a)^n - (1-a)^n} \\ &= \frac{Nu_{\max} - Nu_{\min}}{Nu_{\max}^s - Nu_{\min}^s} \end{aligned} \quad (10)$$

$Nu_{\max}$  and  $Nu_{\min}$  are the maximum and minimum values, respectively, of  $Nu(t)$  in one cycle of fluctuation.  $Nu_{\max}^s$  and  $Nu_{\min}^s$  are quasi-steady Nusselt numbers corresponding to  $U = \bar{U}(1+a)$  and  $\bar{U}(1-a)$ , respectively, and were obtained from the steady state solutions of Equation 5.

The amplitude ratio  $b/a$  (Figure 3) and phase lag  $\phi$  (Figure 4) are plotted against non-dimensional frequency,  $\omega$ , for several Peclet numbers. An interesting result is obtained when the amplitude ratio is plotted against  $\omega/Pe^2$  (Figure 5). This plot indicates that attenuation of about 10% is reached at a frequency roughly given by

$$\begin{aligned} \omega &= 0.031 Pe^2 \\ \text{or } f &= 0.0197 U^2/\alpha \text{ cycles/sec.} \end{aligned} \quad (12)$$

Figure 4 indicates that the phase lag in Nusselt number increases with frequency and tends to attain an asymptotic value. This asymptotic value is higher for higher Peclet numbers.

## CONCLUSIONS

Due to fluctuations in velocity the sensitivity of hot-wire anemometers is considerably attenuated as well as delayed due to thermal

inertia of the large thermal boundary layer. In low Prandtl number fluids, these frequencies of fluctuation are as low as those typically found in turbulent motions. Hence, caution must be taken and due compensation made in such measurements.

Comparison of the results obtained by solving exact Navier-Stokes equations with those obtained by using a potential flow approximation indicates (Figures 3, 4 & 5) that the latter is an adequate approximation for determining frequency response up to Peclet numbers of 1.0.

## SYMBOLS

$a$	amplitude of fluctuation of $U$
$b$	amplitude of fluctuation of $Nu$
$d$	diameter of cylinder (ft)
$f$	frequency of fluctuation (Hz)
$n$	index of power in Equation 7
$r$	normalized radial coordinate ( $r_f/R$ )
$r_f$	radial coordinate (ft)
$t$	normalized time unit, $\alpha t_s/R^2$
$t_s$	time (sec)
$u_r$	radial velocity (fps)
$u$	angular velocity (fps)
$A$	a constant in Equation 7
$B$	a constant in Equation 7
$Nu$	Nusselt number
$Nu(t)$	Nusselt number as a function of time
$Nu^s$	Quasi-steady Nusselt number as defined on after Equation 7
$Pe$	Peclet number, $Ud/\alpha$
$R$	radius of cylinder, $d/2$ (ft)
$T$	excess temperature, $(T_p - T_\infty)/(T_w - T_\infty)$
$T_p$	fluid temperature ( $^\circ C$ )
$T_w$	cylinder temperature ( $^\circ C$ )
$T_\infty$	free stream flow temperature ( $^\circ C$ )
$U$	free stream velocity (fps)
$\bar{U}$	mean $U$
$\alpha$	thermal diffusivity of Hg ( $ft^2/sec$ )
$\theta$	angular coordinate (rads)
$\psi$	normalized stream function defined such that

$$\frac{\partial \psi}{\partial r_f} = -\frac{u_\theta}{UR} \quad \text{and} \quad \frac{\partial \psi}{\partial \theta} = \frac{r_f u_r}{UR}$$

$\zeta$  vorticity,  $\left[ \frac{\partial u_\theta}{\partial r_f} + \frac{u_\theta}{r_f} - \frac{1}{r_f} \frac{\partial u_r}{\partial \theta} \right] \frac{R}{U}$   
 $\nu$  kinematic viscosity of Hg (ft<sup>2</sup>/sec)  
 $\omega$  non-dimensional frequency,  $2 \pi f R^2 / \alpha$   
 $\phi$  phase difference between Nu(t) and U(t), (rads)

#### Subscripts

max maximum value in one cycle  
 min minimum value in one cycle

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#### APPENDIX

Tsinober (13) used the following transformation from  $r, \theta$  coordinates to the new  $x, y$ -coordinates:

$$x = 1/r \quad y = 2\theta/\pi$$

$$r^2 = x^2 + y^2 \quad \theta = \arctan(y/x)$$

A uniform 11 x 21 grid in  $x, y$  system was chosen giving 0.1 spacing between both  $x$ - and  $y$ -grid lines. The new coordinates have the advantage that even with uniform grids, it gives denser grid lines in  $r, \theta$ -coordinates in the large gradient region close to the cylinder surface.

The differential equations to finite difference conversion was done using the following relations.

$$\frac{\partial T}{\partial x} = \frac{T_{i+1,j} - T_{i-1,j}}{2h}; \quad \frac{\partial T}{\partial y} = \frac{T_{i,j+1} - T_{i,j-1}}{2\ell}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{h^2}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\ell^2}$$

$$\frac{\partial T}{\partial t} = \frac{T^{k+1} - T^k}{\tau}$$

Here,  $i$  &  $j$  are grid points along  $x$  &  $y$  axes, respectively;  $h$  &  $\ell$  are grid spacings which were assumed 0.1 each;  $\tau$  is the time step which was assumed to be 1/20th of the time period of fluctuation.



## DISCUSSION

T. J. Hanratty, University of Illinois: I'd like to ask two questions. One is related to the high frequency calculations using potential flow and the other is concerned with whether a perturbation analysis is valid. With respect to the first question: As I understand the potential flow approximation, the thickness of the thermal boundary layer is much greater than the thickness of the hydrodynamic boundary layer. I wonder, whether this assumption breaks down at high frequencies. This would explain why the Oseen approximation gives different and perhaps more accurate results.

As for the second question: I would like to ask either Prof. Sleicher or Dr. Verma whether they have observed higher harmonics in the temperature fluctuations when they put in a given harmonic in the velocity fluctuations. This would give us some indication of how the amplitude of the velocity fluctuations limit the validity of the perturbation analysis which was carried out. I think some further numerical calculations at high amplitudes would be of great interest to see whether as you increase the amplitude of the velocity fluctuations you get higher harmonics in the temperature field.

Sleicher: That was a long question and I'm not sure I can remember all of it. Let me make clear that we did not use any boundary layer approximations here. We didn't calculate, for example, the thickness of the thermal boundary layer. That all came out of the full energy equations. What we did is assume potential flow everywhere. We assume, of course, that potential flow is undisturbed or that potential flow is a constant

property potential flow, so that there were no property variations caused by the heated probe.

Hanratty: I was wondering how good the potential flow approximation is at high frequencies. At high frequencies the temperature field is confined close to the cylinder surface where the velocity field is not described by a potential flow approximation. Have you looked into this?

Sleicher: No, we have not, but I don't think it's going to cause problems. At least it's not going to cause problems at very low Peclet numbers. If Peclet numbers are much above one there may be problems. But at Peclet numbers above one the viscous boundary layer is very small compared with the thermal boundary layer. I actually mean the thermal region that's influenced by the probe. It extends way into potential flow solutions, and potential flow solutions are very good approximations at high frequencies. It's the viscous solutions that are not good. There may be problems caused by a very thin viscous boundary layer near the probe, but I would expect those problems to be small at low Peclet numbers. As it happens, at low Peclet numbers the region of interest is outside the viscous boundary layer and that's most of the flow.

You asked about perturbation analysis and how big a perturbation would be acceptably small for such an analysis. One generally expects a perturbation of 10% is about the limit - maybe 20% at the outside. Of course, 10% is a fairly high turbulence intensity. We have some experiments at very high intensity levels - 50% to 70% - and they don't seem to be any different than at 10%. That caused no harmonics. That's a puzzling result and we're still analyzing our results. Frankly, we've not looked for higher harmonics. We can do this by taking our data that are on tape and processing it in digital computers. If the higher harmonics are there, they will show up in the spectrum. We have not done this yet.

Verma: I just want to mention this point, we do not exactly know the values of the fluctuations at very high frequencies. The numerical solutions are erroneous at high frequencies because of the

very small time steps that must be considered for high frequency. Errors can be caused in numerical methods because of this.

V. W. Goldschmidt, Purdue University: I think many of us are interested to see more comparison with experiments. I think the biggest limitation would be the thermal capacity of the element itself. You push this film of metal as far as you can, but it may not be far enough. Further, there's been a host of literature. Rodriguez, who worked here at UMR, had a loudspeaker hanging on a probe and I think somebody in Prague had a loudspeaker hanging on a probe. I believe Eckelmann at Max Planck Institute has done some of that, too. I would be very curious to see how far we can push your theory into some of those cases more common to us. Coming to the comment that Prof. Hanratty gave, we also were shaking probes at one time, and we found that it doesn't take very long before you begin to sweep back into your own wake, and the solution goes to pot when you sense the wake. The final question I would have is I don't understand the Bellhouse effect. They tell me it's a low frequency, thermal boundary layer response effect, but where does it come into your solution?

Sleicher: Let me add 2 points - I don't think we can push our theory to very high Peclet numbers. We can't push the theory to air, for example. Although the theory would apply to any fluid, I think the frequencies of interest would be applicable only in liquid metals. The problem is that the analysis has some validity to liquid metals because the viscous boundary is very small compared to the thermal boundary layer. In the case that you are talking about, there are viscous boundary layer effects where you get initial lag in the boundary layer itself. Our analysis doesn't have anything to say about that. That is a much more complicated case, and I don't know whether anybody has tried to solve that analytically or not. I think it would be a horrendous job. It was difficult to do the potential flow calculation where the velocity field was very simple. The second point was about the Bellhouse effect. If I remember correctly, that is not a problem for us because that has to do with heat transfer out of the back end of a wedge type probe. You don't have that problem with a cylindrical probe.

K. J. Bullock, University of Queensland: I am not very familiar with the very low Peclet numbers, but I

would like to make some comments about the solutions. Recently, by numerical integration, we obtained solutions for the energy and Navier-Stokes equations for a heated cylinder in air, and some of the results, might be of interest. We used both a normalized amplitude sinusoidal perturbation of 0.1 and also a step function increasing the flow by 50%. We went from Reynolds number of 1 to 1 1/2, 10 to 15 and 26 to 40. The interesting results can be summarized as, there is a highly non-linear, purely algebraic effect which is produced by both the thermal and fluid boundary layers, and a linear weighting function which can be described with a non-dimensional time constant. This may be modelled by a fairly simple first-order system following a non-linearity and the results showed that at Reynolds number of about 40, the time constant associated with the response of the boundary layers is about the order of magnitude of time required for the flow to pass a radius. Thus  $\tau = \frac{tV}{R} = 1$  at Reynolds number of 40 and increased to about 6 at Reynolds number of 1. Solutions certainly become very time consuming near Reynolds number of 1 at a Prandtl number of 1. There is a full report on this work and a paper is in the process of being published.

A. Sesonske, Purdue University: For the benefit of experimentalists, perhaps Prof. Sleicher would like to comment on what advice he would like to give to people who are trying to make measurements in light of these results.

Sleicher: Well, I think my main comment is to be aware of what you're doing. Be aware that there is a frequency response and phase-lag alternation problem. I think if you're aware of that you've gone a long way to solving the problem you work with, that is, as far as the data interpretation is concerned. As far as handling problems are concerned, there's a lot of information on that in the literature. One of the things that's very important in liquid metals is once you have immersed the probe in the metal you should never take it out again. Every time you take it out and put it back in you have to recalibrate it. You collect stuff on the probe when you put it through the surface, no matter how careful you are about the cleanliness of the surface. They do have to be calibrated frequently because of sensitivity changes with time.