1932

Investigation of continuous beams with varying moment of inertia by the method of conjugate points

Russell Arthur Bryant

INVESTIGATION OF CONTINUOUS BEAMS WITH VARYING MOMENT OF INERTIA BY THE METHOD OF CONJUGATE POINTS

by

Russell Arthur Bryant

A

T H E S I S

submitted to the faculty of the

SCHOOL OF MINES AND METALLURGY OF THE UNIVERSITY OF MISSOURI

in partial fulfillment of the work required for the

D E G R E E O F

CIVIL ENGINEER

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Approved by Joe B Butler
Professor of Civil Engineering

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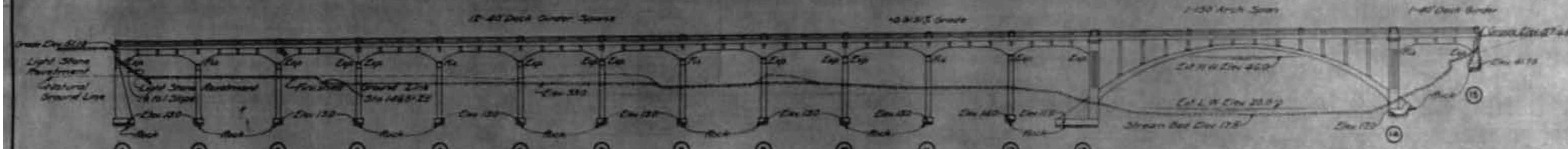
Introduction

The proper treatment of continuous beams is of constantly increasing importance in structural design. Before the general use of reinforced concrete, the application of the Theorem of Three Moments was required only in special structures. Today, concrete buildings and bridges cannot be economically designed without careful analysis of the moments in the continuous beams and slabs. The common methods of such analysis, however, are so involved and tedious that the ordinary practice of designers is to use established, arbitrary coefficients for moments in continuous beams. Needless waste of material naturally results.

This design represents the application to a practical example of a rapid and direct graphic solution of the relation expressed by the Theorem of Three Moments. This method for obtaining moments in continuous beams eliminates the necessity of using arbitrary coefficients and gives the engineer an opportunity to exercise his ingeniuty and judgment toward obtaining efficiency and economy. The use of prescribed coefficients for continuous beams makes the design mechanical and unscientific and results in moments which may be 50% too small or 60% too large.

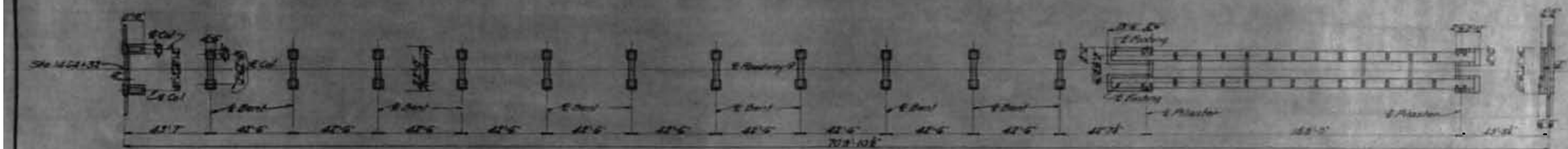
MISSOURI STATE HIGHWAY DEPARTMENT



Note:- Carry Footings under girder bents at least 18" into shale and other soft rock or 6" into solid hard rock. Footings under Piers No. 13 and 14 to be carried at least 18" into and cast against firm undisturbed hard rock.

GENERAL ELEVATION

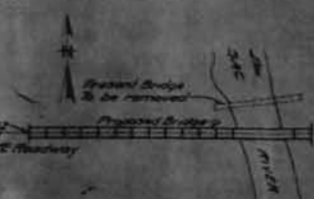
Note:- Channel to be cleared above and below site as shown on Road Plans. Banks to be maintained during construction or restored to natural position.



PLAN

Light stone revetment shall be placed on fill at End Bent No. 1 as shown in sketches. Approximately 200 sq. yds. of revetment work included in road contract.

REVETMENT SKETCHES



LOCATION SKETCH

ESTIMATED QUANTITIES			
Item	Concrete - Cu Yds		
	Class "X"	Class "Y"	Class "Z"
Handrail	69.3		
Deck Girder Superstructures		815.1	
Slab and Curbs over Arch Ring		133.3	
Spandrel Bents			51.9
Pilasters above Elev 29.38			58.7
Piers below Elev 29.38			156.4
Arch Ring between Const Joints at Pils.			146.0
Girder Bents No. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and 15			314.4
Totals	69.3	948.4	727.4
Excavation, Class 1 — Cu Yds			850
Excavation, Class 2 — Cu Yds			767
Reinforcing Steel — Lbs			270,390
Cast Iron Drains — Lbs			4130
Phosphor Bronze Bearings - 16 Sets 8"×17" Expansion @ 3 Plates each			
Phosphor Bronze Bearings - 8 Sets 20"×17" Expansion @ 3 Plates each			
Phosphor Bronze Bearings - 8 Sets 20"×17" Fixed @ 2 Plates each			
Phosphor Bronze Bearings - 2 Sets 10"×20" Expansion @ 3 Plates each			
Phosphor Bronze Bearings - 2 Sets 10"×20" Fixed @ 2 Plates each			

Bridge Excavation above Elev 22.0 will be paid for as Class 1 Bridge Excavation.
Bridge excavation below Elev 22.0 will be paid for as Class 2 Bridge Excavation.

GENERAL NOTES:-

Concrete in handrail and handrail posts shall be Class "X".
Concrete in deck girder spans and slab and curbs on arch span shall be Class "Y".
Concrete in all substructure, including girder bents, pilasters, spandrel columns and arch ring, shall be Class "Z".
All concrete shall be proportioned by the weight proportioning method. See Specifications.
Exposed edges to be beveled ¾" where no other bevel is noted.
Where bituminous felt is used in expansion or partition joints in concrete, stitch felt in vertical joint securely to one face of concrete with copper wire.
Two name plates, type "A" as shown on Std 3.918 to be furnished and placed by contractor. Cost of name plates to be included in price bid for other items.
Bridge excavation in accordance with Section 1 of Standard Specifications issued April 1, 1930, except that quantities paid for will be computed from Ext. L.W. Elev 20.0 where existing ground line is below this elevation, and for Piers No. 13 and 14 will include actual removal of any material lying within such plan limits as shown on Sheet No. 3 instead of allowing quantities to only 18" outside of footings. See Special Provisions.
See Special Provisions in regard to use of admixture, early strength concrete in arch ring, removal of arch centering, finishing concrete surfaces, sequence of pouring arch ribs, bar chairs and spacers, etc.

B.M. Elev 53.94 Spike Nail in root of 24" Post Oak 35' Right of Sta. 1471+63.

INDEX OF SHEETS

BRIDGE OVER SAC RIVER
STATE ROAD FROM STOCKTON TO FAIRPLAY
ABOUT 13.5 MILES WEST OF FAIRPLAY
PROJECT NO. R64-58 STA 1464+32
CEDAR COUNTY

Drawn Dec. 1931 by H.G.
Traced Dec. 1931 by C.A.F.
Checked Feb. 1932 by

Sheet No. 1 of 9

STD. 5-918
H-963

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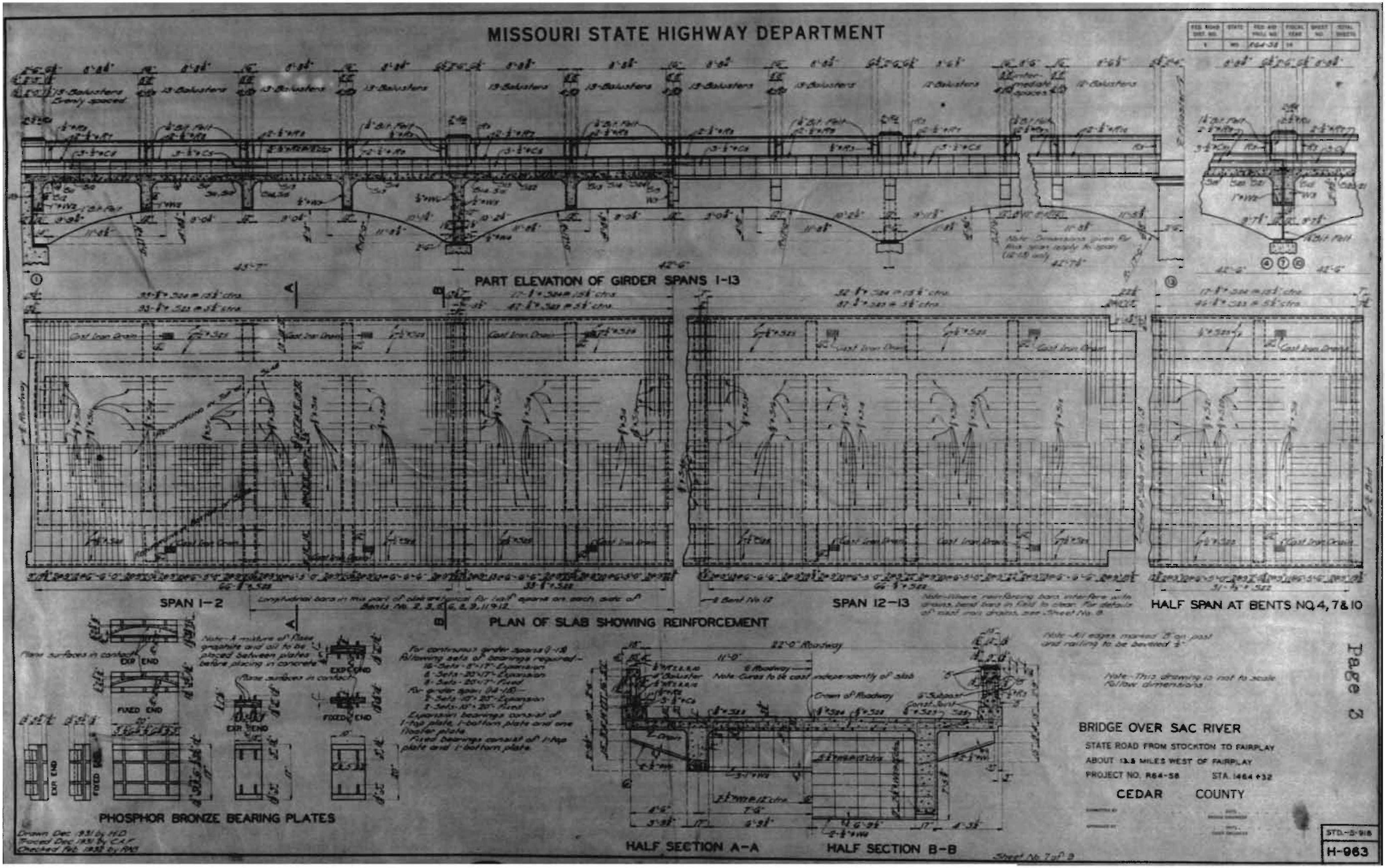
MISSOURI STATE HIGHWAY DEPARTMENT
PART ELEVATION OF GIRDER SPANS 1-13
PLAN OF SLAB SHOWING REINFORCEMENT
SPAN 1-2
SPAN 12-13
HALF SPAN AT BENTS NO. 4, 7 & 10
PHOSPHOR BRONZE BEARING PLATES
HALF SECTION A-A
HALF SECTION B-B
22'-0" Roadway
EXP. END
FIXED END
Note - This drawing is not to scale. Follow dimensions.
BRIDGE OVER SAC RIVER
STATE ROAD FROM STOCKTON TO FAIRPLAY
ABOUT 13.5 MILES WEST OF FAIRPLAY
PROJECT NO. R64-58
STA. 1464+32
CEDAR COUNTY
STD.-S-918
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The following application of the Graphical Method of Conjugate Points to a three span continuous deck girder with varying moments of inertia is an example of the practical usefulness and readability of this method.

DESIGN OF REINFORCED CONCRETE

THREE-SPAN CONTINUOUS DECK GIRDERS

(See Sheet No. 3 for details)

STANDARDS OF DESIGN

Dead Load - 15 lb.per sq.ft.in addition to slab weight

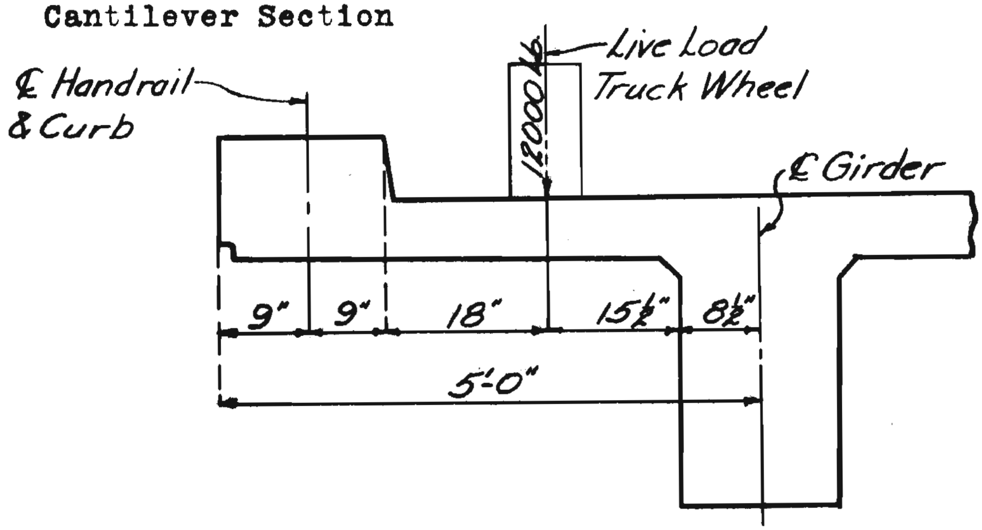
Live Load - A.A.S.H.O. H-15, One lane traffic

Distribution - Missouri State Highway Specifications

Impact - Maximum of 30%

DESIGN OF SLAB

Cantilever Section



Assume wheel load distributed so that 35% of load becomes effective as moment at face of girder. (See "Public Roads" March 1930 pp.21)

Maximum Moment at face of girder

Live Load - 35% of 1,200	= 4,200
+ 30% Impact	= 1,260
Slab - 0.77 x 150 x 4.29 x 2.14	= 1,060
Wearing Surface-15 x 2.54 x 1.27	= 49
Curb and Rail - 325 x 3.54	= 1,150
	= 7,700 ft.lb.

Beam is reinforced for both tension and compression as follows: For balanced reinforcement,

M_1 f_s pj bd^2 = 16,000 x .0097 x .862 x 12 x 7x7

= 78,800 in.lb.

A_1 pbd = .0097x12x7 = 0.815 sq. in.

and $M_2 = M - M_1$ = 91,680 - 78,800 = 12,880 in. lb.

where M_2 equals the moment resisted by the additional compression and tension steel needed and M equals the total external moment.

A_s = total area of tensile steel required

$$= A_1 + \frac{M_2}{(d-d')f_s} = 0.815 + \frac{12,880}{(7-1.87)16,000} = .97 \text{ sq.in.}$$

Use $\frac{3}{4}$ inch round bars at $5\frac{1}{2}$ "cts.

$$f_s' = f_s \times \frac{kd - d'}{d(1-k)} = 16,000 \times \frac{0.414 \times 7-1.87}{7(1-0.414)}$$

= 4,000 lb. per sq. in.

$$A'_s = \frac{M_2}{(d-d')f_s'\left(\frac{n-1}{n}\right)} = \frac{12,880}{(7-1.87)x4,000x\left(\frac{15-1}{15}\right)}$$

$$= 0.67 \text{ sq.in}$$

Use 5/8 inch round bars at $5\frac{1}{2}$"cts.

Design of Slab Panels Between Girders

Assume dead load and live load distributed over entire panel plus 50%.

Live Load 24,000 lb.+ 30% impact = 31,200

31,200 ÷ 150 = 208

208 + 50% - - - - - - - - - - 312 lb/sq.ft.

Dead Load-Slab

150x0.83 - - - - - - - - - - 125 " " "

15 " " "

452 " " "

From Spalding, Hyde & Robinson page 365

Portion of load carried by 10' span is

$\frac{15}{10} - 0.5 = 1.0$

Therefore: Load on slab in 10' length = 452 lb/sq.ft. and load on slab in 15' length = 0 but is assumed at 100 lb/sq ft.

4/3 of average load per sq. ft. is carried by mid-section.

Therefore, 4/3x452 = 603 lb/sq.ft 4/3x100=133 lb/sq.ft.

1 For 10' length

$$M = \frac{wl^2}{12} = \frac{603x12x10x10}{12} = 60,300 \text{ in.lb.}$$

from Table page 205, d = 6.75" or 7"

A_s = 0.64 sq.in. Use 5/8 inch round bars at 6"cts.

2 For 15' length

$$M = \frac{wl^2}{12} = \frac{133x12x15x15}{12} = 30,000 \text{ in.lb.}$$

Navy Bulletin No. 3Yb Solution for concentrated load for 15' span governs in this case.

$$M_p = \frac{15,600 \text{ lb}}{6} (1.32 \log.9 + 0.29)$$

$$= 50,000 \text{ in.lb.}$$

R = 50,000/12 x 6.25 x 6.25 = 108, p .0077

A_s = .0077 x 12 x 6.25 = 0.58 sq.in.

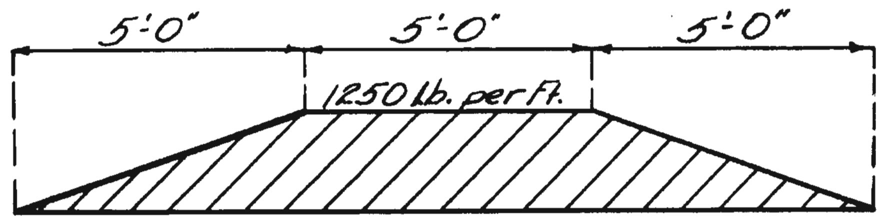
Use 5/8 inch round bars at 7"cts.

Steel has been detailed at 6" & 7" cts. from previous design and since short span detailed is less than 10' this spacing is suitable. Steel to be spaced as designed in mid section with an increase in spacing of each set for edge sections. Use 2/3 this amount of steel for Negative moment over supports.

Design of Webs Under Slab

Dead Load - 1,250 lb.per lin.ft. (slab)

" " - 450 " " " " (beam)



Slab Dead Load Distribution

Dead Load Moment 9650 x 7.5 = 72,400 ft. lbs.

-450 x 7.5 x 3.75 = -12,670

-1,250 x 2.5 x 1.25 = -39,100

-625 x 5.0 x 4.2 = -13,100

7,500 ft.lbs.

Live Load Moment

21,800 x 7.5 = 163,500

15,600 x 6.0 = 93,500

70,000 ft.lbs.

7,500

Total Moment = 77,500 ft.lbs.

$K = \frac{77,500}{41x41} = 46$, $p = .0032$

$A_s = .0032 \times 12 \times 41 = 1.5$ sq.in.

Use 3-1"round bars

Shear

D.L. = 9,625 lb.

L.L. = 21,500

31,125 lbs.

V = 17,000

V_s = 14,125 lbs

Use ½"round at 12"cts.

DESIGN OF GIRDERS - - - 3 SPAN CONTINUOUS

Dead Loads

Rail & Curb		= 3251b/lin.ft.
Slab	81x12.5x150	= 1,520 " " "
15 lb. additional	15x11	= 165 " " "
		2,010 " " "

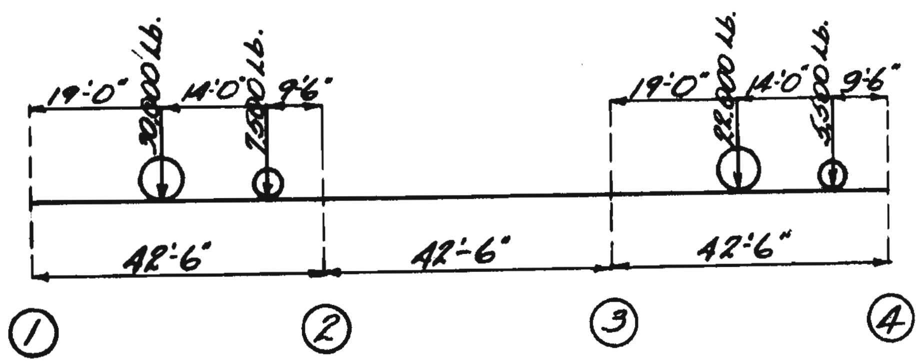
Use 2,000 Lb./Lin.Ft.

Brackets	1.375x.83x3.8x150	= 650 lb.
Webs	6.8x.83x2.8x150	= 2,370 lb.
		3,020 / 42.5 = 71 lb/lin.ft
Girder Stem	2.79x1.417x150	= 593 lb/lin.ft.
Total	2,010 71 593	= 2,700 lb.lin.ft.
		Uniform Load

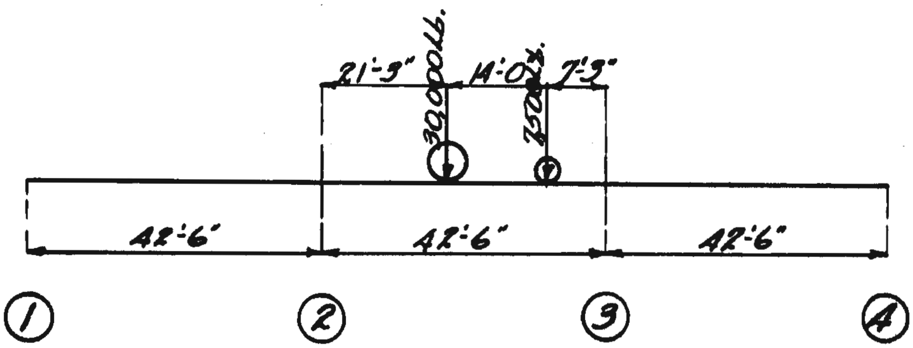
Live Loads

A.A.S.H.O. H-15 Trucks placed as follows:

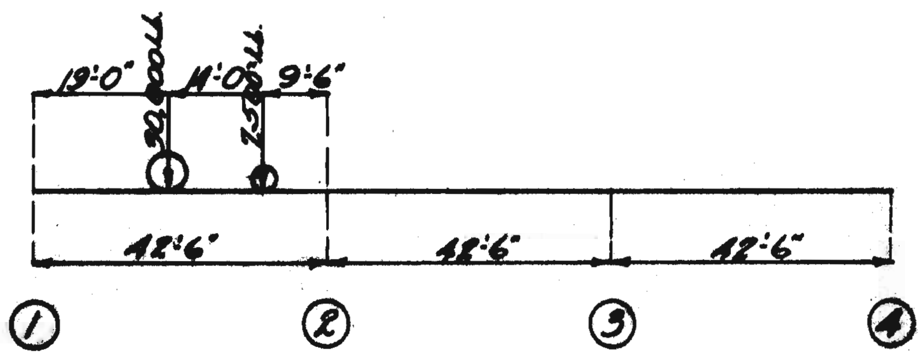
Case I



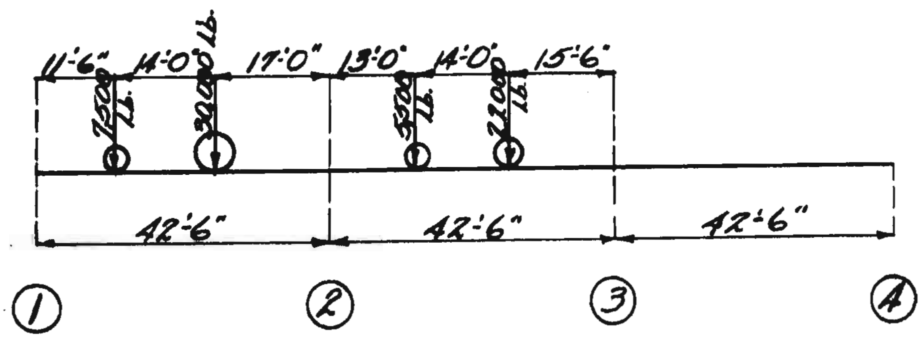
Case II



Case III



Case IV



Moments for Moment Diagrams - Simple Spans

Uniform Load of 2,700 lb/lin.ft.

Dividing girder span of 42'-6" into 1'sections moments at pts. are as follows:

Moment at

5' = 1,350 x 5 x 37.5 =253,000 ft.lbs.

10' = 1,350 x 10 x 32.5=439,000

15' = 1,350 x 15 x 27.5=556,000

20' = 1,350 x 20 x 22.5=607,000

21.25'= 2,700 x 42.25 x 42.25/8 = 610,000 ft.lbs.

25' = 1,350 x 25 x 17.5=590,000

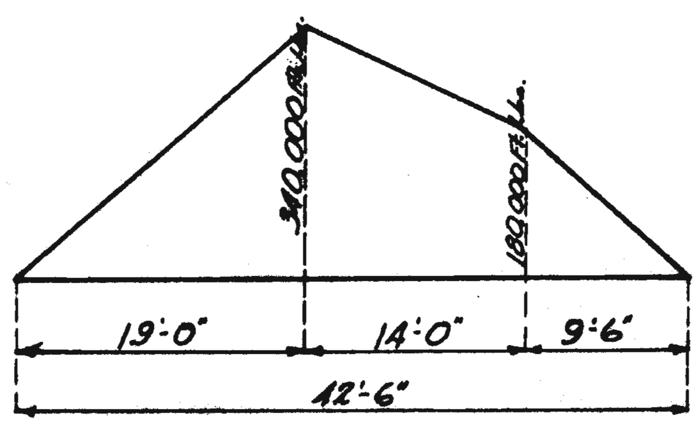
30' = 1,350 x 30 x 12.5=506,000

35' = 1,350 x 35 x 7.5 =354,000

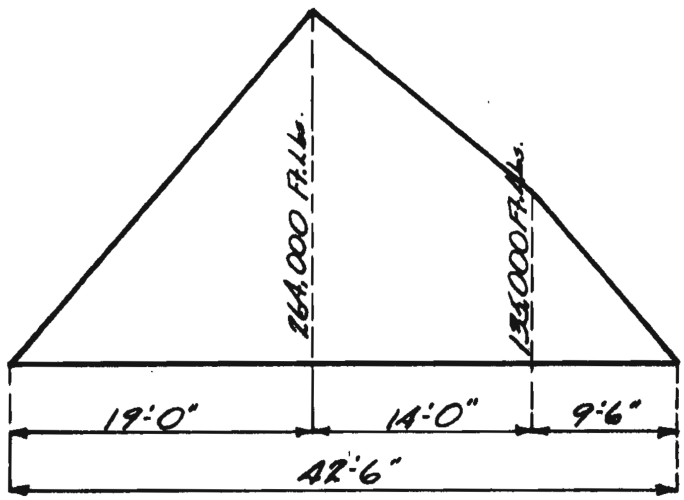
40' = 1,350 x 40 x 2.5 =135,000

Concentrated Live Loads

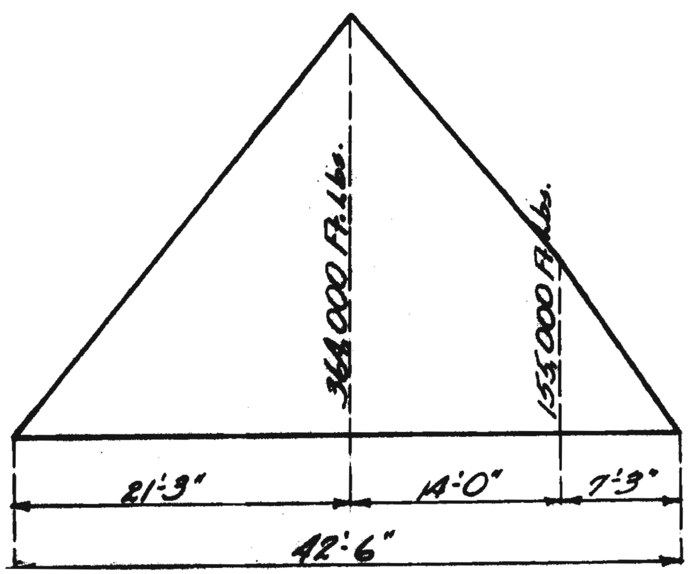
CASE I - Span 1-2 also CASE III Span 1-2



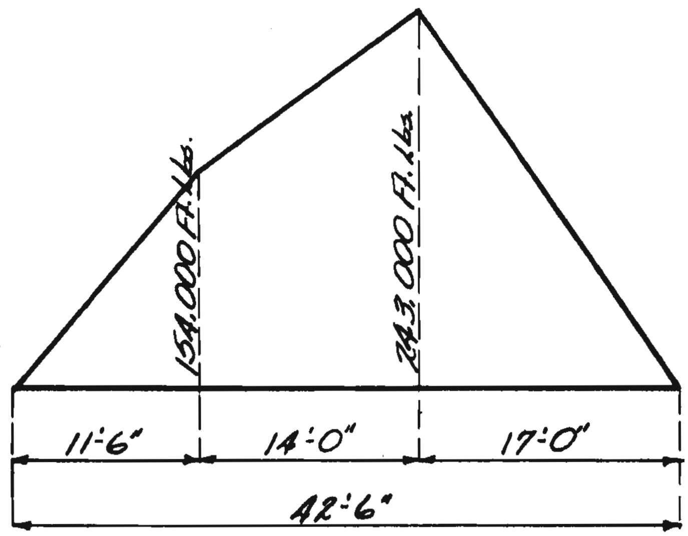
Case I Span 3-4



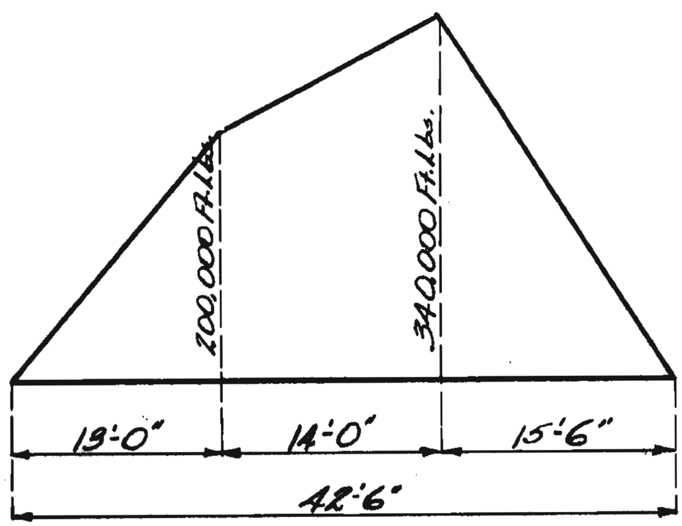
Case II Span 2-3



Case IV Span 1-2



Case IV Span 2-3



The foregoing design and simple beam moments may now be used in the graphical analysis of the girders by the method of conjugate points. For the purpose of quickly determining the behavior of the girders and also for later comparison the graphical solutions are made for each of the four cases considering the girders with a uniform moment of inertia. The method is as follows:

1 Represent the continuous beam to scale and draw on each span the moment graph on the assumption that the individual spans are simple beams. It is now necessary to locate the true datum line (or M-line) for the continuous beam.

2 For freely supported ends the M-line must pass through the extreme left and right supports.

3 On the 1/3 lines of each span locate the U-V points, such that the height of U and V above the base of the moment graphs is equal to the area of the simple beam moment graphs divided by the length of the span.

4 Locate the T - line near each intermediate reaction vertical by transposing the adjoining one-third span segments.

5 Connect V_1 - U_2 and V_2 - U_3. Where these connect-

ing lines cut the T - lines, mark the respective T - points T_{1-2} and T_{2-3}.

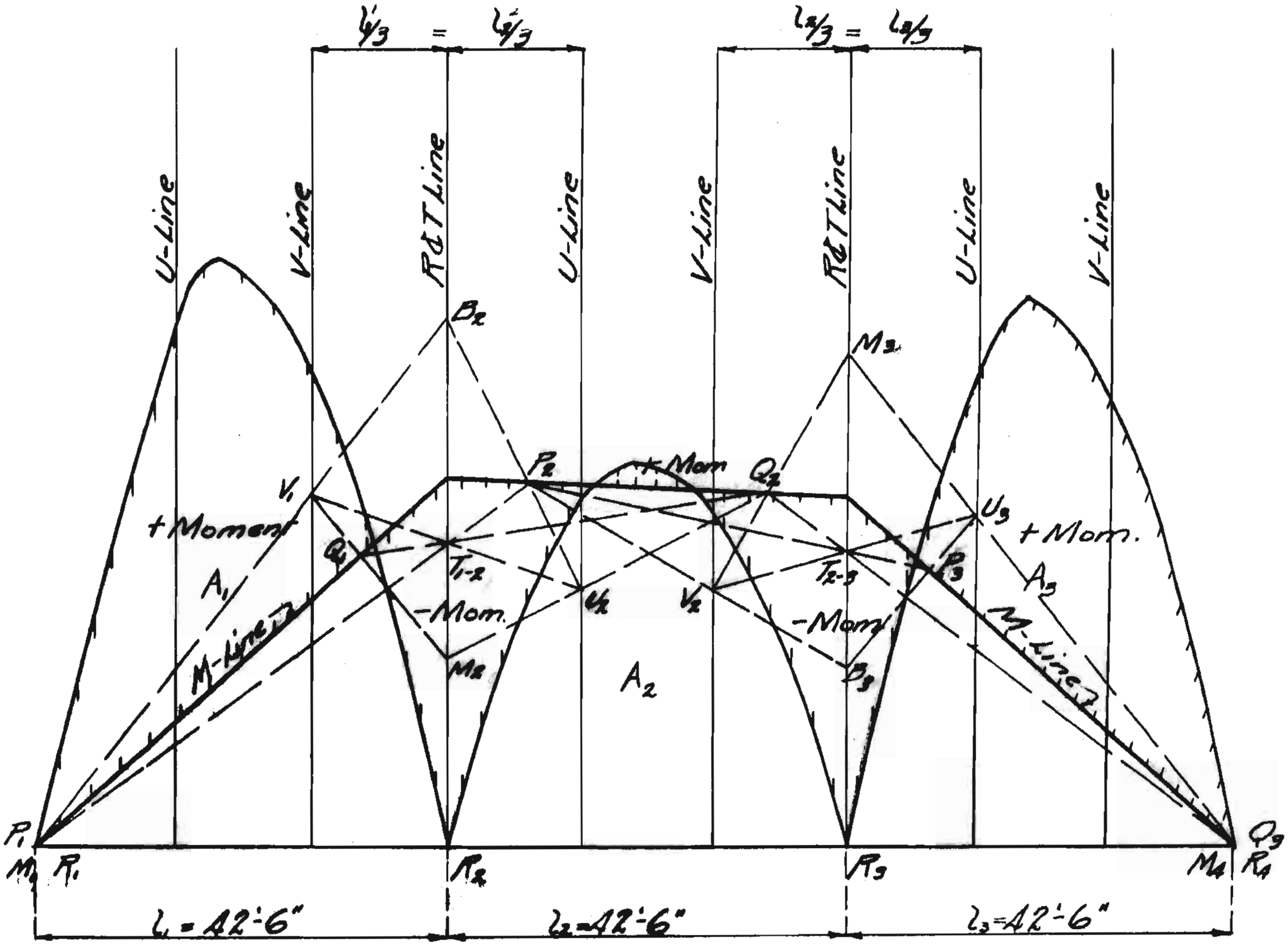
6 Draw P_1 - V_1 intersecting the R_2 - vertical at B_2 Draw B_2 - U_2. A line from P_1 through T_{1-2} intersects B_2-U_2 in the conjugate point, P_2.

7 Draw P_2-V_2 intersecting the R_3-vertical at B_3. Draw B_3-U_3. A line from P_2 through T_{2-3} intersects B_3-U_3 in the conjugate point, P_3.

8 The conjugate points, Q_2 and Q_1 may be located in similar manner by starting at the other end and working toward the left.

9 There are now two points P and Q in each span, through which the M-line, or datum line must pass. If each pair of points P_1 - Q_1, P_2-Q_2 and P_3-Q_3, are connected by a straight line, and this line is extended in each span to the reaction verticals, the broken line thus formed is the required M-line, or true datum line. Common intercepts on the intermediate support verticals will check the correctness of the work.

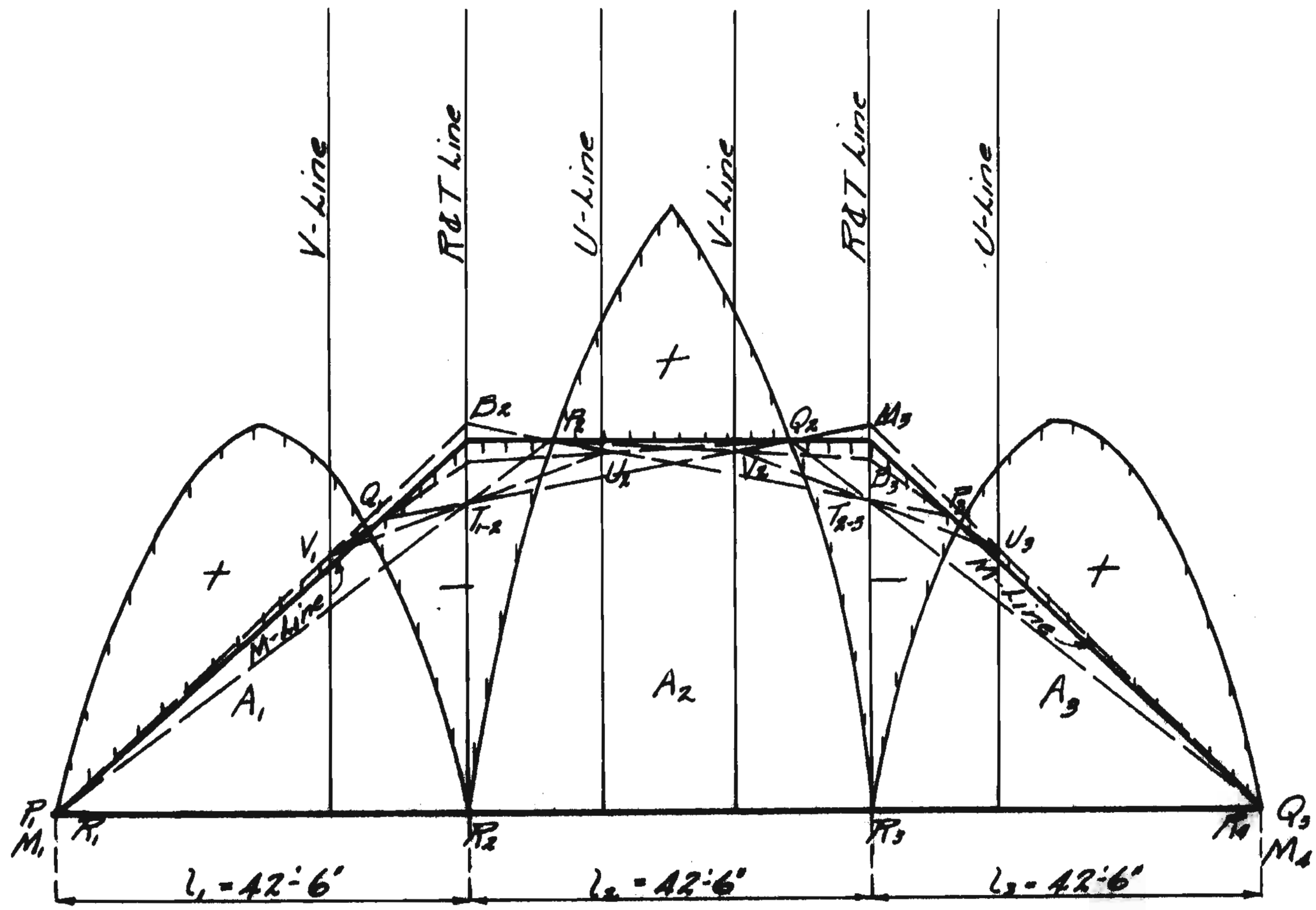
10 Vertical intercepts between the M-line or datum line and the previously drawn simple beam moment graphs represent the bending moments at the respective points of the beam. Ordinates above the



GRAPHICAL SOLUTION

CASE I

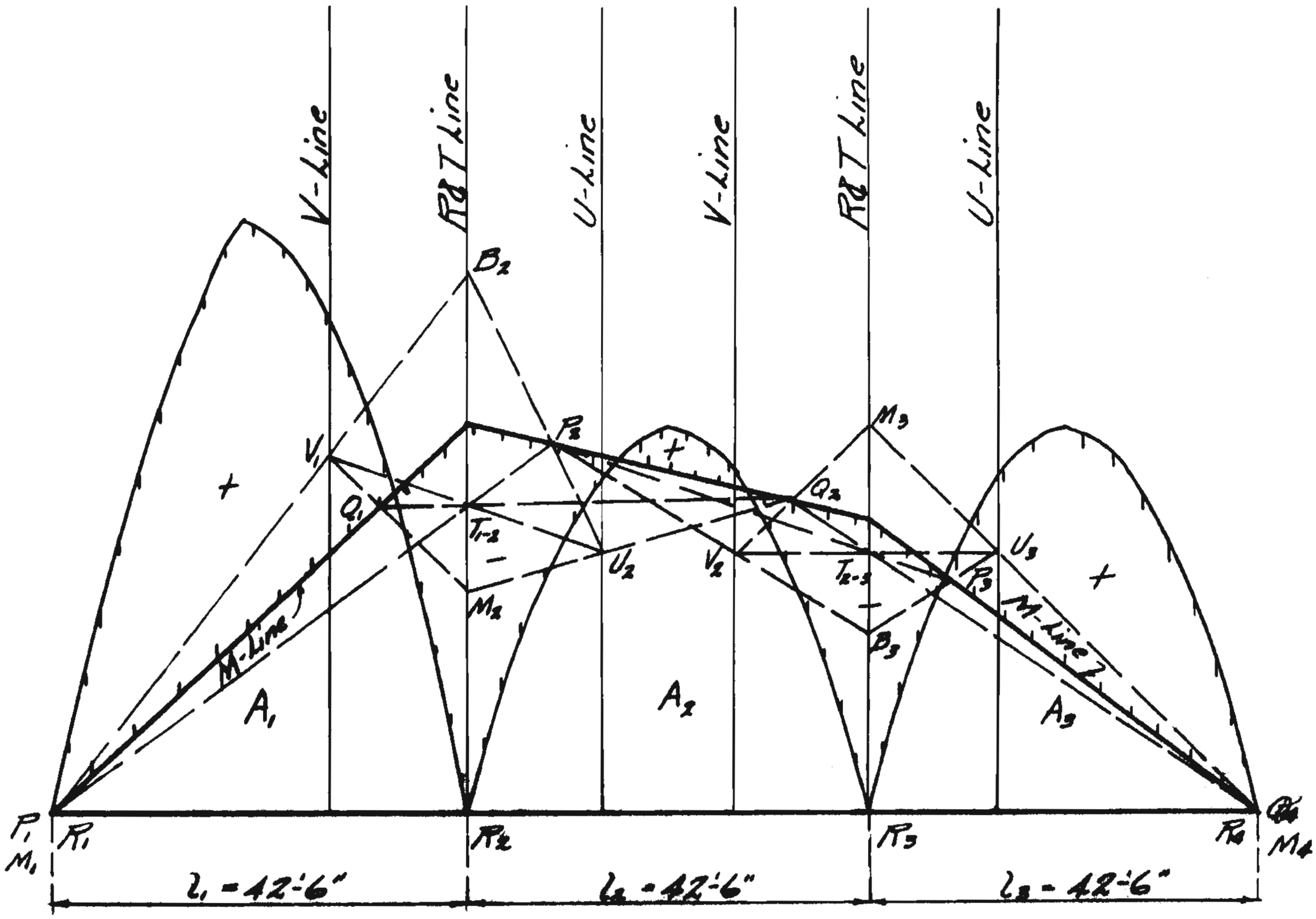
Constant Moment of Inertia



GRAPHICAL SOLUTION

CASE II

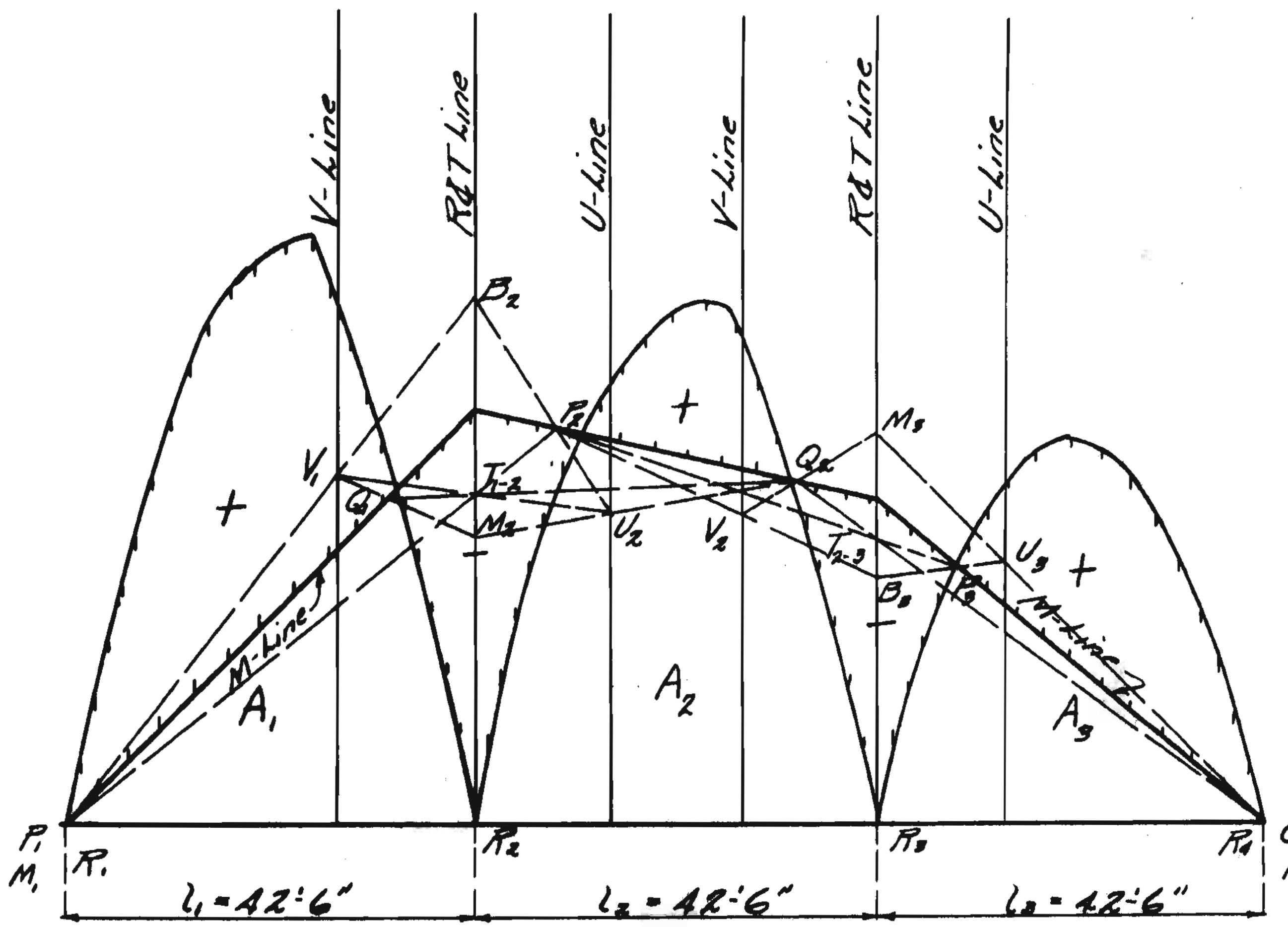
Constant "I"



GRAPHICAL SOLUTION

CASE III

Constant "I"



GRAPHICAL SOLUTION

CASE IV

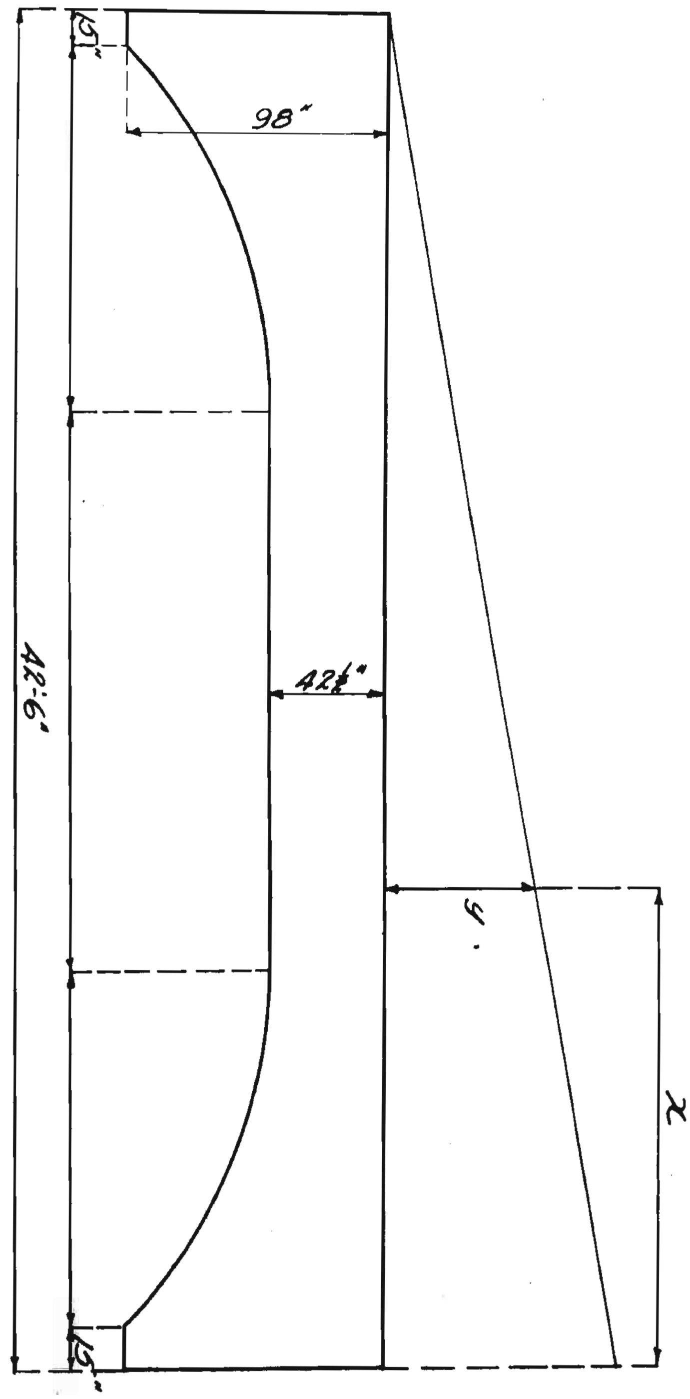
Constant "I"

closing line represent positive bending moments and ordinates below the closing line represent negative bending moments.

It should be noted that the closing line represents the moments of continuity with reversed sign.

The Method of Conjugate Points for varying I will now be used to investigate the common type of girder considered in this design.

The profile of the girder together with the necessary tabulations and summations is given in the following tables. The beam is assumed to be divided into sections 1 ft. long, and all variables refer to the mid-points of these sections. In the tabulated values, x denotes the abscissas, y the straight line ordinates of M-triangles of unit height, and y_a the ordinates of the simple-beam moment areas for A_1 1 and A_2 1. The respective ordinates divided by I give the "modified" ordinates. The applied loading is the same as used for the previous solutions.



TYPICAL BEAM SPAN

CALCULATION OF CONSTANTS

Cases 1 to 4---All Spans

Depth d	I $\frac{d^3}{1000}$	x	y	y/I	$\frac{yx}{I}$
98	941	0.625	0.9800	0.00104	0.00065
94	831	1.625	0.9560	0.00115	0.00184
84	592	2.625	0.9320	0.00157	0.00414
75	422	3.625	0.9080	0.00215	0.00780
68	315	4.625	0.8850	0.00281	0.01300
62	239	5.625	0.8600	0.00360	0.02030
56	176	6.625	0.8370	0.00476	0.03160
52	141	7.625	0.8140	0.00577	0.04400
48	111	8.625	0.7900	0.00712	0.06140
46	97	9.625	0.7670	0.00792	0.07620
44	85	100625	0.7440	0.00875	0.09300
43	80	11.625	0.7200	0.00900	0.10470
42.5	77	12.625	0.6960	0.00904	0.11400
"	"	13.625	0.6730	0.00874	0.11920
"	"	14.625	0.6500	0.00844	0.12360
"	"	15.625	0.6260	0.00813	0.12700
"	"	16.625	0.6020	0.00782	0.12970
"	"	17.625	0.5790	0.00752	0.13260
"	"	18.625	0.5560	0.00723	0.13450
"	"	19.625	0.5320	0.00692	0.13680
"	"	20.625	0.5080	0.00660	0.13610
"	"	21.625	0.4850	0.00630	0.13630
"	"	22.625	0.4610	0.00598	0.13680
"	"	23.625	0.4370	0.00568	0.13400
"	"	24.625	0.4150	0.00540	0.13300
"	"	25.625	0.3910	0.00508	0.13020
"	"	26.625	0.3670	0.00477	0.12700
"	"	27.625	0.3440	0.00447	0.12350
"	"	28.625	0.3200	0.00415	0.11900
"	"	29.625	0.2970	0.00386	0.11420
43	80	30.625	0.2730	0.00341	0.10470
44	85	31.625	0.2500	0.00294	0.09300
46	97	32.625	0.2260	0.00233	0.07620
48	111	33.625	0.2020	0.00182	0.06140
52	141	34.625	0.1790	0000127	0.04400
56	176	35.625	0.1555	0.00088	0.03160
62	239	36.625	0.1320	0.00055	0.02030
68	315	37.625	0.1087	0.00035	0.01300
75	422	38.625	0.0852	0.00020	0.00780
84	592	39.625	0.0617	0.00010	0.00410
94	831	40.625	0.0382	0.00005	0.00180
98	941	41.625	0.0147	0.00002	0.00070
			21.2500	0.18569	3.22470

Case I Span 2-3, Case II Span 1-2 & 3-4, Case III Span 2-3 & 3-4 Case IV Span 3-4

y_a (Uniform load only)	y_a/I	$y_a x/I$
0.0021	0.000002	0.000001
0.0056	0.000007	0.000011
0.0085	0.000014	0.000067
0.0113	0.000027	0.000097
0.0139	0.000044	0.000203
0.0165	0.000069	0.000389
0.0186	0.000106	0.000700
0.0207	0.000147	0.001120
0.0228	0.000206	0.001775
0.0247	0.000255	0.002460
0.0264	0.000311	0.003305
0.0278	0.000350	0.004040
0.0293	0.000381	0.004800
0.0307	0.000398	0.005440
0.0318	0.000413	0.006040
0.0329	0.000427	0.006670
0.0337	0.000438	0.007280
0.0343	0.000445	0.007860
0.0347	0.000451	0.008390
0.0352	0.000457	0.008980
0.0353	0.000458	0.009445
0.0353	0.000458	0.009930
0.0351	0.000456	0.010320
0.0348	0.000452	0.010680
0.0344	0.000447	0.011000
0.0338	0.000439	0.011250
0.0331	0.000430	0.011460
0.0322	0.000418	0.011560
0.0310	0.000403	0.011530
0.0302	0.000392	0.011620
0.0282	0.000352	0.010790
0.0266	0.000313	0.009905
0.0249	0.000256	0.008380
0.0230	0.000207	0.006970
0.0210	0.000149	0.005160
0.0189	0.000107	0.003825
0.0167	0.000070	0.002560
0.0142	0.000045	0.001696
0.0116	0.000027	0.001062
0.0087	0.000015	0.000582
0.0058	0.000007	0.000284
0.0027	0.000003	0.000119
1.0000	0.010852	0.229726

Case I Span 1-2, Case III Span 1-2

y_a	y_a/I	$y_a x/I$
0.0020	0.000002	0.000001
0.0047	0.000006	0.000010
0.0074	0.000011	0.000029
0.0099	0.000024	0.000087
0.0123	0.000039	0.000180
0.0145	0.000061	0.000343
0.0167	0.000095	0.000629
0.0187	0.000133	0.001013
0.0208	0.000188	0.001620
0.0228	0.000235	0.002260
0.0245	0.000288	0.003060
0.0262	0.000328	0.003810
0.0277	0.000359	0.004530
0.0292	0.000379	0.005170
0.0308	0.000400	0.005850
0.0322	0.000418	0.006400
0.0334	0.000434	0.007210
0.0346	0.000450	0.007940
0.0351	0.000456	0.008490
0.0353	0.000458	0.009000
0.0352	0.000457	0.009430
0.0347	0.000451	0.009760
0.0343	0.000446	0.010200
0.0337	0.000438	0.010370
0.0330	0.000429	0.010580
0.0322	0.000418	0.010720
0.0313	0.000407	0.010850
0.0302	0.000392	0.010820
0.0291	0.000378	0.010820
0.0277	0.000360	0.010600
0.0262	0.000328	0.010060
0.0247	0.000291	0.009210
0.0230	0.000237	0.007730
0.0213	0.000192	0.006460
0.0194	0.000138	0.004780
0.0171	0.000097	0.003460
0.0152	0.000064	0.002340
0.0129	0.000041	0.001543
0.0103	0.000024	0.000930
0.0079	0.000013	0.000515
0.0051	0.000006	0.000244
0.0025	0.000003	0.000125
1.0000	0.010374	0.219179

Case I Span 3-4

y_a	y_a/I	$y_a x/I$
0.0022	0.000002	0.000001
0.0051	0.000006	0.000010
0.0079	0.000013	0.000034
0.0108	0.000026	0.000094
0.0134	0.000032	0.000148
0.0160	0.000067	0.000377
0.0182	0.000103	0.000683
0.0203	0.000144	0.001100
0.0226	0.000204	0.001760
0.0246	0.000254	0.002445
0.0265	0.000312	0.003220
0.0282	0.000353	0.004100
0.0298	0.000387	0.004885
0.0314	0.000408	0.005560
0.0329	0.000427	0.006240
0.0342	0.000444	0.006940
0.0354	0.000459	0.007640
0.0365	0.000474	0.008350
0.0375	0.000487	0.009060
0.0375	0.000487	0.009550
0.0375	0.000487	0.010020
0.0370	0.000481	0.010400
0.0365	0.000474	0.010720
0.0359	0.000467	0.011020
0.0351	0.000456	0.011220
0.0342	0.000444	0.011390
0.0332	0.000432	0.011500
0.0323	0.000419	0.011580
0.0310	0.000402	0.011500
0.0296	0.000384	0.011400
0.0279	0.000348	0.010670
0.0263	0.000309	0.009780
0.0246	0.000254	0.008300
0.0227	0.000205	0.006900
0.0206	0.000146	0.005060
0.0184	0.000104	0.003700
0.0158	0.000066	0.002400
0.0137	0.000043	0.001620
0.0110	0.000026	0.001007
0.0083	0.000014	0.000555
0.0054	0.000007	0.000285
0.0026	0.000003	0.000125
1.0000	0.011060	0.233370

Case II Span 2-3

y_a	y_a/I	$y_a x/I$
0.0019	0.000002	0.000001
0.0048	0.000006	0.000010
0.0075	0.000013	0.000034
0.0102	0.000024	0.000087
0.0128	0.000041	0.000190
0.0150	0.000063	0.000355
0.0174	0.000099	0.000655
0.0194	0.000138	0.001053
0.0216	0.000195	0.001682
0.0236	0.000244	0.002348
0.0252	0.000296	0.003150
0.0269	0.000336	0.003910
0.0285	0.000370	0.004670
0.0300	0.000390	0.005320
0.0314	0.000408	0.005960
0.0329	0.000427	0.006680
0.0340	0.000442	0.007350
0.0352	0.000457	0.008050
0.0361	0.000469	0.008740
0.0369	0.000479	0.009400
0.0375	0.000487	0.010020
0.0375	0.000487	0.010400
0.0372	0.000484	0.010940
0.0365	0.000474	0.011200
0.0358	0.000465	0.011460
0.0348	0.000452	0.011590
0.0336	0.000437	0.011630
0.0324	0.000421	0.011620
0.0310	0.000402	0.011500
0.0296	0.000384	0.011400
0.0281	0.000351	0.010750
0.0269	0.000317	0.010020
0.0247	0.000255	0.008330
0.0228	0.000205	0.006900
0.0209	0.000148	0.005120
0.0192	0.000109	0.003880
0.0165	0.000069	0.002530
0.0140	0.000044	0.001655
0.0113	0.000027	0.001043
0.0087	0.000015	0.000595
0.0055	0.000007	0.000285
0.0026	0.000003	0.000125
1.0000	0.010842	0.232640

Case IV Span 1-2

y_a	y_a/I	$y_a x/I$
0.0022	0.000002	0.000001
0.0051	0.000006	0.000010
0.0078	0.000013	0.000034
0.0105	0.000025	0.000091
0.0125	0.000031	0.000143
0.0154	0.000064	0.000360
0.0177	0.000101	0 000669
0.0199	0.000141	0.001076
0.0219	0.000197	0.001700
0.0240	0.000247	0.002380
0.0257	0.000303	0.003220
0.0275	0.000344	0.004000
0.0290	0.000377	0.004760
0.0304	0.000395	0.005380
0.0315	0.000409	0.005980
0.0326	0.000423	0.006610
0.0337	0.000437	0.007270
0.0346	0.000449	0.007850
0.0353	0.000459	0.008550
0.0358	0.000465	0.009130
0.0364	0.000473	0.009760
0.0367	0.000477	0.010300
0.0371	0.000482	0.010900
0.0373	0.000485	0.011450
0.0375	0.000487	0.012000
0.0370	0.000481	0.012320
0.0360	0.000468	0.012460
0.0342	0.000444	0.012290
0.0326	0.000424	0.012150
0.0309	0.000402	0.011890
0.0291	0.000364	0.011150
0.0273	0.000321	0.010170
0.0253	0.000271	0.008850
0.0232	0.000209	0.007040
0.0209	0.000148	0.005120
0.0186	0.000106	0.003780
0.0163	0.000068	0.002490
0.0137	0.000043	0.001620
0.0112	0.000027	0.001043
0.0086	0.000015	0.000595
0.0056	0.000007	0.000285
0.0026	0.000003	0.000125
1.0000	0.011093	0.237002

Case IV Span 2-3

y_a	y_a	$y_a x/I$
0.0021	0.000002	0.000001
0.0053	0.000006	0.000010
0.0082	0.000014	0.000037
0.0109	0.000026	0.000094
0.0135	0.000032	0.000148
0.0161	0.000067	0.000377
0.0183	0.000104	0.000688
0.0206	0.000146	0.001112
0.0228	0.000205	0.001965
0.0247	0.000255	0.002455
0.0266	0.000313	0.003330
0.0284	0.000355	0.004130
0.0300	0.000390	0.004920
0.0314	0.000408	0.005560
0.0326	0.000424	0.006200
0.0337	0.000437	0.006840
0.0346	0.000449	0.007460
0.0354	0.000459	0.008090
0.0359	0.000467	0.008700
0.0364	0.000473	0.009280
0.0369	0.000479	0.009780
0.0372	0.000484	0.010480
0.0375	0.000487	0.011000
0.0375	0.000487	0.011500
0.0375	0.000487	0.012000
0.0373	0.000485	0.012420
0.0369	0.000479	0.012740
0.0359	0.000467	0.012900
0.0342	0.000444	0.012720
0.0324	0.000421	0.012480
0.0305	0.000381	0.011780
0.0287	0.000338	0.010700
0.0265	0.000273	0.008920
0.0242	0.000218	0.007340
0.0220	0.000156	0.005400
0.0196	0.000111	0.003960
0.0173	0.000072	0.002640
0.0147	0.000047	0.001770
0.0121	0.000029	0.001121
0.0089	0.000015	0.000595
0.0058	0.000007	0.000285
0.0027	0.000003	0.000125
1.0000	0.011402	0.243050

By using the constants from the Tables, the following computations can be made:

$m = n = \frac{\Sigma y/I}{42.5}$ $f = \Sigma y_a/I$

$u = v = \frac{\Sigma yx/I}{42.5\ y/I}$ $g = \frac{\Sigma y_a x/I}{42.5\ y_a/I}$

and from these values we have,

$C = mu$ or $C = m(1-u)$

$D = m(1-u)$ or $D = mu$

$E = fg$

then,

$ul = vl = u\ 42.5$

$U = V = E/C+D\ A/l$

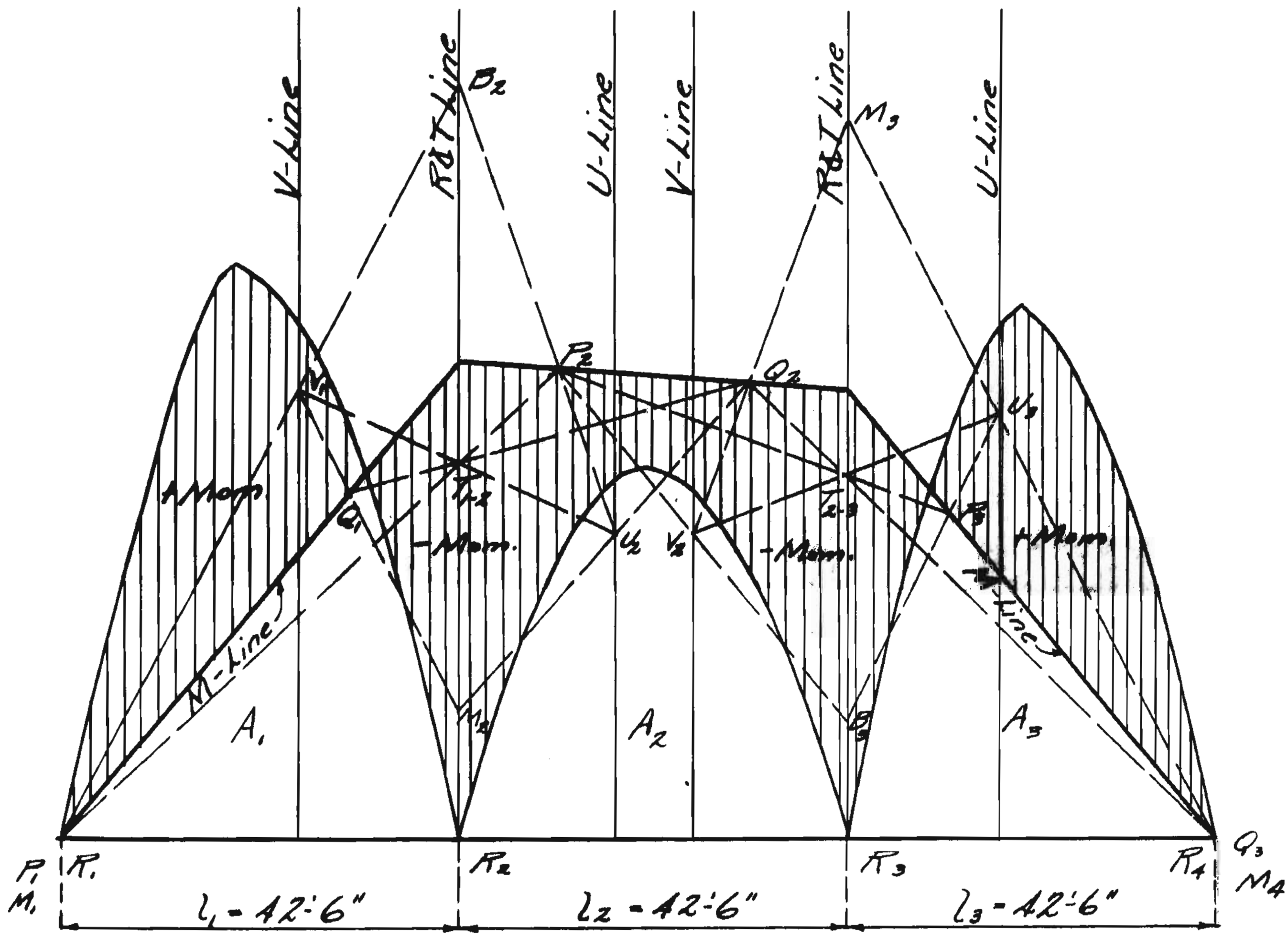
By applying these equations to each of the Cases under consideration, we have,

	m or n	u or v	f	g	C	D
CaseI						
Span 1-2	0.00437	0.409	0.01037	0.5	1.8	2.6
Span 2-3	"	"	0.01085	"	1.8 or	2.6
Span 3-4	"	"	0.01106	"	2.6	1.8
Case II						
Span 1-2	"	"	0.01085	"	1.8	2.6
Span 2-3	"	"	0.01084	"	1.8 or	2.6
Span 3-4	"	"	0.01085	"	2.6	1.8

Case III						
Span 1-2	"	"	0.01037	"	1.8	2.6
Span 2-3	"	"	0.01085	"	2.6	1.8
Span 3-4	"	"	0.01085	"	2.6	1.8
Case IV						
Span 1-2	"	"	0.01109	"	1.8	2.6
Span 2-3	"	"	0.01140	"	1.8 or 2.6	
Span 3-4	"	"	0.01085	"	2.6	1.8

	E	Vl or vl	U or V
Case I			
Span 1-2	5.2	17.35 ft.	733,000 ft.lbs.
Span 2-3	5.4	"	566,666 " "
Span 3-4	5.5	"	700,000 " "
Case II			
Span 1-2	5.4	"	500,000 " "
Span 2-3	5.4	"	760,000 " "
Span 3-4	5.4	"	500,000 " "
Case III			
Span 1-2	5.2	"	733,000 " "
Span 2-3	5.4	"	500,000 " "
Span 3-4	5.4	"	500,000 " "
Case IV			
Span 1-2	5.5	"	760,000 " "
Span 2-3	5.7	"	690,000 " "
Span 3-4	5.4	"	500,000 " "

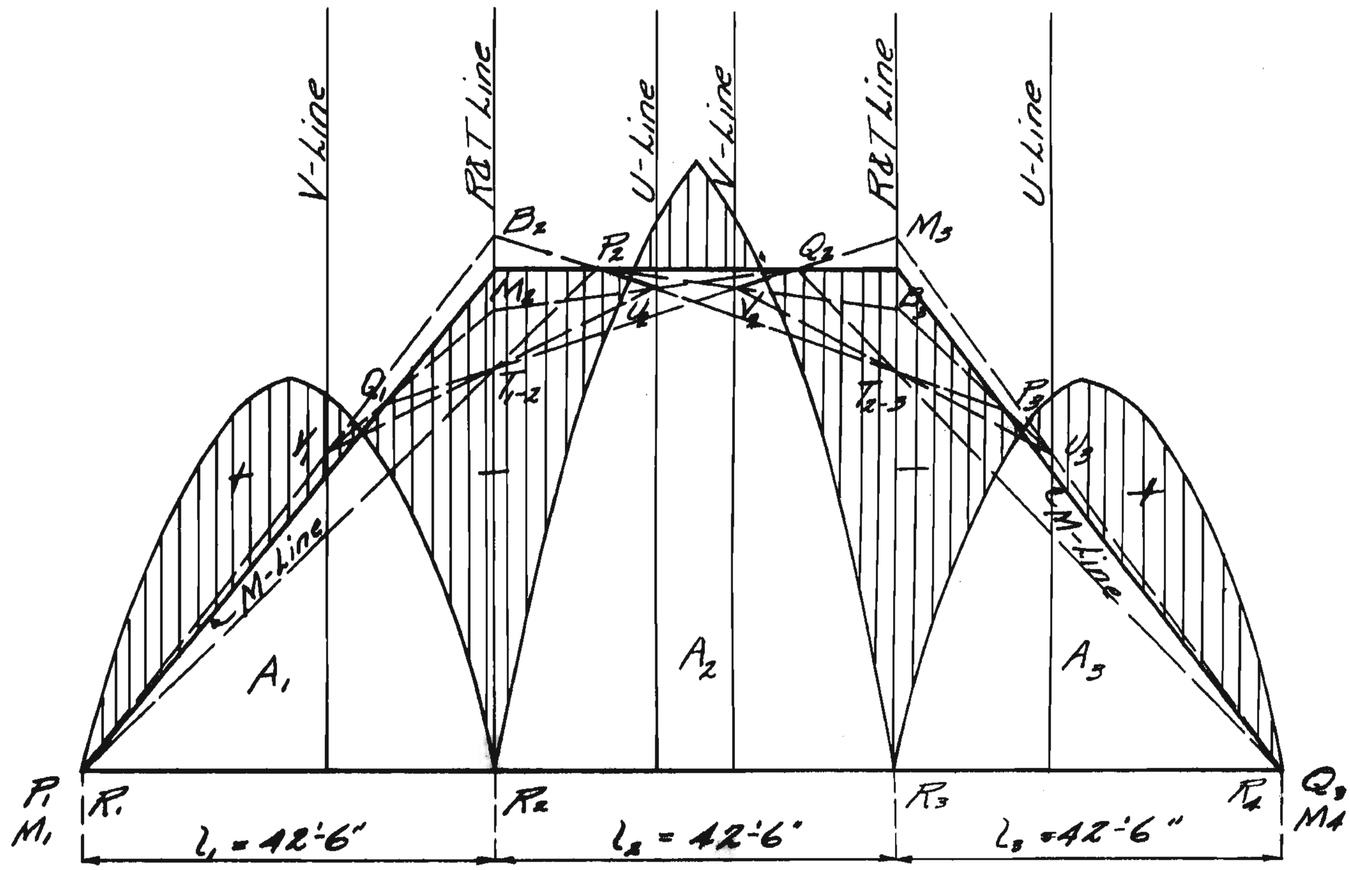
The values uI or vl give the distance from the end of the span to the respective U or V-line of the span and the values U or V give the height of the U or V point above the base line. By applying these values to new graphical solutions we are able to determine the desired reactions of the continuous beam with varying moment of inertia.



GRAPHICAL SOLUTION

CASE I

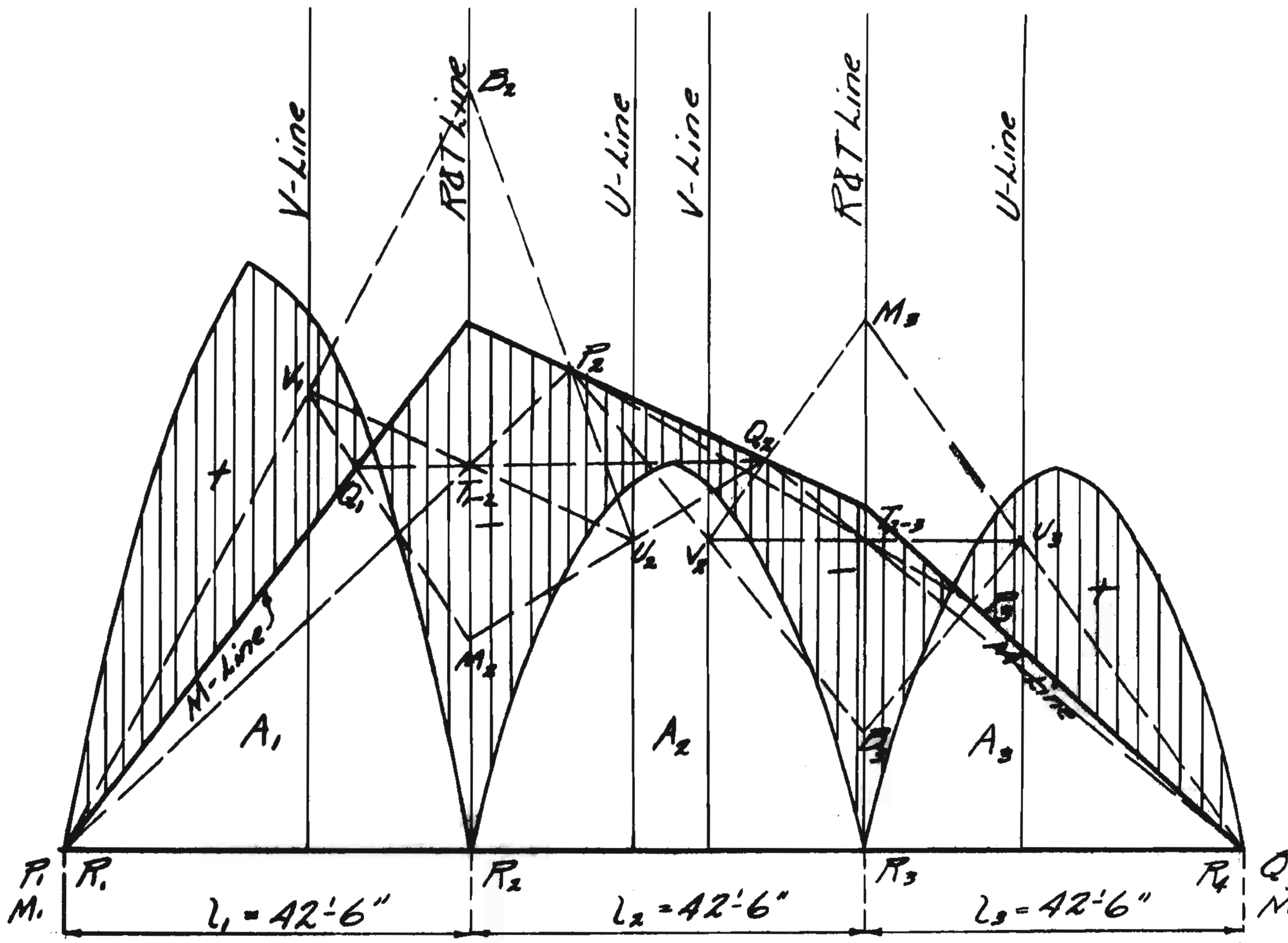
Varying "I"



GRAPHICAL SOLUTION

CASE II

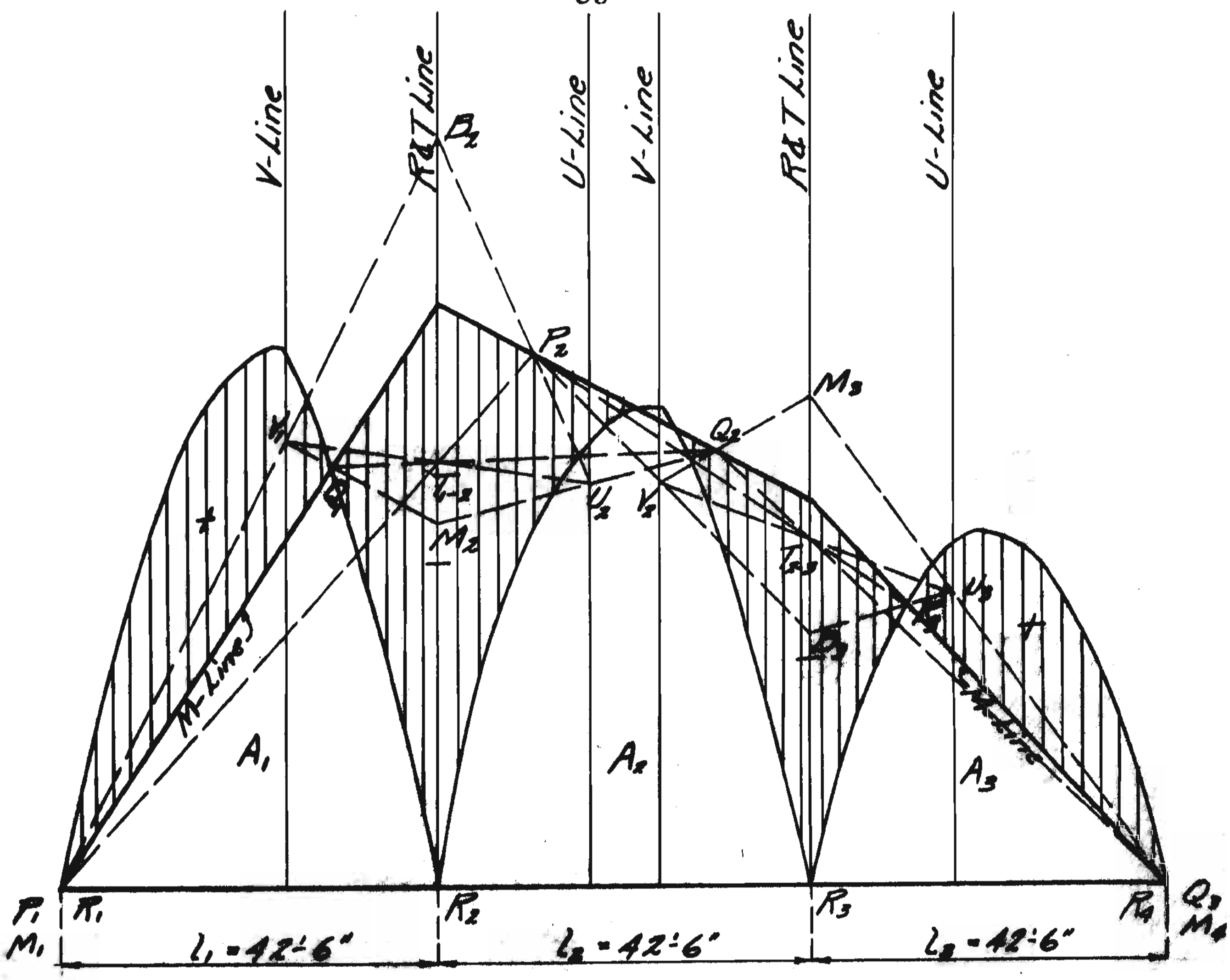
Varying "I"



GRAPHICAL SOLUTION

CASE III

Varying "I"



GRAPHICAL SOLUTION

CASE IV

Varying "I"

From the foregoing graphical solutions of the continuous beam with varying moment of inertia the maximum bending moments at the critical points may be taken by examining the solutions to find the maximum moment at the point occurring in any of the four cases.

Starting from the end of the spans the moments are as follows:

POINT	MOMENT	
12 ft.	+530,000	ft.lbs.
17 ft.	+590,000	" "
21.25 ft.(of span)	+639,000	" "
30 ft.	+180,000 &-50,000	" "
42.5 ft.(over support)	-1,000,000	" "
54 ft.	+270,000	" "
63.75 ft.(of span)	+215,000 &-160,000	" "
72 ft.	-270,000	" "
85 ft. (over support)	-1,000,000	" "
98 ft.	+180,000 &-50,000	" "
106.25 ft.(of span)	+639,000	" "
115 ft.	+530,000	" "

DESIGN OF GIRDERS FOR MOMENTS

First or last end Span --- Section A-A

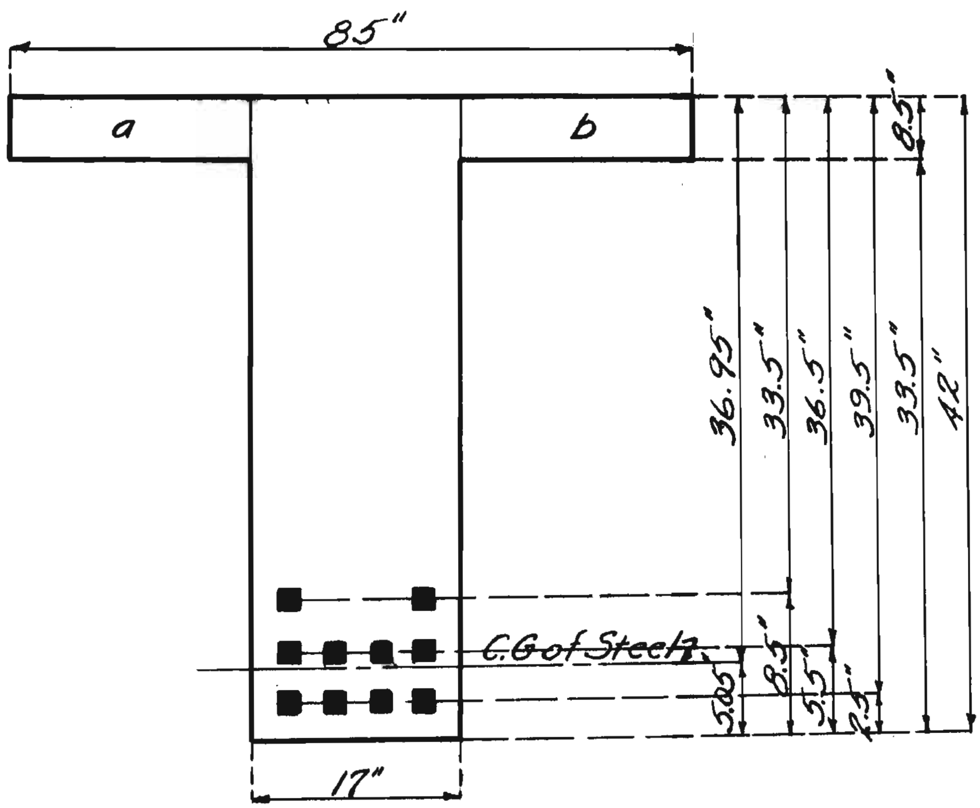
Maximum Moment = + 639,000 ft. lbs.

Girder as shown

Steel 10-1¼ sq. inches

A_s = 15.625 sq. inches

$n A_s$ = 93.75 & 46.80



a+b = 578 sq.inches

SECTION A-A

(See Sheet No.42)

Analysis -

$$578\ (y-4.25)\ \frac{17y^2}{2} = 234.3\ (36.95-y)$$

$$578y - 2460+8.5y^2 = 8,655-234.3y$$

$$8.5y^2+812y-11,115 = 0$$

$$y = \frac{-812+\sqrt{660,000+377,910}}{17} = \frac{1020-812}{17} = 12.2$$

39.5
-12.2
27.3
-3
24.3
-3
21.3

$$nIs = 46.8x21.3^2 = 21,250$$
$$= 93.75x24.3^2 = 55,300$$
$$= 93.75x27.3^2 = 69,850$$
$$I_c = \frac{85}{3}(12.2^3) - \frac{68}{3}(3.7^3) = 50,350$$
$$196,750$$

$$f_c = \frac{639,000x12x12.2}{196,750} = 458 \text{ lbs. per sq. in.}$$

$$f_s = \frac{180x639,000x27.3}{196,750} = 16,000 \text{ lbs. per sq. in.}$$

Section B-B Center of Middle Span

Maximum Moment = +214,000 ft.lbs.

and −160,000 ft.lbs.

Same Section as for Section A-A

Steel 4 - 1 1/8 sq. inches

$A_s = 5.0625$, $nA_s = 76$

Analysis-

$$578(y-4.25)\ \frac{17y^2}{2} = 76(39.5-y)$$

$$578y-2,460\ 8.5y^2 = 3,000-76y$$

$$8.5y^2+654y-5,460 = 0$$

$$y = \frac{-654+\sqrt{421,500+185,800}}{17} - \frac{125}{17} = \begin{array}{r} 39.5 \\ -7.4 \\ \hline 32.1 \end{array}$$

$$nI_s = 93.75\ \overline{32.1}^3 = 96,700$$

$$I_c = \frac{85}{3}(7.4^3) = \begin{array}{r} 11,500 \\ \hline 108,200 \end{array}$$

$$f_c = \frac{214,000 \times 12 \times 7.4}{108,200} = 175 \text{ lbs.per sq. in.}$$

$$f_s = \frac{180 \times 214,000 \times 32.1}{108,200} = 11,400 \text{ lbs.per sq.in.}$$

for -160,000 ft.lbs.@ point 15

$$K = \frac{160,000 \times 12}{70 \times 40 \times 40} = 70,\ p = .005$$

A_s = .005 x 17 x 40 = 3.4 sq. inches

Use 2-1¼ sq.in. bars

for -185,000 ft. lbs @ points 14 & 16

$$K = \frac{185,000 \times 12}{17 \times \overline{39.5}^2} = 84\ p = .0059$$

A_s = .0059 x 17 x 39.5 = 3.97 sq. inches

Use 3-1¼ in. bars

Section C-C Over Intermediate Support

Maximum Moment = -1,000,000 ft.lbs.

$$K = \frac{1,000,000 \times 12}{17 \times 94.5 \times 94.5} = 79 \quad p = .0055$$

A_s = .0055 x 17 x 94.5 = 8.85 sq.in.

Use 4-1¼ in.sq.bars & 2-1 1/8 in.sq. bars as detailed.

Section at end of haunch:-

Maximum Moment = -275,000 ft.lbs.

$K = \frac{275,000 \times 12}{17 \times 39.5 \times 39.5} = 125$ $p = .009$

$A_s = .009 \times 17 \times 39.5 = 6.0$ sq. inches

Design as double reinforced

$K = .379$, $j = .874$, $p = .0077$, $f_s = 16,000$, $f_c = 650$

$M_1 = 16,000 \times .0077 \times .874 \times 17 \times \overline{39.5}^2 = 2,850,000$ in.lbs.

$A_1 = .0077 \times 17 \times 39.5 = 5.16$ sq. inches

$M_2 = 3,300,000 - 2,850,000 = 450,000$ inch lbs.

$A_s = 5.16 + \frac{450,000}{(39.5-2.5)\times 16,000} = 5.16 + .75 = 5.91$ sq. inches

Use 4-1½ inch sq.bars to at least 13' out from intermediate support on each side of support.

DESIGN OF SECTIONS WITH VARYING DEPTH

Sections Through Haunch at Pts. 8,9,11 & 12

$cs^2 15^\circ = .934$

$cs^2 30^\circ = .75$ Use 1.25 x ordinary beam design

$d^2 = \left(\frac{M^1}{f_s pjb}\right)^{①} = \left(\frac{2 M^2}{f_c kjb}\right)^{②}$

① or ② $= \frac{16,000 \quad M}{16,000 \times .0097 \times .862 \times 17} = \frac{M}{2,270} = .000441\ M$

Depth Required:

Pt.8 $d^2 = .000441 \times 250,000 \times 12 = 1,325 = 37\frac{1}{2}$ inches

Pt. 9 d^2 .000441 x 600,000 x 12 = 3,175 = 56½ in.

" 11 d^2 " x 650,000 x 12 = 3,440 = 58½ "

" 12 d^2 " x 400,000 x 12 = 2,115 = 46 "

Steel Required:

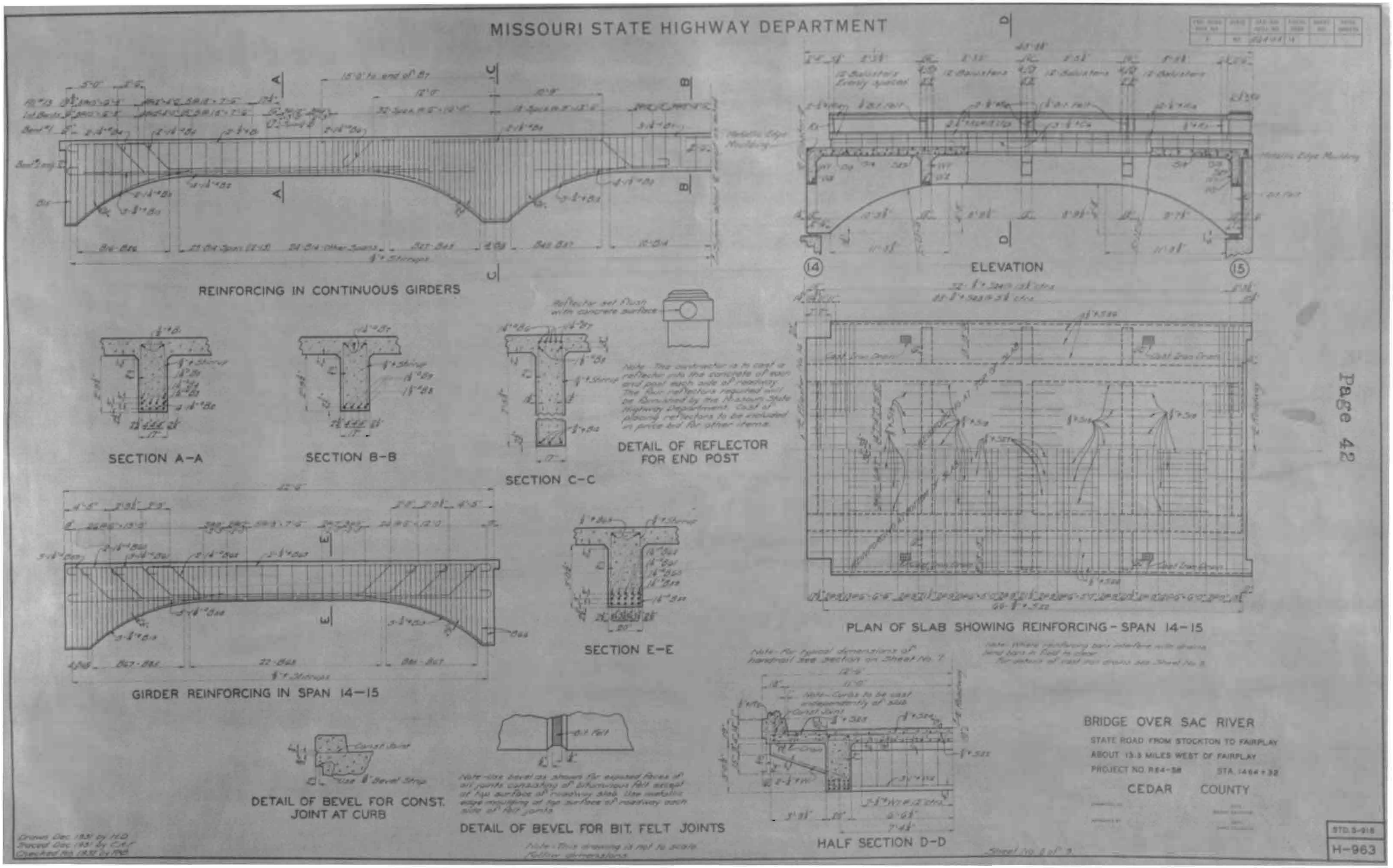
Pt. 8 A_s .0097x17x37.5=6.2 sq.in.= 4-1¼ inch sqs.

" 9 " " x "x56.5=9.32 " " = 4-1¼ " "
& 2-1 1/8 in.sqs.

" 11 " " x "x58.5=9.65 " " = 4-1¼ in.sqs. &
2-1 1/8 in.sqs.

" 12 " " x "x46.0=7.60 " " = 4-1¼ in.sqs. &
2-1 1/8 in.sqs.

Details of the continuous girders are shown on Page 42. These details show the required bars which have been called for in the design of the girders and also the method of placing and spacing the bars to secure an efficient and economical structure.


MISSOURI STATE HIGHWAY DEPARTMENT
REINFORCING IN CONTINUOUS GIRDERS
ELEVATION
SECTION A-A
SECTION B-B
SECTION C-C
DETAIL OF REFLECTOR FOR END POST
PLAN OF SLAB SHOWING REINFORCING - SPAN 14-15
GIRDER REINFORCING IN SPAN 14-15
SECTION E-E
DETAIL OF BEVEL FOR CONST. JOINT AT CURB
DETAIL OF BEVEL FOR BIT. FELT JOINTS
HALF SECTION D-D
BRIDGE OVER SAC RIVER
STATE ROAD FROM STOCKTON TO FAIRPLAY
ABOUT 13.3 MILES WEST OF FAIRPLAY
CEDAR COUNTY
H-963

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Methods of Design

Illustrations

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Moments In Restrained And Continuous Beams By The Method of Conjugate Points

American Society of Civil Engineers

Paper No. 1,598

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