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Two essays on the domain translation from financial options to real options

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TWO ESSAYS ON THE DOMAIN TRANSLATION FROM FINANCIAL OPTIONS TO REAL OPTIONS

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ABSTRACT

Real options (ROs) extend the financial option pricing theory to the valuation of real asset investment and managerial flexibility under uncertainty. However, differences between financial and non-financial markets, and the complex real world environment of applications, build obstacles for the domain translation from financial options to ROs.

This dissertation is motivated by the challenges of domain translation and developed in two essays. The first essay studies the incentive function of ROs (named the RO incentive). The essay develops an option-game framework to model the RO incentive, examines the change of investment behavior caused by the RO incentive, and values the collaboration improvement. A general framework for designing RO incentives is also developed in the essay for different forms of public-private partnerships (PPPs). The second essay focuses on dynamic capacity expansions, a representative RO application, and analyzes important factors of RO practices for the problem. These include economies of scale, capacity expansion mode, opportunity cost of waiting, terminal value of expansions, and capacity cap. Theoretical insights are obtained through the analysis, which are able to efficiently support the dynamic expansion decisions and explain observations from the numerical solution.

The work of this dissertation has reduced the gap between the option theory and RO practices. It also has built a scientific foundation for exploring advanced RO problems such as the incentive design for multiple (more than two) agents and dynamic capacity planning with resource constraints during a mission.

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1. INTRODUCTION

1.1. OVERVIEW OF REAL OPTIONS

Projects, especially those with either long lives or radical innovations, often involve a high degree of uncertainty. Such projects include infrastructure construction and operations, research and development (R&D), and new products introduction, and so on. For example, the price of raw material or the demand for a high-tech product. Uncertainty raises a number of challenges for project management. It may cause decision makers hesitant when investing in high risk projects. Uncertainty is not always a bad thing, however. It makes managerial flexibility valuable, allowing new information to be obtained as a project evolves. The decision maker can use this new information to revise a decision to increase profit, reduce loss, or both. Uncertainty may, in fact, allow a project to produce more value than originally expected.

Traditional project valuation methods, such as the discounted cash flow (DCF) approach, have crucial limitations. They do not consider the uncertainty in decisions and the ability of decision makers to flexibly react to uncertain environments [1]. The real options (ROs) approach is considered to be a more effective tool in assisting decision makers in the face of uncertainty. ROs extend the financial option theory to the valuation of investments in either physical or real assets [2]. An RO gives its owner a right, but not an obligation, to change actions as new information becomes available, thus increasing the project value by either improving its upside potential or limiting its downside losses [3]. Unlike traditional techniques based on the assumption that project cash flows will be certain, ROs enable managerial flexibility to be factored into the valuation model and show that uncertainty itself can generate value that should not be ignored. Therefore ROs provide more accurate estimates of investment returns under uncertainty and better supporting investment decisions.

In practice, decision makers may not have to choose between either investing or not investing. They may have flexibility, such as to wait and see, to expand or contract, to abandon, or to shut down and resume the project, at various points in time. These various flexibilities can be modeled as different types of ROs. Trigeorgis (1996) [4] classified classic real options into seven categories: options to defer, time to build options (staged investment), options to alter operating scale (e.g., either to expand, contract, and shut down and restart), options to abandon, options to switch (e.g., product or process flexibility), growth options, and multiple interacting options. These typical types of ROs are summarized in Table 1.1.

Since Professor Stewart Myers coined the term "real options" in 1977 [5], numerous researchers have valued investment opportunities under uncertainty in an RO approach. RO applications have been extended from nature resource investments (e.g., [6, 7]) to varies others, including manufacturing (e.g., [8, 9]), real estate (e.g., [10, 11]), R&D (e.g.,[12, 13]), and infrastructure (e.g., [14, 15]). The business community also appears to have a growing interest in ROs. Many world famous companies, such as BP [16] and Boeing [17], have adopted the RO technique for both project valuation and investment decision making.

1.2. COMPARISON BETWEEN FINANCIAL OPTIONS AND ROS

The RO valuation extends the option pricing theory to options on real assets. An RO, in many ways, resembles a financial option. For example, the opportunity to invest in a project is often seen as a call option, of which the underling asset is the present value of the project. Similarly, an option to abandon a project is analogous to a put option on the project value. The analogies between financial options and RO are summarized in Table 1.2.

Table 1.1. Typical Types of RO

Types of RO	**Description**	**Corresponding Option**
Option to defer	Hold investment opportunity to the best time	Call option
Time to build option	To commit investment in stages giving rise to a series of valuations and abandonment options	Compound option
Option to alter operating scale	To expand/contract/shut down/restart operation to meet realized demand	Call(to expand or restart)/ put (to contract or shut down) option
Option to abandon	Abandon current operations and realize the salvage value	Put option
Option to switch	Switch between different models of operation	Call option + put option
Growth option	An early investment is a prerequisite to open up future growth opportunities	call options
Multiple interacting options	The value of options affected by other options	Compound option

Source: L. Trigeorgis, 1996, *Real Options: Management Flexibility and Strategy in Resource Allocation*. MIT Press.

ROs, however, are more complicated than financial options. They are generally distinguished from financial options by several major differences as following [4]:

- Non-tradability and preemption.
  - Financial call options are traded with minimal transaction costs. ROs are not generally traded. The non-tradability of ROs may lead to early exercise. For example, a firm anticipating both an increase in demand and a competitive entry may rush to expand its own production/sales capacity early to preempt the competition. In the absence of such competition,

Table 1.2. Analogy between Financial Options and ROs

Financial Option Value Drivers		**RO Value Drivers**
Financial asset price (e.g. stock price)	S	Real asset value (e.g. Project value)
Exercise price	K	Cost to carry out the RO (e.g. capital investment)
Stock price volatility	σ	Asset value uncertainty
Time to expiration	T	Time until the opportunity expires
Risk free interest rate	r	Risk-adjusted growth rate
Dividend	q	Value leakage

Source: M.A. Brach, 2002, *Real Options in Practice*, John Wiley & Sons.

it might prefer to wait for the uncertainty surrounding future demand to resolve itself.

- Non-exclusiveness of ownership and competitive interaction.
  - Financial options on a common stock are proprietary. Only the owner can exercise it without worrying about competition for the underlying assets. Some ROs (patents, licenses) are also proprietary. Others are shared and can be exercised by any firm in the particular industry. For example, the opportunity to introduce a new product is unprotected by the possible introduction of close substitutes.
- Strategic interdependence and option compoundness.
  - Financial options are relatively independent of each other. Multiple ROs, however, may be embedded in a project. For example, a firm may have the flexibility to defer investments, either expand or contract production capacity, switch the output types, and abandon the operation in a single project. ROs are often interdependent, affecting the values of one another. ROs existing early in the decision horizon may be prerequisites for those to

follow. The presence of the later ones may impact the values of the earlier ones.

1.3. FROM FINANCIAL OPTIONS TO ROS

The path from financial options to ROs is not straightforward. It is not a simple domain extension, but rather a domain translation [18]. When compared to financial options, which are usually well-defined and traded in standard, mature financial markets, an RO is often used in non-financial markets. Great effort is required to identify options, develop models, estimate parameters, and probe solutions. The differences between financial options and ROs, and the complex real world environments, build obstacles for applying the option theory to the valuation of investments in real assets.

Assumption violation is a major problem. The options pricing theory is built on strict assumptions. Some of these are often violated in practice in the RO applications in practice (e.g., the complete market assumption and no arbitrage assumption). The no-arbitrage pricing approach in a financial option is based on the use of a portfolio in traded securities that replicates the payoff of an option. The key assumption is that the underlying assets can be traded in an efficient market. Many real assets, however, are not tradable. Thus the no-arbitrage principle seems to lose its foundation. The initial value of the underlying asset, the appropriate rate of return, and the discount rate may all be difficult to be determine [19]. Some researchers developed rectifying assumptions to support the use of the financial option pricing theory for real assets. For example, finding a traded "twin security" that is highly correlated with the real asset value[4], and proposing the market asset disclaimer [20].

The difficulty in parameter estimation is another typical issue. Unlike the values of financial assets which usually follow some well-defined stochastic processes, such as geometric Brownian motion (GMB), the evolution of real asset values may not easily be described by a simple stochastic process. Alternative stochastic processes other than GMB, such as jump processes, mean reverting processes, and combinations of these, have been used to model the processes of underlying assets (e.g., [21, 22, 23]). Estimating the model parameters of ROs is also difficult. For example, value drivers, growth rate, volatility, and interest rate may be time and/or state dependent (e.g., [24, 25]). The volatility of an underlying asset is difficult to properly estimate due to the lack of either historical data or traded option prices (e.g., [26, 20]). The exercise price could include several payments over time or be lumpy (e.g., [27]). The exercise date may be unknown in advance.

Interactions between multiple options complicate the valuation of RO as well. Multiple options may exist in a project, either in a parallel or a sequential manner. They may affect the values of one another, making the values of the multiple options non-additive. Both the pricing and interacting rules of multiple options have been partially studied in the past (e.g., [28, 29]).

Behavior interactions between competitors and collaborators also affect both the values and exercise decisions of ROs and may invalidate the traditional option valuation. Some ROs may be shared by multiple owners. Both the value and exercise strategy of an RO may be affected by the behavior of competitors due to the non-exclusiveness of ownership. For example, the value of the option can be eroded by competitors; the preemption effect reduces the threshold of ROs' exercise (e.g., [30, 4]).

Behavior interactions between collaborators can also significantly affect both the value and exercise of ROs. Some ROs are naturally embedded in the project. For example, the flexibilities in deferring the investment, expanding or contracting the capacity, or abandoning the operation. There are only RO owners but no issuers in such ROs. Other ROs may be offered by the issuers to the owners for some specific purposes. The RO issuers and owners often collaborate as principals and agents, such as a government agency and a concessionaire, or a retailer and a vendor. ROs usually act as incentives in these principal-agent relationships. The behavior dynamics of the participants in the cooperative relationships may impact the value and exercise policy of the ROs. For example, in a highway build-operate-transfer (BOT) project, a government agency offers a concessionaire the option to continue operating the project after concession. The toll cap set by the government agency may influence the decisions of the concessionaire at the toll level and construction investment. Thus affecting the value and exercise of the continuation option. Conversely, the presence of ROs can change both the option issuer and owner's behavior. For example, the continuation option provides an opportunity for the concessionaire to gain profit in a longer time. Thus may stimulate it to improve the construction and maintenance quality of the highway. The effects of behavior dynamics in the RO framework make the traditional option evaluation invalid. A game framework can be introduced to the RO evaluation to address the behavior issue. Additionally, the incentive function of the ROs is unclear and needs to be studied.

Some practical issues in RO applications challenge the option theory as well. For example, the economies of scale impact investment decisions. The increasing economy of scale favors a one-time large investment to benefit from a volume discount. The RO literature yet often suggests a sequential capital investment in that the investment is often irreversible (or at last partially irreversible) under uncertainty. The capacity expansion mode may impact investments as well. The capacity of a project may be built progressively, in infinitesimal units, as most of the RO literature assumes. For example, knowledge often grows continuously over time. Capacity may also be added in large, discrete units. For example, the expansion of highway capacity must be at least one lane. The opportunity cost of waiting also needs to be considered. The RO literature usually suggests a conservative investment (better late than early) due to the value of waiting for more information under high uncertainty. However, they often fail to consider the opportunity cost of waiting: that is, the profit forgone during the waiting time. Opportunity cost of waiting will counteract the benefit of waiting for more information and may change the optimal investment policy. Both the terminal value of the project and the cap of the capacity should also impact the behavior of capital investment.

1.4. OVERVIEW OF DISSERTATION

As mentioned in Section 1.3, the domain translation from financial options to ROs is not straightforward. Obstacles limiting efficient applications of the option theory to real assets have been observed. Some of these have been addressed previously, such as the assumption violation, parameter estimation, multiple options interaction, and behavior interactions between competitors (e.g., [4, 20, 28, 29, 30]). Other important issues, however, have not received sufficient attention. For example, behavior interactions between collaborators and the incentive function of ROs, and important practical issues in applying ROs to the investment in real assets.

This dissertation was motivated by the challenges of domain translation. Two major objectives of this dissertation are as follows.

- **To understand both the incentive function of ROs and the induced behavior issues in cooperative relationships.** This dissertation will introduce the concept of using real options as incentives (termed the RO incentives) to promote more effective collaboration. An option-game framework was built into this dissertation to model the incentive function of the ROs and show how RO incentives benefit the participants in a cooperative relationship. Behavior interaction simulated by RO incentives affect both the value and exercise of the incentives as well. These effects are evaluated in this work as well.

- **To analyze the impact of practical issues on option exercise strategies.** Economies of scale, capacity expansion mode, terminal value of the project, opportunity cost of waiting, and the cap of capacity can affect both the timing and sizing of capital investment, which will be evaluated in this dissertation. Dynamic capacity expansions, representing RO applications, will be used as an example for discussions.

This dissertation is presented in five sections. Section 2 reviews the literature relevant to this research. Section 3 is Essay One. This essay evaluates the incentive function of ROs and the behavior interactions between option issuer and owner in a typical cooperative relationship: public-private-partnerships (PPPs). Section 4 presents Essay Two. This essay addresses the practical issues of RO exercise policies for dynamic capacity expansions. Section 5 summarizes findings from the research and suggests future research.be conducted on advanced ROs.

2. LITERATURE REVIEW

This dissertation benefits greatly from the previous work on RO. The research related, either directly or indirectly, to research fields, such as RO pricing and application, challenges in applying RO in practice, RO as incentives, option game, and capacity planning using RO. This section reviews some of this related research to provide insights for the thesis.

2.1. A BRIEF OVERVIEW OF REAL OPTIONS

RO analysis is a methodology that extends the financial options theory to the valuation of physical or real assets [31]. Different from financial option which provides a right to exercise a certain action upon the financial assets, a real option is a right, but not an obligation, to take some specific action in a real asset, such as a project or business [20]. It provides a tool for evaluating strategic investments, modeling managerial flexibility under high uncertainty, and dynamic decision making in volatile environments. Myers (1977) [5] noted that the principle of financial option theory could be applied to non financial or real assets, and first coined the term real options. Over the past three decades, the RO literature has been formed into a rich repertory. Amram and Kulatilaka (1999) [32], Copeland Antikarov (2001) [20], Copeland and Antikarov (2001) [33], and Nembhard and Aktan (2009) [34] provide comprehensive expositions of this subject.

Trigeorgis (1996) [4] classified common real options into seven categories: option to defer, time-to-build option (staged investment), option to alter operating scale (e.g. to expand, to contract, to shut down and restart), option to abandon, option to switch (e.g. product flexibility or process flexibility), growth options, and multiple interacting options. A number of papers contributed to the RO literature by examining these various types of RO. For example, McDonald and Siegel(1986) [24] and Paddock et al (1988) [35] examined the option to defer and apply it into project evaluation. Myers and Majd(1990) [36] analyzed the option to abandon for salvage value. They evaluated the investment opportunities on the project embedding option to abandon and introduced the salvage value into the valuing function. Trigeorgis and Mason (1987) [37], and Pindyck (1991) [38] examined options to alter operating scale or capacity choice. Baldwin and Ruback (1986) [39] noted that the uncertainty in the future asset price generates an option to switch benefits short- term projects. Kulatilaka and Trigeorgis (1994) [40] proposed the evaluation model for the option to switch in productive factors. For the sequential investment, Carr (1988) [41] and Trigeorgis (1993) [28] dealt with valuing staged (compound) investment. Majd Pindyck (1987) [42] analyzed the option to delay sequential construction for projects that take time to build. Hevert et al (1998) [43] studied the sensitivity of growth options to the changes of interest rate brought on by inflation.

Early literature mainly focuses on valuing individual real options. However, in practice, many investment projects involve several embedded real options. The options might interact and change the value of the project as well as the optimal exercising strategies. Valuing real options in isolation has limited the practical value of real options theory. Brennan Schwartz (1985) [44] examined the combined value of the option to shut down a mine, and to abandon it for salvage. Trigeorgis (1993) [28] showed that the combined value of a collection of real options may differ from the sum of separate option values. Kogut Kulatilaka(1994) [45] analyzed the impact of interactions among a collection of real options on their optimal exercise schedules.

The option pricing theory in financial assets, which was devised by Black Scholes (1973) [46], Merton (1973) [47], and Cox et al (1979) [48], built the quantitative foundation of the real options theory. A comprehensively review on the methods of option pricing is available in Broadie and Detemple (2004)[49].

The Black-Scholes formula is the most important and broadly applied closed-form model for option pricing. It applies the financial option pricing model directly into RO evaluation. Margrabe (1978) [50] generated a analytic solution for an option to exchange one risky asset for another. The major difference between these two models is on the strike price of the option, which is treated as a certain number in Black-Scholes model and as a random variable in Margarbes model. Geske (1979) [51] valued a compound option, which may mainly be applied in sequential investment decisions, such as R& D investment projects. Based on Margarbe and Geskes work, Carr (1988) [41] examined compound exchange option with stochastic strike price.

However, the Black-Scholes formula is not sufficient for pricing some non-standard or complex RO. For example, compound options, American options, or the projects that have multiple uncertainties, state and/or time dependent parameters. Contingent claim is a more general theoretical method for option pricing. This method assumes a given stochastic process for the underlying asset, such as GBM, and then derives and solves an appropriate partial differential equation (e.g., [1, 52]).

The closed-form solution to partial differential equation rarely exists. Therefore, numerical methods, such as lattice methods or simulation, are used to approximate the solution. Lattice/tree methods are based on the seminal work of Cox et al (1979) [48]. They developed the binomial option pricing model which approximates the behavior of an asset price by the upward and downward changes in a particular interval time. Trigeorgis (1996) [4] and Mun (2002) [31] summarized the basic principles of valuing various RO via simple binomial trees. Other lattice methods include trinomial tree, adaptive mesh model, etc. The lattice/tree approach provides a more simplified method to value options. It has been widely used to price both vanilla and some exotic options.

Monte Carlo simulation, initially used by Boyle (1977) [53], is used to approximate the continuous-time stochastic process by generating discretely sampled paths.

It is a very useful technique to value American-type options, especially when more than one factor affects the value of the options. Hulland White (1987) [54] suggested a control variate technique to improve computational efficiency when there is a derivative similar to the one being valued and has an analytic solution available. Broadie and Glasserman (1997) [55] designed a Monte Carlo method to value the American financial option that incorporated early exercise, multiple-state variables, multi-choice decisions and temporal optimality. Maung and Foster (2002) [56] used Broadie and Glassermans method to simulate the option values under two marketing alternatives in the hog industry.

2.2. CHALLENGES OF REAL OPTIONS IN PRACTICE

It is challenging to translate the financial option into RO. Some researchers have identified difficulties in applying RO in practice when they establish the path from financial options to RO. For example, Lander and Pinches (1998) [19] discussed three major difficulties in applying option-based model in corporate decision-making. First, existing RO models are not well understood by practitioners. Also, using these models requires high mathematical skills. Second, many of the required assumptions in option theory are often violated in the RO application in practice. Third, mathematical tractability limits the scope of application. Miller and Park (2002) [57] summarized the drawbacks of RO assumption in the six parameters that impact the option value: underlying asset, risk, exercise price, expiration date, interest rate, and dividends.

To defend the use of financial option pricing for real assets, rectifying assumptions are developed. For example, with respect to the non-tradability of the real assets, Mason and Merton (1985) [58] argued that the real asset contributes to the market value of the publicly traded firm, and thus the real asset can be treated as if it were traded by itself. Trigeorgis (1996) [4] claimed that the returns of an RO can be replicated by a portfolio including shares of its twin security and risk-free bond. Therefore the RO value must be the no-arbitrage value of the option on its twin traded security. Copeland Antikarov (2001) [20] proposed the marketed asset disclaimer, stating that the real asset value is perfectly correlated with itself and is the best unbiased estimate of the market value of the real asset if it were traded.

To more accurately describe the evolution of the underlying assets, alternative stochastic processes other than GBM are used in the RO literature, for example, Poisson jump process, mean-reverting process, and the combination of them and the GBM. Brach and Paxson (2001) [59] provided a good survey on jump process in RO literature. Hull (2008) [2] and Copeland Antikarov (2001) [20] discussed how interest rates, commodity prices, and costs are better modeled with a mean reverting process.

Towards the discussion on the risk and the appropriate discount rate, Hull and White (1987) [54] and Hull (2008) [2] introduced the parameter of market price of risk for the underlying asset and adjusted the expected growth rate of the underlying asset by the market price of risk. Then the risk neutral valuation framework can be used in RO. Smith and Nau (1995) [60] defined a project where the market risks can be hedged and the private risks cannot be mitigated as a "partially complete market." The RO value is dependent upon the ratio of private to market risk.

To estimate a reasonable volatility, several approaches are identified in the literature, for example, twin security information and Monte Carlo simulation. Kelly (1998) [26] and Smit (1997) [7] used the futures market to estimate volatility for natural resource projects. The historical return of the twin security can be used as a proxy for the real asset volatility. Copeland and Antikarov (2001) [20] estimated the volatility of a project by Monte Carlo simulation. Miller and Park (2002) [57] applied the simulation approach in a manufacturing RO application. Mun (2002) [31] summarized the methods for volatility estimation.

2.3. OPTION GAME

Traditional RO strategies are invalid under conditions of uncertainty and competition; therefore, "true" optimal strategies should be derived from the RO models that are embedded in a game framework.

The term "option games" first appeared in Lambrecht and Perraudin (1994) [61]. In this paper, the authors develop a model under incomplete competition (duopoly) in the incomplete information conditions. Smit and Ankum (1993) [30] cast the option game approach for project timing in the different setting of market competition. This paper, considered an investment strategy, encompasses a sequence of tactical investment projects, which yield different level of returns. The results suggested that an exclusive project with large present value creates a tendency to invest early. Dixit and Pindyck (1994) [1] studied a perpetual option in a symmetric duopoly context using continuous time analysis. The two firms decide their optimal time to invest under price uncertainty. They showed that the leader will enter the new market earlier if there is no competition. Trigeorgis (1996) [4] incorporated the preemption effect on the threshold. The effect of competition is modeled as an additional dividend which is lost for the owner of the real option. This parameter significantly reduces the threshold. Grenadier (1996) [62] developed an option-game model for real estate investment timing. It considers the time to build effect on option value. A two-builders sub-game perfect equilibrium is developed, finding a pair of symmetric Markovian exercise strategies. Smit and Trigeorgis (1997) [63] analyzed two stage games where investment opportunity value depends on endogenous competitive interactions using real options approach combined with game theory. They study the optimal decisions of timing and output levels of firms with asymmetric production costs, illustrate the trade- off between the value of waiting and the strategic commitment value under different competition structures. Kulatilaka and Perotti (1998) [64] studied the investment strategies under uncertainty and imperfect competition where there is a first mover advantage brought by investing in a strategic growth option. Huisman and Kort (2003) [65] analyzed the new technology adoption strategies in a duopoly setting. It was shown that, under a certain scenario, the strategies of the two firms turn from competition into joint adoption. Perotti and Kulatilaka (1999) [66] considered the decision to invest in a time-to-market option under Cournot quantity competition with the first mover benefit. They conclude that the value of such option is unambiguously increasing in demand uncertainty, and higher uncertainty level justifies earlier exercise of the option. Grenadier (1999) [67] took the situation of asymmetric information into account to develop a more general equilibrium framework for option exercise games. It is found that an informational cascade can arise endogenously when all the agents exercise immediately. Pawlina and Kort (2001) [68] examined the impact of investment cost asymmetric on the value of firm and on the optimal strategies of exercising real options under imperfect competition. Different levels of cost asymmetry result in different type of equilibriums. Weeds (2002) [69] derived a continuous time, duopoly option- game framework to study optimal investment strategies for firms competing for a patent with uncertainties in the probability of technological success of the project and in the economic value of the patent. Economic uncertainty generates a tendency of waiting. However, the fear of preemption counteracts the incentive of delay. Grenadier (2002) [70] provided a general and tractable solution approach for deriving equilibrium investment strategies in a continuous-time Cournot-Nash oligopolistic setting. It finds an equilibrium that is analytically simple and potentially widely applicable. The impact of competition on exercise strategies leads to a rapid erosion in the option to wait and brings the investment trigger to a level that is very near the zero net present value. Lambrecht and Perraudin (2003) [71] incorporated incomplete information and preemption into an equilibrium model in which groups of firms invest strategically. It suggested that the optimal investment strategy may lie anywhere between the zero-NPV trigger level and the optimal strategy of a monopolist, depending on the distribution of competitors costs and the implied fear of preemption. Huisman and Kort (2003) [65] treated the technology adoption decision of a firm in a duopoly framework. Outcomes ranged from preemption equilibrium to equilibrium with second mover advantages, depending on the time of the new technology comes and the level of advantage of producing with new technology comparing with the monopoly profits are gained by adopting the current technology. Huisman (2004) [72] extended the model of Dixit and Pindyck (1994) [1] by introducing a new technology coming in an uncertain time of the future. Results showed that taking into account the possible occurrence of a new technology, the preemption game in Dixit and Pindyck (1994) [1] could be turned into a war of attrition, which is a game where the second mover gets the highest payoff. Bouis et al (2009) [73] extended the duopoly model of Dixit and Pindyck (1994) [1] to the oligopoly context with three or more symmetric firms. This is the first study that contributes to the problem of strategic real option with more than two competitors. Smit and Trigeorgis (2009) [74] proposed a methodology for valuing infrastructure investment using option games approach and illustrate it by a case of evaluating airport infrastructure expansion investments. They take the infrastructure of each airport as an asset with sequential expansion options in a competitive environment and developed an option game for modeling European airport expansion.

The joint analysis of real options and game theory is also suitable for deriving RO strategies for collaborators under uncertainty (e.g., the private sector and the public sector in a PPP project); however, this topic has attracted little attention among researchers.

2.4. REAL OPTIONS AS INCENTIVES

Previous literature has identified and modeled connections between incentives and RO. Some researchers modeled and valued the incentives, such as subsidies and guarantee offered by the public sector to the private sector in PPPs, as RO.

Mason and Baldwin (1988) [14] claimed that many subsidies and guarantee have features of options, and thus they modeled and valued government subsidies to large-scale energy projects as put options. Using the Taiwan High-Speed Rail Project as a case study, Huang and Chou (2006) [75] illustrated that the minimum revenue guarantee (MRG) can be modeled as a series of European style call options and evaluated an option to abandon, as well. Results from their study showed that both the option to abandon and MRG create values, which are reduced if they were combined. Cheah and Liu (2006) [76] modeled the MRG as a put option and the governments right of repayment as a call option in a bridge project, and used a Monte Carlo simulation to price the options. Alonso-Conde et al (2007) [77] evaluated the incentives of options to defer payments, options to delay payments, and options to terminate the concession period early, for a large toll road project. They further illustrated the ways in which real options affect the incentives to invest and measured the value that the public entity may transfer to the private entity through government guarantee. Liu and Cheah (2009) [15] treated the guarantee on production volumes of a waste water treatment plant as a put option written by the government and the cap on the tariff as a call option owned by the government. Their study showed that incentives for a PPP project, if they have option features, will expand the feasible negotiation range for both the public and private entities. Wang and Liu (2008) [78] designed an option contract to coordinate a retailer-led supply chain. The option contract motivates the supplier to produce more products than that in the benchmark situation to satisfy the potential extra orders from retailer when the market demand is realized. Results show that such an option incentive can coordinate the retailer and supplier to act in the best interest of the channel. The profit in the entire channel is improved and the two parties are brought to a win-win situation.

These previous studies focused on modeling existing types of incentives as real options, and valued them solely from the viewpoint of option owner. Little work has designed real options for incentive creation. The behavioral dynamics between the option writer and option owner are also rarely discussed in the literature.

2.5. CAPACITY PLANNING USING REAL OPTIONS

The RO literature on capacity related problems is rich. Intensive discussions on this topic were provided by, for example, Dixit and Pindyck (1994) [1], Trigeorgis (1996) [4], Amram and Kulatilaka (1999) [32], and Schwartz and Trigeorgis (2001) [33]. Typical literature is summarized in Table 2.1. In early RO literature, capacity investment mainly focused on determining the optimal timing to invest in a certain project. For example, McDonald and Siegel (1985) [79] derived the optimal timing to shut down a plant to maximize the expected production profit when the demand followed a Wiener process. Majd and Pindyck (1989) [80] considered a competitive firm whose costs decline with the cumulative output and the price of the firm's output evolves stochastically. An optimal decision strategy that maximizes the firm's market value was found: to produce when the price exceeds a critical level, which is a declining function of cumulative output. Dixit (1995) [81] examined the thresholds of investing incremental irreversible capital when profit is diffusing and the marginal return first increases and then decreases. A review of Dixit and Pindyck (1994) [1] by Hubbard (1994) [82] pointed out that the RO theory had focused more on the timing of the investment and did not offer specific predictions about the level of investment. Clearly, the size of capacity investment is also an important issue. In practice, firms usually face a range of capacity choices, but not just the "invest or not" choice.

Table 2.1. The RO Literature on Capacity Planning

Category	**Literature (e.g.)**
Timing of investment	McDonald and Siegel(1985)[79]; Majd and Pindyck (1989) [80]; Dixit and Pindyck (1994) [1]; Benavides et al (1999) [83]; Dangl (1999)[84]; Bar-Ilan and Strange (1999) [85]; Harchaoui and Lasserre (2001) [86]; Driouchi et al (2006) [87]; Chronopoulos et al (2011) [88]; Hagspiel et al (2011) [89]
Capacity choice	Pindyck (1988) [90]; Fine and Freund (1990) [91]; He and Pindyck (1992) [92]; Dixit (1993) [93]; Abel et al (1996) [94]; Dangl (1999) [84]; Benavides et al (1999) [83]; Bar-Ilan and Strange (1999) [85]; Dixit Pindyck (2000) [52]; Birge (2000) [95]; Liang and Chou (2003) [96]; Decamps et al (2006) [97]; Chou et al (2007) [98]; Qin and Nembhard (2010) [25]; Chronopoulos et al (2011) [88]; Hagspiel et al (2011) [89]; Qin and Nembhard (2012) [99]

A strand of RO studies, especially more recent ones, considered the size of investment besides the timing. For example, Pindyck (1988) [90] examined the initial capacity choice considering irreversible incremental investment opportunities, uncertain returns, and opportunity costs. He and Pindyck (1992) [92] extended this analysis to include flexible capacity and compare this to the situation when only dedicated equipment is used. Fine and Freund (1990) [91] presented a two-stage stochastic model of the tradeoff between flexible capacity and the increased cost of acquiring it, as compared with dedicated or non-flexible capacity. Dixit (1993) [93] evaluated a model with irreversible choice among mutually exclusive projects with different levels of capacity. Decamps et al (2006) [97] reduced this model to two alternative projects and introduced parameter restrictions to the model. Abel et al (1996) [94] discussed a two-period model where the expandability and the reversibility of investment were completely available within the first period but were restricted within the second period. The flexibilities in capacity expansion and contraction were modeled as call options and put options respectively. Benavides et al (1999) [83] studied the optimal scale, type, and timing of IC manufacturing capacity expansion with the demand following a geometric Brownian motion process. They found that the deployment policy should be conservative because of the presence of uncertainty and larger, more efficient facilities. Dangl (1999) [84] used ROA to determine optimal timing and capacity choice of a once and for all investment under uncertainty. Results showed uncertainty in future demand leads to an increase in optimal installed capacity but delay of the investment. Bar-Ilan and Strange (1999) [85] considered both the timing and intensity of investment under incremental and lumpy investment. Birge (2000) [95] applied the results of option theory to capacity planning problems with constrained resources. Risk was incorporated into planning models by adjusting capacity and resource levels. Harchaoui and Lasserre (2001) [86] statistically tested the validation of option theory of irreversible investment. They derived the value of options to invest in capacity using contingent claims valuation and proved that this model explained investment size and timing satisfactorily from both the statistical and the economic points of view. Chronopoulos et al (2011) [88] also took into account both timing and size of investment by analyzing the impact of risk aversion as well as operational flexibility in the form of suspension and resumption options on these decisions. Hagspiel et al (2011) [89] compared the optimal capacity decisions between firms with and without production flexibility. They found that the flexible firm invests in higher capacity than the inflexible firm and the capacity difference increases with uncertainty.

Although the above literature studied the optimal size of capacity, it considered only the initial capacity choice. A few RO researches addressed the dynamic capacity planning issue. For example, Liang and Chou (2003) [96] utilized the RO theory in determining dynamic capacity choice. They validated that the geometric Brownian motion model can reasonable represent the demand process of the semiconductor industry using the historical data and showed that the option based approach, in long-term, could generate a capacity plan that requires less investment and generates higher operating income. Chou et al (2007) [98] modeled the highly volatile demand of semiconductor industry as a geometric Brownian motion process. Based on this assumption, they provide a framework for formulating long-term capacity strategy and integrating capacity planning with business planning. Qin and Nembhard (2010) [25] modeled the workforce planning problem as sequential investments in workforce capacity during the product life cycle. They illustrated that the RO-based workforce agility could reduce the sensitivity of production quality to market risks, allowing manufacturers to rapidly and economically adapt to the unexpected changes in the market. The dynamic capacity strategy is very complex thus was solved in these studies using numerical methods. However, the numerical results may not be able to get much theoretical insights. Theoretical analysis may be needed to reveal more essential features of the optimal dynamic capacity policy.

Some of the RO literature on capacity planning assumes the capacity changes continuously, for example, the capacity is differentiable as respect to time (e.g., [84, 25]). However, the capacity is usually non-differentiable, say, can only be added by large discrete units. For example, in manufacturing industry, capacity often expanded by plants; at least one lane should be added once in highway expansion. How to address the capacity planning problem when highly diffused demand and non-differentiable capacity occur jointly would be challenging.

As to the timing of investment, RO literature usually suggests that the capacity policy should be conservative (better late than early) because of the value of waiting under high uncertainty (e.g.,[83, 84, 89]). However, they often fail to consider the opportunity cost of waiting, that is the profit loss during the waiting time. An exception is Dixit and Pindyck (2000) [52], which assumed the cost of the incremental capacity increased with time. A more common case in practice is, the capacity is adjusted dynamically after the product is on the market and making profit. Therefore, the capacity installed in a late time can serve and generate profit only for a short period when the product life is limited. Opportunity cost of waiting will counteract the benefit of waiting for more information and may change the optimal capacity policy.

3. ESSAY ONE: THE INCENTIVE FUNCTION OF REAL OPTIONS

3.1. PROBLEM STATEMENT

Public infrastructure projects such as the construction and operation of highways, railways, or airports usually have long lifetimes and require a great deal of capital. Future economic and operating conditions can change substantially over a project's life. Public-private partnerships (PPPs) provide a means to finance large infrastructure projects in a way that gives private enterprises attractive business opportunities while allowing governments to acquire financial resources, transfer risks, and increase service efficiency. However, many popular PPP forms, such as build-operate-transfer (BOT), consider risks within the concession period, which is shorter than the service life of infrastructure. The public sector is exposed to great risks during the post-concession period. Moreover, excluding the post-concession period from revenue management is contrary to the growing attention to sustainability.

Recently, a method of risk mitigation, which involves using incentives offered by the public sector to the private sector, captures particular interest. Incentives such as subsidies, guarantees, and rights of expansion or abandonment have shown to alleviate the private sector concern with risks associated with PPPs. However, little work has been done to assess the benefits that the public sector may gain from offering incentives in PPPs. Also, the mechanism of incentives in PPPs is still unclear. ROs valuation provides a way to model flexibility-type incentives and optimizes the incentive functionality. The public sector is the option writer if it offers the private sector some flexibility in operating a PPP project (e.g., an option to abandon the operation before the expiration of the concession period). This flexibility encourages the private sector to collaborate in the PPP by alleviating its concern with risks. If it decides to take the option, the private sector pays a premium and become the option owner. Moreover, when used as incentives, ROs may change the behaviors of both parties to an option contract. The behavior dynamics of option owner and writer are rarely modeled in the RO literature.

In this essay, an option game framework is built to examine the incentive function of RO in the cooperative relationships and the effects of behavior interaction of option issuer and owner on the value of RO. A highway build-operate-transfer (BOT) project is used as an example to demonstrate the proposed framework. Designing specific RO incentives for different PPP forms is suggested to promote better PPPs.

3.2. RO INCENTIVE SCHEME: AN EXAMPLE IN A BOT PROJECT

3.2.1. A BOT Contract Without Options. BOT is a PPP agreement often applied to transportation infrastructure projects. Figure 3.1 illustrates change in ownership in a typical BOT project. The private concessionaire (PRI) is responsible for infrastructure construction. As a reward, it retains ownership of the infrastructure for the concession period, $(0, T_c]$, and gains profits from operating it. The governmental agency (GOV) takes over the infrastructure after the concession period and continues to operate it until the end of its service life, T.

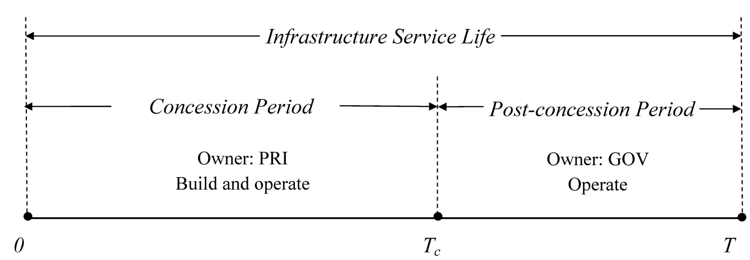


Figure 3.1. Change in Ownership in a BOT Project Without Options

To measure and demonstrate the desired effects of RO incentives, a highway BOT project without options is first modeled. The design of a BOT contract is a complex process. This paper assumes that the contract negotiation has reached the stage at which some issues have already been settled through negotiations between the GOV and the PRI. These issues include the length of the concession period, T_c, the toll price, P, the minimum requirements for construction quality, h, and the PRI's minimum required return, π_c. Now, the GOV is planning the highway capacity to maximize the social welfare offered by this project. As the highway builder and owner of the highway during the concession period, the PRI is the appropriate party to control construction quality. To inform the decision of the PRI, the total construction cost, I, and the maintenance cost, M, are defined as follows.

Assumption 1: *The total construction cost increases linearly with highway capacity and construction quality; that is,*

$$I(k, C) = (h + k)C, \tag{1}$$

where h is the minimum requirement of construction quality and k, the quality improvement factor, is the unit cost of quality improvement.

Assumption 1 is consistent with empirical findings, for example, in Levinson and Karamalaputi (2003) [100].

Assumption 2: *The annual maintenance costs are proportional to the highway capacity, and the unit maintenance cost for each year is relevant to the construction quality and road age; that is,*

$$M(k, C, t) = mk^{-\theta}e^{\lambda t}C, \quad m, \theta, \lambda > 0, \tag{2}$$

where m is the capacity coefficient, θ is the quality improvement factor, and λ is the aging factor.

Maintenance costs usually increase linearly with highway capacity. Practical experience indicates that increasing the investment in quality can reduce maintenance costs, but at a reduced rate because of diminishing marginal returns. Therefore, the unit maintenance cost is assumed to be a decreasing convex function of k. Moreover, the aging of a highway can quickly increase the difficulty of maintenance, making the unit maintenance cost an increasing convex function of highway age.

The decision process for BOT without options is a two-stage game of complete and perfect information. The players are the PRI and the GOV, as illustrated by Figure 3.2.

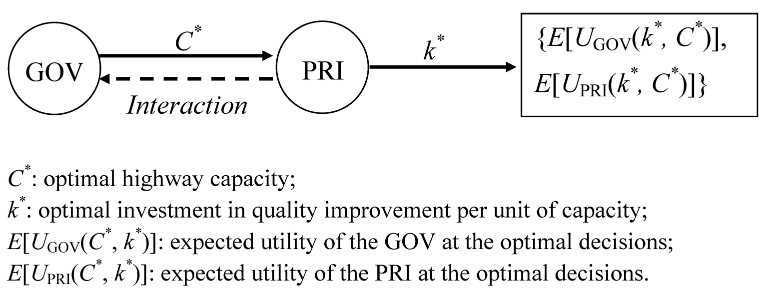


Figure 3.2. Contracting Process for the BOT Without Options: A Two-stage Game of Complete and Perfect Information.

The PRI wants to maximize the expected profit from the concession by optimizing the investment in quality improvement, k, for any given capacity, C; that is,

$$E[U_{PRI}(k^*, C)] = \max_k \left\{ -I(k, C) + \sum_{t=0}^{T_c - 1} e^{-rt} \left[R(\hat{Q}_t) - M(k, C, t) \right] \right\}, \tag{3}$$

where r is the discount rate, $\hat{Q}_t$ denotes the expected traffic volume during $[t, t+1)$, and $R(\hat{Q}_t) = P\hat{Q}_t$ calculates the expected toll revenue during that period. By solving equation (3), the PRI determines the optimal investment in quality improvement:

$$k^* = \left(\theta m \sum_{t=0}^{T_c-1} e^{(\lambda-r)t}\right)^{\frac{1}{\theta+1}}. \tag{4}$$

Equation (4) shows that the PRI's optimal action is independent of that of the GOV.

The GOV's objective is to maximize the social welfare created by the project. The general social welfare, W, is defined as the sum of the consumers' and producers' surplus realized over the entire service life, T, of the highway ([101]). Therefore, it is calculated as

$$W(k, C) = -I(k, C) + \sum_{t=0}^{T-1} e^{-rt} \left[B\left(\hat{Q}_t\right) - T\left(\hat{Q}_t, C\right) - M(k, C, t)\right]. \tag{5}$$

where $B(\hat{Q}_t)$ represents the expected benefit of travelers who use the highway during $[t, t+1)$, which is an increasing function of the expected traffic volume, and $T(\hat{Q}_t, C)$ is the travel time cost, calculated as

$$T\left(\hat{Q}_t, C\right) = \beta \hat{Q}_t t^0 \left[1 + a(\hat{Q}_t/C)^b\right], \tag{6}$$

where β is the average time value per traveler per unit time, t^0 is the travel time under free flow conditions, and $t^0\left[1 + a(\hat{Q}_t/C)^b\right]$ is the traditional BPR (Bureau of Public Roads) travel time function that measures the time needed to travel a certain route ([102]).

To make the BOT contract attractive to the PRI, the GOV must ensure the profitability of the PRI by planning highway capacity, a tactic called the second-best social optimum problem ([103]). Knowing the PRI's action, k^*, the GOV chooses the optimal highway capacity:

$$
\begin{aligned}
&E[U_{GOV}(k^*, C^*)] = \max_{C} W(k^*, C) \\
&s.t. \quad E[U_{PRI}(k^*, C)] \geq \pi_c.
\end{aligned} \tag{7}
$$

Solution of this function yields the optimal capacity, C^* (see Appendix A). Consequently, the payoffs for the GOV and the PRI depend on the decisions of both, as shown in Figure 3.2.

3.2.2. Uncertainties in Traffic Volume and Toll Revenue. The two-stage game framework for determining the BOT contract is based on expected traffic volume. If the actual annual traffic volume, Q_t, deviates significantly from expectations, the realized toll revenue may be insufficient to pay off the initial investment and cover maintenance (e.g., [75, 104]). Like the work of Lara Galera and Sánchez Soliño (2010) [105], this paper assumes that demand evolves stochastically over time, but is independent of travel costs (toll price and travel time cost):

$$
\frac{dQ_t}{Q_t} = \mu dt + \sigma dW_t. \tag{8}
$$

In Equation (8), the annual growth rate of traffic volume has a normal distribution, $N(\mu, \sigma^2)$. The term σ is commonly known as volatility, which measures the scale of traffic volume uncertainty. The stochastic movement of Q_t is modeled by W_t, a standard Wiener process ([2]). Therefore, the traffic volume follows a GBM process, and its variance grows linearly over time. Equation (8) specifically models the diffusion of traffic volume in a region experiencing fast development.

It should be remarked that the assumption of rigid traffic demand is not universally valid. Demand could be elastic, decreasing with increased travel costs ([103, 106, 107]). However, rigid travel demand can often be observed. For example, such demand is common in developing regions where the construction of transportation infrastructure lags behind an exploding economy, and in urban areas, especially during rush hours and when only one route is available to reach a particular destination. In these cases, travel demand is not affected by toll price or congestion. Using an elastic demand will be an extension of this paper and a stochastic differential game can be considered to solve the problem.

Considering that decisions in infrastructure development and operations are often made at discrete time, a binomial tree, shown in Figure 3.3, represents the layout of Q_t throughout the highway's service life ([2]). The binomial tree illustrates the annual traffic volume at discrete time, $t = 0, 1, \ldots, T$, and at any time t it shows $t+1$ unique levels of annual travel volume, $Q_{ti}(i = 0, 1, \ldots, t)$. Q_{ti} indicates the traffic volume at the note t, i on the binomial tree. Q_{ti} may increase to $Q_{(t+1)i}$ with a probability of p; with a probability of $1-p$, it may decrease to $Q_{(t+1)(i+1)}$.

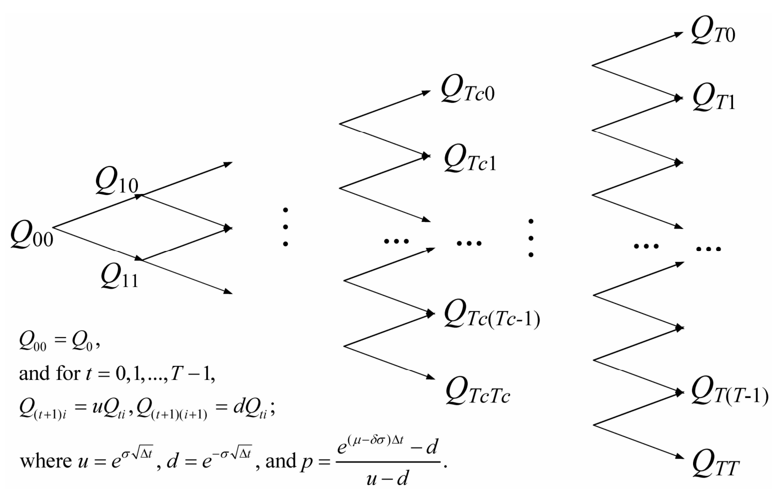


Figure 3.3. Evolution of Annual Traffic Volume With Time

Figure 3.3 shows that annual traffic volume, Q_{ti}, can be very different from its expected value, $\hat{Q}_t$. Consequently, toll revenue may fall far short of the levels necessary to pay off the initial investment and maintain the highway throughout its service life. For instance, if actual revenue is lower than expected, the GOV will have to find additional funds to make up the shortfall. At node (t, i) on the binomial tree, the extra funding needed by the GOV is represented by $\max[M(k^*, C^*, t) - R(Q_{ti}), 0]$. The present value of the expected total shortages during the post-concession period, $F_{BS}(k^*, C^*)$, is given by

$$F_{BS}(k^*, C^*) = \sum_{t=T_c}^{T-1} \sum_{i=0}^{t} e^{-rt} P\{Q_{ti}\} \max\left[M(k^*, C^*, t) - R\left(Q_{ti}\right), 0\right]. \tag{9}$$

If traffic volume is higher than expected, the GOV is likely to generate revenue in excess of the required maintenance costs, yet travelers suffer heavy traffic congestion. Excess toll revenue cannot easily be applied to alleviate traffic congestion or gain other benefits in practice.

3.2.3. A BOT Contract With Options. The GOV can address the aforementioned risk by adding options to the BOT contract. Besides the rights and obligations specified for the concession, the GOV can offer the PRI an option to continue operating the project after the concession expires. If it exercises the option, the PRI can still choose to terminate operation of the highway at any time during the post-concession period. The first option is a European-style (i.e., the option can be exercised only at maturity) continuation option; the second option is an American-style (i.e., the option can be exercised on or before its maturity) abandonment option. This abandonment option is compounded with the continuation option because the former is valid only if the PRI exercises the latter. These two options allow the PRI to capture potential opportunities for additional profits when future conditions appear favorable, and to avoid possible losses if conditions appear unfavorable. Clearly, the options are value-added to the PRI. The GOV prices the options to determine the option premium, which allows it to cover the possible shortage in maintenance funds or to create more social welfare from the project. The options have potential to improve the BOT scheme presented in Section 3.2.1.

When the PRI exercises either of the two options, ownership of the highway is transferred from one party to the other, as shown in Table 3.1. In the BOT contract without options, the GOV is the owner of the highway after the concession expires. In a BOT contract with options, however, the PRI can now choose to exercise the continuation option, in which case ownership of the highway is transferred from the GOV back to the PRI. It is returned to the GOV permanently if the PRI exercises the abandonment option.

Table 3.1. Change in Highway Ownership in the BOT With Options

<table>
<tr><th>Time</th><th>Concession Period</th><th colspan="2">Post-concession Period</th></tr>
<tr><td rowspan="2">Highway Ownership</td><td rowspan="2">PRI</td><td colspan="2">GOV
(if the continuation option is expired)</td></tr>
<tr><td>PRI
(if the continuation option is exercised; until the abandonment option is exercised)</td><td>GOV
(once the abandonment option is exercised)</td></tr>
</table>

Ownership of the highway is different from the options; the PRI is the owner of the continuation and the abandonment options, whereas the GOV is the writer of these options.

With the RO incentive to continue the operation, the PRI faces decisions about option acquisition and exercise. The GOV must price the options and, if it sells them to the PRI, decide how to use the premium. A valuation of the options in a game-like framework helps them derive their optimal decisions.

This work evaluates the project in a risk-neutral world after the continuation and abandonment options are added to the BOT contract. Risk-neutral valuation is essential in option pricing. In a risk-neutral world, all individuals are indifferent to risk ([2]). A problem can be transformed from the real world to the risk-neutral world by measuring the uncertainty in the underlying asset using a risk-neutral probability, p, and discounting cash flows using the risk-free rate, r_{rf}. Decision outcomes in a risk-neutral world are the same as those in the real world; however, the risk-neutral decision making process is easier because no risk-adjusted rate need be estimated for each party and for various times in the life of the project ([2]). The estimation of the risk-neutral probability is shown in Figure 3.3.

The GOV determines the premium by pricing the RO incentive. Since the abandonment option is compounded with the continuation option, the value of the continuation option is dependent on that of the abandonment option. Therefore, the abandonment option is evaluated first. Let A_t be the PRI's action at $t = T_c, T_c{+}1, ..., T{-}1$. Whereas A_{T_c} is derived from the valuation of the continuation option, subsequent actions, $\{A_t | t = T_c + 1, T_c + 2, \ldots, T - 1\}$, are derived from the abandonment option. At $t = T_c, T_c + 1, \ldots, T - 1$, the beginning of each year, the PRI determines whether terminating the operation starting the next year will maximize the expected value-to-go:

$$\begin{aligned} Z(Q_{ti}, A_t) = \max_{A_{t+1}} \{ & R(Q_{ti}, A_t) - M(k^*, C^*, t, A_t) \\ & + e^{-r_{rf}\Delta t} \left[pZ(Q_{(t+1)i}, A_{t+1}) + (1-p)Z(Q_{(t+1)(i+1)}, A_{t+1}) \right] \}, \end{aligned} \tag{10}$$

where $A_{t+1} \in \{$"*operate*", "*abandon*"$\}$. Equation (10) shows that the decision in A_{t+1} is made only if A_t is "*operate*" because the PRI can no longer operate the highway after it has abandoned the right. Therefore, $Z(Q_{ti}$, "*abandon*"$)$ is equal to 0. The expected value-to-go for any time t is determined by dynamic programming (DP) ([108]). The backward recursion process of DP stops at T_c when the PRI must decide whether to exercise the continuation option:

$$Z(Q_{T_c i}) = \max_{A_{T_c}} \left\{ Z(Q_{T_c i}, A_{T_c}) \right\}, \tag{11}$$

where $A_{T_c} \in \{$"*continue*", "*expire*"$\}$. If the PRI decides to exercise the continuation option, it obtains the abandonment option for the post-concession period, and $Z(Q_{T_c i}) = Z(Q_{T_c i}, \text{"}continue\text{"}) = Z(Q_{T_c i}, \text{"}operate\text{"})$. Otherwise, the continuation option expires and $Z(Q_{T_c i}) = Z(Q_{T_c i}, \text{"}expire\text{"}) = 0$.

The expected present value of $Z(Q_{T_c})$ is the value of options to the PRI:

$$V_{PRI}(k^*, C^*) = e^{-r_{rf} T_c} \sum_{i=0}^{T_c} Z(Q_{T_c i}) P\{Q_{T_c i}\}. \tag{12}$$

Here, the term $V_{PRI}(k^*, C^*)$ represents the options value for the PRI because it can vary with the decisions of the GOV and the PRI according to Equation (10).

The GOV will operate the highway only in two situations: first, if the PRI decides not to exercise the European option of continuation and, second, if the PRI terminates the continued operation early. Therefore, at decisions k^* and C^*, the expected shortage of maintenance funding for the GOV becomes:

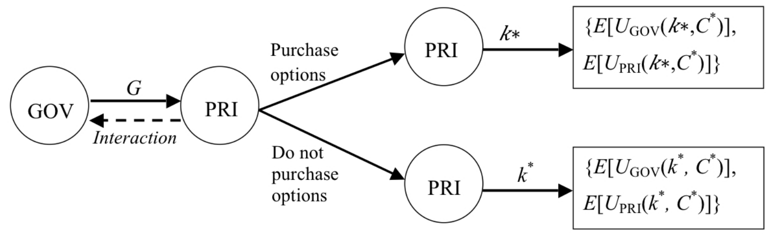
$$F_{OS}(k^*, C^*) = \sum_{t=T_c}^{T-1} \sum_{i=0}^{t} e^{-r_{rf} t} P\{Q_{ti}\} \max \begin{bmatrix} M(k^*, C^*, t, \text{"}abandon\text{"}) \\ -R(Q_{ti}, \text{"}abandon\text{"}), 0 \end{bmatrix}. \tag{13}$$

3.2.4. Change in PRI Behavior Motivated by the RO Incentive. Given the benefits associated with the options, the previous decisions on highway capacity and quality improvement may no longer be optimal. This possibility is indicated by Equation (12). For example, the PRI may further increase the option's value by changing the investment decision. Options may influence the behaviors of both the option owner and the option writer; therefore, a standard RO valuation cannot determine the optimal decisions. The option-related decisions must be put in a game-like framework to formulate the interaction between the GOV and the PRI.

Since the RO incentive is an add-on to the BOT contract, the GOV's decision on capacity, C^*, remains unchanged, although it may not be theoretically optimal. The PRI is more flexible: The BOT contract without options defines only the minimum requirement for construction quality, and the PRI itself can determine the level of its investment in quality beyond the minimum. Therefore, this paper discusses only the change in the PRI's investment behavior stimulated by the RO incentive. As shown in Figure 3.4, in a complete and perfect information dynamic game, the GOV determines the premium of the RO incentive first. Then, on the basis of the premium, the PRI decides on the options purchase. If the PRI decides not to buy the options, the decision regarding quality improvement remains the same as that for the BOT without options. If the PRI buys the options, it will determine a new investment in quality improvement to maximize the expected profit from the entire project.

After the options for the post-concession period are added to the BOT contract, the PRI's evaluation function for the project changes:

$$E[\widetilde{U}_{PRI}(\tilde{k}^*, C^*)] = \max_k \left\{ \begin{array}{l} -I(k, C^*) \\ + \sum_{t=0}^{T_c-1} e^{-r_{rf}t} \left[R(\hat{Q}_t) - M(k, C^*, t) \right] \\ + V_{PRI}(k, C^*) - G \end{array} \right\}, \tag{14}$$



G: option premium;
k∗: new optimal quality improving investment when options are purchased.

Figure 3.4. Contracting Process for the BOT With Options

where G is the premium determined by the GOV. The value of G should be no greater than $V_{PRI}(\tilde{k}^*, C^*)$ to persuade the PRI to purchase the options.

The options that the GOV offers to the PRI yield several benefits, which are summarized in the following lemma and propositions (See Appendices B-D for the proofs).

Lemma 1: *The optimal investment in quality improvement when the RO incentive is offered, $\tilde{k}^*$, is no less than that when no RO incentive is offered. That is, $\tilde{k}^* \geq k^*$.*

Lemma 1 suggests that by offering the options, the GOV motivates the PRI to augment its investment in quality improvement, thus reducing maintenance costs. This change further increases the social welfare produced by the project and reduces the expected shortfall in maintenance funds facing the GOV. This effect is expressed as follows:

Proposition 1: *The RO incentive that the GOV offers to the PRI increases the expected social welfare produced by the project. That is, $W(\tilde{k}^*, C^*) \geq W(k^*, C^*)$.*

Proposition 2: *The RO incentive that the GOV offers to the PRI reduces the expected shortfall in maintenance funds facing the GOV. That is, $F_{OS}(\tilde{k}^*, C^*) \leq F_{BS}(k^*, C^*)$.*

3.2.5. An Expansion Option as a Means to Improve Social Welfare. The benefit of the option premium to the GOV is not limited to provision of a financing resource when toll revenue is insufficient to pay for highway operation. If future traffic volume is significantly higher than expected, the planned highway capacity, C^*, may not be sufficient to ensure a reasonable travel time, causing heavy traffic congestion. The GOV may consider using the premium to generate more social welfare from the project. This paper assumes that travelers utility increases with expanded capacity. Therefore, the GOV considers using the premium as a resource to finance expansion of highway capacity and maintain the added capacity throughout the service life of the highway. The toll from the added capacity is collected by the PRI, so the PRI's revenue remains unchanged (due to the rigid demand assumption). This paper assumes that capacity expansions can be properly managed to minimize the impact on the operation of the existing highway; therefore, capacity expansions will be no disadvantage to the PRI.

To ensure that the premium is properly used to support capacity expansions, the GOV must evaluate a series of decisions: whether to expand the highway, when to expand the highway, and how much capacity to add. These decisions are formulated as a DP problem with the objective to maximize the expected increment in social welfare during the entire project service life. The term C_t represents the highway capacity at time t, and $C_t \in \{c_0, c_1, c_2, \ldots, c_m\}$, with $c_0 < c_1 < \ldots < c_m$. The initial capacity, C_0, is equal to C^*; that is, $C_0 = C^* = c_0$. The expansion decision is made at the end of each year, and new lanes become available the following year.

Let $V^*(Q_{ti}, C_t)$ designate the expected maximum increment in social welfare at time t and thereafter, which is produced by capacity expansions. Then,

$$\begin{aligned} V^*(Q_{ti}, C_t) = \max_{C_{t+1}} \Big\{ & v(Q_{ti}, C_t, C_{t+1}) + e^{-r_{rf}\Delta t} \big[pV^*(Q_{(t+1)i}, C_{t+1}) \\ & +(1-p)V^*(Q_{(t+1)(i+1)}, C_{t+1}) \big] \Big\}, \end{aligned} \tag{15}$$

where $v(Q_{ti}, C_t, C_{t+1})$ is the improvement in social welfare during $(t, t+1]$, calculated as

$$\begin{aligned} v(Q_{ti}, C_t, C_{t+1}) = & \left[T(Q_{ti}, C^*) - T(Q_{ti}, C_t) \right] \\ & - \left[M(\tilde{k}^*, C_t, t) - M(\tilde{k}^*, C^*, t) \right] \\ & - \left[I(\tilde{k}^*, C_{t+1}) - I(\tilde{k}^*, C_t) \right]. \end{aligned} \tag{16}$$

The optimization problem in (15) is solved through a backward recursion, and at time zero the expected improvement in social welfare, $V^*(Q_0, C^*)$, is found, which is $V^*(Q_{00}, C_0)$.

By retrieving the optimal expansion strategy from the solution of (15), the expected present value of expansion-related costs, F_E, is determined. This value includes the construction investment and maintenance costs for the added capacity. The option premium may not be equal to the expansion-related costs; therefore, the social welfare added by the option premium is calculated as

$$\min[G/F_E, 1]V^*(Q_0, C^*). \tag{17}$$

3.2.6. Total Social Welfare Added by the RO Incentive. The RO incentive produces additional social welfare from two channels, as calculated in Sections 3.2.4 and 3.2.5 respectively. First, the RO incentive stimulates the PRI to increase the investment in quality improvement, thus producing ΔW_k. Second, the premium of the RO incentive finances possible capacity expansions, which produces ΔW_C. That is,

$$\begin{aligned}\Delta W &= \Delta W_k + \Delta W_C \\ &= \Big[W(\tilde{k}^*, C^*) - W(k^*, C^*)\Big] + \min[G/F_E, 1]V^*(Q_0, C^*).\end{aligned} \tag{18}$$

The value of ΔW_k is nonnegative according to Proposition 1, and that of ΔW_C is nonnegative since $V^*(Q_0, C^*)$ is the value of the expansion option.

3.2.7. The Options Premium. The premium charged for the continuation and abandonment options is the key factor affecting implementation of the RO incentive scheme. The GOV takes a variety of objectives into account when it determines the option premium. In particular, it considers the need to secure the desired public benefit from the project, to increase the possibility that the project will be self-liquidating, and to ensure the profitability of the private sector.

The PRI expects a reasonable profit from continuation of the project; therefore, $G \leq V_{PRI}(\tilde{k}^*, C^*) - \pi_o$, where π_o is the minimum required return of the PRI for the post-concession period. The GOV hopes that the option premium is an effective financing resource for possible highway maintenance and capacity expansions; therefore, $G \geq F_{OS}(\tilde{k}^*, C^*) + F_E$. If $F_{OS}(\tilde{k}^*, C^*) + F_E \leq V_{PRI}(\tilde{k}^*, C^*) - \pi_o$, there is a room to negotiate the option premium; otherwise, $G = V_{PRI}(\tilde{k}^*, C^*) - \pi_o$. Therefore, the option premium, G, is set to

$$\min\left[F_{OS}(\tilde{k}^*, C^*) + F_E, V_{PRI}(\tilde{k}^*, C^*) - \pi_o\right] \leq G \leq V_{PRI}(\tilde{k}^*, C^*) - \pi_o. \tag{19}$$

3.2.8. Numerical Studies. This paper provides a numerical example to demonstrate the analytical results described above. The data in the example is not from a real case, but is set very close to the reality. For example, the toll, 10 yuan, is set according to the toll level of most Chinese highways, 0.34 0.5 yuan per kilometer; the construction cost of the 30-kilometer highway in this example is 925 million yuan, matching the average construction cost of highway in China, which is 30 40 million yuan per kilometer. In addition, a sensitivity analysis is performed to generalize the example.

3.2.8.1. A numerical example. The numerical example is based on a highway project in western China. To support economic development, city A is developing a new industrial zone in its exurb. City B is close to the new industrial zone; however, it is accessible only by an old provincial highway passing by it. To meet the exploding commuting and freight transport demands, city B's department of transportation (GOV) decides to construct a 30-kilometer expressway connecting the urban area of city B and the industrial zone. The expressway is expected to have a useful life of 30 years. This expressway can cut travel distance by 25 kilometers and reduce travel time from 50 to 20 minutes. Faced with a large investment in construction and deep uncertainty over the lengthy service life, the GOV authorizes a private company (PRI) to develop the expressway under a BOT agreement. After several rounds of negotiation, the BOT contract is drawn up with an average toll rate of 10 yuan per vehicle, a 20-year concession period, and a minimum construction quality requirement of 6.2 yuan per unit of capacity.

Initial projections forecast approximately 20 million vehicles in the first year of operation. The growth rate of the travel volume is estimated to be 4% per year, with a standard deviation of 15%. Clearly, the new expressway is shorter than the existing provincial highway, has a higher speed limit, and charges a reasonable toll; therefore, demand will not be affected significantly by the degree of congestion. The high volatility in traffic volume promises great uncertainty in toll revenues. To better manage future revenue, improve social welfare, and motivate the PRI to enhance construction quality, the GOV is considering offering the PRI a right to continue operating the expressway after the concession period and, if it takes the options, to terminate the operation early with a one-year notice. Table 3.2 summarizes the parameter values for this example.

Table 3.2. Parameter Values and Formulas for the Numerical Example

Parameter	**Value**	**Unit**
Q_0	20	million vehicle/year
T	30	year
T_c	20	year
μ	0.04	–
σ	0.15	–
λ	0.05	–
r_{rf}	0.05	–
P	10	yuan/vehicle
δ	0.05	–
$T(\hat{Q}_t, C)$	$5\hat{Q}_t\,[1 + 0.15(Q_t/C)^4]$	yuan/year
$M(k, C, t)$	$2k^{-0.2}e^{0.06t}C$	yuan
$I(k, C)$	$(20 + k)C$	yuan
C_t	$\{C^*, C^* + 17.4\}$	million vehicle/year

The minimum required profit of the PRI during the concession period, π_c, is set to maintain the PRI's equivalent annual rate of return at 10% during the concession period:

$$\pi_c = \left(e^{(0.1 - r_{rf})T_c} - 1\right) I. \tag{20}$$

Initially, the PRI passes the option premium, G, to the GOV; however, the option will yield no return until the concession period expires in 20 years. The PRI requests an 8% annual rate of return for the opportunity cost of waiting, plus a reasonable profit from the options, for example, a 10% annual rate of return from the premium during the post-concession period. The marginal return from the options investment, π_o, is determined by

$$\begin{aligned} &G = V_{PRI} e^{-[0.08T_c + 0.1(T - T_c) - r_{rf}T]}, \\ &\pi_o = V_{PRI} - G. \end{aligned} \tag{21}$$

3.2.8.2. Results for the numerical example. Table 3.3 summarizes the results for the numerical example. Without an RO incentive, the optimal capacity is 32.1 million vehicles per year (about 4 lanes, assuming the average capacity per lane is 1000 vehicles per hour), and the optimal quality factor is 6.2 yuan per capacity unit.

Offered the RO incentive, the PRI increases the quality factor from 6.2 to 8.8 yuan per capacity unit. Consequently, the initial investment is increased by 10.1%, from 839.8 to 925.0 million yuan. The RO incentive is then worth 776.9 million yuan to the PRI.

Table 3.3. Numerical Results for the Example

		With Options		**Without Options**		
		Variable	**Value**	**Variable**	**Value**	**Unit**
Decisions	PRI	$\tilde{k}^*$	8.8	k^*	6.2	yuan/veh year
	GOV	C^*	32.1×10^6	C^*	32.1×10^6	vehicles/year
		G	258.6	–	–	million yuan
Outcomes	PRI	$I(\tilde{k}^*, C^*)$	925.0	$I(k^*, C^*)$	839.8	million yuan
		$E[\widetilde{U}_{PRI}]$	2027.6	$E[U_{PRI}]$	1514.3	million yuan
		V_{PRI}	776.9	–	–	million yuan
	GOV	F_{OS}	35.7	F_{BS}	49.4	million yuan
		F_E	287.8	–	–	million yuan
		ΔW_k	28.4	–	–	million yuan
		ΔW_C	3.50×10^4	–	–	million yuan

By offering the RO incentive, the GOV anticipates the following benefits: First, because the RO incentive motivates the PRI to invest in higher quality, the social welfare is increased by 28.4 million yuan, and the shortfall in maintenance funds is reduced by 38.4%, from 49.4 to 35.7 million yuan. Second, the GOV receives a premium of 258.6 million yuan. This upfront income can be used to finance capacity expansions if heavy traffic congestion is anticipated. Each additional lane is assumed to have a capacity of 8.7 million vehicles per year. When two lanes are added, the premium can provide 91% of the expected expansion-related expenses and effectively add 3.50×10^4 million yuan of social welfare over the 30-year lifetime of the project(near 40 yuan per vehicle). Table 3.3 demonstrates that ΔW_C is significantly higher than ΔW_k. Therefore, the social welfare added by the improved construction quality can be seen as a favorable side effect of the RO incentive. The premium can also effectively fill the shortfall in maintenance costs if the PRI abandons operation of the expressway during the post-concession period, which it will likely do should traffic volume be too low. The results show that the RO incentive allows both parties to better manage revenues for the BOT project under high uncertainty.

3.2.8.3. Effectiveness of the RO incentive. The effectiveness of the RO incentive can vary depending on in project conditions. This paper finds that the effectiveness of RO incentive is strong when the volatility of traffic volume is high and when the concession period is short, which are typical conditions where the development of a BOT agreement is difficult.

Volatility, σ, measures the uncertainty in annual traffic volume. Figure 3.5 illustrates how volatility influences the effectiveness of the RO incentive. Normally, options become more valuable when the volatility of the underlying asset increases. However, Figure 3.5(c) demonstrates that the value of the RO incentive decreases as volatility increases. This unusual circumstance can be interpreted as a result of the change in investment behaviors due to uncertainty. Figures 3.5(a) and 3.5(b) indicate that the GOV and the PRI tend to be conservative in their investments; the GOV will reduce highway capacity when uncertainty in future traffic volume is high, and the PRI will also substantially reduce its investment in quality improvement. These changes may increase maintenance costs, thus also increasing the GOV's the expected shortage in maintenance funds (Figure 3.5(e)) and limiting the social welfare added by improving highway quality (Figure 3.5(g)). This study demonstrates that the behavioral dynamics between the option issuer and option owner affect the option value; therefore, a standard RO valuation has limitations for supporting appropriate decisions. Nevertheless, the decrease in the value of RO incentive is not large (see Figure 3.5(c)), and the RO incentive is still attractive for the PRI, as Figure 3.5(d) shows.

As the option value, V_{PRI}, decreases with increasing volatility, σ, the GOV reduces the premium, G, but not to an extremely low point, as Figure 3.5(c) shows.

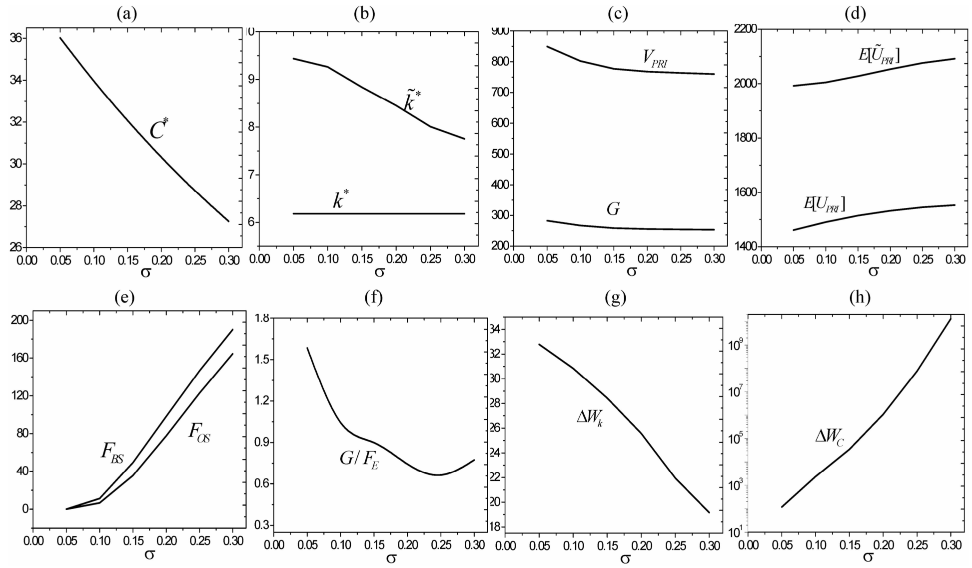


Figure 3.5. Impacts of Traffic Volume Volatility on the Effectiveness of RO Incentive

Therefore, its ability to finance capacity expansions is not compromised. The GOV could still expect greater social welfare as a result of the capacity expansions despite the volatility increase, as shown in Figure 3.5(h). Although increased volatility can reduce ΔW_k, compared to ΔW_C, ΔW_k is small and the advantage of the RO incentive will not be depressed by a high volatility. Therefore, as the writer of the continuation and abandonment options, the GOV is also in favor of high volatility.

Figure 3.6 illustrates how the effectiveness of the RO incentive can be affected by the length of the concession period. The RO incentive improves a BOT scheme with a short concession period. The PRI is discouraged from participating in a BOT if it is given only a short period of ownership. Although the PRI would reduce investment in quality, k^*, as the concession length decreases (shown in Figure 3.6(b)), it is unable to maintain the level of expected utility, $E[U_{PRI}]$ (shown in Figure 3.6(d)). If the GOV offers a longer concession period in order to encourage the PRI to participate, it may have to lower the capacity requirement to meet the profitability constraint (see Figure 3.6(a)), thus hurting the benefit to travelers.

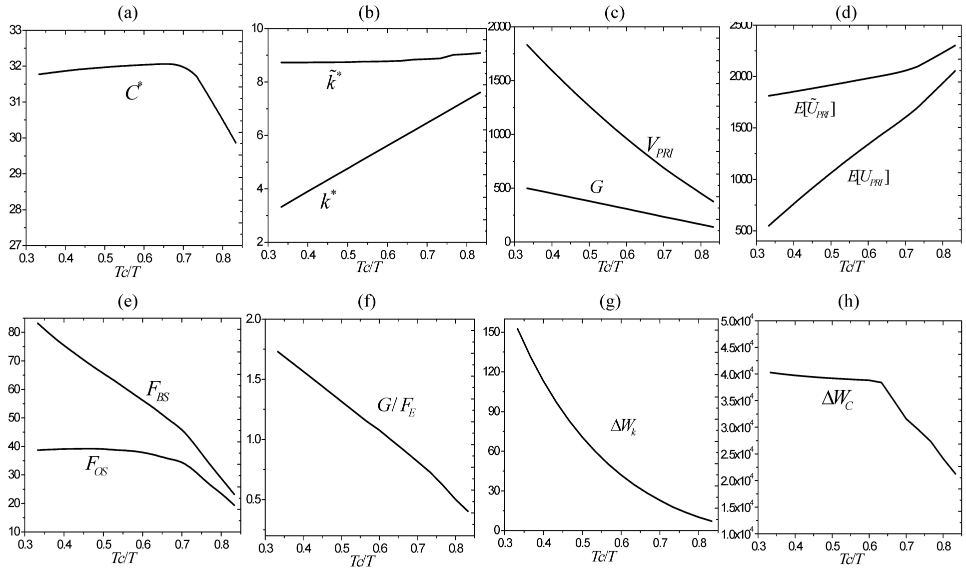


Figure 3.6. Impacts of Concession Period on the Effectiveness of RO Incentive

The RO incentive can address this dilemma. The RO valuation reveals that the options value will increase if the post-concession period becomes longer. Figure 3.6(c) also shows that V_{PRI} increases as the portion of concession period decreases, which compensates for the loss in the PRI's profit due to the shortened concession period. Therefore, Figure 3.6(d) shows that the PRI's expected profit with options, $E[\tilde{U}_{PRI}]$, decreases slowly with the decreased concession period. In addition, when a shorter concession period is selected for BOT, the PRI generates a larger portion of profit from the post-concession period with options, which is less risky than that from the concession period. Therefore, with the options, the PRI will not significantly reduce its investment in quality improvement although the concession period is short, as Figure 3.6(b) shows. Although the GOV reduces the concession period, it gives the

PRI a longer period with options; consequently, the GOV asks a higher premium (see Figure 3.6(c)) and uses it to produce greater social welfare (see Figure 3.6(h)).

3.3. DESIGN RO INCENTIVE IN PPPS

Public-private partnerships (PPPs) are becoming one of the major delivery methods of public infrastructure in recent years. PPPs are arrangements for public and private sectors to cooperate in developing large-scale projects [109]. With PPPs the public sector is able to alleviate financial burden [110], share risks and revenues with the private sector [111], increase the value for money spent on public projects by improving services efficiency [112], and reduce lifecycle costs [113]. The private sector has widely participated in financing, construction, and operations in PPPs [114, 115]. Particularly, private financing in public projects had been rapidly increasing during the past two decades to meet the emerging demands for public facilities. By 2010 the private investment commitments in PPP infrastructure projects in developing countries had reached $100 billion, over 8 times of the amount in 1990, as Figure 3.7 shows. Private financing in energy infrastructures had especially substantial growth. The investment commitment increased from $0.11 billion to $62.15 billion, more than 500 times.

Despite that PPPs have broad benefits and increasing usages, obstacles to PPPs applications are often reported. PPPs projects often have a long service life that associates with high uncertainty. Poor risk management and unrealistic projections often lead to the failure of PPP projects in highly uncertain conditions [116, 117]. Interest conflicts between the public and private sectors in PPPs projects also raise problems. The goal of pursuing profit of the private sector may lead to low performance in public projects and thus reduce public welfare. Therefore, incentives are often used to promote better collaborations in PPPs.

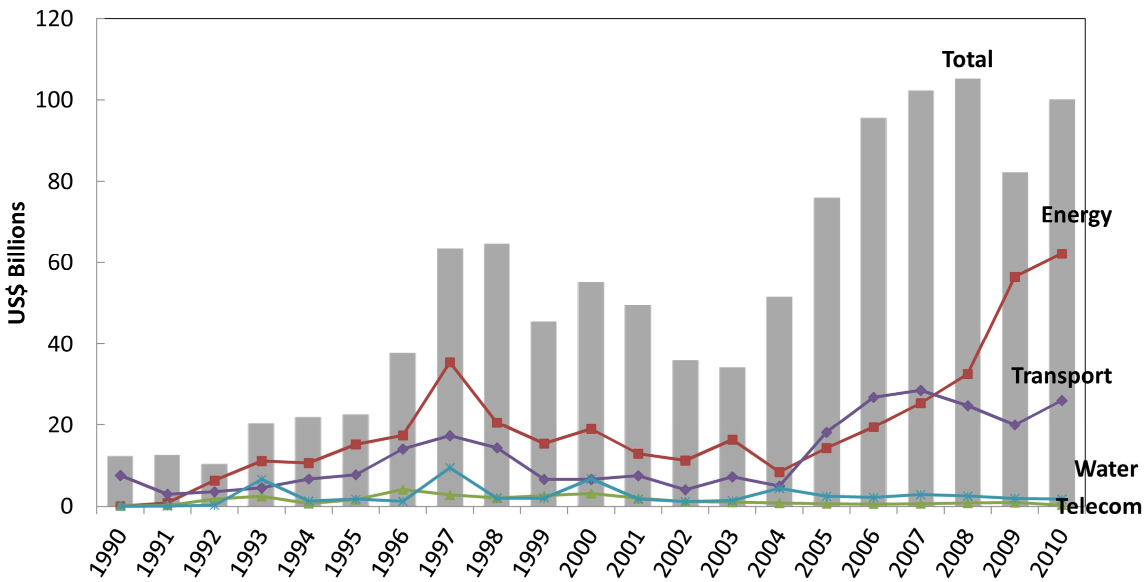


Figure 3.7. Private Investment Commitments in PPP Infrastructure Projects in Developing Countries by Cectors, 1990 to 2010 (Data sources: World Bank and PPIAF, PPI Project Database. http://ppi.worldbank.org)

ROA provides a way of modeling flexibility-type incentives for PPP projects and is able to optimize the functionality of the incentives. Connections between incentives and real options (ROs) have been discussed and modeled by previous research. For instance, the minimum revenue guarantee (MRG) to the private sector can be modeled as a put option, and the right to extend the concession can be seen as a continuation option [76, 118] However, PPP forms are diverse. Each form has a different degree of private involvement, sources of uncertainty, and major concerns to be alleviated. There is no RO model that is universally effective for all forms of PPPs. This paper assesses the contractual features and risks of major PPP forms and accordingly, proposes a general framework of selecting incentive-type RO models for PPPs projects.

3.3.1. An Overview of PPPs. PPPs are available for different project objectives. They can vary in the level of private sector involvement, contractual

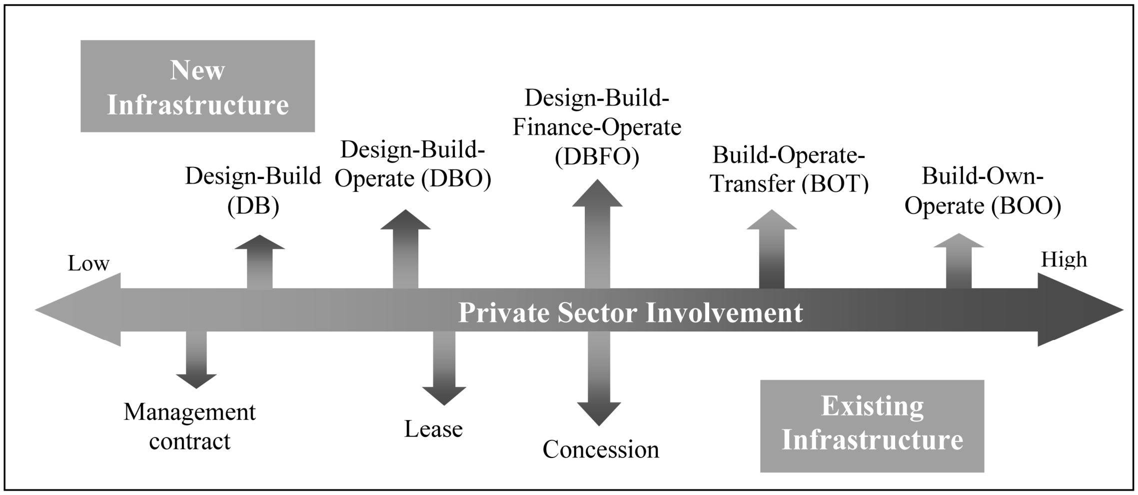


Figure 3.8. Major Forms of PPPs.

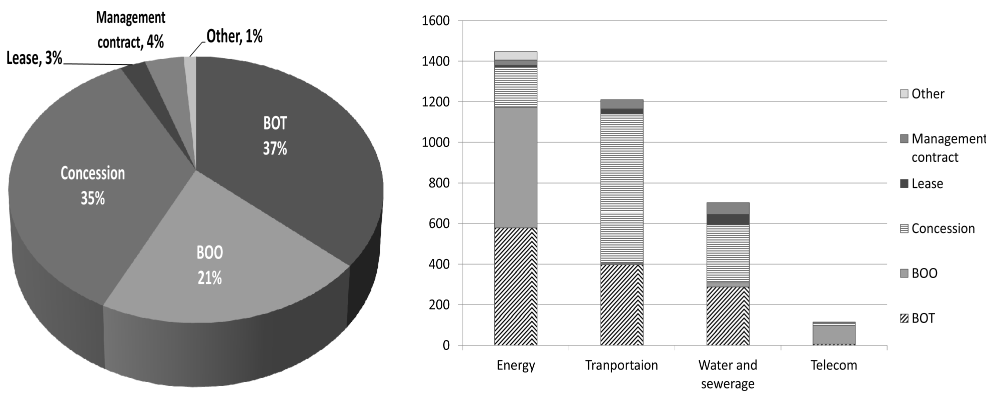
features, asset ownership, risk allocation between public and private sectors, and lifecycle stages. Figure 3.8 lists major PPP forms according to the level of private sector involvement and categorizes these by lifecycle stages of infrastructure. Further descriptions of the PPPs are in Table 3.4.

Selection of a PPP form for a specific project needs to consider the circumstance of the project, including regulatory constraints, public funding availability, project scale, project type, and the main purpose of the project. For example, in developing countries, the demand for public infrastructures grows rapidly yet the public budget sometimes is insufficient. Therefore, PPP forms involving private financing are commonly adopted. Figure 3.9 summarizes the major forms of PPP contracts implemented in infrastructure projects in developing countries during 1990-2010. The figure shows that the most commonly adopted PPP forms in the developing counties are BOT, concession, and BOO. These account for 37%, 35%, and 21% of PPP infrastructure projects, respectively. The popularity of each PPP form varies in different industries. For example, over 80% of telecom PPP projects and 40% of energy PPP projects adopted BOO. In the sectors of transportation as well as water and sewerage,

Table 3.4. Descriptions of Major Forms of PPPs

Typical Forms of PPPs		**Descriptions**	**Asset Owner**	**Effective Life Cycle Stage**
New Infrastructure	Design-Build (DB)	The private sector is responsible for the design and construction of infrastructure, and paid by a fixed fee.	Public	Design Construction
	Design-Build-Operate (DBO)	The private sector is responsible for the design and construction of infrastructure, and operates and maintains it for a specified period.	Public	Design Construction Operation
	Design-Build-Finance-Operate (DBFO)	The private sector is responsible for the financing, design, and construction of infrastructure, and operates and maintains it for a specified period.	Public	Design Construction Operation
	Build-Operate-Transfer (BOT)	The private sector finances, builds the infrastructure; owns, operates, and maintains it during the concession period; then transfers it back to the public sector.	Private (before transfer) Public (after transfer)	Design Construction Operation
	Build-Own-Operate (BOO)	Similar to BOT, but the private sector retains the infrastructure ownership in perpetuity.	Private	Design, Construction Operation
Existing Infrastructure	Management Contract	The private sector is paid a predetermined rate for managing the public facility. Operational risks remains with the public sector.	Public	Operation
	Lease	The private sector leases an existing infrastructure from the public sector, operates and maintains the infrastructure for a specified leasing period.	Public	Operation
	Concession	The private sector rehabilitates, and/or builds an add-on to an existing facility; then operates it during the concession period.	Public	Operation

however, projects using BOO were very few. Instead, concession and BOT are the most popular ways of delivering infrastructure projects. In the United States, Design-Build (DB) is the major PPP method of delivering transportation infrastructure in that the public funding is relatively sufficient and tolls were forbidden on federal-funded roads before the Intermodal Surface Transportation Efficiency Act (ISTEA) had been issued in 1991.



(a) (b)

Figure 3.9. PPP Contracts Implemented in Infrastructure Projects in Developing Countries by Types, 1990 to 2010 (Data sources: World Bank and PPIAF, PPI Project Database. http://ppi.worldbank.org)

Although applications of PPPs in public projects are growing, obstacles that reduce the effectiveness of PPPs are observed. For example, the demand for, and operating and maintenance (O& M) costs of, public infrastructure can substantially change during long service lives; therefore, the private sector may hesitate to invest in these projects due to the uncertainties. Moreover, unlike the public sector primarily aiming at social welfare, the private sector is profit seekers. Their different objectives may lead to an outcome deviating from the original expectation of PPPs.

3.3.2. PPP Incentives as ROA. Incentives such as guarantees and subsides are used in PPPs to alleviate the private sectors concerns with risks, or to motivate the private sector to improve service performance. Incentive pricing is important for PPPs projects. ROA is considered an effective tool for valuing flexibility-type incentives. An option gives its owner a right, but not an obligation, to buy (call option) or sell (put option) a certain amount of assets at a certain price by a certain date [2]. ROs extend the financial options to the valuation of investment in real assets and managerial flexibilities. The value of ROs stems from option owners ability to asymmetrically react to opportunities of gaining greater profit and potential of loss as investment environment changes.

The incentives offered by the public sector are similar to ROs in that they both provide downside protection or opportunities of gaining greater profit under uncertainty [119]. For example, when demand drops below a certain level, the MRG secures a fixed level of income to the private sector. It can be considered a right to sell the operation revenue at a fixed price; therefore, it can be modeled as a put option. Some regulatory flexibilities held by the public sector can also be valued as ROs. For instance, while offering the MRG, the public sector usually set a revenue cap (RCP) as well, to prevent the private sector from gaining too much profit from public projects. RCP provides the public sector the right to receive the revenue exceeding a certain level, similar to buying the actual revenue at a fixed price. Therefore, the RCP can be modeled as a call option owned by the public sector.

A simple example that models MRG as an RO is provided here for illustration. Suppose the current revenue is \$10 million and the public sector promises a MRG of \$8 million. The revenue of next period, S, is uncertain and can vary between 0 million and \$20 million. Because of the MRG, the private sector can receive a revenue, max(S, 8), instead of S in the next period. Therefore, the payoff of the MRG, G, is $\max(8 - S, 0)$, similar to the payoff of a put option with the value of underlying asset at maturity of S and an exercise price of \$8 million, as shown in Figure 3.10.

The value of the MRG can be calculated as a put option value using option pricing. It should be remarked that this example employs a single period MRG for straightforward illustration. In practice, projects have multiple periods of revenues and MRG should be modeled as a series of put options or a multiple exercisable option.

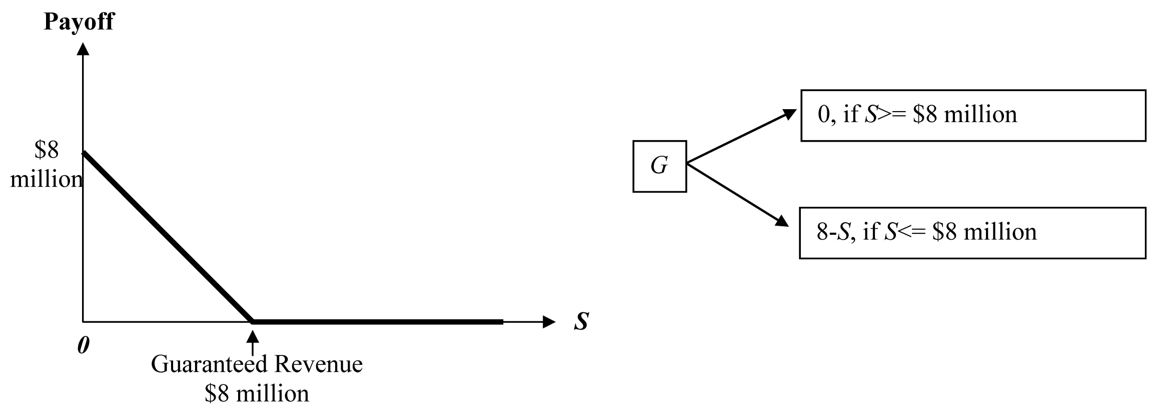


Figure 3.10. The Payoff of MRG

3.3.3. Modeling Incentive as ROs for Major Types of PPPs. In this subsection, a general framework for modeling incentive as ROs for a specific type of PPP will be introduced.

3.3.3.1. Design of RO incentives for major type of PPPs. The design of RO incentives primarily considers two factors: objectives of incentives and sources of uncertainties. Purposes of offering an incentive in PPPs are usually as follows. The first purpose is to attract the private sector. Due to the high uncertainty of PPP projects, the private sector may hesitate to invest in some urgent public projects. Incentives such as guarantees and subsides are offered to the private sector to alleviate their concern of risks, thus attracting private investment in public projects. Regulation is the second purpose. The private sector may make excessively high profit from public projects. This situation, if happens, will reduce public welfare. The public sector, therefore, set regulations such as revenue or tariff cap to PPPs projects. These regulation methods can be considered incentives for the public sector. Stimulation is the third purpose. The private sector has interest conflicts with the public sector. Therefore, the public sector always has concerns with project quality control, efficiency, and public satisfaction of services provided by the private sector. In this case, another kind of incentives is needed, which can stimulate the private sector to behave as the public sector would like it to. The sources of uncertainties in a project determine the underlying assets of the ROs and kinds of risks that need to be eliminated or transferred. For example, the risk of losing profits from operations may be pertaining to the uncertainty in revenue. Therefore, the public sector offers incentives regarding revenue uncertainty, such as MRG, to share this risk with the private sector, in order to attract them to participate in project operations. The revenue is the underlying asset of the options when valuing these incentives.

Incentives should be designed specifically for different forms of PPPs according to their unique features. For PPP forms with low private involvement, such as management contracts and DB, the main financial and operational uncertainties remain with the public sector. The major concern on these PPPs is how to ensure the quality of services provided by the private sector. Therefore, incentives for stimulation, such as revenue sharing or quality warranty, instead of that for attraction and regulation, are needed. In the PPPs that the private sector takes the responsibilities of financing, construction, operation and maintenance, such as DBFO and BOT, the RO incentives regarding interests, demands, and costs are offered to alleviate the private sectors concerns with uncertainty in these or to protect public welfares. In addition, an option to extend concession period can change the private sectors goal from a short-term profit into long-term profit. The change may motivate the private sector to improve quality of construction, maintenance, and services. However, the incentive of concession extension is not effective in the PPPs without concession, such as BOO. Table 3.5 analyzes major uncertainties, risk sharing, and concerns of representative PPPs. RO incentives that can be implemented by these PPP forms are suggested based on the analysis.

3.3.3.2. An example of designing and valuing RO incentives. This section shows the design and valuation of a concession option as an RO incentive for

Table 3.5. RO Incentives Suggested for Major PPP Forms

		Types of PPPs				
		Management Contracts	**DB**	**Lease & Concession**	**DBO& DBFO& BOT**	**BOO**
Risk Analysis	**Major uncertainties**	demand	construction cost	demand, O&M cost	construction cost, demand, O&M cost, financial cost	demand, O&M cost, financial cost
	Risk allocated to (in contract period)	public	private	private	private	private
	The private sector's concerns	--	--	high operating uncertainty	high operating uncertainty, financial cost	high operating uncertainty, Financial cost
	The public sector's concerns	service performance	construction time & quality	excessive profit, service performance, operating risk after contract period	excessive profit, service performance, operating risk after contract period	super profit, service, performance
RO Incentive Design	**Attraction**	--	--	MRG, MDG, subsides, adjustable tariff, option to abandon operation	MRG, MDG, adjustable tariff, option to abandon operation, debt guarantee, maximum interest rate	MRG, MDG, adjustable tariff, debt guarantee, maximum interest rate,
	Regulation	--	--	RCP, DCP, TCP, concession callable (call)	RCP, DCP, TCP, concession callable (call)	RCP, DCP, TCP
	Stimulation	revenue share option (call), contract extension (continuation option)	warranty (put owned by the public)	concession extension (continuation option)	concession extension (continuation option)	--

BOT projects. Under a BOT contract, the private sector is responsible for financing and building a transportation infrastructure. The private sector thereby obtains the ownership of the infrastructure and can operate it to gain profit during the concession period as rewards. The private sector may not be interested in quality investment that will benefit the infrastructure throughout the service life, because the infrastructure will be transferred back to the public sector after the concession. The maintenance near the end of the concession may also be kept at a minimum level. The high uncertainty in revenue is another concern of the public sector. To address these problems, the public sector can offer a concession extension option to the private sector. With the option, the private sector can choose to extend the concession period if the market condition is favorable. The concession extension option is a right to buy an asset (the revenue from the extended concession) at a certain price (the O& M cost during the extended concession) on a certain time (at the concession expires); therefore, it can be modeled as a European call option. The payoff of the option is

$$\max(\hat{E}_\tau[R] - K, 0), \tag{22}$$

Where τ is the end of concession, K is the total O& M cost during $(\tau, T]$ and $\hat{E}_\tau[R]$ is the expected total revenues from the extended concession using risk-neutral valuation at τ. If the annual revenue, S_t , follows a geometric Brownian motion (GBM) process,

$$dS_t = \mu S_t + \sigma dw_t, \tag{23}$$

where μ is the growth rate of annual revenue, σ is the volatility of annual revenue, and w_t is a random walk process, then

$$\begin{aligned} \hat{E}_\tau[R] &= \sum_{i=\tau}^{T-1} \hat{E}(S_i) = \sum_{i=\tau}^{T-1} e^{r(i-\tau)} S_\tau \\ &= e^r[1 - e^{r(T-\tau-1)}]/(1 - e^r) S_\tau. \end{aligned} \tag{24}$$

Let $\alpha = e^r[1 - e^{r(T-\tau-1)}]/(1 - e^r)$,

$$\hat{E}_\tau[R] = \alpha S_\tau. \tag{25}$$

The value of the concession extension option is

$$c = \alpha \hat{E}_0 \left[\max \left(S_\tau - \frac{K}{\alpha}, 0 \right) \right]. \tag{26}$$

Equation 26 shows that the value of this option, valued at present, is dependent of the annual revenue in year τ. Therefore, the option value, c, can be directly determined by the Black-Scholes formula [2]:

$$c = \alpha S_0 N(d_1) - K e^{-r\tau} N(d_2), \tag{27}$$

In equation 27 N designates the cumulative distribution function of standard normal distribution, and d_1 and d_2 are calculated as

$$d_1 = \tfrac{\ln[S_0/(K/\alpha)] + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}; \quad d_2 = d_1 - \sigma\sqrt{\tau}. \tag{28}$$

The public sector can ask for a premium between $[0, c]$ for selling this option, and use this upfront outlay to either cover the potential O&M cost shortage after the concession, or improve the social welfare by, for example, capacity expansions.

For instance, a BOT project has a concession, τ, of 20 years and the initial annual revenue, S_0, is \$10 million. The concession extension option provides the private sector a right to extend the concession for 10 years and the total O& M costs during the extended concession, K, is estimated as \$150 million. The option value, c, will be \$62 million if the risk free rate, r, and the volatility of the revenue, σ, are 5% and 10%, respectively. The option premium can be set from 0 to \$62 million. However, in practice, the premium should not be too high so that the private sector can expect a reasonable return from purchasing the option.

Because the concession may be extended, the private sector has a motivation to improve the construction quality and the maintenance level during the concession to reduce the potential maintenance cost during the extended period. For example, the private sector will be willing to invest \$30 million in construction quality and maintenance improvement if it can reduce the expected total O& M cost in the extended period from \$150 million to \$100 million. The interests of the public and private sectors are, therefore, unitized by the RO incentive, and both parties benefit from it.

This example demonstrates how to design and evaluate a single RO incentive in PPP projects. PPP agreements usually include multiple options. These options can interact and affect the exercise and value of each other. The interaction between multiple options may enhance or reduce the effectiveness of incentives. The BOT project in the example, for instance, may also have MRG and RCP besides the concession extension option. The MRG, if it is effective during the extended concession, can bring more profit to the private sector and increase the value of the concession extension option. The RCP, to the contrast, makes the concession extension option less attractive. Therefore, designing RO incentives in PPP projects should consider interactions between different options. The interactions may increase the complexity of valuing RO incentives.

3.4. SUMMARY OF ESSAY ONE

An option-game framework was built in Essay One to model the incentive function of the RO and show how RO incentives benefit the participants of a cooperative relationship. A highway BOT project was presented for demonstration. In addition to the concession, the public sector offers an RO incentive to the private sector at a premium. The RO incentive gives the private sector a right to continue the operation after the concession period expires (i.e., a continuation option) as well as a right to terminate the continued operation early (i.e., an abandonment option). With the options, the private sector has opportunities to gain additional profit during the post-concession period, with well managed risks. Also, it further increases the total return from the project by increasing its investment in quality improvement. By offering the RO incentive, the public sector successfully converts the uncertain revenue from the post-concession period to an immediate income, that is, the premium. The public sector can use the premium to make up the potential shortfall in maintenance funds if the private sector terminates the operation early. If traffic volume is significantly higher than expected, the premium is also an effective financing resource for capacity expansions. Sensitivity analysis showed that the RO incentive makes both parties more robust to the great uncertainty in revenue management and ensures that a short concession period is still attractive to the private sector. That is, both parties can benefit from the RO incentive. A better PPP therefore is achieved.

A general framework for designing RO incentives for different forms of PPPs was also proposed. Uncertainties in PPP projects and interest conflicts between public and private sectors often raise problems that prevent projects from successful execution. Incentives can be offered to resolve these problems. Commonly used incentives that were modeled as ROs were summarized. This dissertation analyzed the features of each PPP form, identifies major uncertainties facing public and private sectors and their concerns with the PPP form, and designs effective RO incentives for it to promote a better PPP.

4. ESSAY TWO: PRACTICAL ISSUES IN RO APPLICATIONS

4.1. PROBLEM STATEMENT

Capacity expansions under stochastic demand diffusion is a representative application of ROs. Dynamic capacity expansions share similarities with ROs due to the presence of flexibilities in the timing and sizes of expansions. Therefore, the RO models for dynamic capacity expansions are generally multiple exercisable American call options. The RO valuation would answer the questions of "when" and "how much" capacities to add during the decision horizon. In the review of literature, it is further noticed that some important factors add varieties to the dynamic capacity expansions in that they may impact the timing and sizing of capacity expansions. The economies of scale is the first factor. Increasing economy of scale favors one time, large investment to benefit from the volume discount. The RO literature yet often suggests sequential investment in capacity in that the investment is often irreversible (or at last partially irreversible) and under uncertainty. The expansion mode may impact expansion strategies as well. Capacity may be added progressively, in infinitesimal units; for example, knowledge often grows continuously over time. Capacity may be added in large, discrete units; for example, the expansion of highway capacity has to be at least one lane. Opportunity cost of waiting is the third factor. The RO literature usually suggests that capacity expansions should be conservative (better late than early) because of the value of waiting under high uncertainty. However, they often fail to consider the opportunity cost of waiting, that is the profit loss during the waiting time. Opportunity cost of waiting will counteract the benefit of waiting for more information and may change the optimal capacity policy. Terminal value of the expansion phase and the cap of capacity expansion should also impact the behavior of expansions.

This essay performs a systematic analysis of dynamic capacity expansion problem under stochastic demand diffusion. The dynamic capacity expansion problem is modeled as a multiple exercisable option. The optimal strategy of expansion is presented as thresholds of option exercises. This essay will show how the thresholds are affected by the economies of scale, expansion mode, capacity cap, opportunity costs of waiting, and terminal value of expansion, respectively.

4.2. RO VALUATION OF DYNAMIC CAPACITY EXPANSIONS

4.2.1. Demand Processes. Stochastic diffusion of demand usually occurs when the new product or service is just spreading into the market. It has a finite time horizon and is often followed by a stationary phase during which the demand process is random but relatively stable. Figure 4.1 illustrates a sample path of the demand over the diffusion and stationary phases. The diffusion phase may end with a very high demand under successful marketing or favorable market environment, while bad marketing or an unfavorable market condition can take the sales of the new product to the deep freeze. After the diffusion process is completed, the demand process becomes stationary. Affected by the varying market conditions, the demand is still uncertain. However, it will be stabilized within a range that is close to the demand level at the end of the growing stage.

Consider a firm that faces stochastic diffusion of the demand for a new product in the next T years. The demand is assumed to follow the geometric Brownian motion

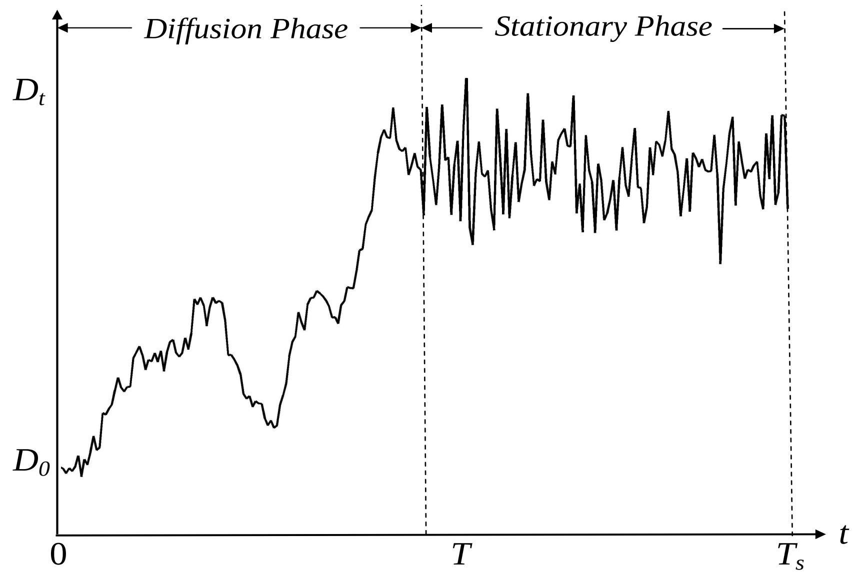


Figure 4.1. A Sample Path of Demand During the Product Life

(GBM) process [83]. That is, the annual growth rate of the demand has a normal distribution, yielding the following equation of demand dynamics:

$$dD_t = \mu D_t dt + \sigma D_t dW_t, \ 0 \leq t \leq T \text{ and } D_0 >= 0 \text{ is known,} \tag{29}$$

where D_t is the demand per year at time t ($0 \leq t \leq T$), μ denotes the expected drift rate of the demand, and the term σ is the volatility, which is the standard deviation of drift rates and measures the scale of demand uncertainty. W_t, a standard Wiener process, models the stochastic movement of D_t.

During the stationary phase, $(T, T_s]$, the annual demand, D_t, is a random process defined in $[D_{smin}, D_{smax}]$. The expected value of D_t is assumed to be equal to D_T and the standard deviation of it is σ_s. It can be described by the following equation:

$$D_t = H_t + D_T, \; T < t \leq T_s. \tag{30}$$

H_t in equation (30) is a white noise process, which has the features of:

$$\begin{aligned} &E\{H_t\} = 0 \text{ and } Var\{H_t H_t^T\} = \sigma_s^2, \; \forall t \\ &E\{H_t H_\tau\} = 0 \text{ for } \forall t \neq s. \end{aligned} \tag{31}$$

4.2.2. Valuation Model. The firm has the flexibility to build production capacity progressively according to the progress customer acquisition. During the diffusion phase, the firm observes the current demand, reviews the available capacity, and then decides if capacity expansion at this time is needed and how much the capacity should be added to maximize the expected remaining profit. This decision can be made repeatedly until the demand becomes stationary or the capacity reaches its limit (if any). The flexibilities of dynamically expanding capacity is modeled as multiple exercisable call options, of which the benefit is the added profits during the remaining periods and the exercise price is the installation cost of the new capacity.

The profit flow is a function of capacity, C_t, and demand, D_t, denoted as $\pi(D_t, C_t)$,

$$\pi(D_t, C_t) = P \min[D_t, C_t] - mC_t. \tag{32}$$

where P is the market price of the product and m is the marginal production cost. The firm is assumed to be a price taker, that is, the supply of the firm does not affect the product price. To simplify the problem, m is assumed to be a constant.

The initial production capacity of this firm is C_0. Denote ξ_t the expansion amount at t, then the capacity dynamics is define as

$$dC_t \triangleq C_{t+dt} - C_t = \xi_t, \tag{33}$$

where $\xi_t \in \mathscr{A}_t$ is nonnegative ($\mathscr{A}_t$ is the action space given C_t).

The cost of installing additional capacity, $g(\xi_t)$, is an increasing function of ξ_t and starts at zero, that is

$$g(\xi_t + \epsilon) \geq g(\xi_t), \ \forall \epsilon \geq 0, \text{ and } g(0) = 0. \tag{34}$$

The firm continuously monitors the realized demand and chooses the optimal ξ_t throughout the diffusion phase, $[0, T]$, to maximize the expected remaining profit. The objective function is

$$J = \max_{dC_{t(0\to T)}} E\left[e^{-rT} S(D_T, C_T) + \int_0^T e^{-rt} \pi(D_t, C_t) dt - \int_0^T e^{-rt} g(dC_t) \right]. \tag{35}$$

$S(D_T, C_T)$ in Equation (35) is the terminal value at T. The optimal path, $\{\xi_t^* | 0 \leq t < T\}$, is found by maximizing the expected profit-to-go, defined below, at any value of C_t and D_t for $0 \leq t < T$. The valuation function at any state (t, D_t, C_t) is:

$$\begin{aligned} V(t, D_t, C_t) = \max_{dC_t \in \mathscr{A}_t} E\Bigg[& \int_t^{t+dt} e^{-rs} \pi(D_s, C_s) ds - g(dC_t) \\ & + e^{-rdt} V(t + dt, D_{t+dt}, C_{t+dt}) \Bigg]. \end{aligned} \tag{36}$$

where $\mathscr{A}_t$ is the action space given the state at time t, and the terminal condition is

$$V(T, D_T, C_T) = S(D_T, C_T). \tag{37}$$

In Equation (36), $D_{t+dt} = D_t + dD_t$ and $C_{t+dt} = C_t + dC_t$; the dynamics dD_t and dC_t are determined by Equations (29) and (33), respectively.

The terminal value, $S(D_T, C_T)$, is the expected total profits during the stationary phase, which is evaluated at the beginning of the stationary phase (it is also the end of the diffusion phase). Through the stationary phase, the capacity keeps at the level at the end of diffusion phase, C_T. The annual demand is D_t, as described in Equation (30). The firm requires a discount rate of r_s. Then the expected profit of the stationary period, that is, the terminal value at T, is

$$\begin{aligned} S(D_T, C_T) &= E\left[\int_T^{T_s} e^{-r_s(s-T)}\left[P\min(D_t, C_T) - mC_T\right]ds\right] \\ &= \frac{1-e^{-r_s(T_s-T)}}{r_s}\left\{(P-m)C_T + PE\left[min(D_t - C_T, 0)\right]\right\} \end{aligned} \tag{38}$$

Let $F(.)$ represents the cumulative distribution function of D_t during the stationary phase, $f(.)$ be the probability density function, then

$$E\left[\min(D_t - C_T, 0)\right] = \int_{D_{smin}}^{D_T}[D_t - C_T]f(D_t)dD_t = \int_{D_{smin}}^{D_T} F(D_t)dD_t, \tag{39}$$

where D_{smin} is the lower bound of D_t during the stationary phase.

4.2.3. Numerical Schemes. The dynamic capacity expansion problem in Equation (35) is often solved numerically using Dynamic Programming (DP). Through the numerical solution, the threshold of exercise and the optimal amount of expansion at any state, (t, D_t, C_t), can be found.

A binomial tree, shown in Figure 4.2, can be a discrete approximation of the GBM process that D_t follows during time $[0, T]$. Let $M = T/\Delta t$ be an integer, where Δt is the size of the time step. At any discrete time i there are $i + 1$ unique levels of demand, D_{ij}, where

$$D_{ij} = D_0 u^{i-j} d^j, \text{ for } i = 0, 1, ..., M, \; j = 0, 1, ..., i,$$
$$u = e^{\sigma\sqrt{\Delta t}}, \; d = 1/u, \; p = \frac{e^{\hat{\mu}\Delta t} - d}{u - d}, \tag{40}$$

where u and d are the up-movement and down-movement factors, respectively; p is the possibility that an up-movement occurs; $\hat{\mu}$ is the risk neutral growth rate of demand, which equals $\mu - \lambda\sigma$, where λ is the market price of risk on demand ([2]).

Let $\Delta\xi$ be the step of expansion amount dC_t and define $K = \left[\frac{C_{max} - C_{min}}{\Delta\xi}\right]$, then a capacity level, C_k, is

$$C_k = C_{min} + k\Delta\xi, \text{ for } k = 0, 1, ..., K. \tag{41}$$

Thus, the continuous states, $(t,\, D_t,\, C_t)$, can be represented in a 2.5-D discrete system, as shown in Figure 4.2. Any node, (i, j, k), in the 2.5-D system indicates a unique combination of t, D_t, and C_t.

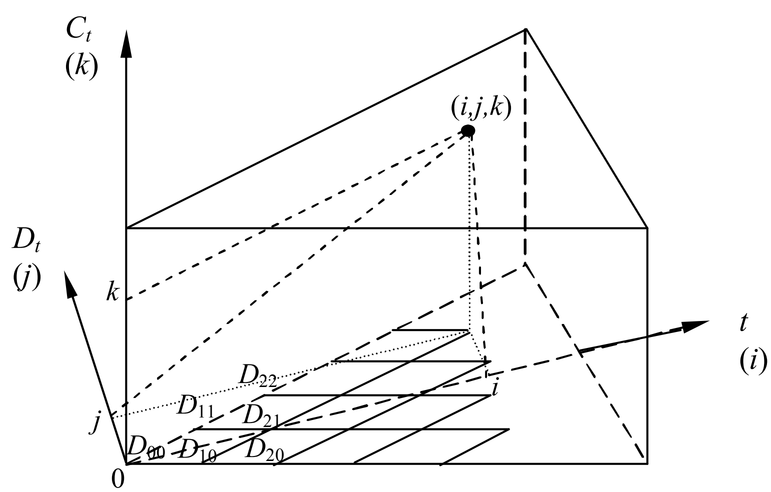


Figure 4.2. 2.5-D Binomial Tree that Discretely Approximates $(t,\, D_t,\, C_t)$

The 2.5-D state system is storage consuming if it is saved as a 3-D matrix. In addition, the three states plus the action, dC_t, increase the dimensions to 3.5-D. It is hard for a system that is more than 3-D to be visualized. Fortunately, D_{ij} is a function of i, thus a 2-D grid, shown in Figure 4.3, can be built to replace the 2.5-D system.

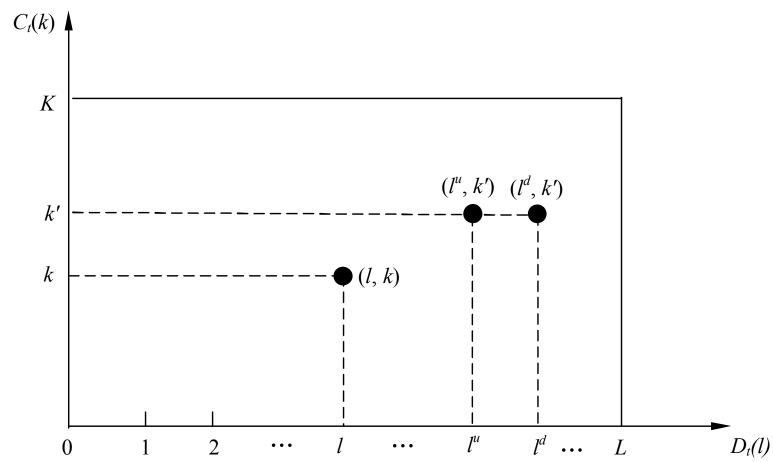


Figure 4.3. 2-D Grid that Discretely Approximates (t, D_t, C_t)

The 2-D grid is defined as (l, k). l is the index of discrete demand levels during the diffusion phase and $L = (M+1)(M+2)/2 - 1$ is the largest index value of l. k is still the index of capacity levels. For $l = 0, 1, \ldots, L$,

$$D_l = D_0 u^{i-j} d^j, \tag{42}$$

where

$$\begin{aligned} i &= \left\lceil \frac{-3 + \sqrt{9 - 4(1 - 2l)}}{2} \right\rceil, \\ j &= l - \frac{i(i+1)}{2}. \end{aligned} \tag{43}$$

Therefore, the valuation procedures on the 2-D grid are as follows:

When $l \geq \frac{M(M+1)}{2}$, $\forall k$,

$$V_{lk} = S(D_l, C_k); \tag{44}$$

when $l = \frac{M(M+1)}{2} - 1, \frac{M(M+1)}{2} - 2, \ldots, 0$, and $\forall k$,

$$V_{lk} = \max_{\xi_{lk} \in \mathscr{A}_k} \left\{ \pi_{lk}\Delta t - g(\xi_{lk}) + e^{-r\Delta t}[pV_{l^u k'} + (1-p)V_{l^d k'}] \right\}, \tag{45}$$

where

$$\begin{aligned} l^u &= l + (i+1) = \frac{(i+1)(i+2)}{2} + j, \\ l^d &= l + (i+1) + 1 = \frac{(i+1)(i+2)}{2} + j + 1, \\ k' &= k + \frac{\xi_{lk}}{\Delta \xi}, \end{aligned} \tag{46}$$

with i and j defined in (43). π_{lk} in Equation (45) designates the profit flow, $\pi(D_l, C_k)$. The completion of the backward recursion defined in Equations (44-45) gives V_{00}, which is the equal to the expected net present value (NPV) of production plus the option value.

4.3. EXPANSION POLICY

Although the numerical method can solve the problem, it is not able to provide much insights of the optimal expansion policy. Theoretical analysis is, therefore, performed in this section to develop better understanding of the optimal policy.

4.3.1. Capacity Expanded in Infinitesimal Units. First, let us consider the scenario that C_t is continuous at any $t \in [0, T]$, which indicates that capacity expansions can be in infinitesimal units as the time interval, dt, approaches 0:

$$\lim_{dt \to 0} dC_t = \lim_{dt \to 0} (C_{t+dt} - C_t) = 0. \tag{47}$$

Approximate conditions are commonly observed. For example, factories usually increase their capacity by adding laborers, working overtime. The service capacity also can be adjusted flexibly, such as increasing servers, adding a phone line, and so on. Further assume the derivative defined below exists at any time t,

$$\lim_{dt \to 0} \frac{dC_t}{dt} = \lim_{dt \to 0} \frac{C_{t+dt} - C_t}{dt} = q_t, \tag{48}$$

where q_t ($0 \leq q_t \leq q_{max}$) is the instantaneous annual growth rate of capacity. Therefore,

$$\xi_t = dC_t \approx q_t dt, \tag{49}$$

In the remaining of the paper, ξ_t and dC_t are used exchangeably. The cost of adding new capacity, $g(dC_t)$, becomes a function of q_t and dt, $g(q_t, dt)$. Define dg/ξ_t as g' and d^2g/ξ_t^2 as g'', and assume they exist, then applying Taylor's expansion, Equation (36) leads to the following differential equation:

$$\begin{aligned} V(t, D_t, C_t) = \max_{dC_t \in \mathscr{A}_t} E \Big\{ & \pi(D_t, C_t)dt - g'(dC_t)dC_t + (1 - rdt)V + \frac{\partial V}{\partial t}dt \\ & + \frac{\partial V}{\partial D_t}dD_t + \frac{1}{2}\frac{\partial^2 V}{\partial D_t^2}(dD_t)^2 + \frac{\partial V}{\partial C_t}dC_t + o(dt) \Big\}, \end{aligned} \tag{50}$$

Given the demand dynamics and capacity dynamics in Equations (29) and (49), Equation (50) becomes the following when $dt \to 0$:

$$\begin{aligned} 0 = \max_{dC_t \in \mathscr{A}_t} \Bigg\{ & \left[\frac{\partial V}{\partial C_t} - g'(dC_t) \right] q_t \\ & + \pi(D_t, C_t) - rV + \frac{\partial V}{\partial t} + \hat{\mu} D_t \frac{\partial V}{\partial D_t} + \frac{1}{2}\sigma^2 D_t^2 \frac{\partial^2 V}{\partial D_t^2} \Bigg\}. \end{aligned} \tag{51}$$

Define L_1 as follows,

$$L_1 \triangleq \frac{\partial V}{\partial t} + \mu D_t \frac{\partial V}{\partial D_t} + \frac{1}{2}\sigma^2 D_t^2 \frac{\partial^2 V}{\partial D_t^2}, \tag{52}$$

then Equation (51) becomes

$$rV - L_1 - \pi = \max_{q_t \in \mathscr{A}_t'} \left\{ \left[\frac{\partial V}{\partial C_t} - g'(q_t dt) \right] q_t \right\}. \tag{53}$$

The economies of scale may play an important role in the features of the optimal policy. If g is a linear function of dC_t, that is, $g(dC_t) = hq_t dt$ (h is a constant), then $\frac{\partial V}{\partial C_t} q_t - \frac{g}{dt}$ is also a linear function of q_t with a slope $\frac{\partial V}{\partial C_t} - h$. The solution of q_t^* is therefore

$$q_t^* = \begin{cases} q_{max}, & \text{if } \frac{\partial V}{\partial C_t} - h > 0 \\ 0, & \text{if } \frac{\partial V}{\partial C_t} - h \leq 0. \end{cases} \tag{54}$$

When $\frac{\partial V}{\partial C_t} - h \leq 0$, the firm will choose not to exercise the expansion option and keep the current capacity level; when $\frac{\partial V}{\partial C_t} - h > 0$, the firm will exercise the option and expand the capacity at the maximum expansion rate. Intuitively, people may think choosing the most suitable expansion rate between $[0, q_{max}]$ would be optimal. However, Equation (54) shows that when the expansion cost is linear, even though there are a wide range of expansion rates available, the optimal policy is still binary - either do not expand or expand at the maximum expansion rate. That means, like a standard option, the decision maker only needs to decide to exercise the option or not, but not how much the capacity should be added.

If the expansion cost, g, is a concave function of dC_t, displaying an increasing economy of scale. Then $\frac{\partial V}{\partial C_t} - g'$ is increasing on the range of u_t is a convex function of dC_t. Therefore, q_t^* is

$$q_t^* = \begin{cases} q_{max}, & \text{if } \frac{\partial V}{\partial C_t} - g' \geq 0 \; \forall q_t \\ 0, & \text{otherwise.} \end{cases} \tag{55}$$

Similar to the linear expansion cost case, the optimal policy, q_t^*, is also binary when the expansion cost has a increasing economy of scale.

If the expansion cost, g, is a convex function of dC, showing a decreasing economy of scale, $\frac{\partial V}{\partial C_t} - g'$ is decreasing on the range of q_t. Therefore, q_t^* has the following formula:

$$q_t^* = \begin{cases} q_{max}, & \text{if } \frac{\partial V}{\partial C_t} - g' > 0 \; \forall q_t \\ \frac{g'^{-1}(\frac{\partial V}{\partial C_t})}{dt}, & \text{if } \exists \; \frac{\partial V}{\partial C_t} - g' = 0 \\ 0, & \text{if } \frac{\partial V}{\partial C_t} - g' < 0 \; \forall q_t \end{cases} \tag{56}$$

In this case, the binary feature of the optimal policy does not hold.

Theorem 1. *When the expansion cost has a constant or increasing economy of scale, the optimal capacity expansion policy is binary - either not to expand, or to expand at the maximum expansion rate - even there are multiple expansion rates available. When the expansion cost has decreasing economy of scale, the binary feature of the optimal policy does not hold.*

4.3.2. Capacity Expanded in Large, Discrete Units. The previous analysis assumes that the capacity is continuous over time. The assumption may not be hold. For example, capacity may only be expanded in large, discrete units. This situation are more commonly observed. For instance, in the manufacturing, when an existing plant is fully loaded, the capacity may only be added in the units of machines or production lines. In highway expansions, at least one lane should be added. In these cases, the features of optimal expansion policy would be changed. Under this condition, dC_t may not approach zero as $dt \to 0$ at any time t.

The Taylor's expansion of the valuation function when $dt \to 0$ becomes

$$rV - L_1 - \pi = \max_{dC_t \in \mathscr{A}_t} \left\{ \frac{\Delta V(t, D_t, C_t, dC_t) - g(dC_t)}{dt} + L_2 \right\}, \tag{57}$$

where $\Delta(t, D_t, C_t, dC_t)$ represents the change in $V(t, D_t, C_t)$ if capacity is increased by dC_t (i.e., $V(t, D_t, C_t + dC_t)$-$V(t, D_t, C_t)$). Since capacity is added in large, discrete units, $\lim_{dt\to 0} \frac{dC_t}{dt}$ may not exists or go to infinity. Therefore, when dt approaches 0, $\Delta(t, D_t, C_t, dC_t)$ may no longer be reasonably approximated by $\frac{\partial V}{\partial C_t} dC_t$ because the higher order items are not negligible. This makes L_2 as follows are not negligible too:

$$L_2 \triangleq \frac{\partial \Delta V}{\partial t} + \hat{\mu} D_t \frac{\partial \Delta V}{\partial D_t} + \frac{1}{2}\sigma^2 D_t^2 \frac{\partial^2 \Delta V}{\partial {D_t}^2}. \tag{58}$$

Let y denote $\Delta V - g + L_2 dt$ in Equation (57). The optimal value of dC_t, dC_t^*, is the one that maximizes y. The solution thus rests with the structure of y. Because the y is a higher order function of dC_t, it may have multiple vertexes within the range of dC_t. dC_t^* can be any available value of ξ_t shown in Figure 4.4, depending on the coefficients in y, i.e., the relationships between V and the current states, t, D_t, C_t, and the cost structure of expansion.

4.4. NUMERICAL STUDIES

4.4.1. A Numerical Example. In this section, a numerical example is used to illustrate the methodology and verify the theoretical results. This example is used as a benchmark in the numerical studies. Then the effects of the economies of scale,

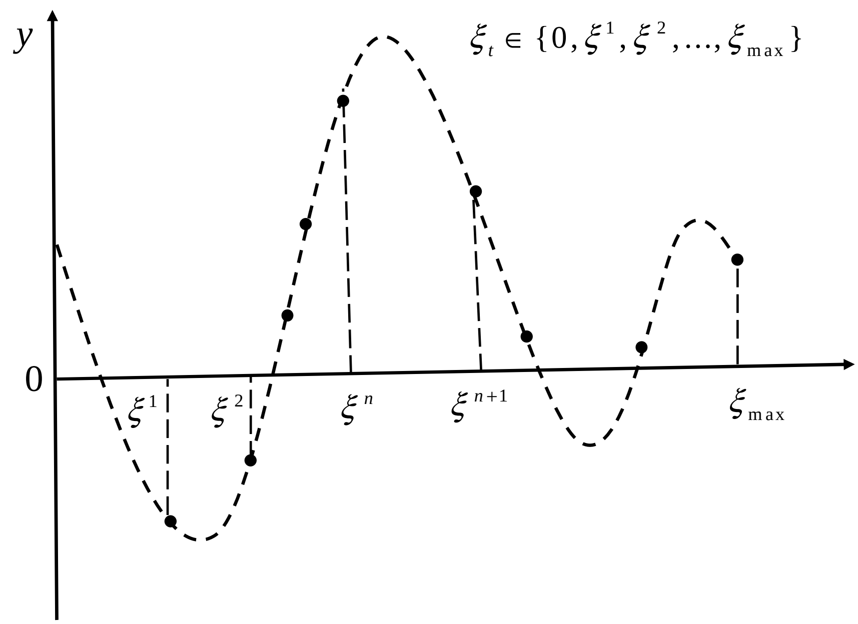


Figure 4.4. Optimal Solution dC_t^* When dC_t Is in Large, Discrete Units

capacity cap, expansion mode, waiting cost, and the terminal value will be examined comparing to the benchmark example.

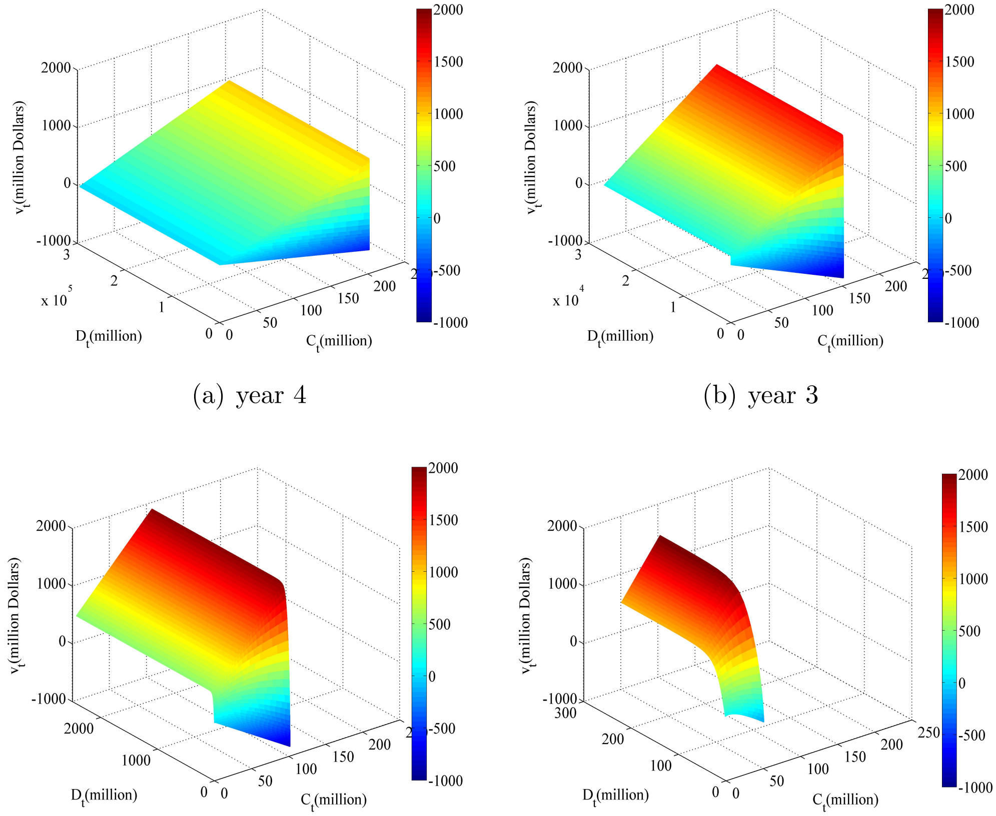
A new product is spreading in the market. The diffusion phase is expected to last 5 years and followed by a 5-year stationary phase. The initial annual demand is estimated to be 20 million units. During the diffusion stage, the demand follows a GBM process with a drift rate of 4% and volatility of 10%. The price of the product is 8 dollars per unit and the marginal cost is 3 dollars. The firm has flexibility to build the production capacity dynamically during the diffusion phase, starting with 0. The capacity of the new production can be expanded continuously by adding labors, with a maximum annual expansion rate of 50 million units. The decision of capacity expansion is made semi-monthly. The economies of scale of the capacity installation is estimated to be constant, that is, the function of the expansion cost is linear, $g(dC_t) = 4q_t dt$. The risk free rate is 5%. The profit gained in the stationary phase, that is the terminal value at the end of diffusion phase, is not considered here. The parameter values are summarized in Table 4.1.

Table 4.1. Parameter Values for the Example

Parameter	**Value**	**Parameter**	**Value**
D_0	20 million	C_0	0 million
T	5 years	q_{max}	50 million/year
μ	4%	P	8 dollars
λ	0.05	m	3 dollars
σ	10%	h	4 dollars
r	5%	S	0 million
Δt	1/24 years		

Figure 4.4.1 shows the evolution of the expected value-to-go, $V(t, D_t, C_t)$, at all values of D_t and C_t, and four selected time spots, $t = 4$, 3, 2, and 1 year during the backward recursion. With the optimization progress, the range of negative values is shrinking and the cumulative positive value is increasing. The expected NPV with options is obtained at $t = 0$, which is equal to 258.19 million dollars.

Visualization of the optimal policy is difficult when the number of states is more than two (there are three states in this example). It is difficult to obtain a direct impression from a 3-D or 4-D figure. Therefore, some time spots, for instance, t=1,2,3, and 4, are selected instead of the entire time horizon, to observe the changes of optimal policy at the current demand and capacity level. Figure 4.6 shows the optimal policy of expansion at the end of year 1, 2, 3, 4. The X and Y axes represent the current capacity, C_t, and the demand, D_t, respectively. Colors are used to indicate the value of optimal expansion rate, q_t^*. The numerical results from dynamic programming show that, when time step is small enough, the optimal policy trends to be binary, even



(a) year 4

(b) year 3

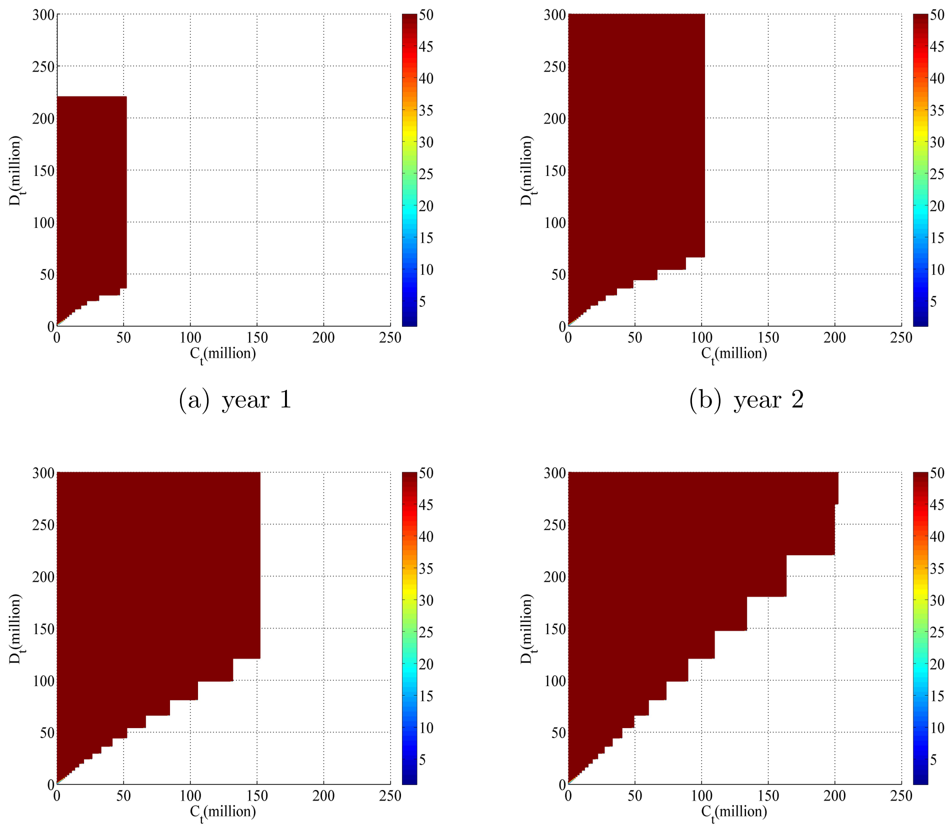
(c) year 2

(d) year 1

Figure 4.5. Evolution of $V(t, D_t, C_t)$ at t=4, 3, 2, and 1 Year

though the expansion rates are available from [0,50]. This observation is consistent with the theoretical analysis in Section 4.3.

Because of the binary feature, a free boundary, which is the lowest demand level that the option should be exercised (at q_{max} for this example) at any given capacity, can be used to illustrate the optimal policies for different time spots within a single figure. In the area that is below the free boundary the capacity will keep at the current level. Figure 4.7 displays the free boundaries of exercising the expansion option when t = year 1, 2, 3, and 4. For example, at the end of year 3, if the current capacity is 100 million, the expansion option should be exercised when the demand is equal or higher than about 80 million, at the maximum expansion rate of 50 million per year.



(a) year 1 (b) year 2

(c) year 3 (d) year 4

Figure 4.6. Optimal Policy at Selected Time Spots

To illustrate how the optimal policy is applied to a decision scenario, a random path of demand and the corresponding optimal control of capacity is provided in Figure 4.8. At the beginning, the initial demand is 20 million units and capacity is 0 million units. During the early period, the capacity is increasing continuously at the maximum expansion rate to meet the demand level. By $t = 0.38$ year, the capacity has exceeded the demand. However, because of the positive expectation for the future, the optimal action is still to exercise the expansion option until the capacity reach a relatively high level compared to the demand. For example, At $t = 0.63$ year, C_t is 25.7 million while D_t is only 22.2 million, the optimal decision is still to expand the capacity. Although the capacity may keep the current level when

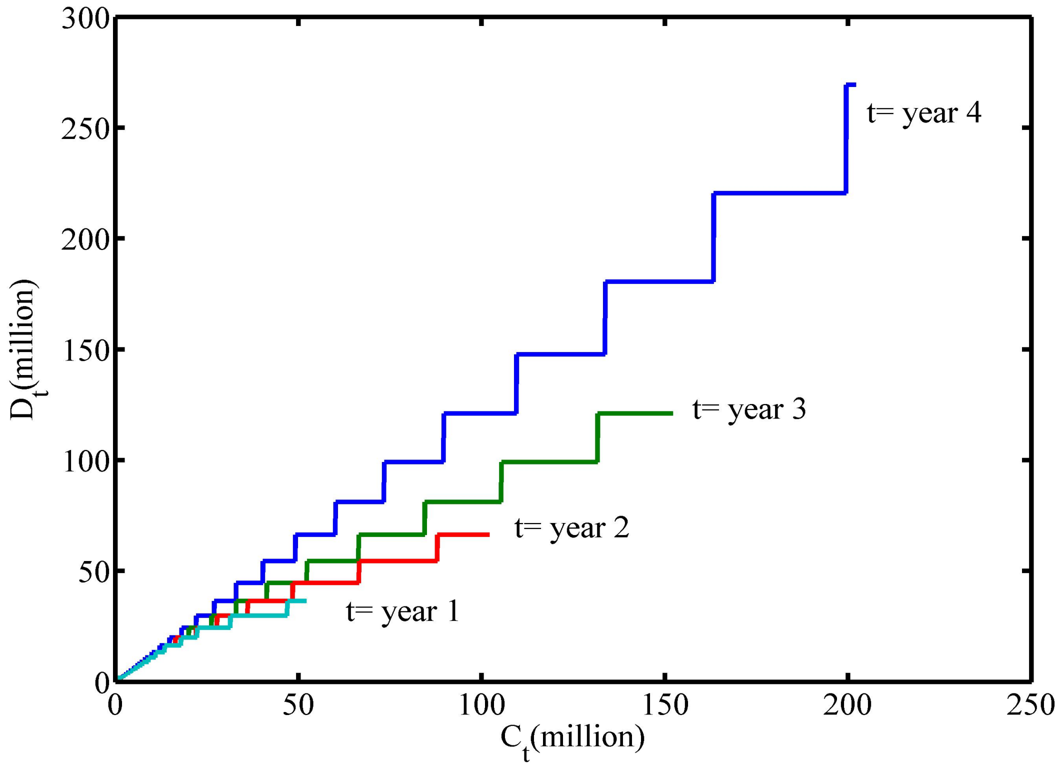


Figure 4.7. Free Boundaries at Selected Time Spots.

it is high enough (e.g. during $t \in [0.83, 1.67)$ and $t \in [1.83, 2.88)$), some times rapid increase in the demand still encourages capacity expansion. For example, at time $t = 1.67$ and $t = 2.88$ year. The policy tends to be more conservative during the late period than the early years. Even the demand is higher than the capacity, the firm choose not to expand. For example, at $t = 4.5$ year, the capacity keeps at 30 million while the demand reaches 32 million.

4.4.2. Waiting Cost. Figures 4.7 and 4.8 show that the expansion policy during late years is more conservative than in early periods. This observation is different than some previous studies. In the literature on capacity expansion using RO, investment in capacity is often better late than early. (e.g.,[83, 84]). For the high uncertainty in future demand, holding the option until more information is

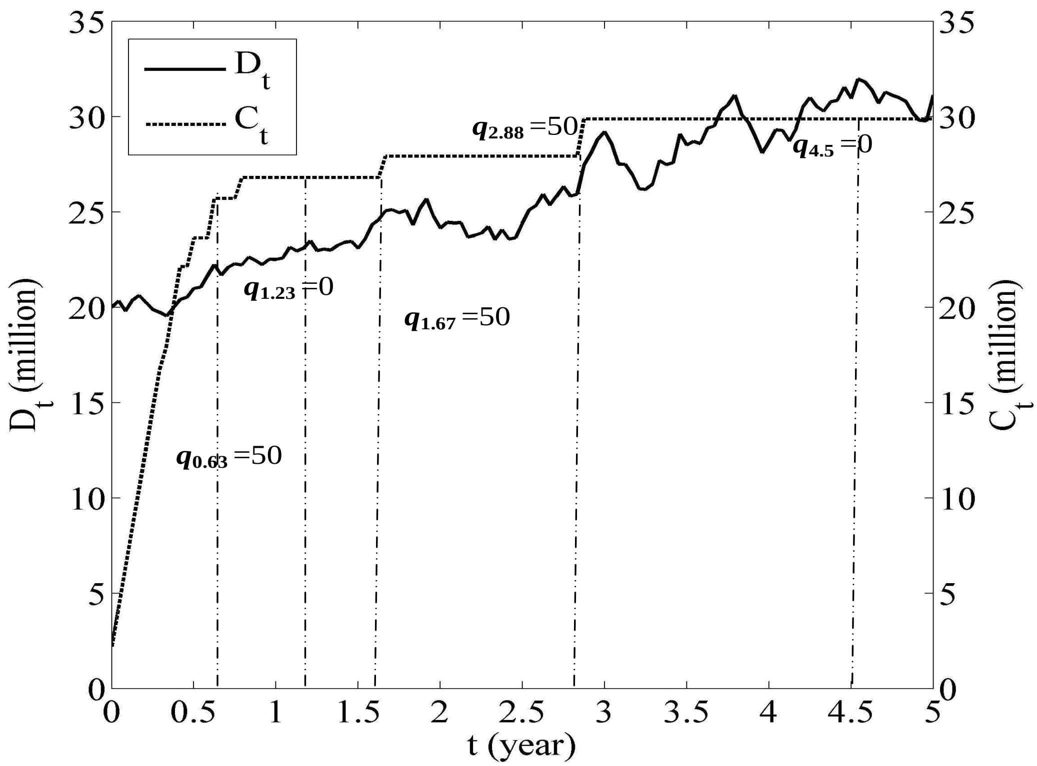


Figure 4.8. A Random Path of Demand and Corresponding Optimal Capacity Control

revealed is valuable because it can make the adjusted capacity better match the demand. However, when considering the cost of waiting (or diminishing value from options), the optimal policy may change. The free boundary in Figure 4.7 shows that the optimal policy is more aggressive in earlier years than those of the later years. That is, the thresholds of exercising the option are lower in the earlier years than those of the later years. For example, Figure 4.7 shows that the threshold demand for triggering expansion at the end of year 1 is about 38 million when the current capacity is 50 million. However, the threshold increases to about 60 million at the end of year 4. Although the uncertainty in demand is better resolved in year 4, the decision in year 4 is more conservative than year 1. It is because the capacity installed later will serve for a shorter time and generate less cumulative profit with the same installation cost. That is, the rate of return from expansion investment decreases over time. The decreasing rate of return counteracts the benefit of uncertainty resolving, making investing aggressively in the early years a better choice than waiting.

4.4.3. Economies of Scale. As analyzed in 4.3.1, the economies of scale of the expansion cost will affect the optimal policy. To verify the results of theoretical results, two situations are examined respectively. One is that the expansion cost is a concave function, $g(\xi_t) = h\xi_t^{2/3}$, and the other is the expansion cost is a convex function, $g(\xi_t) = h\xi_t^{3/2}$.

Figure 4.9 shows the free boundaries when the capacity expansion has an increasing economy of scale (i.e., the cost function is an decreasing function of capacity expansion). The result is similar to that in the situation of constant economy of scale: the optimal policy is binary. In addition, the rate of return on expansion increases under the increasing economy of scale. Thus the free boundaries of exercising is lower than the situation of linear expansion cost. For instance, when the cost function is constant, the threshold to exercise the option is about 120 million when the current capacity is 100 million at the end of year 4; the threshold is decreased to about 80 million when the cost function is concave. Furthermore, the differences between the boundaries at different time spots are reduced. It means the optimal policy becomes less time sensitive. The reason is that the increasing rate of return from expansion under an increasing scale of economy reduces the cost of waiting.

The optimal policy when the economy of scale is decreasing is shown in Figure 4.10. Unlike the situation with constant or increasing economies of scale, the optimal policy is not binary. The optimal expansion rate at a certain capacity grows gradually with the increasing current demand level. The maximum expansion rate, q_{max}, may be never reached. Therefore, free boundaries cannot be used to display the optimal policies in this case. For example, at the end of year 1, when the capacity is 50 million, the expansion rate would be 1 million when the demand is 16 $\sim$ 20 million, 2

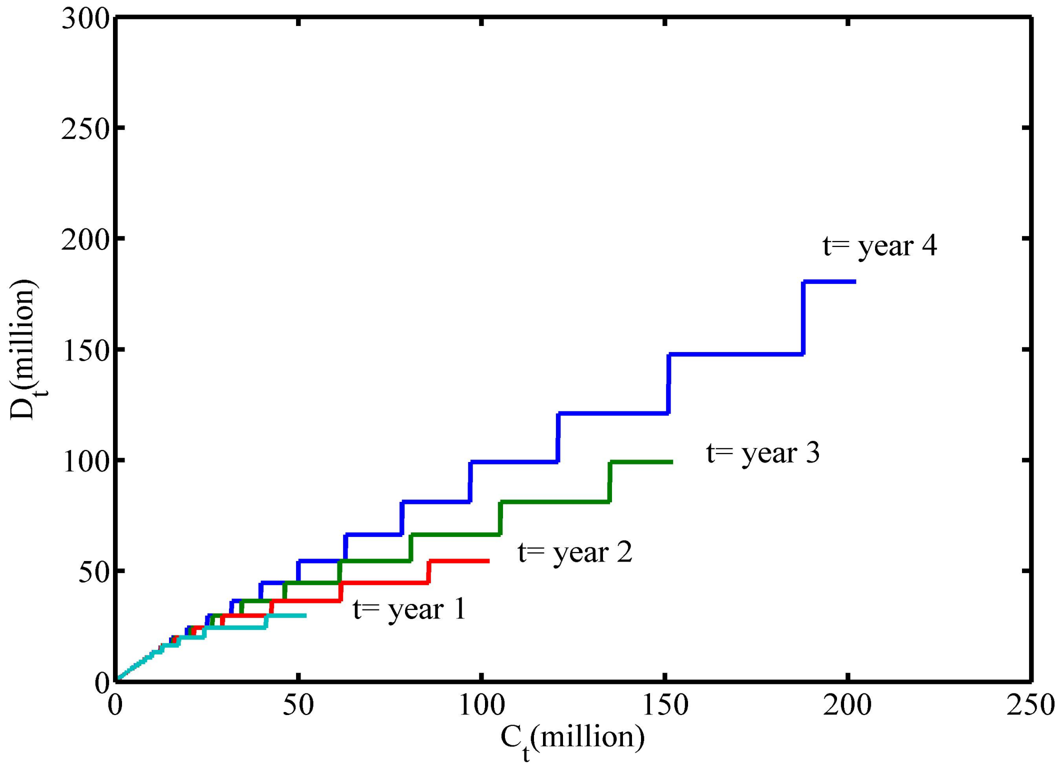
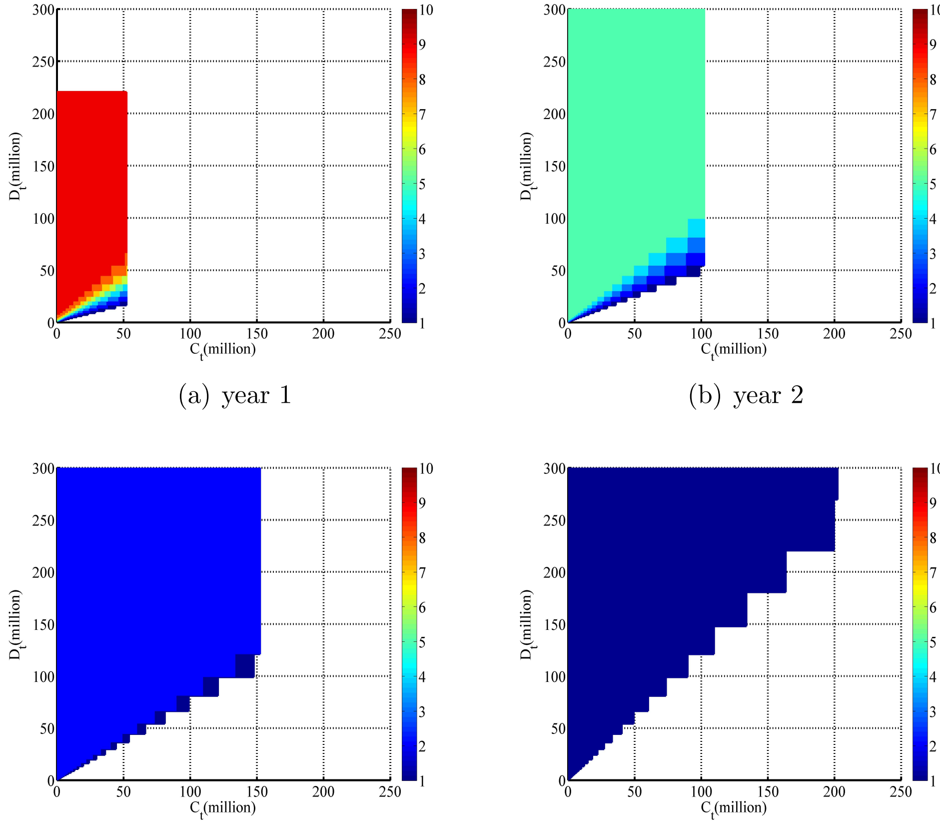


Figure 4.9. Free Boundary When the Economies of Scale Is Increasing

million when the demand is $20 \sim 24$ million, ..., 9 million when the demand is no less than 81 million. The decreasing economy of scale on the expansion cost also make the optimal policy more time sensitive, in that the decreasing rate of return from expansion further raises the cost of waiting. Figure 4.10 shows that the maximum chosen expansion rate decreases over time. It is 9 million/per year at the end of year 1, but only 1 million at the end of year 4. In addition, the threshold of expansion increases rapidly. For example, at the end of year 1, the threshold of expanding the capacity at 1 million/year is only about 18 million of demand. This number increases to about 55 million at the end of year 4.

4.4.4. Expansion Policy When Capacity is Expanded in Large, Discrete Units. Then, the producing capacity of the new product is assumed must



(a) year 1 (b) year 2

(c) year 3 (d) year 4

Figure 4.10. Optimal Policy When the Economies of Scale Is Decreasing

be expanded in large, discrete lots. Each lot is a production line with throughput 10 million units of product. The maximum expansion amount at one time is five lots with the total production capacity of 50 million units, as well as the total maximum capacity. In this case, the expansion amount per time, ξ_t, is discrete with a step of 10 million and a maximum value of 50 million. Figure 4.11 shows the optimal policies in selective time spots. The feature of the optimal policies is completely different from that when the capacity is changed continuously over time. Other than the binary policy, each expansion amount available can be chosen as the optimal action, according to the current states, t, D_t, C_t. For example, at the end of year 4, when the current capacity is 20 million, one production line (10 million) will be added when the demand reaches 30 million, two production lines should be added if demand increases to 37 million units, and three production lines should be added if demand is increased to 50 million. This result verifies the analysis in Section 4.3.2. The policies are also unlike those with convex cost function and continuously capacity function (see Figure 4.9), although neither of them is binary. The capacity in Figure 4.11 has only discrete values. Expansion policies thus appear as lines, corresponding to the discrete capacity levels, instead of areas in Figure 4.9. It should be remarked that the policy lines for each capacity level is supposed to be defined on any value of D_t. The gaps in Figure 4.11 are caused by the discretization of D_t using binomial lattice. A smaller time step or a more accurate method of discrete approximation, such as trinomial lattice, can narrow these gaps and will be an extension of current work.

4.4.5. Capacity Cap. The analysis in the benchmark case assumes that the total amount of capacity expansions can be unlimited. In practice the maximum capacity is usually constrained by the availability of resources, such as financial budget, labor, machines, or spaces. Given a capacity cap, C_{max}, the expansion rate, q_t, may not be able to reach the maximum q_{max} when the current capacity has already been at a relatively high level although q_{max} is optimal, theoretically. The maximum rate that can actually be applied becomes $\min[q_{max}, (C_{max} - C_t)/dt]$, no longer q_{max}. Therefore, the binary feature of the optimal policy when $g(\xi_t)$ is linear or concave will not hold when C_t is approaching C_{max}.

As an numerical example, the capacity cap is set at 50 million. The optimal policy is either no expansion or expansion at the rate $\min[q_{max}, (C_{max} - C_t)/dt]$. Thus the optimal policy is not binary and decreases gradually from q_{max} to 0 when the current capacity is approaching C_{max}, as shown in Figure 4.12. For example, when the current capacity reaches 49 million, the maximum expansion rate can only be 24 million per year; that is, the expansion amount during a 1/24 year interval is either 0 or 1 million under this circumstance.

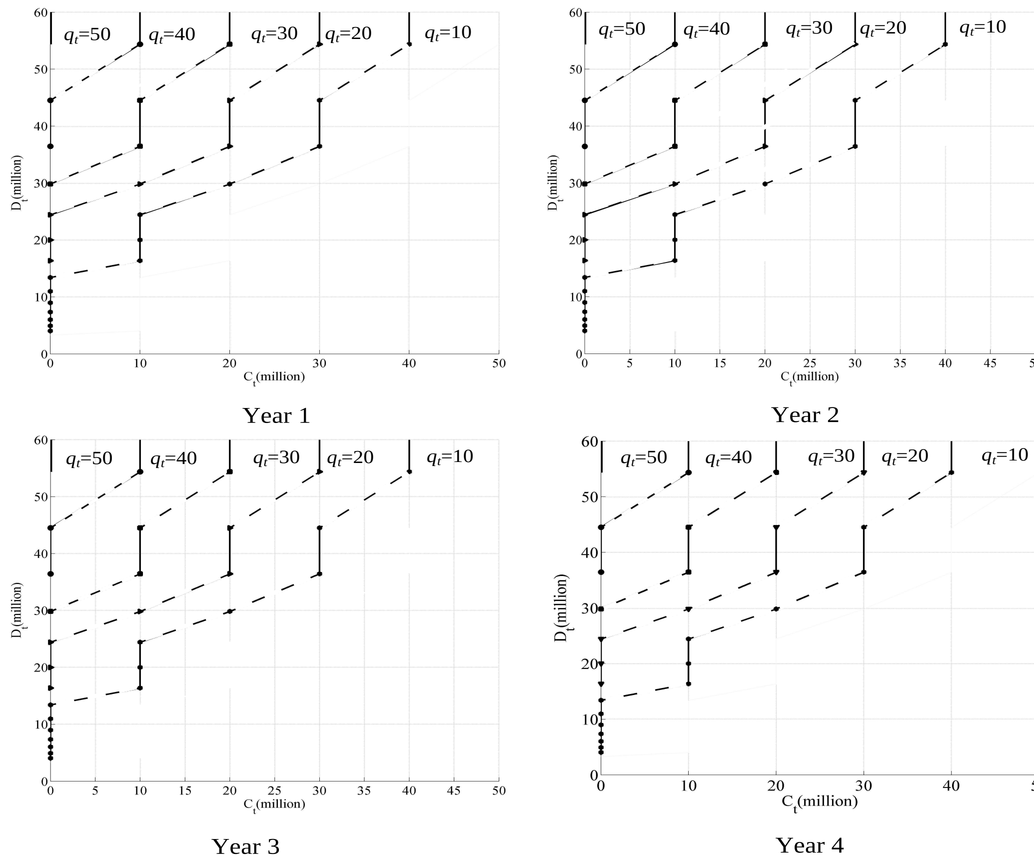
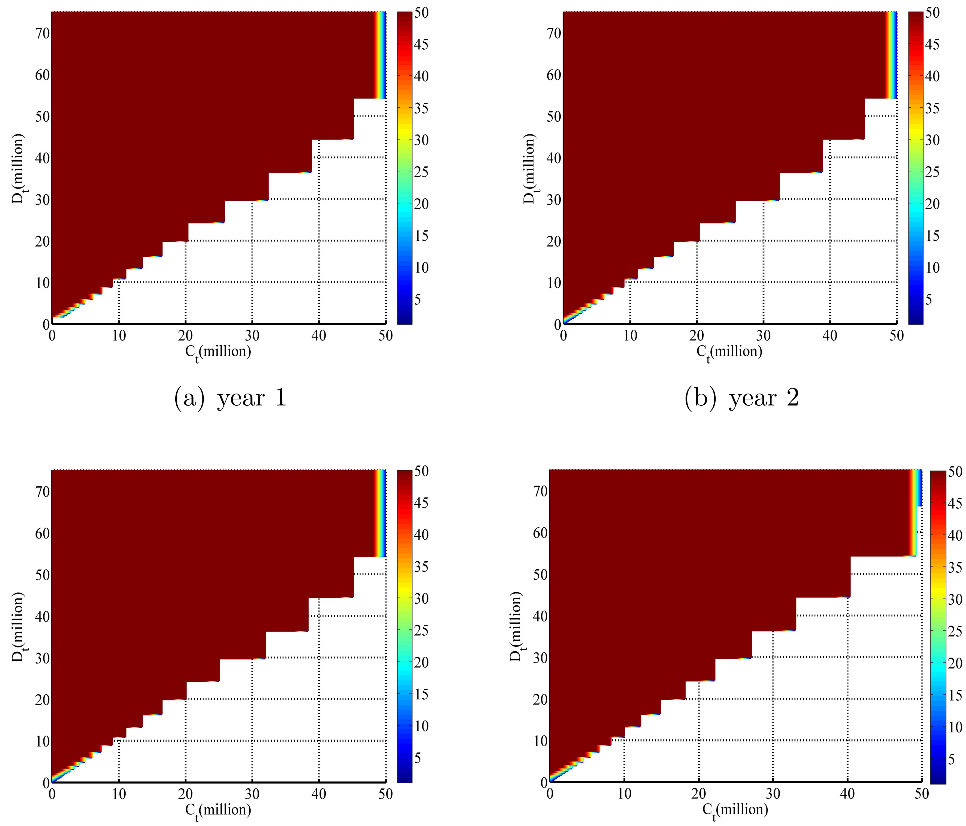


Figure 4.11. Optimal Policy When Capacity Is Added in Large, Discrete Lots

4.4.6. Terminal Value of the Diffusion Phase. The benchmark example assumes a 0 terminal value at the end of diffusion phase. Actually, the profit of stationary phase should be considered into the decision as the terminal value of the diffusion phase.

In this example, D_t during the stationary phase is assumed to follow the log-normal distribution with a mean of D_T and standard deviation σ_s. That is, $\ln D_t$



(a) year 1 (b) year 2

(c) year 3 (d) year 4

Figure 4.12. Optimal Policy Given a Capacity Cap

follows a normal distribution. This assumption assures the demand is non-negative. Therefore, Equation (38) has a closed form solution,

$$
\begin{aligned}
S(D_T, C_T) =& \frac{e^{-r_s dt} - e^{-r_s(T_s - T)}}{r_s} \Bigg\{ (P-m)C_T + \\
& PD_T \left[\Phi\left(\frac{\mu_l + \sigma_l^2 - 1}{\sigma_l}\right) - \Phi\left(\frac{\mu_l + \sigma_l^2 - \ln C_T}{\sigma_l}\right) \right] \\
& - C_T \Phi\left(\frac{\ln C_T - \mu_l}{\sigma_l}\right) \Bigg\},
\end{aligned} \tag{59}
$$

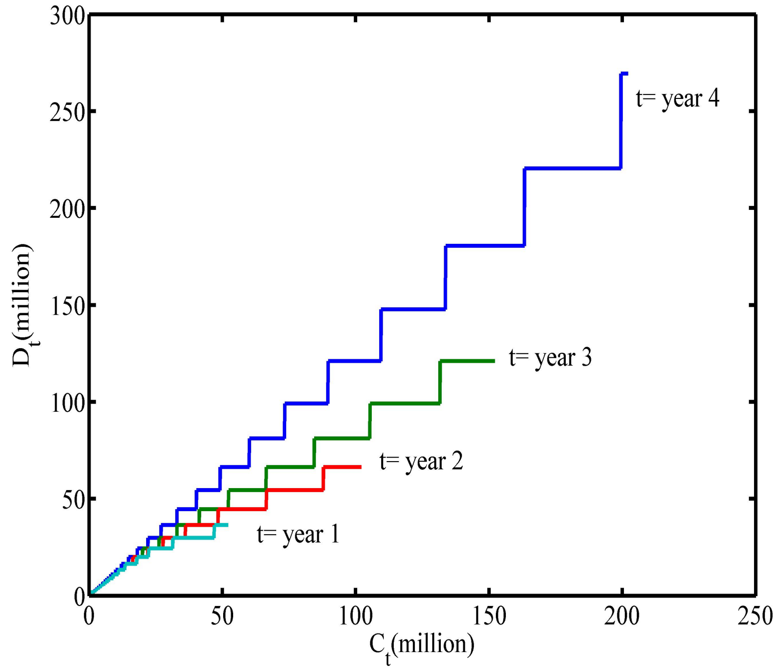
where r_s is the risky discount rate during the stationary phase; Φ is the cumulative distribution function of a standard normal distribution; μ_l and σ_l are the mean and standard deviation of $\ln D_t$, respectively, and have the values of

$$\begin{aligned}\mu_l &= \ln D_T - \frac{1}{2}\ln(1+\frac{\sigma_s^2}{D_T^2}),\\ \sigma_l &= \sqrt{\ln(1+\frac{\sigma_s^2}{D_T^2})}.\end{aligned} \tag{60}$$

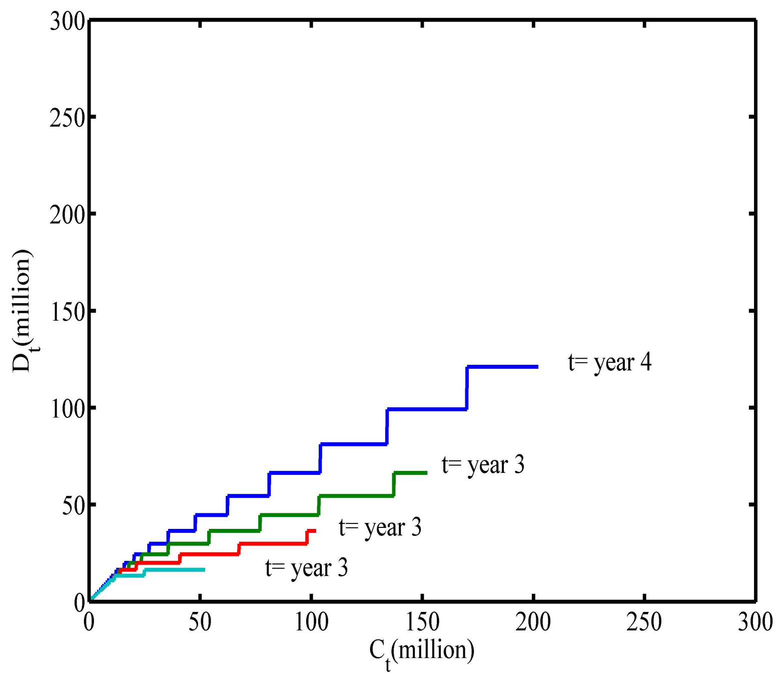
Assume that the stationary phase lasts for 5 years, that is $T_s - T = 5$ years. The standard deviation of D_t during the stationary phase, σ_s, is 5% of D_T. The firm requires a discount rate of $r_s = 15\%$ during this phase. Then the value of the expected NPV becomes 575.35 million, which is 2.2 times of the value in the benchmark example.

The free boundary of exercise when the terminal value is positive is shown in Figure 4.13. First, the free boundaries in Figure 4.13(b) is much lower than those in Figure 4.13(a). This change is because the return of the capacity expansion is increased by the positive terminal value, thus leading to more aggressive investment behavior. Second, the optimal policy is less time sensitive than that without terminal value. The positive terminal value augments both the positive and negative potentials of the project value. It makes the value of waiting for the demand uncertainty resolved increase because the capacity adjustment that is accurately match the realized demand will gain a larger profit or prevent from loss more. The increased waiting value makes the decision maker prefer to invest later, thus further reduces the threshold of option exercise in the later years.

At the end of the numerical study, I remark that data changes in the numerical case would not change the effects of the factors. The strength of the effects might be influenced by the data, but the essence of them would not be changed. For example,



(a) Without terminal value



(b) With terminal value

Figure 4.13. Comparison Between the Free Boundaries When It Is With and Without Terminal Value

waiting cost, as long as it exists, would counteract the benefit of waiting. The amount of waiting cost only changes the degree of the benefit of waiting being counteracted.

4.5. SUMMARY OF ESSAY TWO

Essay Two performed a systematic analysis of dynamic capacity expansion problem. It addressed several important issues in RO practices, including economies of scale, capacity expansion mode, opportunity cost of waiting, terminal value of project, and the cap of capacity. The problem of dynamic capacity expansions under the stochastic diffusion of demand was modeled as multiple exercisable options. Theoretical analysis and numerical results showed that when the capacity is added in infinitesimal units, and the expansion cost has a constant or increasing economy of scale, the optimal policy of expansion is binary -either not to expand or to expand at the maximum expansion rate - despite there are multiple expansion rates can be chosen. The binary feature does not hold when the expansion cost presents a decreasing economy of scale or when the capacity cap exists. In addition, increasing economy in the expansion cost lowers the threshold of expansion, while decreasing economy increases the threshold and reduces the size of expansion. When the capacity is added in large, discrete units, the optimal size of expansion can be any available values, depending on the relationships between the expected profit-to-go and the current state of the system (including the time left, realized demand, and current capacity). When the opportunity cost of waiting is not negligible, the benefit of waiting for the uncertainty resolved is counteracted by the opportunity cost, leading to more aggressive investment behavior: the thresholds of expansion in earlier years is lower than those in later years. Increasing economy of scale in the expansion cost and a significant terminal value weaken the effect of opportunity waiting cost by increasing the rate of return on expansion and the value of waiting, respectively.

5. CONCLUSION AND FUTURE WORK

5.1. SUMMARY OF THE DISSERTATION RESEARCH

The domain translation from financial options to ROs is challenging. Differences between financial and non-financial markets and the complex real world environments build obstacles for applying the option theory to the valuation of real asset investments. These include, but are not limited to, assumption violations, difficulty in parameter estimation, multiple options interaction, behavior interactions between competitors and cooperators, and the practical issues in RO applications. Researchers have made efforts to overcome the obstacles and promote the use of option pricing in valuing the investment in real assets. However, some important issues, for example, behavior interactions between cooperators and the incentive function of ROs, have not received sufficient attention. This dissertation made efforts to fill the gaps. This dissertation reported a study of the incentive function of RO, examined the effects of behavior interaction in cooperative relationships on ROs, and analyzed the influence of some practical issues on the timing and sizing of capital investments.

An option-game framework was built in Essay One to model the incentive function of ROs. It examined how the RO incentives change behaviors of the decision maker, as well as the effects of behavior interaction in cooperative relationships on the evaluation and exercise of ROs.

Based on the incentive function of ROs, Essay One introduced a new thinking in improving cooperative relationships under uncertain environments, which involves the design of suitable RO incentives to accomplish better cooperation. An option-game framework was built to model the incentive function of the ROs. Results showed both parties can benefit from the RO incentive.

In addition, a general framework for designing RO incentives for different forms of PPPs was also proposed to apply the incentive function of ROs in practice.

Essay Two built a bridge between RO practices and theoretical development by performing a systematic analysis of dynamic capacity expansion problem using ROs. It provided the closed form solutions of optimal policy for different economies of scale when capacity is expanded continuously and systematically examined the important factors for RO practices, including economies of scale, capacity expansion mode, opportunity cost of waiting, terminal value of project, and the capacity cap, on the exercising strategy of ROs.

The problem of dynamic capacity expansions under the stochastic diffusion of demand was modeled as a multiple exercisable call option. A 2-D numerical scheme, instead of the traditional 2.5-D data structure, for solving dynamic capacity expansion problem was also proposed to save the storage space and make the visualization of results easier.

Theoretical analysis and numerical results showed that, the optimal policy of expansion is binary, like the standard option, when the capacity is added in infinitesimal units and the expansion cost has a constant or increasing economy of scale, despite there are a range of expansion rate available. In addition, the practical factors affect the expansion strategies, such as the timing of expansion, the amount of expansion, and the threshold of exercise, significantly.

5.2. CONTRIBUTIONS

The research reported in this dissertation filled some of the gaps in the RO literature and smoothed the process of "domain translation." It provided a scientific understanding of the incentive function of ROs and the effects of behavior interaction between the cooperators on ROs. In addition, it identified the impacts of some important issues of RO practices on option exercise strategies. More specifically, the major contributions of this dissertation were summarized as follows:

- Introduced a new thinking in improving cooperative relationships under uncertain conditions, which involves the design of suitable RO incentives to accomplish better cooperation relationship.
- Built an option-game framework to model RO incentives and the behavior dynamics promoted by it, which provides implications in the usage of RO incentives.
- Provided comprehensive guidelines to the design and valuation of RO incentives for different forms of PPPs and different objectives.
- Illustrated an approach to bridge RO practices and theoretical development, and to complement each other.
- Analyzed important factors of RO practices for the problem of dynamic capacity expansions and examined the impacts of these on the exercise strategy.

5.3. FUTURE WORK

The research reported in this dissertation has built a foundation for advanced RO problems that subject to future research. Possible extensions of this work are discussed below, but not limited to these.

- **Alternative models for the underlying variables.** In this dissertation, the underlying variables were assumed to follow the GBM process. Alternative stochastic processes, such as Poisson jump process and mean reverting process, may be more realistic. Different numerical schemes and research outcomes may be associated with alternative stochastic processes.

- **Endogenous price and demand.** The price and demand were assumed as exogenous in this dissertation. This assumption is often made in the option literature and in line with the reality in many cases. However, endogenous price and demand are more common in practice. The decisions in capacity can influence the price and demand. Therefore, relaxation of the assumption of rigid demand and fixed price is a practical extension of this work, which will broaden the application of the research reported in this dissertation.

- **Resource constraints.** This work did not consider the resource constraints such as limited capital budget or nature resources, which commonly exist in practice. The allocation of limited resource under uncertainty is an emerging research topic and pose new challenges on current RO models, algorithms, and numerical schemes. Resource constraints would make dynamic capacity planning path dependent, thus significantly increasing the computational complexity of solving the problem.

- **Multiple sources of uncertainty.** This work assumed a single source of uncertainty to simplify the analysis and focus on the major research objectives. In fact, multiple sources of uncertainty are very common to most projects, such as demand, price, construction cost, and completion time. Considering multiple sources of uncertainty in the RO valuation would be a practical extension.

APPENDIX A

SOLUTION OF THE OPTIMAL INITIAL CAPACITY IN ESSAY ONE

The optimal initial capacity, C^*, is obtained by solving the inequality constrained maximization problem in (7). The Lagrangian for the problem in (7) is

$$L(C, \rho) = W(k^*, C) + \rho(E[U_{PRI}(k^*, C)] - \pi_c). \tag{61}$$

Let C be a local optimum to the problem in (.61) and (C^*, ρ^*) be the corresponding Kuhn-Tucker point that satisfies the following conditions:

$$\begin{cases} \nabla_C L(C^*, \rho^*) = \nabla_C W(k^*, C^*) + \rho^* \nabla_C \left(E[U_{PRI}(k^*, C^*)] - \pi_c\right) = 0, \\ \rho^*(E[U_{PRI}(k^*, C^*)] - \pi_c) = 0, \\ E[U_{PRI}(k^*, C^*)] - \pi_c \geq 0, \\ \rho^* \geq 0. \end{cases} \tag{62}$$

If $\rho^* = 0$, (.62) yields:

$$\begin{aligned} \nabla_C W(k^*, C^*) =& ab\beta t^0 \sum_{t=0}^{T-1} \left(e^{-r_{rf}t} \hat{Q}_t^{b+1} \right) C^{*-(b+1)} - (h + k^*) \\ & - mk^{*-\theta} \sum_{t=0}^{T-1} e^{(\lambda - r_{rf})t}, \end{aligned} \tag{63}$$

and

$$C^* = \left[\frac{ab\beta t^0 \sum_{t=0}^{T-1} \left(e^{-r_{rf}t} \hat{Q}_t^{b+1} \right)}{h + k^* + mk^{*-\theta} \sum_{t=0}^{T-1} e^{(\lambda - r_{rf})t}} \right]^{\frac{1}{b+1}}. \tag{64}$$

The third condition in (.62) is satisfied when

$$\pi_c \leq \sum_{t=0}^{T_c-1} e^{-r_{rf}t} P\hat{Q}_t - \left[\frac{ab\beta t^0 \sum_{t=0}^{T-1} \left(e^{-r_{rf}t} \hat{Q}_t^{b+1} \right)}{h + k^* + mk^{*-\theta} \sum_{t=0}^{T-1} e^{(\lambda - r_{rf})t}} \right]^{\frac{1}{b+1}} \left[h + k^* + mk^{*-\theta} \sum_{t=0}^{T_c-1} e^{(\lambda - r_{rf})t} \right]. \tag{65}$$

If $\rho^* > 0$, the second condition in (.62) yields $E[U_{PRI}(k^*, C^*)] - \pi_c = 0$; therefore,

$$C^* = \frac{\sum_{t=0}^{T_c-1} e^{-r_{rf}t} P\hat{Q}_t - \pi_c}{h + k^* + mk^{*-\theta} \sum_{t=0}^{T_c-1} e^{(\lambda - r_{rf})t}}. \tag{66}$$

The first condition in (.62) yields

$$\rho^* = \frac{\left[\begin{array}{l} ab\beta t^0 \sum_{t=0}^{T-1} \left(e^{-r_{rf}t} \hat{Q}_t^{b+1} \right) C^{*-(b+1)} - (h + k^*) \\ - mk^{*-\theta} \sum_{t=0}^{T-1} e^{(\lambda - r_{rf})t} \end{array} \right]}{h + k^* + mk^{*-\theta} \sum_{t=0}^{T_c-1} e^{(\lambda - r_{rf})t}}. \tag{67}$$

Substituting (.66) for C^* in (.67), $\rho^* > 0$ is satisfied when

$$\pi_c > \sum_{t=0}^{T_c-1} e^{-r_{rf}t} P\hat{Q}_t - \left[\frac{ab\beta t^0 \sum_{t=0}^{T-1} \left(e^{-r_{rf}t} \hat{Q}_t^{b+1} \right)}{h + k^* + mk^{*-\theta} \sum_{t=0}^{T-1} e^{(\lambda - r_{rf})t}} \right]^{\frac{1}{b+1}} \left[h + k^* + mk^{*-\theta} \sum_{t=0}^{T_c-1} e^{(\lambda - r_{rf})t} \right]. \tag{68}$$

Substituting (4) for k^* in (.64) and (.66) yields the solution of the inequality constrained optimization problem:

$$C^* = \begin{cases} C_1^*, \pi_c \in [0, \omega] \\ C_2^*, \pi_c \in (\omega, \infty], \end{cases} \tag{69}$$

where

$$C_1^* = \left\{ \frac{ab\beta t^0 \sum_{t=0}^{T-1} \left(e^{-r_{rf}t} \hat{Q}_t^{b+1} \right)}{h + \left[\theta m \sum_{t=0}^{T_c-1} e^{(\lambda - r_{rf})t} \right]^{\frac{1}{\theta+1}} + m \left[\theta m \sum_{t=0}^{T_c-1} e^{(\lambda - r_{rf})t} \right]^{\frac{-\theta}{\theta+1}} \sum_{t=0}^{T-1} e^{(\lambda - r_{rf})t}} \right\}^{\frac{1}{b+1}}, \tag{70}$$

$$C_2^* = \frac{\sum_{t=0}^{T_c-1} e^{-r_{rf}t} P \hat{Q}_t - \pi_c}{h + (1 + \frac{1}{\theta}) \left[\theta m \sum_{t=0}^{T_c-1} e^{(\lambda - r_{rf})t} \right]^{\frac{1}{\theta+1}}}, \tag{71}$$

and

$$\omega = \sum_{t=0}^{T_c-1} e^{-r_{rf}t} P \hat{Q}_t - \left\{ \frac{ab\beta t^0 \sum_{t=0}^{T-1} \left(e^{-r_{rf}t} \hat{Q}_t^{b+1} \right)}{h + \left[\theta m \sum_{t=0}^{T_c-1} e^{(\lambda - r_{rf})t} \right]^{\frac{1}{\theta+1}} + m \left[\theta m \sum_{t=0}^{T_c-1} e^{(\lambda - r_{rf})t} \right]^{\frac{-\theta}{\theta+1}} \sum_{t=0}^{T-1} e^{(\lambda - r_{rf})t}} \right\}^{\frac{1}{b+1}} \left\{ h + (1 + \frac{1}{\theta}) \left[\theta m \sum_{t=0}^{T_c-1} e^{(\lambda - r_{rf})t} \right]^{\frac{1}{\theta+1}} \right\}, \tag{72}$$

$\square$

APPENDIX B

PROOF OF LEMMA 1 IN ESSAY ONE

Because $\tilde{k}^*$ solves the problem in (14), the following is true:

$$\frac{\partial E[\widetilde{U}_{PRI}(\tilde{k}^*, C^*)]}{\partial k} = \frac{\partial E[U_{PRI}(\tilde{k}^*, C^*)]}{\partial k} + \frac{\partial V_{PRI}(\tilde{k}^*, C^*)}{\partial k} = 0. \tag{73}$$

The options evaluating process in section 3.2.4 demonstrates that $V_{PRI}(k, C^*)$ is the expected net present value gained by the PRI from the option of operating the highway during the post-concession period. That is,

$$\begin{aligned} V_{PRI}(k, C^*) = & \sum_{t=T_c}^{T-1} \sum_{i=0}^{t} e^{-r_{rf}t} P\{Q_{ti}\} R(Q_{ti}, \text{“}operate\text{”}) \\ & - \sum_{t=T_c}^{T-1} \sum_{i=0}^{t} e^{-r_{rf}t} P\{Q_{ti}\} M(k, C^*, t, \text{“}operate\text{”}). \end{aligned} \tag{74}$$

Therefore,

$$\frac{\partial V_{PRI}}{\partial M} \leq 0. \tag{75}$$

Because $\frac{\partial M}{\partial k} = -\theta m C^* k^{-\theta-1} e^{\lambda t} < 0$,

$$\frac{\partial V_{PRI}}{\partial k} \geq 0. \tag{76}$$

Equations (.76) and (.73) yield $\frac{\partial E[U_{PRI}(\tilde{k}^*, C^*)]}{\partial k} \leq 0$. If $\frac{\partial E[U_{PRI}(\tilde{k}^*, C^*)]}{\partial k}$ denotes $f(k)$, (3) yields

$$\frac{\partial f(k)}{\partial k} = -\theta(\theta+1) m k^{-\theta-2} C^* \sum_{t=0}^{T_c-1} e^{-r_{rf}t} \leq 0; \tag{77}$$

that is, $f(k)$ is a decreasing function of k. Because k^* maximizes $E[U_{PRI}(k, C^*)]$, $f(k^*) = 0$. Since $f(\tilde{k}^*) \leq f(k^*) = 0$, and $f(k)$ is a decreasing function of k,

$$\tilde{k}^* \geq k^*. \tag{78}$$

APPENDIX C

PROOF OF PROPOSITION 1 IN ESSAY ONE

Because $\tilde{k}^*$ maximizes $E[\widetilde{U}_{PRI}(k, C^*)]$, $E[\widetilde{U}_{PRI}(\tilde{k}^*, C^*)] > E[\widetilde{U}_{PRI}(k^*, C^*)]$; that is,

$$\begin{aligned} &V_{PRI}(\tilde{k}^*, C^*) - I(\tilde{k}^*, C^*) - \sum_{t=0}^{T_c-1} e^{-r_{rf}t} M(\tilde{k}^*, C^*, t) \\ &\geq V_{PRI}(k^*, C^*) - I(k^*, C^*) - \sum_{t=0}^{T_c-1} e^{-r_{rf}t} M(k^*, C^*, t). \end{aligned} \tag{79}$$

Equation (.74) then yields

$$\begin{aligned} &\left\{ \begin{aligned} &I(\tilde{k}^*, C^*) + \sum_{t=0}^{T_c-1} e^{-r_{rf}t} M(\tilde{k}^*, C^*, t) \\ &+ \sum_{t=T_c}^{T-1} \sum_{i=0}^{t} e^{-r_{rf}t} P\{Q_{ti}\} M(\tilde{k}^*, C^*, t, \text{“}operate\text{”}) \end{aligned} \right\} \\ &\leq \left\{ \begin{aligned} &I(k^*, C^*) + \sum_{t=0}^{T_c-1} e^{-r_{rf}t} M(k^*, C^*, t) \\ &+ \sum_{t=T_c}^{T-1} \sum_{i=0}^{t} e^{-r_{rf}t} P\{Q_{ti}\} M(k^*, C^*, t, \text{“}operate\text{”}) \end{aligned} \right\}. \end{aligned} \tag{80}$$

In addition,

$$\begin{aligned} \sum_{t=T_c}^{T-1} e^{-r_{rf}t} M(k, C^*, t) = &\sum_{t=T_c}^{T-1} \sum_{i=0}^{t} e^{-r_{rf}t} P\{Q_{ti}\} M(k, C^*, t, \text{“}operate\text{”}) \\ &+ \sum_{t=T_c}^{T-1} \sum_{i=0}^{t} e^{-r_{rf}t} P\{Q_{ti}\} M(k, C^*, t, \text{“}abandon\text{”}); \end{aligned} \tag{81}$$

therefore,

$$
\begin{aligned}
&W(\tilde{k}^*, C^*) - W(k^*, C^*) = \\
&\left[\begin{array}{l}
I(k^*, C^*) + \sum_{t=0}^{T_c-1} e^{-r_{rf}t} M(k^*, C^*, t) \\
+ \sum_{t=T_c}^{T-1} \sum_{i=0}^{t} e^{-r_{rf}t} P\{Q_{ti}\} M(k^*, C^*, t, \text{“}operate\text{”}) \\
+ \sum_{t=T_c}^{T-1} \sum_{i=0}^{t} e^{-r_{rf}t} P\{Q_{ti}\} M(k^*, C^*, t, \text{“}abandon\text{”})
\end{array}\right] \\
&- \left[\begin{array}{l}
I(\tilde{k}^*, C^*) + \sum_{t=0}^{T_c-1} e^{-r_{rf}t} M(\tilde{k}^*, C^*, t) \\
+ \sum_{t=T_c}^{T-1} \sum_{i=0}^{t} e^{-r_{rf}t} P\{Q_{ti}\} M(\tilde{k}^*, C^*, t, \text{“}operate\text{”}) \\
+ \sum_{t=T_c}^{T-1} \sum_{i=0}^{t} e^{-r_{rf}t} P\{Q_{ti}\} M(\tilde{k}^*, C^*, t, \text{“}abandon\text{”})
\end{array}\right].
\end{aligned} \tag{82}
$$

Because $\frac{\partial M}{\partial k} < 0$ and $\tilde{k}^* \geq k^*$,

$$
\begin{aligned}
&\sum_{t=T_c}^{T-1} \sum_{i=0}^{t} e^{-r_{rf}t} P\{Q_{ti}\} M(\tilde{k}^*, C^*, t, \text{“}abandon\text{”}) \\
&\leq \sum_{t=T_c}^{T-1} \sum_{i=0}^{t} e^{-r_{rf}t} P\{Q_{ti}\} M(k^*, C^*, t, \text{“}abandon\text{”}).
\end{aligned} \tag{83}
$$

Equations (.80) and (.83) together give the following relationship:

$$
W(\tilde{k}^*, C^*) \geq W(k^*, C^*). \tag{84}
$$

$\square$

APPENDIX D

PROOF OF PROPOSITION 2 IN ESSAY ONE

The potential shortfall in the GOV's maintenance funds is reduced from $F_{BS}(k^*, C^*)$ to $F_{OS}(\tilde{k}^*, C^*)$ for two reasons: First, the PRI may operate the highway during the post-concession period if it exercises the options. Under those circumstances, the GOV has no maintenance commitment; therefore,

$$
\begin{aligned}
F_{OS}(k^*, C^*) &= F_{BS}(k^*, C^*) \\
&\quad - \sum_{t=T_c}^{T-1} \sum_{i=0}^{t} e^{-r_f t} P\{Q_{ti}\} \max \begin{bmatrix} M(k^*, C^*, t, \text{``}operate\text{''}) \\ -R(Q_{ti}, \text{``}operate\text{''}), 0 \end{bmatrix},
\end{aligned} \tag{85}
$$

which indicates that

$$F_{OS}(k^*, C^*) \leq F_{BS}(k^*, C^*). \tag{86}$$

Second, (2) indicates that $\partial M / \partial k < 0$; that is, higher construction quality reduces annual maintenance costs. According to (13), $\partial F_{OS}(k, C^*)/\partial M > 0$. Therefore, $\partial F_{OS}(k, C^*)/\partial k < 0$. Lemma 1 indicates that $\tilde{k}^* > k$; hence,

$$F_{OS}(\tilde{k}^*, C^*) \leq F_{OS}(k^*, C^*), \tag{87}$$

Finally, the following relationship is derived:

$$F_{OS}(\tilde{k}^*, C^*) \leq F_{BS}(k^*, C^*). \tag{88}$$

$\square$

APPENDIX E

TABLE OF NOTATION IN ESSAY ONE

Notation in Section 3.2

Symbol	Explanation
A_t	PRI's action at year t
$B(\hat{Q}_t)$	Travelers benefit for traveling on the highway during $[t, t+1)$
C	Highway capacity
C^*	Optimal initial highway capacity
C_t	Highway capacity at year t
$E(U_{GOV})$	Expected utility of the GOV
$E(U_{PRI})$	Expected utility of the PRI without the RO incentive
$E(\widetilde{U}_{PRI})$	Expected utility of the PRI with the RO incentive
F_{BS}	Expected maintenance shortages of the GOV without offering the RO incentive
F_E	Expected expansion-related costs
F_{OS}	Expected maintenance shortages of the GOV if offering the RO incentive
G	Premium of the RO incentive
$I(k, C)$	Construction investment
$M(k, C, t)$	Maintenance costs for year t
P	Toll level
$P\{Q_{ti}\}$	Probability of the traffic volume being Q_{ti}
Q_t	Traffic volume during $[t, t+1)$
Q_{ti}	Traffic volume at node (t, i) on the binomial tree
$\hat{Q}_t$	Expected traffic volume during $[t, t+1)$
$R(Q_t)$	Toll revenue during $[t, t+1)$
T	Highway service life

T_c	Length of concession
$T(Q_t, C)$	Total travel time cost to highway users at year t
V_{PRI}	Expected value of the RO incentive for the PRI
$V^*(Q_0, C^*)$	Value of the expansion option
W	Expected social welfare produced by the project
ΔW	Additional social welfare added by the RO incentive
ΔW_k	Additional social welfare associated with the change in k
ΔW_C	Additional social welfare produced by capacity expansions using the option premium
a, b	Coefficients of volume/capacity ratio in the travel time cost
h	Unit construction cost that can meet the minimum quality requirement
k	Unit cost of quality improvement beyond h
k^*	Optimal investment in quality improvement without the RO incentive
$\tilde{k}^*$	Optimal investment in quality improvement with the RO incentive
m	Capacity coefficient
p	Risk-neutral probability
r	Discount rate
r_{rf}	Risk-free rate
t^0	Travel time through the highway under free flow condition
β	Average time value per traveler per unit time
δ	Market price of risk for Q_t
θ	Quality improvement factor

λ	Aging factor
μ	Expected annual growth rate of traffic volume
π_c	Minimum profit request of the PRI from concession
π_o	Minimum profit request of the PRI from purchasing the RO incentive
σ	Volatility of annual traffic volume

Notation in Section 3.3

Symbol	**Explanation**
$\hat{E}_\tau(R)$	Expected total revenue from the extended concession at time τ
K	Total O& M cost during the extended concession
S_t	Annual revenue at year t
T	Highway service life
c	option value
w_t	A random walk process
μ	Expected annual growth rate of revenue
σ	Volatility of annual revenue
τ	End of concession

APPENDIX F

TABLE OF NOTATION IN ESSAY TWO

Symbol	Explanation
$\mathscr{A}_t$	Admissible space of ξ_t
C_0	Initial capacity
C_t	Capacity at time t
C_{min}	The lowest capacity level
C_{max}	The highest capacity level
D_0	Initial demand
D_{ij}	Demand at node (i, j) on the binomial tree
D_{smax}	Upper bound of demand during the stationary phase
D_{smin}	Lower bound of demand during the stationary phase
D_t	Demand at time t
H_t	A white noise process
K	Maximum index of capacity
L	Maximum index of time and demand combination in the 2-D grid
M	Maximum index of time
P	Unit price of the product
S	Terminal value
T	The end of diffusion phase
T_s	The end of stationary phase
W_t	A standard Wiener process
V	Expected value-to-go
d	down-movement ratio in binomial tree
$g(\xi_t)$	Expansion cost function
h	Constant coefficient of the linear expansion cost function

m	marginal production cost
p	Possibility of an up-movement occurs
q_t	expansion rate per year
q_{max}	Maximum expansion rate
r	Risk free rate
r_s	Risky discount rate during the stationary phase
dt	Time interval
Δt	Discrete time interval
u	up-movement factor in binomial tree
Φ	Cumulative distribution function of the standard normal distribution
λ	Market price of risk on demand
μ	Drift rate of the demand
$\hat{\mu}$	Risk neutral growth rate of D_t
μ_l	Mean of $\ln D^s$
ξ_t	Expansion amount at time t
ξ_{max}	Maximum expansion amount at time t
π	Profit flow of the firm
σ	volatility of the demand
σ_l	Standard deviation of $\ln D_t$ during the stationary phase
σ_s	standard deviation of D_t during the stationary phase
ΔV	The change of V due to the capacity increase by dC_t
$\Delta\xi$	Unit expansion amount

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