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ABSTRACT

This paper discusses the consequence of a continuum analysis of turbulence using the Reynolds convention, on the cascade of energy to internal thermal energy. It is observed that there are two distinct dissipative paths or traps that the energy follows, neither of which involves vorticity. It is observed that the so-called "Reynolds' Stresses" are not involved in these irreversible dissipative paths, but are in the reversible bridge between the mean and fluctuating flows.

An effort is made to generate a consistent physical interpretation of all of the terms in the equations used rather than selecting isolated terms for explanation. Some suggestions are made for modeling the dissipative terms and an appendix is included to illustrate how information can be lost in an integration process that results in erroneously ascribing dissipative roles to perfectly reversible terms and equations.

INTRODUCTION

The energetics of turbulent motion is a subject which has long excited the interest of fluid mechanicists, both theoretician and experimentalist alike.[3,5,8,18] Because of the chaotic nature of turbulence the statistical theory of turbulence of Taylor[17] and, in particular, Kolmogoroff's theory of isotropic turbulence,[10a,b,c] have received wide attention and interest. The energetics of turbulence are usually discussed[5,8] in terms of energy spectra derived from Fourier transforms of turbulent velocity fluctuation correlations. However, in dealing with the energetics of nonisotropic turbulence in this manner it is not always clear (Reference 5, p. 284) in which direction the energy flows when one considers the Fourier transformed differential equation of turbulent energy flow.

The present paper presents a thermodynamic analysis of the energetics of turbulent flow in an effort to identify the path (or paths) by which energy in the turbulent field is transferred from one form to another and ultimately dissipated irreversibly as internal thermal energy. In performing this analysis we shall deal directly with the turbulent velocity averages using the Reynolds convention[8] rather than with the Fourier transformed spectral equations because we believe that the physical significance and thermodynamic role of various terms in the equations is more easily identified in this format.

In order to limit somewhat the complexity and bulk of the equations to be analyzed and in the interest of a specific example, we shall consider here only the energetics of flow of incompressible, nonpolar (in the sense of Dahler and Scriven[6]) fluids. This category covers the majority of Newtonian and non-Newtonian fluids of practical interest.

THEORETICAL ANALYSIS

A. The Balance Equations: General

Before considering turbulent motions directly it will prove useful to write down the general balance equations for momentum and energy and the auxiliary conditions for mechanical energy and specific entropy production.[19] In the following we shall use standard Cartesian indicial notation because of its convenience. Generalization to more general tensorial notation or to Gibbs' dyadic *notation is easily* accomplished but adds nothing essential to the discussion.

The momentum balance equation, also known as Cauchy's first law of motion (Reference 19, p. 545) in 'material coordinates' is:

$$\rho \frac{dv_k}{dt} = \rho F_k + T_{jk,j} \tag{1}$$

where ρ, v_k, F_k, T_{jk} are, respectively, the density, velocity vector, body force vector, and stress tensor. In terms of the mean normal stress, $p = -(1/3)T_{kk}$, and the stress deviator tensor P_{jk} defined by:

$$P_{jk} = T_{jk} + p\delta_{jk} \tag{2}$$

this becomes the more familiar form:

$$\rho \frac{dv_k}{dt} = \rho F_k - p_{,k} + P_{jk,j} \tag{3}$$

In the following analysis it is convenient to express the acceleration dv_k/dt in terms of Lagrange's form (Reference 19, p. 377-8):

$$\frac{dv_k}{dt} = \frac{\partial v_k}{\partial t} + \tfrac{1}{2}(v^2)_{,k} + \epsilon_{kpq} w_p v_q \tag{4}$$

where $v^2 = \underset{\sim}{v} \cdot \underset{\sim}{v} = v_j v_j$ and $w_p = \epsilon_{prs} v_{s,r} = (\underset{\sim}{\nabla} \times \underset{\sim}{v})_p$ is the vorticity of the flow. The reason for choosing to express the acceleration vector in this form is that Lagrange's decomposition permits the isolation of the effect of fluid particle spin to a single vector, $\underset{\sim}{w} \times \underset{\sim}{v}$, which Truesdell and Toupin[19] (p. 378) call the Lamb vector. Furthermore, this decomposition is accomplished entirely in terms of vectors and does not involve dyadics, thus permitting discussion of the acceleration in terms of lines, tubes, and the other properties of vector fields. In particular the Lamb vector, $\underset{\sim}{w} \times \underset{\sim}{v}$, is that part of the acceleration imparted to a fluid particle by virtue of its spin. Introduction of Eq. 4 into Eq. 3 gives the following form of Cauchy's first law which we shall utilize here:

$$\rho \frac{\partial v_k}{\partial t} + \tfrac{1}{2}\rho(v^2)_{,k} + \rho\epsilon_{kpq} w_p v_q = \rho F_k - p_{,k} + P_{jk,j} \tag{5}$$

The general energy balance equation may be written for this case as (Reference 19, p. 609):

$$\rho \frac{d}{dt}(\tfrac{1}{2}v^2) + \rho \frac{d\epsilon}{dt} = \rho v_k F_k - v_k p_{,k} + v_k P_{jk,j} + P_{jk} e_{jk} - q_{k,k} \tag{6}$$

where ϵ, e_{jk}, and q_k are, respectively, the specific internal thermal energy, the symmetric deformation rate tensor, $\tfrac{1}{2}(v_{j,k} + v_{k,j})$, and the heat flux vector, and we have set Q=0.

The mechanical energy equation, obtained by multiplying Cauchy's first law, Eq. 3, by v_k is:

$$\rho \frac{d}{dt}(\tfrac{1}{2}v^2) = \rho v_k F_k - v_k p_{,k} + v_k P_{jk,j} \tag{7}$$

When this result is subtracted from Eq. 6 one obtains the thermal energy equation or energy balance (Reference 19, p. 609):

$$\rho \frac{d\epsilon}{dt} = P_{jk} e_{jk} - q_{k,k} \tag{8}$$

Clearly, Eqs. 6, 7, and 8 are not all independent, any one being derivable from a combination of the other two. For the purposes of our present analysis it is convenient, but not essential to choose Eqs. 7 and 8 as the *independent* pair, although we shall have occasion later to consider the independent pair as Eqs. 6 and 7.

The last of the basic equations which we require is the expression for the production of specific entropy (Reference 19, p. 642, with appropriate nomenclature change):

$$\rho \frac{ds}{dt} = -\left[\frac{q_k}{\theta}\right]_{,k} - \frac{q_k \theta_{,k}}{\theta^2} + \rho \frac{Q}{\theta} + \frac{1}{\theta} P_{jk} e_{jk} \tag{9}$$

where s, θ, and Q are, respectively, the specific entropy, temperature, and volume source of energy (from chemical reactions, etc.). We shall consider in the following that Q = 0. From considerations of the integral of Eq. 9 over the volume of the fluid element Truesdell and Toupin[19] (p. 642-644) show that:

$$P_{jk} e_{jk} - \frac{q_k \theta_{,k}}{\theta} \geq 0 \tag{10}$$

which they call the postulate of irreversibility.

Equations 6, 7, 8, and 10 provide the structure for a thermodynamic analysis of the energetics of turbulent motion of incompressible, nonpolar fluids. Before specializing the equations to consider turbulent motions, however, it is possible to make certain observations regarding the general problem of the thermodynamics of deforming continua. For example, from Eq. 10 it is evident that the only nonthermal term in any of the equations which participates in the production of entropy is $P_{jk}e_{jk}$. This term is the surplus of external work over inner work mentioned by Truesdell and Toupin[19] (p. 639) and in an adiabatic system constitutes the essential thermodynamic irreversibility of the energy flow in the fluid. It is especially important at this point that one distinguish between thermodynamic irreversibility (positive semi-definite entropy production) and mechanical reversibility (invariance of terms to a change of variable $t \rightarrow -t$, or equivalently $\underset{\sim}{v} \rightarrow -\underset{\sim}{v}$ (Reference 14, Chap. 3)). In particular we note that setting $\underset{\sim}{v} = -\underset{\sim}{v}$ in $P_{jk}e_{jk}$ leaves its sign unchanged whereas Eqs. 9 and 10 clearly identify this term as being the term responsible for entropy production and hence for the essential thermodynamic irreversibility in the system. Invariance of sign under the transformation $t \rightarrow -t$ is not germain to the question of thermodynamic irreversibility.

It may be well to also observe here that the equations presented apply to a continuum for both thermostatics and thermodynamics as discussed by Truesdell and Toupin.[19] The equations, therefore, apply to irreversible thermodynamics as well as equilibrium analysis, and for as long as the domain can be considered a continuum must handle the nonequilibrium or irreversible effects of fluctuation as introduced by the Reynolds convention. Thus, for a Fourier transformed approach where the dissipation is treated by wave numbers, frequencies, and eddy sizes, so long as the eddies are of such a size as to be considered in the continuum domain the question of non-equilibrium in high frequency eddies does not invalidate the analysis. What happens beyond this point is certainly open to question, but the question of what point or condition energy crosses from the domain of continuum to the statistical concepts of molecular thermodynamics is not addressed here. The Reynolds' convention and the equations presented in this analysis remain constrained to the domain of continuum mechanics.

Having thus identified the essential thermodynamic irreversibility in the system we can draw several conclusions regarding the thermodynamic reversibility of various energy transfer terms* in the several equations. For example, Eq. 8 for the balance of internal energy considered together with Eq. 10 clearly shows that energy flows irreversibly from mechanically dissipative power, $P_{jk}e_{jk}$, and heat influx, $-q_{k,k}$, into internal thermal energy. Thus, these equations describe a one-way transfer process since clearly a decrease $(\frac{-d\epsilon}{dt})$ in internal energy cannot effect a decrease in mechanical power because Eq. 10 shows that for all conditions $P_{jk}e_{jk} \geq 0$.

On the other hand, however, we observe that for incompressible flow, the total rate of work against the stresses, $(v_k T_{jk})_{,k}$, is equal to:

$$(v_k T_{jk})_{,k} = -v_k P_{,k} + v_k P_{jk,j} + P_{jk} e_{jk} \tag{11}$$

That is, the total power expended is divided into two parts, the recoverable power $-v_k P_{,k} + v_k P_{jk,j}$, and the thermodynamically irreversible power dissipation, $P_{jk}e_{jk}$. Thus, since only the dissipative power $P_{jk}e_{jk}$ is thermodynamically irreversible, one concludes that the recoverable power terms, $-v_k P_{,k} + v_k P_{jk,j}$, represent energy transfer processes which are thermodynamically reversible. However, since both of these latter terms appear in the mechanical energy equation, Eq. 7, it seems clear that this latter equation describes a thermodynamically reversible transfer or exchange of energy between kinetic energy and recoverable mechanical work.

Two other observations of some interest also follow from the above thermodynamic analysis. We may write the velocity gradient tensor as:

$$v_{k,j} = \tfrac{1}{2}(v_{k,j} + v_{j,k}) + \tfrac{1}{2}(v_{k,j} - v_{j,k}) = e_{jk} + \Omega_{jk} \tag{12}$$

where Ω_{jk} is the skew symmetric angular velocity tensor related to the vorticity vector by the relation:

$$\Omega_{jk} = -\tfrac{1}{2}\,\epsilon_{jmk}\, w_m \tag{13}$$

Thus, since $P_{jk} = P_{kj}$ is symmetric, it follows that $P_{jk}e_{jk} = P_{jk}(e_{jk} + \Omega_{jk}) = P_{jk}v_{j,k}$ and so the production of entropy is not influenced by the vorticity of the flow. This means that even though a fluid particle rotates, and experiences an acceleration thereby (given by the Lamb vector), vorticity does not contribute directly to the production of entropy.

The other observation of interest in this connection also involves the vorticity vector. We note that in calculating the increase in kinetic energy, $\rho \frac{d}{dt}(\tfrac{1}{2}v^2)$, from Eqs. 3 and 4 one finds:

$$\rho \frac{d}{dt}(\tfrac{1}{2}v^2) = \rho v_k \frac{dv_k}{dt} = \rho \frac{\partial}{\partial t}(\tfrac{1}{2}v^2) + \rho v_k (\tfrac{1}{2}v^2)_{,k} \tag{14}$$

That is, the term involving the Lamb vector, $\rho v_k \epsilon_{kpq} w_p v_q = \rho \underset{\sim}{v} \cdot (\underset{\sim}{w} \times \underset{\sim}{v}) = 0$, does not contribute. This occurs because the Lamb vector, which is the part of the acceleration imparted to the fluid particle by virtue of its spin, is oriented normal to the velocity vector. Therefore, when one computes v·a this term makes no contribution. Consequently, not only does the vorticity (and hence local fluid particle spin) not contribute to the irreversible entropy production, but it also does not contribute to the energy balance of the particle. It will be shown below that these same results are equally valid for the fluctuations of vorticity in a turbulent motion.

B. The Balance Equations: Turbulent

In order to examine the above results and conclusions in relation to turbulent motions, we shall adopt the familiar Reynolds convention[8] of expressing physical quantities ϕ as a mean $\bar{\phi}$ plus an instantaneous fluctuation ϕ'. Although this device admittedly divides the flow into two somewhat arbitrary parts, it provides a useful and convenient method for the study of the effects of turbulent motion. At this point it is important to realize the implication of what one does in choosing to use the Reynolds convention. The concept of a mean plus deviation is convenient for statistical analysis of erratic motions. However, when one uses such a mathematical model, he can introduce results which follow directly from the model and not from the real

*Because of the multitude of ways vector equations may be transformed the assignment of physical meaning to any isolated form is meaningless. Only when all terms in a given equation are considered together in the context of the particular equation can meaningful physical significance be attached to particular terms.

physical system. Such is the case with the Reynolds convention. Later we shall observe a dichotomy of terms which causes us to think as if there were two separate flows (a mean flow and a fluctuating flow) simultaneously occurring in the field. Obviously such a concept is not physically realistic. However, if we wish to involve this model because of its convenience in one area, we must be consistent in its use and follow to their logical consequence all results implied by the model. Therefore, the reader is urged to keep in mind that many of the results to follow are a direct consequence of modeling the turbulent motion through the Reynolds convention. Choosing some other convention may well lead to different interpretations, but such interpretations are not valid in the context of the model here chosen.

Introduction of the Reynolds convention into Equations 5, 7, 8, and 10 followed by Eulerian time averaging gives the following equations:

1) turbulent momentum balance or Cauchy's first law

$$\rho \frac{\partial \bar v_k}{\partial t} + \tfrac12\rho(\bar v^2)_{,k} + \rho\epsilon_{kpq}\,\bar w_p \bar v_q + [\rho(\overline{v'^2})_{,k} + \rho\epsilon_{kpq}\,\overline{w'_p v'_q}] = \rho F_k - \bar p_{,k} + \bar P_{jk,j} \qquad (15)$$

2) turbulent mechanical energy

$$\rho\frac{\partial}{\partial t}(\tfrac12\bar v^2) + \rho\bar v_k(\tfrac12\bar v^2)_{,k} + [\rho\frac{\partial}{\partial t}(\tfrac12\overline{v'^2}) + \overline{\rho v'_k(v'^2)_{,k}}] + \{\rho\bar v_k(\tfrac12\overline{v'^2})_{,k} + \overline{\rho v'_k(v'_j\bar v_j)_{,k}}\} = \bar v_k\bar F_k - \bar v_k\bar p_{,k} + \bar v_k\bar P_{jk,j} + [-\overline{v'_k p'_{,k}} + \overline{v'_k P'_{jk,j}}] \qquad (16)$$

3) turbulent thermal energy

$$\rho\frac{\partial\bar\epsilon}{\partial t} + \rho\bar v_k\bar\epsilon_{,k} + [\rho\overline{v'_k\epsilon'_{,k}}] = -\bar q_{k,k} + \bar P_{jk}\bar e_{jk} + [\overline{P'_{jk}e'_{jk}}] \qquad (17)$$

4) adiabatic turbulent postulate of irreversibility

$$\bar P_{jk}\bar e_{jk} + [\overline{P'_{jk}e'_{jk}}] \geq 0 \qquad (18)$$

We note that the cases of adiabatic flow ($q_k = 0$) and diabatic flow ($q_k \neq 0$) must be considered separately. Since even in the case of liquids (or gases behaving as incompressible fluids) we have $\rho=\rho(\theta)$, the simultaneous occurrence of heat transfer in diabatic flow will result in density fluctuations, ρ', in direct proportion to temperature fluctuations, θ'. Thus, the analysis becomes considerably more complicated. We shall consider here only the adiabatic incompressible case and leave the diabatic flow case for later consideration.

C. Energetics of Incompressible, Adiabatic, Turbulent Flow of Nonpolar Fluids

Following the example of the general considerations given above, we look first at the production of entropy in the turbulent flow. From Eq. 18 a number of significant consequences are immediately evident. As in the previous case, only the dissipative mechanical power produces entropy. In the present case, however, it is evident that the mean flow and the fluctuating flow separately and simultaneously produce entropy.* Furthermore, no interaction occurs between the mean and fluctuating flows with regard to entropy production. That is, no terms of the form $\bar P_{jk}e'_{jk}$ or $P'_{jk}\bar e_{jk}$ appear in Eq. 18. The significance of this result is that two separate sources of entropy production are identified by the thermodynamic analysis. Thus, the gross or mean motion produces entropy and separately the fluctuating motion simultaneously produces entropy. This picture of the energy dissipation or entropy production process in a turbulent field (based on the Reynolds convention) differs significantly from the conventional physical conceptualization of dissipation being the end result of an energy cascade from large eddies to small eddies (Reference 5, p. 261; Reference 8, p. 6; and Reference 13, p. 13-14).

Examination of Eq. 17, the turbulent energy balance, reveals another significant feature of the energy cascade process. It is quite evident from this equation (which we saw in the previous section corresponds to the thermodynamically irreversible conversion of mechanical power to internal energy) that a distinct dichotomy of energy cascade paths exists. All terms in this equation consist of products of mean quantities or correlated products of fluctuating quantities (terms enclosed in brackets) but no mixed terms involving products of mean with fluctuating terms. Thus, it appears that the dual entropy production mechanism identified by the thermodynamic postulate of irreversibility corresponds to a dual cascade path for conversion of mechanical power to internal energy. Thus, the mean flow converts mechanical power $\bar P_{jk}\bar e_{jk}$ irreversibly to internal energy through the path $\rho\partial\bar\epsilon/\partial t + \rho\bar v_k\bar\epsilon_{,k}$ while simultaneously the fluctuating motion converts mechanical power $\overline{P'_{jk}e'_{jk}}$ irreversibly to internal energy through the path $\overline{\rho v'_k\epsilon'_{,k}}$ but neither interacts with the other.

The thermodynamic analysis appears to be definite on the following point: once mechanical power is dissipated producing entropy, two separate, simultaneous energy cascade paths convert it to internal energy. Two obvious questions are raised by this result: (1) how does energy get from one form of motion (say mean) to the other (say fluctuating); and (2) how does the energy get into one or the other of the mechanical power dissipation forms? The answers to these two questions must be contained in the mechanical energy balance (Eq. 16) and the total work conditions (Eq. 11).

Consider the second question first. The time-averaged form of Eq. 11, the total power condition, is:

$$(\bar v_k\bar T_{jk})_{,j} + [\overline{v'_k T'_{jk}}]_{,j} = -\bar v_k\bar p_{,k} + \bar v_k\bar P_{jk,j} + [-\overline{v'_k p'_{,k}} + \overline{v'_k P'_{jk,j}}] + \bar P_{jk}\bar e_{jk} + [\overline{P'_{jk}e'_{jk}}] \qquad (19)$$

Evidently the dichotomy of energy cascade to internal energy starts here since there are two clearly identified paths for partitioning the total power between recoverable and dissipative forms for both the mean flow and fluctuating flow (bracketed terms). Therefore, the mean to fluctuating transfer must occur in the mechanical energy balance, which we saw earlier corresponds to thermodynamically reversible energy transfer processes. In order to make identification of this transfer process clearer, it is desirable to rewrite the two transfer terms (enclosed in braces {}) in Eq. 16.

By using the equation of continuity for the fluctuating flow ($v'_{k,k}=0$) and the symmetry of $v_j v_k$ we can rewrite these terms as:

$$\rho\bar v_k(\tfrac12\overline{v'^2})_{,k} + \overline{\rho v'_k(v'_j\bar v_k)}_{,j} = \rho\overline{v'_j v'_k}\,\bar e_{jk} + 2\rho\bar v\,(\overline{v'_j e'_{jk}}) \qquad (20)$$

Similarly, we can write $\rho\bar v_k(\tfrac12\bar v^2)_{,k} = \rho\bar v_j\bar v_k\bar e_{jk}$ and $\overline{\rho v'_k(\tfrac12 v'^2)_{,k}} = \rho\overline{v'_k v'_j e'_{jk}}$. With these changes and the results of Eq. 20 we can rewrite Eq. 16 as:

$$\underset{(a)}{\rho\frac{\partial}{\partial t}(\tfrac12\bar v^2)} + \underset{(b)}{\rho\bar v_j\bar v_k\bar e_{jk}} + [\underset{(a')}{\rho\frac{\partial}{\partial t}(\tfrac12\overline{v'^2})} + \underset{(b')}{\rho\overline{v'_j v'_k e'_{jk}}}]$$

$$+ \{\underset{(c)}{\rho\overline{v'_j v'_k}\,\bar e_{jk}} + \underset{(d)}{2\rho\bar v_k(\overline{v'_j e'_{jk}})}\} = \underset{(e)}{\rho\bar v_k\bar F_k} - \underset{(f)}{\bar v_k\bar p_{,k}} + \underset{(g)}{\bar v_k\bar P_{jk,j}}$$

$$+ [\underset{(f')}{-\overline{v'_k p'_{,k}}} + \underset{(g')}{\overline{v'_k P'_{jk,j}}}] \qquad (21)$$

*This is one of the consequences of the Reynolds convention model mentioned above.

In Equation 21 terms (a), (b), (e), (f) and (g) represent the reversible conversion of mean flow energy between kinetic energy (terms (a), (b)) and recoverable power (terms (e), (f), and (g)). Terms (a'), (b'), (f'), and (g') represent the same thing for the fluctuating flow. The remaining two terms, (c) and (d), represent the interaction between the mean and fluctuating flow and as such are the bridge by means of which energy is converted reversibly from mean to fluctuating form.

We observe that term (c) in Eq. 21 represents the tensorial generalization of the term which Lin[12] (p. 58-63) shows converts energy from the mean to the fluctuating flow. This term thus appears to represent the thermodynamically reversible mechanism of transfer between the mean and fluctuating flows and the means by which the turbulence is sustained or suppressed. If term (c) is negative, energy is transferred from mean to fluctuating flow and the turbulence is sustained. Conversely, if term (c) is positive, the energy flow is from the fluctuating to the mean motion and the turbulence is suppressed.[2]

Term (d), the other interaction term between mean and fluctuating flows, is quite interesting and represents some rather curious interactions. Since $e'_{jk} = \frac{1}{2}(v'_{j,k} + v'_{k,j})$, we may express the vector $2\overline{v'_k e'_{jk}}$ as:

$$2\overline{v'_k e'_{jk}} = (\overline{v'_k v'_j})_{,k} + (\tfrac{1}{2}\overline{v'_k v'_k})_{,j} \tag{22}$$

The first term on the right side of Eq. 22 is the divergence of the fluctuating momentum flux tensor, $\underset{\sim}{\nabla}\cdot\overline{\underset{\sim}{v}'\underset{\sim}{v}'}$ while the second term is the gradient of the turbulent kinetic energy, $\underset{\sim}{\nabla}(\tfrac{1}{2}\overline{v'^2})$. Eq. 22 implies partition of the energy between these two terms. Since the vector $\rho\overline{v}_i$ is the mean mass flux vector, or equivalently the mean momentum vector, the term $2\rho\overline{v}_j\ \overline{v'_k e'_{jk}}$ represents the way the energy associated with spatial nonuniformities of the turbulent fluctuation field are converted, stretched or contracted, and redistributed by the mean momentum field. This is analogous to the effect of the primary vortex stretching interaction with the disturbed flow oscillations observed by Klebanoff, et al.,[9] Kovasznay, et al.[11] and Stuart[15] in the generation of turbulent bursts in the transition process. Consequently, we suggest these terms describe the anisotropic nature of the turbulent shear flow due to the existence of nonuniform velocity fields. In particular, when term (c) corresponds to the generation of turbulent motion, term (d) corresponds to the generation of anisotropy in the turbulent fluctuations. However, since the thermodynamic analysis has shown that the mechanical energy balance represents only thermodynamically reversible energy transfer processes, it follows that term (d) also serves the function of destroying anisotropy of the turbulence when term (c) represents decay rather than generation of turbulence. In a completely isotropic turbulent field, both terms (c) and (d) vanish identically. Therefore, it appears that the coupling between the mean and fluctuating motions as represented by terms (c) and (d) is of such a nature that both terms act in concert to generate or destroy anisotropic turbulence.

D. Recapitulation

The above analysis can best be visualized graphically in terms of the energy flow diagram shown in Figure 1. The blocks in the upper part of the figure (above the horizontal dashed line) correspond to the three sets of reversible energy transfer terms of the mechanical energy equation (mean, fluctuating, and coupling). Since all of these terms represent thermodynamically reversible processes, all of the arrows indicating direction of energy flow are double ended. For the sake of definiteness, we have assumed that the flow is driven by gravity flow so that the term $\rho\overline{v}_k\overline{F}_k$ is shown as an energy source term. Clearly this is an arbitrary selection and several other choices could equally well have been made without making any essential changes in the analysis.

The blocks below the horizontal dashed line in Figure 1 correspond to the terms of the thermal energy balance, Eq. 17. The dual cascade paths are clearly shown representing the distinct mean-fluctuating entropy production dichotomy. The energy flows in this section of the diagram are thermodynamically irreversible and consequently the directional arrows are only single-ended indicating the unidirectionality of the flow.

Finally, the effect of the total power distribution relation (Eq. 19) in partitioning the mechanical power between recoverable (thermodynamically reversible) and dissipative (thermodynamically irreversible) forms is displayed by means of dashed directional arrows crossing the horizontal dashed line representing the coupling between the reversible and irreversible processes. Thus, Figure 1 displays the postulated energetics of adiabatic,

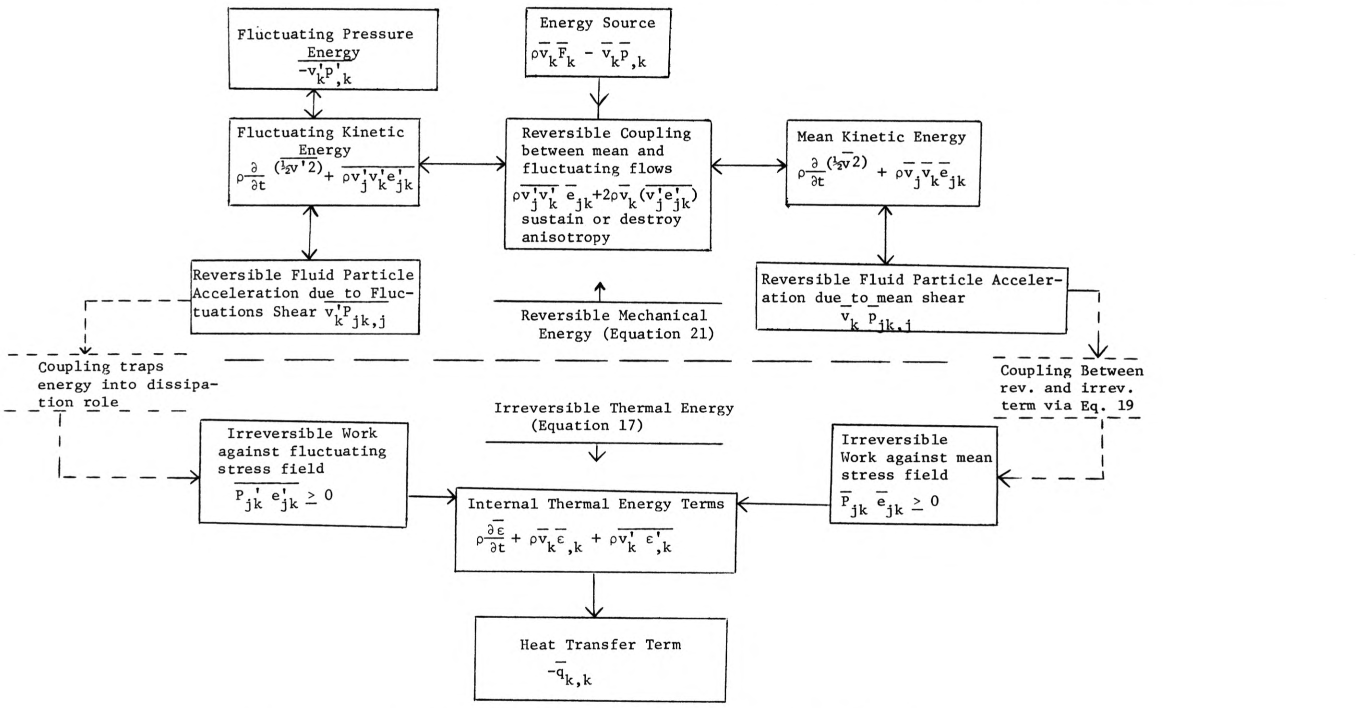

Figure 1. Schematic illustration of the two separate energy flow paths for a flow in a fixed boundary system driven by an external pressure source.

incompressible, turbulent flow of nonpolar fluids in a simple graphical form.

E. Additional Observations

The velocity fluctuation momentum flux tensor, $\rho\overline{v_j'v_k'}$, is usually known by the name "Reynolds' stress" tensor.[5,8] This name arises because when the Reynolds convention is used in developing a turbulent momentum balance from Eq. 3 (without using Eq. 4 as we have done here) one finds that the resultant equation is identical with the original with all terms replaced by mean quantities except for a term $(\rho\overline{v_j'v_k'})_{,j}$. If this new term is taken across the equality sign and written with the viscous stress divergence as $(\overline{P}_{jk}-\rho\overline{v_j'v_k'})_{,j}$ and the resultant combined divergence is called a turbulent stress tensor $\tilde{P}_{jk,j}$, then the turbulent momentum equation is formally identical with the usual laminar momentum balance and all known integrations of the latter can be taken over directly to the former. By this artifice, the turbulent momentum flux tensor $\rho\overline{v_j'v_k'}$ is absorbed into the stress system as an apparent stress arising from the turbulent fluctuations.

The thermodynamic energy analysis presented above has shown some interesting facts about this turbulent momentum flux tensor. The most significant fact is that, unlike the viscous stress tensors $\overline{P}_{jk}$ and P_{jk}' with which it is conventionally (we believe spuriously) lumped, this tensor does not contribute to the thermodynamically irreversible production of entropy. That is, none of the turbulent energy associated with $\rho\overline{v_j'v_k'}$ is dissipative. Its absence from the turbulent postulate of irreversibility, Eq. 18, is conspicuous. A similar observation was made by Irmay.[20]

Indeed, Eq. 21 clearly reveals the true nature of the turbulent momentum flux tensor. In term (c) of this equation, we see it combined with $\overline{e}_{jk}$ to transfer energy from the fluctuating kinetic form into mean distortional form, or vice versa, in a thermodynamically reversible manner. We note especially the form of terms (b), (b'), (c), and (d) of Eq. 21 in comparison with the two entropy-producing terms of Eq. 18. It is apparent from this comparison that stress tensors contracted with distortion tensors e_{jk} (either mean or fluctuating) produce entropy irreversibly while momentum flux tensors $(\rho\overline{v}_k\overline{v}_j, \rho\overline{v_j'v_k'}, \rho\overline{v}_jv_k')$ contracted with the distortion tensors e_{jk} (either mean or fluctuating) convert energy reversibly between kinetic and distortional forms.

Another interesting result of the preceding energetic analysis is the persistence of the result that vorticity does not enter directly into the energy cascade process even in turbulent flows. This is easily seen by observing that since $\overline{P}_{jk}$ and p_{jk}' are both symmetric, their contraction with $\overline{v}_{j,k}$ and $v_{j,k}'$ eliminate the rotational contributions $\overline{\Omega}_{jk}$ and Ω_{jk}' from either the entropy production equation or the energy balance equation. The same is true of all the acceleration tensors in the mechanical energy equation (i.e., $\rho\overline{v}_j\overline{v}_k$, $\rho\overline{v_j'v_k'}$, etc.). Thus, the energy cascade system of turbulent motion is not influenced by the local spin (either mean or fluctuating) of the fluid particles. This is an interesting result and one which probably would not be guessed intuitively. This does not suggest that the vorticity and vortex motion are unimportant in turbulent flows. On the contrary, they are extremely important in determining momentum flux distributions and the dynamic stability of viscous types of motion. All we have shown here is that the vorticity does not effect the energy of the flow.

F. Turbulence Constitutive Relations: Some Speculations

The above results concerning the role of the turbulent momentum flux tensor $\rho\overline{v_j'v_k'}$ (the "Reynolds' stress" tensor) raises a rather fundamental question concerning the nature of turbulence constitutive relations. The problem is the following. If, as is almost universally done (Reference 8, p. 13-20, for example), the tensor $\rho\overline{v_j'v_k'}$ is combined with the viscous stress tensor $\overline{P}_{jk}$ and called a turbulent "stress" so as to facilitate partial integration of the turbulent momentum balance equation, then in order to proceed further one requires a relation of some sort between $\rho\overline{v_j'v_k'}$ and some property of the mean flow. Since $\rho\overline{v_j'v_k'}$ appears with $\overline{P}_{jk}$, which is known to depend upon $\overline{e}_{pq}$ through the rheological constitutive equation, the temptation (Reference 8, p. 20) is very great to assume that $\rho\overline{v_j'v_k'}$ likewise depends upon $\overline{e}_{pq}$ through some sort of a turbulent constitutive relation. Thus, the notion of an eddy viscosity has become strongly entrenched in the phenomenological literature of turbulence. However, we have seen that $\rho\overline{v_j'v_k'}$ does not interact with $\overline{e}_{jk}$ (or e_{jk}' either) in any dissipative entropy producing fashion whereas both $\overline{P}_{jk}$ and P_{jk}' do. Consequently, the soundness of attempting to relate $\rho\overline{v_j'v_k'}$ to $\overline{e}_{pq}$ constitutively is fundamentally questionable, a well-known but often ignored fact. If this procedure is not proper, then, we ask what is the proper course to follow. This is a basic question and one not easily or simply answered. In this paper we do not attempt to answer it in detail as this will no doubt require a great deal of labor. Rather, we offer below a few speculations which we hope may stimulate others besides us to explore this problem from another viewpoint.

From Eq. 15 we see that $\overline{P}_{jk}$ but not P_{jk}' affects the balance of momentum in turbulent motion. It seems physically reasonable that the intrinsic rheological response of the fluid $P_{jk} = f_{jk}(e_{pq})$, where $f_{jk}(\;)$ represents the formal rheological equation of state, does not suddenly cease to be valid upon introduction of erratic turbulent secondary motions. Therefore, we suggest that the formal rheological constitutive relation $f_{jk}(\;)$ applies to the turbulent stress fields in the forms:

$$\overline{P}_{jk} = f_{jk}(\overline{e}_{pq}) \tag{23}$$

$$P_{jk}' = f_{jk}(e_{pq}') \tag{24}$$

We do not include e_{pq}' in Eq. 23 nor $\overline{e}_{pq}$ in Eq. 24 because such inclusions could give rise to mixed correlation terms in the entropy production condition. This possibility was clearly excluded by the earlier energetics analysis of bifurcated dissipation paths.

In order to suggest some possible avenues to follow in seeking turbulent constitutive relations, it may be helpful to consider the following reformulation of Eq. 15. Since we can formally write the vorticity vector as $w_k = \varepsilon_{kpq}\Omega_{pq}$ and $\varepsilon_{kpq}w_k = 2\Omega_{pq}$ where Ω_{pq} is the spin tensor, we can rewrite the turbulent Lamb vector as:

$$\rho\varepsilon_{kpq}\overline{w}_p\overline{v}_q + \rho\varepsilon_{kpq}\overline{w_p'v_q'} = -2\rho\,[\overline{\Omega}_{kq}\overline{v}_q + \overline{\Omega_{kq}'v_q'}\,] \tag{25}$$

Furthermore, we choose to assume that $\overline{F}_k$ is derivable from some scalar potential field Φ by the relation $F_k = -\Phi_{,k}$. Finally we recognize that $\overline{P}_{jk,j} - \overline{P}_{,k} = \overline{T}_{jk,j}$. Utilizing these results we may rewrite Eq. 15 in the following equivalent form:

$$\rho\frac{\partial\overline{v}_k}{\partial t} + \rho(\tfrac{1}{2}\overline{v}^2)_{,k} - 2\rho\,\overline{\Omega}_{kq}\overline{v}_q - 2\rho\,\overline{\Omega_{kq}'v_q'}$$

$$= \{-[\rho\overline{\Phi} + \tfrac{1}{2}\rho\overline{v'^2}]\,\delta_{jk} + \overline{T}_{jk}\}_{,j} \tag{26}$$

This equation is suggestive of possible new ways to approach the problem of formulating phenomenological turbulence constitutive relations. For example, it can be shown how the term $-2\rho\overline{\Omega}_{kq}\overline{v}_q$ is responsible for driving a secondary but nonturbulent mean flow in certain circumstances when the basic laminar flow becomes dynamically unstable and undergoes transition. A second instability in the nonturbulent three-dimensional flow results in transition to turbulence[7,9,11] thus activating the terms $-2\rho\overline{\Omega_{kq}'v_q'}$ and $-\frac{\rho}{2}(\overline{v'^2})_{,k}$ as driving forces.

Since the turbulent fluctuating Lamb vector ($\rho\epsilon_{kpq}\overline{w'_p v'_q} = -2\rho\overline{\Omega'_{kq}v'_q}$) seems to be responsible for the turbulence, we speculate that perhaps a proper turbulence contitutive relation might be of the form:*

$$\epsilon_{kpq}\overline{w'_p v'_q} = G_k(\overline{\Omega}_{pq}\overline{v}_q). \tag{27}$$

If we examine the right hand side of Eq. 26 we observe a marked formal similarity to the Newtonian compressible fluid form of the equations of motion in which a second or bulk coefficient of viscosity appears (Reference 1, Chap. 5). There, this second viscosity coefficient serves to generate a deviation between the arithmetic mean normal stress and the thermodynamic pressure. In Eq. 26 the term $\{-\rho(\frac{1}{2}\overline{v'^2})\delta_{jk}\}_{,j}$ corresponds analogously to a deviation from the mean normal stress. Thus, by analogy to the bulk viscosity concept above we speculate that possibly a proper eddy "viscosity" might arise from a second turbulence constitutive relation of the form:

$$\rho\overline{v_j'v_j'} = \eta(\overline{I}_e, \overline{II}_e, \overline{III}_e) \tag{28}$$

where $\overline{I}_e$, $\overline{II}_e$, and $\overline{III}_e$ are the principal scalar invariants of $\overline{e}_{jk}$.

Clearly, the above forms are highly speculative, tentative and subject to a great deal of interpretation. We propose these forms only in an effort to stimulate further inquiry into this whole subject. We fully realize that either or both of these speculations may prove to be fruitless, but on the other hand they may also lead to new models of turbulence and insight into its complexities.

SUMMARY AND CONCLUSIONS

A thermodynamic analysis of the energetics of adiabatic, incompressible turbulent flow of nonpolar fluids in terms of the Reynolds convention has resulted in identification of a number of characteristic features of turbulence. The main conclusions drawn from this analysis are the following:

1. Entropy is produced simultaneously by dissipative mechanical power generation by both the mean flow and the fluctuating flow. There is no entropy production as a result of interaction between mean and fluctuating flow quantities.

2. A clear-cut dichotomy of energy cascade paths exists with regard to the conversion of mechanical power to thermal internal energy. One cascade path involves only mean flow terms while the other involves only fluctuating flow terms. There is no conversion of mean power by fluctuating quantities or vice versa.

3. This dichotomy between mean and fluctuating energy forms extends to the production of total mechanical power. That is, the total power expended, part of which is thermodynamically reversible and part of which is thermodynamically irreversible, is divided between mean forms and fluctuating forms with no interaction.

4. The mechanical energy equation describes energy transfer processes which are thermodynamically reversible.

5. The energy balance or "thermal" energy equation describes the thermodynamically irreversible conversion of mechanical and thermal energy to internal thermal energy.

6. The energetics of turbulent motion are not effected by local fluid particle spin (vorticity) with the consequence that the energy cascade process of turbulent motion is independent of vorticity, mean and fluctuating.

7. The turbulent momentum flux tensor, $\rho\overline{v_j'v_k'}$, commonly called the "Reynolds' stress" tensor, does not participate in the thermodynamically irreversible production of entropy or in either of the cascade paths by means of which this entropy producing mechanical power dissipation converts energy to internal energy.

8. The function of the Reynolds' stress tensor is to transfer mean distortional energy to fluctuating kinetic energy and vice versa by thermodynamically reversible means.

9. The generation or destruction of anisotropy in the turbulent fluctuation field is accomplished by the interaction between the mean momentum $\rho\overline{v}_j$ and the vector $2\overline{v_k'e_{jk}'}$. This interaction apparently serves to perpetuate anisotropy when turbulence is being generated or to eliminate anisotropy of the fluctuations when turbulence is decaying.

10. In general it does not seem proper to seek phenomenological turbulence constitutive equations of the form $\rho\overline{v_j'v_k'} = g_{jk}(\overline{e}_{pq})$ because of the nondissipative nature of the turbulent momentum flux tensor $\rho\overline{v_j'v_k'}$. Therefore, some alternative possibilities for formulating more proper relations are postulated.

The thermodynamic analysis presented in this paper was couched in terms of the more easily (physically) interpretable fluctuating properties rather than in terms of the Fourier transforms thereof. Thus, while we have clearly identified directions of energy flow and the thermodynamic reversibility or irreversibility of various parts of the energy equations, we have not related these terms to their wave number transforms. Consequently, while we have identified the energy cascade paths in real space, we have not done so in wave number space. This now appears as the logical next step. However, that problem is outside the intended scope of the present paper and must be treated separately. In transferring the present results into wave number formalism one must choose a particular rheological equation of state (probably Newtonian to begin with) so as to be able to make specific calculations.

APPENDIX

In the text of the paper we stressed the fact that the mechanical energy equation represents the thermodynamically reversible transfer of energy between distortional and kinetic forms. Surely the reader has wondered how we square such a statement with the time-honored traditional use of the macroscopic or "engineering" mechanical energy equation (Reference 4, p. 213-214) in pipeline design problems when this latter equation contains a term for the irreversible rate of conversion of mechanical to internal energy. To see the relation between these two seemingly incompatible statements we consider here briefly the derivation of the macroscopic mechanical energy equation following Bird, et al.[4] For comparison purposes we shall revert here to Gibbs' dyadic notation.

We write Eq. 7 above in this notation as:

$$\frac{\partial}{\partial t}(\tfrac{1}{2}\rho v^2) + \underset{\sim}{\nabla}\cdot(\tfrac{1}{2}v^2\underset{\sim}{v}) = -\underset{\sim}{\nabla}\cdot(\rho\phi\underset{\sim}{v}) - \underset{\sim}{\nabla}\cdot(p\underset{\sim}{v}) + \underset{\sim}{v}\cdot(\underset{\sim}{\nabla}\cdot\underset{\approx}{P}) \tag{A1}$$

where we have used the fact that ρ = constant (and hence $\underset{\sim}{\nabla}\cdot\underset{\sim}{v} = 0$) to rewrite the various terms as divergences and have written $\underset{\sim}{F} = -\underset{\sim}{\nabla}\phi$ for the body force term. To obtain the macroscopic mechanical energy equation from Eq. A1 we must average it over the entire volume of the flow system (say a pipeline). This requires a volume integration over a fixed volume V_o. We can conveniently handle each term in Eq. A1 except the term $\underset{\sim}{v}\cdot(\underset{\sim}{\nabla}\cdot\underset{\approx}{P})$. For example:

$$\int_{V_o}\frac{\partial}{\partial t}(\tfrac{1}{2}\rho v^2)dV = \frac{d}{dt}\int_{V_o}\tfrac{1}{2}\rho v^2 dV = \frac{\partial K}{\partial t} \tag{A2}$$

where K is the total kinetic energy of the system. All of the divergence terms follow the same pattern as the following:

*The form chosen for $G_k(\ )$ in Eq. 27 might well be such that in the special case it reduces to G. I. Taylor's vorticity transport theory.[16]

$$\int_{V_o} \nabla\cdot(\tfrac{1}{2}\rho v^2 \mathbf{v})dV = \int_{S_o} \tfrac{1}{2}\rho v^2\ \mathbf{v}\cdot\mathbf{n}dS =$$

$$\int_{S_1} \tfrac{1}{2}\rho v^2\ \mathbf{v}\cdot\mathbf{n}dS + \int_{S_w} \tfrac{1}{2}\rho v^2\ \mathbf{v}\cdot\mathbf{n}dS + \int_{S_2} \tfrac{1}{2}\rho v^2 \mathbf{v}\cdot\mathbf{n}dS \qquad \text{(A3)}$$

where we have used the divergence theorem and observed that the boundary of the total flow system, S_o, is composed of the cross sections S_1 and S_2 of the flow channel plus the wetted boundary surface, S_w. Since $\mathbf{v} \equiv 0$ on S_w this integral vanishes identically and:

$$\int_{V_o} \nabla\cdot(\tfrac{1}{2}\rho v^2\ \mathbf{v})dS = \tfrac{1}{2}\rho\Delta <v^3> \qquad \text{(A4)}$$

where $<v^3>_1 = \int_{S_1} v^2\ \mathbf{v}\cdot\mathbf{n}dS$ and $\Delta<v^3> = <v^3>_2 - <v^3>_1$.(The minus sign arises because $n_2 = -n_1$.) Thus, the averaged mechanical equation becomes:

$$\frac{\partial K}{\partial t} + \tfrac{1}{2}\rho\Delta<v^3> = -\ \rho\Delta\phi<v> - \Delta p<v> + \int_{V_o} \mathbf{v}\cdot(\nabla\cdot\mathbf{P})dV \qquad \text{(A5)}$$

where we assumed $<\phi> = \phi$ and $<p> = p$.

At this point we must face the problem of evaluating the final integral. What is conventionally done to evaluate this term is to introduce the identity:

$$\nabla\cdot(\mathbf{v}\cdot\mathbf{P}) = \mathbf{v}\cdot(\nabla\cdot\mathbf{P}) + \mathbf{P}:\nabla\mathbf{v} \qquad \text{(A6)}$$

thus obtaining:

$$\int_{V_o} \mathbf{v}\cdot(\nabla\cdot\mathbf{P})dV = \int_{V_o} \nabla\cdot(\mathbf{v}\cdot\mathbf{P})dV - \int_{V_o} \mathbf{P}:\nabla\mathbf{v}\ dV \qquad \text{(A7)}$$

$$= \int_{S_o} (\mathbf{v}\cdot\mathbf{P})\cdot\mathbf{n}dS - \int_{V_o} \mathbf{P}:\nabla\mathbf{v}dV \qquad \text{(A7')}$$

Now, setting $S_o = S_1 + S_w + S_2$ as before and noting that $\mathbf{v} = 0$ on S_w:

$$\int_{V_o} \mathbf{v}\cdot(\nabla\cdot\mathbf{P})dV = \Delta<\mathbf{v}\cdot\mathbf{P}> - \int_{V_o} \mathbf{P}:\mathbf{e}dV \qquad \text{(A8)}$$

$$= \Delta<\mathbf{v}\cdot\mathbf{P}> - E_D \qquad \text{(A8')}$$

where we have noted that because of the symmetry of $\mathbf{P}$ the term $\mathbf{P}:\nabla\mathbf{V} = \mathbf{P}:(\mathbf{e} + \Omega) = \mathbf{P}:\mathbf{e}$. The thermodynamic analysis given in the main text showed us that $\mathbf{P}:\mathbf{e}$ is the thermodynamically irreversible energy dissipation and hence its integral is called E_D, the viscous energy dissipation or friction loss. Bird, et al.[4] (p. 214) argue that the term $\Delta<\mathbf{v}\cdot\mathbf{P}>$ represents "the work being done by viscous forces to push fluid into or out of the system and this contribution can be safely neglected." They thus set:

$$\int_{V_o} \mathbf{v}\cdot(\nabla\cdot\mathbf{P})dV \simeq - \int_{V_o} \mathbf{P}:\mathbf{e}\ dV \equiv - E_D \qquad \text{(A9)}$$

with the consequence that Eq. A5 finally becomes:

$$\frac{\partial\hat{K}}{\partial t} + \Delta\tfrac{1}{2}\ \frac{<v^3>}{<v>} + \Delta\phi + \frac{1}{\rho}\ \Delta p + \hat{E}_D = 0\ . \qquad \text{(A10)}$$

where $\hat{K} = K/\rho<v>$ and $\hat{E}_D = E_D/\rho<v>$. Eq. A10 is the familiar engineering or macroscopic mechanical energy balance and contains the term E_D representing the total viscous energy dissipation or friction loss.

From the above derivation we can plainly see what has happened. It is a well known fact that when we average a differential equation we generally lose some information. At Eq. A6 we see that the device of replacing $\mathbf{v}\cdot(\nabla\cdot\mathbf{P})$ by $\nabla\cdot(\mathbf{v}\cdot\mathbf{P}) - \mathbf{P}:\mathbf{e}$ is equivalent to adding and subtracting $\mathbf{P}:\mathbf{e}$ and is an identity. Thus, at this point, the <u>differential</u> result shows no net contribution due to $\mathbf{P}:\mathbf{e}$ since it appears twice with different signs and so adds to zero. However, at Eq. A7' we have lost this cancellation effect because the averaging of the positive part $+\ \mathbf{P}:\mathbf{e}$ with $\mathbf{v}\cdot(\nabla\cdot\mathbf{P})$ has resulted in a term which vanishes on S_w while no compensating cancellation occurs for the negative part $-\mathbf{P}:\mathbf{e}$. In fact, this means that the price paid for introducing a divergence into the volume integral of $\mathbf{v}\cdot(\nabla\cdot\mathbf{P})$ so it could be transformed to a surface integral and evaluated was the introduction of a spurious dissipative term which did not average to zero. Thus, one takes a thermodynamically reversible <u>differential</u> equation and creates from it a thermodynamically irreversible <u>averaged</u> result. The apparent disparity mentioned earlier is thus clearly seen to arise from the expediency of transforming the volume integral of the reversible stress power. Perhaps it is not all that safe to neglect the worrisome term $\Delta<\mathbf{v}\cdot\mathbf{P}>$.

SYMBOLS

E_D	Viscous energy dissipation
e_{jk}	Symmetric deformation rate tensor = $\tfrac{1}{2}(v_{j,k} + v_{k,j})$
F_k	External force in k direction
f_{jk} or g_{jk}	Denotes a tensor function
G_k	A vector function
K	Total kinetic energy of the system
P_{jk}	Stress deviator tensor
p	Pressure or negative of the trace of the stress tensor
Q	Volume source of energy
q_k	Heat flux vector in direction k
$S_{(subscript)}$	Surface indicated by subscript
S	Surface
s	Specific entropy
T_{jk}	Stress tensor in k direction with j being the normal to the plane of action
t	Time
V	Volume
V_o	A fixed volume
v	Scalar velocity (having been averaged)
v_k	Velocity in k = 1, 2, 3 directions
w_p	Vorticity = $\varepsilon_{prs}v_{s,r}$
δ_{ij}	Kroniker delta
ε	Specific internal thermal energy
ε_{prs}	Alternating tensor
θ	Temperature
ρ	Density
Φ	A scalar potential field
Ω_{jk}	Skew symmetric angular velocity tensor = $\tfrac{1}{2}\varepsilon_{jmk}W_m$
$\overline{(\ )}$	Time average = $\frac{1}{t}\int_0^t(\ )dt$
( )'	(Prime) denotes a <u>fl</u>uctuating quantity in Reynolds convention where $(\ ) = \overline{(\ )} + (\ )'$
< >	An average as defined in the text (usually over a surface or a volume rather than time)
$(\underset{\sim}{\ })$	Vector = $(\ )_k$
$(\underset{\approx}{\ })$	Tensor = $(\ )_{kl}$
$\overline{I}_e$, $\overline{II}_e$ & $\overline{III}_e$	Principle scalar invariants of $\overline{e}_{jk}$

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DISCUSSION

J. L. ZAKIN (University of Missouri-Rolla): Professor Irmay from the Technion gave a seminar here earlier this year in which he urged us to keep the Reynolds stresses over on the left side of the equations of motion with the accelerative terms, where they belong and where they came from, rather than putting them over on the right hand side. Maybe that would cast some light on why they are not dissipated.

CANNON: That's right, we feel very strongly that they belong with the inertia terms and not the dissipative terms. One should not try to form a constitutive relation that is common between the dissipative parts and the accelerative parts.

S. KLINE (Stanford University): I would like to say two things. First of all in the paper of 1967 we did point out that the two trains were separated and the cascade theory, in terms of homogeneous theory at least, is talking only about the second half of what you call one train. There's one problem in here that bothers me. If you have a coherent process then you can simplify that energy train you have. You have mechanical energies in the flow stream and then some of it goes into turbulence kinetic energy and that's production. Then you have dissipation of turbulence. Down here you have thermal energy in your sink and then you've got mean strain energy. The mean dissipation is there. That's really what you had in much more detail. So the cascade theory in terms of homogeneous turbulence really only talks about that and that's clear. I don't see how you can prove that this part is non-dissipative. I don't know how you assign an entropy to that state and I don't know anybody else that knows either. If you have a coherent state then we know the entropy of it. And you can't do this for electromagnetic radiation either. And the electrical engineers can't do it and the physicists can't do it. Presumably in theory you could use ideas about the entropy of information acquisition and work out what the entropy of this state is. In principle that's one thing, but, in fact nobody knows how to carry out those operations. I don't know how you assign an entropy to that state.

CANNON: The only thing we're trying to point out is that the mechanical energy equation is completely reversible. All of the terms in it are reversible. The thermal energy equation contains the irreversibility and you have to have some means of trapping the reversible energy into irreversible means. We're not really trying to say how it happens other than to note that it goes through the total shear work terms by two separate paths.