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## CHOOSING ALTERNATIVE ENERGY SYSTEMS UNDER CONDITIONS OF UNCERTAINTY

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### Abstract

A methodology for simulating the decision process of an investor deciding between alternative energy systems is presented. The approach assumes the investor bases his decision on cost (or rate of return) and risk. Risk is treated directly in the model and not reduced to a certainty equivalent. The rate of return-risk characteristics of many system combinations allows them to be eliminated as viable choices to the investor without reference to his personal attitude toward risk.

### 1. INTRODUCTION

The future supply of energy in the United States has recently been receiving a great deal of political, scientific and public attention. Research organizations all over the country have been making projections of possible energy supply and demand conditions into the future as far as the year 2020. Almost all of these forecasts and accompanying models are at the macroeconomic level. They deal with the whole nation, large regions or, at best, states.

This paper deviates from the usual approach in that it deals with a single individual. It presents a microeconomic model of the energy investor's decision process. An energy investor is an individual in a position to decide which alternative type of energy system will be installed in a municipality, private utility or building to meet future demands for energy. Perhaps the best example of this individual is an executive of an investor-owned utility who is formulating plans for capacity expansion. The choices open to the decision maker include: coal-fired systems, gas-fired units, nuclear plants, possibly hydro plants, or some of the more exotic energy systems such as solar, wind or geothermal. It is the independent decisions of numerous energy investors which will dictate the nation's future mix of energy generation and, subsequently, the nation's derived demand for energy-related resources. Given the importance of these individuals, it is worthwhile to investigate their decision-making processes in more detail.

The two major factors entering the energy investor's decision are cost and risk. A large amount of research has been aimed at estimating the cost of alternative systems.\* Very little work has been completed that deals with the latter subject. This study attempts to take an initial step in quantitatively evaluating uncertainty and its effect on the decision process. A methodology for dealing with uncertainty is presented. It is hoped that this framework will help stimulate additional research in this important area.

### 2. GENERAL APPROACH

The individual decision model utilized throughout the discussion is adapted from the Sharpe-Markowitz model of portfolio theory.\*\* The original model was intended to simulate decisions concerning the optimal mix of stocks and bonds in a portfolio. A major advantage of the Sharpe-Markowitz approach over previous work is the explicit incorporation of uncertainty of return into the decision process. The model also eliminates most feasible alternative portfolio combinations without the necessity of evaluating interpersonal attitudes toward risk and rate of return trade-offs. Under some additional assumptions, a unique optimal combination of risky assets can be determined without the use of any subjective comparisons.

The model presented parallels the Sharpe-Markowitz approach very closely. The following discussion will,

\* An article on evaluating the total cost of an energy system by J. Bradley and D. Costello appears in these proceedings.

\*\* William F. Sharpe, Portfolio Theory and Capital Markets (McGraw Hill Co., 1970).

therefore, be limited to the adaptations of the model to the energy investment decision process.

Throughout the discussion, the term "energy system" will refer to an organized method of producing energy characterized by the type of fuel used as the major input. An "energy mix" is a combination of energy systems which together meet the entire demand facing the investor. For a utility, the "energy mix" represents the company's generation mix. When the cost of an energy system is mentioned, it refers to the total cost of the unit realized by the owner. The cost includes all generation, fuel handling, land and required pollution control equipment and any other costs incurred in meeting governmental safety, health and environmental regulations.

The energy investment decision model can be divided into three distinct phases.\* The first step involves predicting the future return and risks associated with individual energy systems. This phase requires subjective evaluations of the future developments and trends in each of these systems. The interrelationship of these various systems must also be approximated in this phase. The second step is to compute all the possible energy mixes that can be derived by combining systems. The return and risk of each mix is then calculated and compared to other mixes. The final step involves selecting a mix based on the investor's preferences toward risk and return. These three phases will act as a guideline in the discussion that follows (Sections 3-5). Following that discussion, the incorporation of a riskless asset or system will be examined (Section 6). The last section contains a summary of the approach.

### 3. INDIVIDUAL ENERGY SYSTEMS ANALYSIS

The energy investor is assumed to make his decision based on the expected return of the investment and the uncertainty associated with that return. All relevant factors that affect the investor's decision are assumed to be summarized by these two parameters. The expected rate of return on conventional energy sources can be obtained from historical information. The rate of return on solar and other "new" forms of energy must be gathered by indirect means, including expert opinion and preliminary cost estimates.

The variance of the rate of return will be used to approximate the risk variable. The calculation is somewhat straightforward for conventional energy systems. Some modification in the risk variable may have to be made to incorporate future developments, such as fossil fuel availability, additional pollution control requirements, and/or safety regulations. The risk associated with nonconventional systems can be approximated by again using expert opinion, projected future trends in capital costs, consumer acceptance, storage capabilities and available practical experience with the systems.

\* William F. Sharpe, Portfolio Theory and Capital Markets, p. 31 (McGraw-Hill Co., 1970).

\*\* Standard deviation of return is merely the square root of the variance. It is portrayed in the figures for convenience of presentation.

The interrelationship of the rates of return for different systems is also required for the analysis. Measures of covariance will be used to estimate these interrelationships. The historical covariance between conventional energy systems can be used as a first approximation for some of the alternatives. Continued work will be necessary to approximate such relationships for unconventional systems. One possible solution involves the use of the expert opinion concerning expectations of returns on different systems. The covariance of each pair of systems could be calculated from this sample. These results would be used as a proxy for the required covariance terms.

### 4. ENERGY MIX ANALYSIS

Once the characteristics of each alternative energy system have been defined, the investor must choose one or a combination of systems to meet his total demand. The analysis that follows assumes that the investor prefers a larger rate of return to less and prefers less risk to more. In other words, return is considered desirable and risk is undesirable. Based on these assumptions, many combinations of assets can immediately be disregarded. Figure 1 illustrates this point.

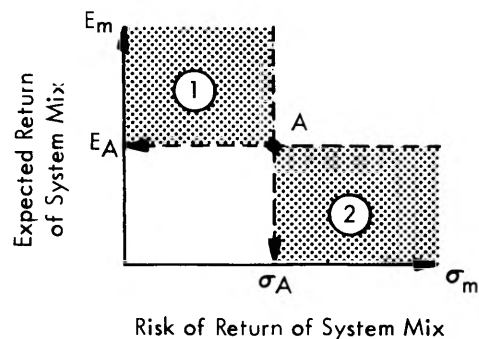


Figure 1 - Preferred and Undesirable Risk-Return Combinations

The combination of energy systems represented by point A in Figure 1 is characterized by an expected rate of return  $E_A$  and a risk (i.e., the standard deviation of return) of  $\sigma_A$ .\*\* Combinations of systems which lie in area ① are all preferred to A. Any combination in area ① will either (a) yield a higher expected return than A with the same risk ( $\sigma_A$ ) or (b) yield a lower risk than  $\sigma_A$  with the same return or (c) yield both a higher return and a lower risk than A. All combinations of systems which lie in area ② are less preferred than A. Any combination in this area will either (a) yield a higher degree of risk with no increase in return or (b) yield a lower expected return with the same amount of risk or (c) result in a lower rate of return and a higher risk than A.

#### 4.1 ALGEBRAIC RELATIONSHIPS OF THE MIX ANALYSIS

The expected rate of return of the entire mix of energy systems represented by point A is comprised of the sum of returns in each component. That is, the expected return of the mix is a linear combination of the expected returns of each system that is part of the mix. Algebraically,

$$E_m = \sum_{i=1}^n E_i X_i \quad \dots \quad (1)$$

where  $E_m$  = the expected rate of return on the entire energy mix

$E_i$  = the expected rate of return of energy system  $i$

$X_i$  = the percent of the total energy mix that is invested in system  $i$  (expressed as a decimal fraction of the total)

$n$  = the number of systems in the energy mix

and  $\sum_{i=1}^n X_i = 1$  and  $0 \leq X_i \leq 1$  for all  $i$

The expected return of the entire energy mix will usually be greater than the individual system with the lowest return and less than the return expected from the highest yielding system. If  $X_i = 1$  for any one system, the expected return of the mix will equal the expected return of the one system that comprises the entire mix.

In analyzing the uncertainty associated with an energy mix, one must consider the risk associated with each component system and the interaction of these systems. The variance of expected returns of each system will be used to represent individual system risk. The covariance or correlation coefficient will be used as an approximation of the interaction between any two systems in the mix. The variance of system  $i$  is the squared deviation of each possible outcome from its expected value, weighted by its probability. Algebraically,

$$\sigma_i^2 = \sum_{k=1}^{\ell} P_k (R_k - E_i)^2 \quad \dots \quad (2)$$

where  $\sigma_i^2$  = the variance of system  $i$

$P_k$  = the probability of outcome  $k$

$R_k$  = the rate of return of outcome  $k$

$E_i$  = the expected return of system  $i$ ,

$$E_i = \sum_{k=1}^{\ell} P_k R_k$$

$\ell$  = the total number of possible outcomes.

The covariance between the return of system  $j$  and system  $k$  is the product of the deviations of the two systems from their respective expected returns, weighted by the joint probability of each set of outcomes. Algebraically,\*

$$C_{jk} = \sum_{j,k} \Pr(R_j R_k) (R_k - E_k) (R_j - E_j) \dots (3)$$

where  $C_{jk}$  = the covariance between system  $j$  and  $k$

$\Pr(R_j R_k)$  = the probability of outcome  $R_k$  and  $R_j$  occurring together

The correlation coefficient between energy systems  $j$  and  $k$  is given by:

$$\rho_{jk} = \frac{C_{jk}}{\sigma_j \sigma_k} \quad \dots \quad (4)$$

where  $\rho_{jk}$  = the correlation coefficient between system  $j$  and  $k$

$\sigma_j$  = the standard deviation of system  $j$   
( $\sigma_j = \sqrt{\sigma_j^2}$ )

$\sigma_k$  = the standard deviation of system  $k$   
( $\sigma_k = \sqrt{\sigma_k^2}$ )

The uncertainty (variance) associated with the entire energy mix is related to the uncertainty of each system in the mix. However, unlike the expected return, this relationship is not linear. The variance of the mix is represented by the following general form:

$$\sigma_m^2 = \sum_{i=1}^n \sum_{j=1}^n X_i X_j \sigma_i \sigma_j \rho_{ij}$$

$$\sigma_m^2 = \sum_{i=1}^n \sum_{j=1}^n X_i X_j C_{ij} \quad \dots \quad (5)$$

where  $\sigma_m^2$  = the variance of the energy mix  $m$ .

#### 4.2 MIX ANALYSIS ASSUMING ONLY TWO CHOICES

To gain some insight into how the interaction of systems affect the risk of the entire mix, it is helpful to consider the special case of only two systems (i.e.,  $n=2$ ). In this case the expected return and variance of the mix simplify to:

\* William F. Sharpe, *Portfolio Theory and Capital Markets*, p. 41 (McGraw Hill Co., 1970).

$$E_m = X_1 E_1 + X_2 E_2 \quad (6)$$

$$\sigma_m^2 = X_1^2 \sigma_1^2 + X_2^2 \sigma_2^2 + (2X_1 X_2 \sigma_1 \sigma_2) \rho_{12} \quad (7)$$

where  $X_1$  = the percentage of the total invested in system 1

$E_1$  = the rate of return of system 1

$\sigma_1^2$  = the variance of system 1

and  $X_1 + X_2 = 1$

The expected return of the mix is a linear combination of the expected returns of the two systems. The variance of the mix depends on the variances of each system, the percentage invested in each system and the correlation of the systems. If  $X_1$  equaled 1 (i.e.,  $X_2 = 0$ ), then  $\sigma_m^2$  would equal  $\sigma_1^2$ . Other combinations of  $X_1$  and  $X_2$  will give a mix of risk that is a combination of  $\sigma_1^2$  and  $\sigma_2^2$ . The uncertainty of the entire mix will then depend on how the risks of the two systems are correlated. To analyze these situations we will consider the effect of the alternative values of the correlation coefficient ( $\rho_{12}$ ). Three cases will be examined:  $\rho_{12} = +1$ ,  $\rho_{12} = -1$  and  $\rho_{12} = 0$ .

#### 4.2.1 Case 1; Correlation Coefficient Equal to + 1

If  $\rho_{12}$  equals + 1 the systems are perfectly correlated. In other words, whenever the return on one system changes the return on the other moves proportionally in the same direction. The advantage of diversifying your investment between these two systems is somewhat reduced because they both always move in the same direction. The variance of the energy mix can be expressed as: (assuming  $\rho_{12} = + 1$ )

$$\sigma_m^2 = X_1^2 \sigma_1^2 + X_2^2 \sigma_2^2 + 2X_1 X_2 \sigma_1 \sigma_2$$

$$\sigma_m^2 = (X_1 \sigma_1 + X_2 \sigma_2)^2$$

$$\sigma_m = X_1 \sigma_1 + X_2 \sigma_2 \quad \dots (8)$$

In other words, if  $\rho_{12} = + 1$  the standard deviation of the energy mix is a linear combination of the standard deviation of the two components.

This situation can be depicted graphically. The expected return of the entire energy mix is graphed vertically and the risk of standard deviation of the mix is on the horizontal axis in Figure 2. Point A in Figure 2 represents the mix made up entirely of system 1, while B represents a mix comprised entirely of system 2. The line AB represents the possible combinations of  $E_m$  and  $\sigma_m$  attainable by combining systems 1 and 2 in different amounts. That is, each point along the line AB represents different values of  $X_1$  and therefore  $X_2$ , since  $X_2 = 1 - X_1$ .

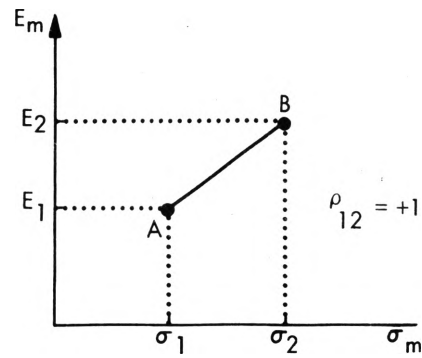


Figure 2 - Possible Combinations of Energy Mix and Return--Two Energy Systems, Perfect Positive Correlation

#### 4.2.2 Case 2; Correlation Coefficient Equal to - 1

The second case we will consider assumes that the two systems are perfectly negatively correlated (i.e.,  $\rho_{12} = - 1$ ). In this situation if the return on one system declines, the return on the other system will increase by a proportional amount. This makes diversification extremely appealing because investing in both of these systems will insure that the level of risk of the entire mix can be reduced below the risk of any one system. In fact, in the case of a correlation of - 1, risk can be totally eliminated. The following formulation will illustrate this point.

The variance of the mix under  $\rho_{12} = - 1$  is given by:

$$\sigma_m^2 = X_1^2 \sigma_1^2 + X_2^2 \sigma_2^2 - 2X_1 X_2 \sigma_1 \sigma_2$$

$$\sigma_m^2 = (X_1 \sigma_1 - X_2 \sigma_2)^2$$

$$\sigma_m = X_1 \sigma_1 - X_2 \sigma_2 \quad \dots (9)$$

The value of  $X_1$  and therefore  $X_2$  can be set so that  $\sigma_m$  equals 0. The relationship between  $E_m$  and  $\sigma_m$  in this case will be represented by the line segment ABC in Figure 3.

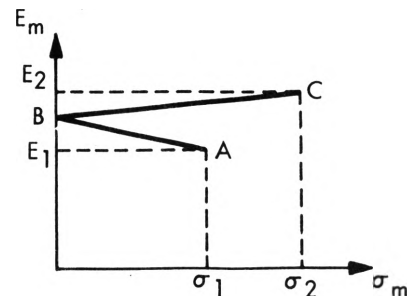


Figure 3 - Possible Combinations of Risk and Return--Two Energy Systems, Perfect Negative Correlation

Point B in Figure 3 represents a combination of systems 1 and 2 which yield no risk and an expected return greater than zero. It should be noted that the investor would never choose a point along line segment BA. Although any point along this segment is feasible, the investor can always find another combination of systems 1 and 2 that will yield a higher expected return for the same amount of risk. These combinations lie along line segment BC. The line segment BC dominates AB and an energy mix along AB would never be chosen. The line segment BC is therefore termed the "efficient frontier" of the feasible set. This concept is explained more fully later in the analysis.

#### 4.2.3 Case 3; Correlation Coefficient Equal to Zero

The third alternative under investigation assumes a correlation coefficient equal to 0. In this case, the expected return of the mix takes on its characteristic form but, unlike cases I and II, the variance does not reduce to a perfect squared term. Algebraically, the mix is characterized by:

$$E_m = X_1 E_1 + X_2 E_2 \quad (10)$$

$$\sigma_m^2 = X_1^2 \sigma_1^2 + X_2^2 \sigma_2^2 \quad (11)$$

The feasible\* (i.e., attainable) set of system mixes that can be obtained by varying  $X_i$  is graphed in Figure 4. Note that it is possible to reduce the risk of the mix below the risk of system 1. However, in this case it is not possible to reduce the energy mix uncertainty to zero. It should also be noted that the line segment AB is dominated by segment BC and can therefore be disregarded.

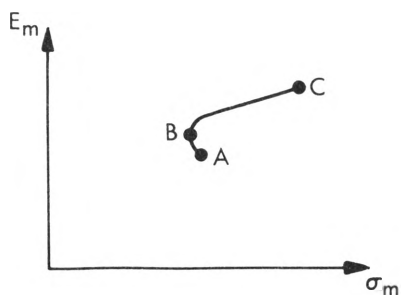


Figure 4 - Possible Combinations of Risk and Return--Two Energy Systems, Zero Correlation

In general, the correlation of the rates of return of the component energy systems will have an effect on the overall risk of the energy mix selected. The risk associated with the energy mix can usually be reduced

by diversifying into more than one energy system.\*\* In other words, real economic benefits can be derived from diversification.

#### 4.3 GENERALIZING THE MIX ANALYSIS TO NUMEROUS CHOICES

Analyzing the energy mixes comprised of only two energy systems is useful for explanatory purposes and generalizing to more than two systems is straightforward. For example, if the energy mix only contained three possible systems one could first construct the feasible set for two of the three systems. The third system is incorporated by combining it with all possible combinations of the first two systems. Each point on the feasible set consisting of only two systems can be considered a new system. The new system is then combined with the third system. Figure 5 illustrates this approach.

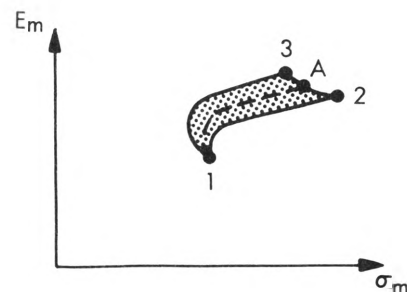


Figure 5 - Possible Combinations of Risk and Return--Three Energy Systems

Combinations of systems 1 and 2 in Figure 5 yield the feasible set designated by the line segment (1,2). Combinations of 1 and 3 yield segment (1,3). Combinations of systems 3 and 2 yield line segment (3,2). Point A represents some combination of systems 3 and 2. If those systems were combined with varying amounts of system 1, the feasible set would be given by the line (A,1). That is, energy mix A can be treated as a single system and combined with other systems. When all possible combinations are considered, the feasible set becomes an area rather than a line. This is the shaded area in Figure 5. The same approach is used to determine the feasible set for more than three systems.

#### 4.4 THE EFFICIENT FRONTIER

Using the assumption that  $E_m$  is desirable and  $\sigma_m$  is not desirable, many of the feasible combinations can be eliminated from consideration. A combination of systems would be disregarded if another feasible

\* The feasible set contains all possible combinations of rates of return and risk that can be obtained by combining the available systems in different ways.

\*\* The risk associated with the mix will be less than the risk of the least risky system (if that risk is greater than 0) if  $\rho_{12} < \sigma_1/\sigma_2$ .

combination existed that had a higher rate of return with the same variance or a lower variance and the same rate of return. After this test is performed, each remaining energy mix will lie along a line that represents the north-west boundary of the feasible set. In other words, in order to get a larger rate of return one must take on additional risk. Similarly, in order to reduce risk one must accept a lower rate of return. This locus of remaining energy mixes is called the efficient frontier.\* Figure 6 illustrates the relationship between the efficient frontier and the feasible set. The entire shaded area in the figure represents the feasible set. The line AECB represents the efficient frontier. Any point on the line AECB represents the highest return for each standard deviation or the lowest  $\sigma_m$  for each attainable level of expected return. For example, the mix D lies within the feasible set but energy mix C (on the frontier) yields a higher expected return and the same variance. Similarly, energy mix E yields a lower risk and the same return. Any point along the efficient frontier between E and C yields a higher return and a lower variance than D.

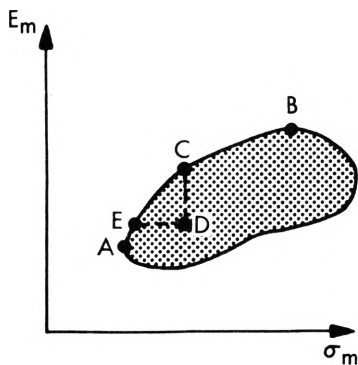


Figure 6 - The Feasible Set and Efficient Frontier

In general, the efficient frontier can be generated using a nonlinear programming approach. The problem can be stated in terms of a constrained maximization. The variables that can be manipulated to obtain this maximization are the percentage of the total invested in each system ( $X_i$ ). Mathematically, the general problem can be stated as choosing  $X_1, X_2, \dots, X_n$  to:\*\*

\* William F. Sharpe, Portfolio Theory and Capital Markets, p. 33 (McGraw Hill Co., 1970).

\*\* William F. Sharpe, Portfolio Theory and Capital Markets, p. 58 (McGraw Hill Co., 1970).

$$\max [\lambda E_m - \sigma_m^2] \quad \text{for all } \lambda \geq 0$$

$$= \max \left[ \lambda \left( \sum_{i=1}^n X_i E_i \right) - \left( \sum_{i=1}^n \sum_{j=1}^n X_i X_j C_{ij} \right) \right], \quad \text{for all}$$

$$\lambda \geq 0$$

subject to:  $\sum_{j=1}^n X_i = 1, 0 \leq X_i \leq 1, \text{ for all } X_i$

and: any other constraints on  $X_i \dots (12)$

If every system under consideration had a variance greater than zero (i.e., some risk) then the efficient frontier, would be a set similar to line AB in Figure 7. The analysis of the energy mix would be complete. In other words, no energy mix along the efficient frontier can be eliminated on an objective basis. Along the efficient frontier, the only way to achieve a higher expected return is to accept more risk.

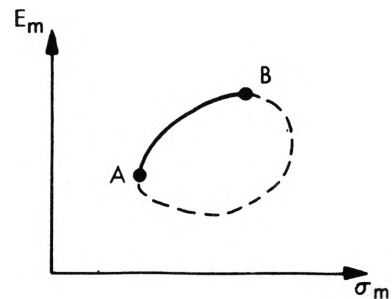


Figure 7 - The Efficient Frontier of Combinations of Hypothetical Energy Mixes

### 5. SELECTION OF ENERGY MIX BY THE INVESTOR

The final step in the methodology is to allow the investor to choose a mix that lies along the efficient frontier. The mix he chooses will depend on his attitudes toward risk and rate of return. He has to decide how much additional risk he is willing to take on to increase his expected return. If he is not too concerned with risk he will choose a point near mix B in Figure 7. If he is more averse to risk he will choose an energy mix near point A in the figure.

## 6. ADDING A RISKLESS CHOICE TO THE ENERGY MIX

Additional energy mixes can be eliminated from consideration if a riskless asset is introduced. This new alternative can be interpreted as the choice of not investing in energy systems at all but rather in some government secured bond or Treasury bill. One could also conceive of this as an energy system that the government subsidizes in such a way as to insure some positive return. For a private individual considering energy for his residence, the riskless alternative could be construed as obtaining energy from the existing power grid.

The riskless alternatives available to an investor will depend on whether the investor is an individual, a corporation, or a public utility. If an investor has more than one riskless alternative before him it is relatively easy to reduce his alternatives to only one. Since more return is preferred to less and all these alternatives have no risk he will choose the alternative with the highest return and disregard the others. This is represented by point P in Figure 8.

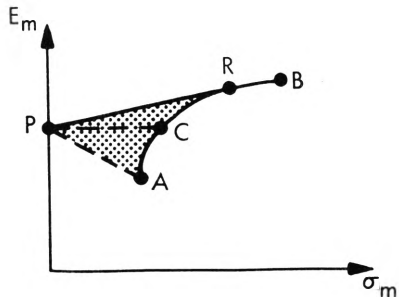


Figure 8 - Feasible Set of Energy Mixes with a Riskless Asset

As in the previous analysis, the investor is not restricted to putting all his investible funds in only one alternative. The new alternative can be combined with any energy mix along the efficient frontier. The result will be to increase his feasible set. One can consider any mix along the existing frontier just as the two systems were combined in the development of the two-system feasible set in Section 4.2. For example, alternative P can be combined with energy mix A in Figure 8 to yield a new set of possible combinations represented by the line PA. Similarly, alternative P can be combined with energy mix C to yield the new combinations along PC. In general, the new alternative can be combined with each energy mix along the efficient frontier. The total addition to the feasible set is represented by the shaded area in Figure 8.

The efficient frontier is also altered by the introduction of the riskless alternative. Using the assumption that  $E_m$  is a desired good and  $\sigma_m$  is not desired (i.e., a "bad") most of the new possible mixes can be eliminated. Even some of the energy mixes that were on the original efficient frontier are no longer desirable. For example, energy mix A is now dominated by all energy mixes on the ray PD in Figure 9. In fact, all the energy mixes between A and R on the old frontier are now dominated by points along the ray PDR. The new efficient frontier is made of the line PDRB. All points between P and R are comprised of varying amounts of energy mix R and the riskless asset P. The line segment RB represents different mixes of energy systems and no funds in P.

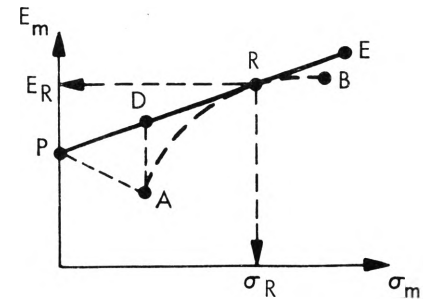


Figure 9 - Alterations in the Efficient Frontier with the Addition of a Riskless Asset

If the investor is allowed to borrow at the riskless rate P the risky energy mixes beyond point R can also be eliminated. However, energy mix R does remain in the efficient frontier. If the investor were allowed to borrow at rate P he could invest the additional funds in energy mix R and lever his expected return (and risk) above  $E_R$ . Since combinations along the ray RE dominate energy mixes along RB, the efficient frontier becomes a straight line with intercept P tangent to the original efficient frontier ARB at point R.

The new efficient frontier contains only one point (R, that is made up entirely of a risky mix of energy systems. Points between P and R represent combinations of mix R and the riskless alternative and points between R and E represent combinations of mix R and borrowing at rate P. The energy mix R is the optimal mix of energy systems since it is the only energy mix remaining on the efficient frontier.\*

\* This terminology closely parallels Sharpe's concept of the optimal portfolio of risky assets (see Sharpe, p. 69).



The choice left to the investor is now reduced to choosing what combinations of the riskless and the optimal mix R he wishes to purchase. He does not have to choose between different risky energy mixes. The actual combination of the riskless alternative and mix R will be determined by the investor's subjective preference for risk relative to expected return (see Section 4).

#### 7. SUMMARY

The energy investor is assumed to choose between competing energy systems based on two factors--expected rate of return and risk. The rate of return is equal to the difference between the expected revenue and the system's cost. This difference is then divided by the cost. Risk or uncertainty is represented by the variance of the return from its average value. The investor selects a combination of alternative systems, one of which may be riskless, to maximize the difference between the return of the mix and its risk. Using the Lagrange multipliers the problem can be stated algebraically. The investor chooses  $X_1, X_2, \dots, X_n$  to:

$$\begin{aligned} & \max_{X_i} [\lambda E_m - \sigma_m^2] \text{ for all } \lambda \geq 0 \\ = & \max \left[ \lambda \left( \sum_{j=1}^n X_i E_j \right) - \left( \sum_{i=1}^n \sum_{j=1}^n X_i X_j C_{ij} \right) \right], \text{ for all} \\ & \lambda \geq 0 \end{aligned}$$

subject to the constraints:

$$\sum_{j=1}^n X_i = 1, \quad 0 \leq X_i \leq 1, \quad \text{for all } X_i$$

and: any other relevant constraints ... (13)

This general framework can be used to simulate the decision process of many diverse types of energy investors. The additional constraints facing each investor (such as regulation, availability of fuels or diversification requirements), should be incorporated when the model is exercised.

The methodology presented is only a small step in understanding the decision-making process of energy investors across the U.S. Additional research aimed at estimating the parameters outlined by the approach should add significant amounts to our understanding.

Mr. Costello is an energy economist at Midwest Research Institute, Kansas City, Missouri. He specializes in the application of micro and macro economic theory to the development and analysis of energy policy. Recently, he led the socioeconomic analysis tasks in a major study of the impacts of commercial development of solar energy for the U.S. Congress. Mr. Costello received the B.A. in Economics (1972) from the State University of New York where he graduated Magna Cum Laude and the M.A. in Economics (1973) from Ohio State University.